QCD renormalization constants through the Loop -Tree Duality

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Motivation

The standard approach to perform perturbative calculation in QCD relies in the application of the subtraction formalism. There are several variants of the subtraction method at NLO, which involve:

- Treat separately real and virtual contributions.
- Computational difficulty related with the final-state phase-space of the different contributions.
- Building counter-terms.

Motivation

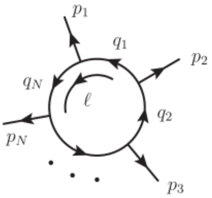
An alternative to elude the introduction of IR counter-terms, it's the application of the loop–tree duality (LTD).

- The LTD theorem establishes a direct connection among loop and phase-space integrals.
- Dual integrals and real-radiation contributions exhibit a similar structure, and can be combined at integrand level.

To build a complete loop-tree duality representation of the cross section, it is crucial to include the renormalised self-energy correction, what implies calculate the renormalization constants.

Considering a generic N- particle scalar one-loop integral,

$$L^{(1)}\left(p_1, \dots, p_N\right) = \int_\ell \prod_{i \in \alpha_1} G_F\left(q_i\right),$$



over Feynman propagators $G_F\left(q_i\right) = \left(q_i^2 - m_i^2 + \imath 0\right)^{-1}$, where it's corresponding dual representation is

$$L^{(1)}\left(p_1, \dots, p_N\right) = -\sum_{i \in \alpha_1} \int_\ell \widetilde{\delta}\left(q_i\right) \prod_{j \in \alpha_1, j \neq i} G_D\left(q_i; q_j\right).$$

Review of the loop-tree duality

The corresponding dual representation is

$$L^{(1)}\left(p_1, \ldots, p_N\right) = -\sum_{i\in\alpha_1} \int_\ell \widetilde{\delta}\left(q_i\right) \prod_{j\in\alpha_1, j\neq i} G_D\left(q_i; q_j\right).$$

- $G_D\left(q_i; q_j\right) = \left(q_j^2 - m_j^2 - \imath 0 \eta \cdot k_{ji}\right)^{-1}$ are dual propagators.
- $i, j \in \alpha_1 = \{1, 2, \ldots, N\}$ label the internal lines.
- The sign of prescription $\imath 0$ depends on $k_{ji} = q_j - q_i$.
- m_i and $q_{i,\mu} = \left(q_{0,\mu}, \mathbf{q}_i\right)$, masses and momenta of the internal lines.
- $\mathbf{q}_i = \ell + k_i$, $k_i = p_1 + \cdots + p_i$ and $k_N = 0$.
- ℓ is the loop momentum.

On the other hand

$$\int_{\ell} \bullet = -\imath \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \bullet$$

where $d = 4 - 2\epsilon$, and

$$\widetilde{\delta}(q_i) \equiv 2\pi\imath\theta(q_{i,0})\,\delta\left(q_i^2 - m_i^2\right),$$

is used to set the internal lines on-shell in

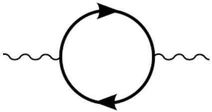
$$L^{(1)}(p_1, \ldots, p_N) = -\sum_{i \in \alpha_1} \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \widetilde{\delta}(q_i) \prod_{j \in \alpha_1, j \neq i} G_D(q_i; q_j),$$

where $\theta(q_{i,0})$ restricts the integration domain to $q_{i,0} > 0$.

Photon self-energy

The photon one-loop self energy has the form,

$$\imath\Pi_{\mu\nu} = -\int \frac{d^d\ell}{(2\pi)^d} Tr\left[\imath e_0\gamma_\mu\left(\imath\frac{\not{\ell}+\not{p}+m_0}{(\ell+p)^2-m_0^2}\right)\imath e_0\gamma_\nu\left(\imath\frac{\not{\ell}+m_0}{\ell^2-m_0^2}\right)\right].$$



In the massless approx, relabelling $\ell = q_1, \ell + p = q_2$ and contracting,

$$\Pi\left(p^2\right) = -8\left(1-\epsilon\right)\frac{\imath e_0^2}{\left(3-2\epsilon\right)p^2}\imath\mu^{-2\epsilon}\int_\ell \frac{q_1\cdot q_2}{q_1^2 q_2^2}.$$

Loop-Tree Duality

Applying the LTD to the integral $\Pi\left(p^2\right)$,

$$\Pi\left(p^2\right) = -\frac{8(1-\epsilon)\mu^{-2\epsilon}}{(d-1)p^2} e_0^2 \left[\int_\ell \frac{\widetilde{\delta}(q_1) q_1\cdot p}{2q_1\cdot p + p^2 - \imath 0} - \int_\ell \frac{\widetilde{\delta}(q_2) p\cdot q_2}{-2q_2\cdot p + p^2 + \imath 0}\right].$$

Taking the parametrization

$$p^\mu = (p_0, \mathbf{0}), \, q_i^\mu = p_0 \xi_{i,0}\left(1, 2\sqrt{v_i\left(1-v_i\right)} e_{i,\perp}, 1-2v_i\right),$$

$$\Pi\left(p^2\right) = -\frac{2^5\left(1-\epsilon\right) e_0^2}{\left(d-1\right) p^2 \mu^{2\epsilon}}\left[\int \frac{d\left[\xi_{1,0}\right] d\left[v_1\right] \xi_{1,0}^2}{1+2\xi_{1,0}-\imath 0} - \int \frac{d\left[\xi_{2,0}\right] d\left[v_2\right] \xi_{2,0}^2}{1-2\xi_{2,0}+\imath 0}\right],$$

where,

$$d\left[\xi_{i,0}\right] = \frac{\left(4\pi\right)^{\epsilon-2}}{\Gamma\left(1-\epsilon\right)}\left(\frac{4p^2}{\mu^2}\right)^{-\epsilon} \xi_{i,0}^{-2\epsilon} d\xi_{i,0} \text{ and } d\left[v_i\right] = \left(v_i\left(1-v_i\right)\right)^{-\epsilon} dv_i.$$

Explore the behavior of $\Pi\left(p^2\right)$.

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [0,\omega)$ and expanding on series around ϵ we have

$$e_0^2\left[-\frac{4\omega^2+\log\left(1+2\omega-\imath 0\right)+\log\left(1-2\omega+\imath 0\right)}{12\pi^2}+O\left(\epsilon\right)\right].$$

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [\omega,\infty)$ and expanding on series around ϵ we have

$$e_0^2\left[-\frac{1}{12\pi^2\epsilon}-\frac{5-3\gamma+3\imath\pi+3\log\left(-\imath 0-\frac{p^2}{4\pi}\right)}{36\pi^2}+O\left(\epsilon\right)\right].$$

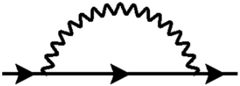
- On the finite region, we find a four dimensional realization

$$-\frac{e_0^2}{6\pi^2}\int_0^1 dv_1\int_0^\omega \frac{d\xi_{1,0}4\xi_{1,0}^2}{1+2\xi_{1,0}-\imath 0}\left[1+\log\left(\xi_{1,0}^2\right)\left(v_1\delta\left(v_1\right)+\left(1-v_1\right)\delta\left(1-v_1\right)\right)\right]$$
$$+\frac{e_0^2}{6\pi^2}\int_0^1 dv_2\int_0^\omega \frac{d\xi_{2,0}4\xi_{2,0}^2}{1-2\xi_{1,0}+\imath 0}\left[1+\log\left(\xi_{2,0}^2\right)\left(v_2\delta\left(v_2\right)+\left(1-v_2\right)\delta\left(1-v_2\right)\right)\right].$$

Quark self-energy

In massless, the quark self energy has the form,

$$\Sigma(p) = \not{p}\Sigma_v\left(p^2\right).$$



We have that

$$\imath\not{p}\Sigma_v\left(p^2\right) = C_F \int \frac{d^d\ell}{(2\pi)^d}\left[\imath g_0\gamma^\mu\right]\left[-\imath D^0_{\mu\nu}(\ell)\right]\left[\imath g_0\gamma^\nu\right]\left[\imath S_0(\ell+p)\right].$$

Multiplying $4p^2\Sigma_v\left(p^2\right)$ by $\frac{\not{p}}{4p^2}$ and taking the trace of both sides,

$$\Sigma_v\left(p^2\right) = -\mu^{-2\epsilon}(2-2\epsilon)\, C_F g_0^2 \int_\ell \frac{1}{q_1^2 q_2^2}\left(1+\frac{p\cdot\ell}{p^2}\right).$$

Loop-Tree Duality

Applying the LTD to the integral $\Sigma_v\left(p^2\right)$, we have

$$\Sigma_v\left(p^2\right) = 2\mu^{-2\epsilon}\left(1-\epsilon\right)C_F g_0^2\left[\int_\ell \frac{\widetilde{\delta}(q_1)\left(1+\frac{p\cdot q_1}{p^2}\right)}{2p\cdot q_1+p^2-\imath 0} + \frac{1}{p^2}\int_\ell \frac{\widetilde{\delta}(q_2)\,p\cdot q_2}{-2p\cdot q_2+p^2+\imath 0}\right].$$

Taking the same parametrisation, we get

$$\Sigma_v\left(p^2\right) = -\frac{2^3(1-\epsilon)C_F g_0^2}{\mu^{2\epsilon}}\left[\int \frac{d\left[\xi_{1,0}\right]d[v_1]\left(\xi_{1,0}+\xi_{1,0}^2\right)}{1+2\xi_{1,0}-\imath 0} - \int \frac{d\left[\xi_{2,0}\right]d[v_2]\xi_{2,0}^2}{1-2\xi_{2,0}+\imath 0}\right].$$

Loop-Tree Duality

Explore the behavior of $\Sigma_v\left(p^2\right)$.

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [0,\omega)$ and expanding on series around ϵ we have

$$e_0^2\left[\frac{-\log\left(1+2\omega-\imath 0\right)-\log\left(1-2\omega+\imath 0\right)}{16\pi^2}+O\left(\epsilon\right)\right].$$

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [\omega,\infty)$ and expanding on series around ϵ we have

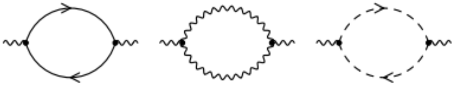
$$e_0^2\left[-\frac{1}{16\pi^2\epsilon}+\frac{-1+\gamma+\log\left[1-\left(2\omega-\imath 0\right)^2\right]+\log\left(-\imath 0-\frac{p^2}{4\pi}\right)}{16\pi^2}+O\left(\epsilon\right)\right].$$

- Also we get a four dimensional representation for the finite region:

$$\frac{e_0^2}{8\pi^2}\int_0^1 dv_1\int_0^\omega \frac{d\xi_{1,0}4\xi_{1,0}\left(1+\xi_{1,0}\right)}{1+2\xi_{1,0}-\imath 0}\left[1+\log\left(\xi_{1,0}^2\right)\left[v_1\delta\left(v_1\right)+\left(1-v_1\right)\delta\left(1-v_1\right)\right]\right]$$
$$+\frac{e_0^2}{8\pi^2}\int_0^1 dv_2\int_0^\omega \frac{d\xi_{2,0}4\xi_{2,0}^2}{1-2\xi_{1,0}+\imath 0}\left[1+\log\left(\xi_{2,0}^2\right)\left[v_2\delta\left(v_2\right)+\left(1-v_2\right)\delta\left(1-v_2\right)\right]\right].$$

Gluon self-energy

Gluon propagator receives contributions from:



- The fermion-loop is the same as in the photon propagator with just including the color factor n_f, and the replace of e_0 by g_0.
- The contribution to the gluon-loop is

$$(\Pi_1)^{\mu}_{\mu} = -\imath \frac{1}{2} C_A g_0^2 \int \frac{d^d \ell}{(2\pi)^d} \frac{N}{q_1^2 q_2^2} = \frac{1}{2} C_A g_0^2 \mu^{-2\epsilon} \int_{\ell} \frac{N}{q_1^2 q_2^2},$$

where $N = d\left((2\ell + p)^2 + (\ell - p)^2 + (\ell + 2p)^2\right) - 2\,(2\ell + p) \cdot (\ell - p) - 2\,(\ell + p) \cdot (\ell + 2p) + 2\,(\ell + 2p) \cdot (\ell - p)$.

- The contribution to the gluon-loop is

$$(\Pi_2)^{\mu}_{\mu} = \imath C_A g_0^2 \int \frac{d^d \ell}{(2\pi)^d} \frac{q_1 \cdot q_2}{q_1^2 q_2^2} = -C_A g_0^2 \mu^{-2\epsilon} \int_{\ell} \frac{q_1 \cdot q_2}{q_1^2 q_2^2}.$$

Loop-Tree Duality

Applying the LTD,

$$(\Pi_1)^\mu_\mu = -\frac{1}{2} C_A g_0^2 \mu^{-2\epsilon} \left[\int_\ell \frac{\widetilde{\delta}(q_1)\, N_1}{(q_1+p)^2 - \imath 0} + \int_\ell \frac{\widetilde{\delta}(q_2)\, N_2}{(q_2-p)^2 + \imath 0} \right],$$

with

$$\begin{aligned} N_1 &= 6(d-1)\left(q_1^2 + p\cdot q_1 + p^2\right) \text{ and} \\ N_2 &= 6(d-1)\left(q_2^2 - p\cdot q_2 + p^2\right), \end{aligned}$$

then

$$(\Pi_1)^\mu_\mu = -12(d-1)\frac{C_A p^2 g_0^2}{\mu^{2\epsilon}} \left[\int \frac{d[\xi_{1,0}]\, d[v_1] \left(\xi_{1,0} + \xi_{1,0}^2\right)}{1 + 2\xi_{1,0} - \imath 0} + \int \frac{d[\xi_{2,0}]\, d[v_2] \left(\xi_{2,0} - \xi_{2,0}^2\right)}{1 - 2\xi_{2,0} + \imath 0} \right].$$

Explore the behavior of $(\Pi_1)^{\mu}_{\mu}$.

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [0,\omega)$ and expanding on series around ϵ we have

$$-\frac{9}{2} C_A p^2 g_0^2 \left[\frac{4\omega^2 - \log\left(1+2\omega-\imath 0\right) - \log\left(1-2\omega+\imath 0\right)}{16\pi^2} + O\left(\epsilon\right)\right].$$

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [\omega,\infty)$ and expanding on series around ϵ we have

$$-\frac{9}{2} C_A p^2 g_0^2 \left[-\frac{1}{16\pi^2\epsilon} + \frac{-4-12\omega^2+3\gamma+3\log\left[1-(2\omega-\imath 0)^2\right]+3\log\left(-\imath 0-\frac{p^2}{4\pi}\right)}{48\pi^2} + O\left(\epsilon\right)\right].$$

- The four dimensional representation for the finite region:

$$-\frac{9}{2}\frac{C_A g_0^2 p^2}{8\pi^2}\int_0^1 dv_1 \int_0^{\omega} \frac{d\xi_{1,0} 4\xi_{1,0}\left(1+\xi_{1,0}\right)}{1+2\xi_{1,0}-\imath 0}\left[1+\log\left(\xi_{1,0}^2\right)\right.$$
$$\left.\times\left[v_1\delta\left(v_1\right)+\left(1-v_1\right)\delta\left(1-v_1\right)\right]\right] - \frac{9}{2}\frac{C_A g_0^2 p^2}{8\pi^2}\int_0^1 dv_2 \int_0^{\omega}\frac{d\xi_{2,0}4\xi_{2,0}\left(1-\xi_{2,0}\right)}{1-2\xi_{1,0}+\imath 0}\times$$
$$\left[1+\log\left(\xi_{2,0}^2\right)\left[v_2\delta\left(v_2\right)+\left(1-v_2\right)\delta\left(1-v_2\right)\right]\right].$$

The ghost contribution is given by

$$
\begin{aligned}
(\Pi_2)^{\mu}_{\mu} &= \mu^{-2\epsilon} C_A g_0^2 \left(\int_\ell \frac{\widetilde{\delta}(q_1)\, q_1 \cdot p}{2p\cdot q_1 + p^2 - \imath 0} - \int_\ell \frac{\widetilde{\delta}(q_2)\, p \cdot q_2}{-2p\cdot q_2 + p^2 + \imath 0} \right), \\
&= \frac{4 C_A g_0^2 p^2}{\mu^{2\epsilon}} \left(\int \frac{d\left[\xi_{1,0}\right] d\left[v_1\right] \xi_{1,0}^2}{1 + 2\xi_{1,0} - \imath 0} - \int \frac{d\left[\xi_{2,0}\right] d\left[v_2\right] \xi_{2,0}^2}{1 - 2\xi_{2,0} + \imath 0} \right).
\end{aligned}
$$

Explore the behavior of $(\Pi_2)^\mu_\mu$.

- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [0,\omega)$ and expanding on series around ϵ we have
$$4 C_A g_0^2 p^2 \left[\frac{4\omega^2 + \log\left(1 - 2\omega + \imath 0\right) + \log\left(1 + 2\omega - \imath 0\right)}{128\pi^2} + O\left(\epsilon\right) \right].$$
- Integrating on $v_1, v_2 \in [0,1]$, $\xi_{1,0}, \xi_{2,0} \in [\omega,\infty)$ and expanding on series around ϵ we have
$$4 C_A g_0^2 p^2 \left[\frac{1}{128\pi^2\epsilon} - \frac{-2+4\omega^2+\gamma+\log\left[1-(2\omega-\imath 0)^2\right]+\log\left(-\imath 0-\frac{p^2}{4\pi}\right)}{128\pi^2} + O\left(\epsilon\right) \right].$$
- The four dimensional representation for the finite region:

$$\frac{C_A g_0^2 p^2}{16\pi^2} \int_0^1 dv_1 \int_0^\omega \frac{d\xi_{1,0} 4\xi_{1,0}^2}{1+2\xi_{1,0}-\imath 0} \left[1 + \log\left(\xi_{1,0}^2\right)\left[v_1\delta\left(v_1\right) + \left(1 - v_1\right)\delta\left(1 - v_1\right)\right]\right]$$
$$\frac{C_A g_0^2 p^2}{16\pi^2} \int_0^1 dv_2 \int_0^\omega \frac{d\xi_{2,0} 4\xi_{2,0}^2}{1-2\xi_{1,0}+\imath 0} \left[1 + \log\left(\xi_{2,0}^2\right)\left[v_2\delta\left(v_2\right) + \left(1 - v_2\right)\delta\left(1 - v_2\right)\right]\right].$$

Summary

The possibility to performing purely four dimensional implementations could lead to

- Major improvements in the computation of higher-order corrections in QFT.
- Allows a better understanding of the mathematical structures behind scattering amplitudes.
- LTD let to perform an integrand-level combination of real and virtual terms, which leads to a fully local cancellation of singularities, allows to implement the calculation without making use of DREG and shows the nature of the singularities.

The next step in order to build a full four dimensional representation, is to build a UV-counterterm in order to cancel locally the UV-divergence.


Thank You