Modern Aspects of Perturbative QFT and Gravity

Bryan Larios
in collaboration with
J. Lorenzo Díaz-Cruz

BUAP

XXXI Annual Meeting DPyC-SMF

May 24, 2017

Overview

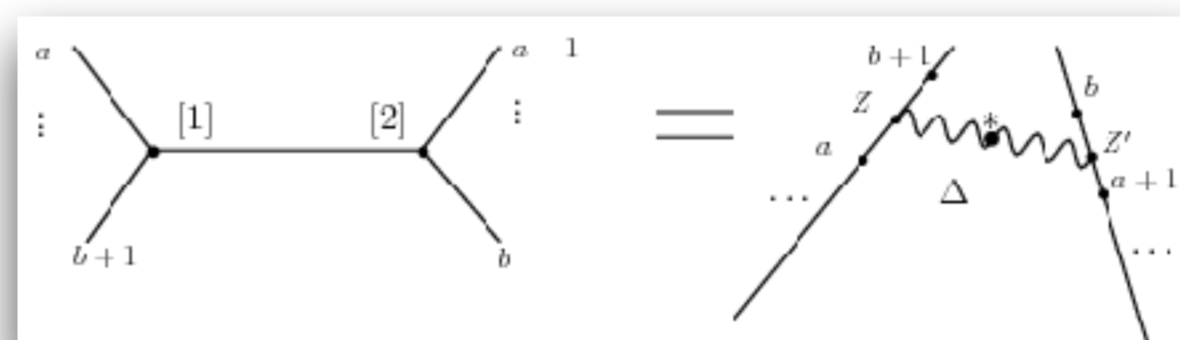
- Introduction / Motivation.
- Basics of the *Spinor Helicity Formalism* (SHF).
- Frontier in *Scattering Amplitudes* (SA).
- Our contributions.
- Final comments.

Motivation

- **Particle Physics**: The key observable measured in particle scattering experiments is the ***scattering cross section.***

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{A}|^2$$

- **Mathematics:** It has been realized in recent years that *amplitudes* **themselves** have a very interesting mathematical structure.



Motivation

The calculation with Feynman diagrams are cumbersome, however final results often strikingly simple.

Brice S. DeWitt, 1967

It is well known that the number of Feynman diagrams tends to grow very quickly with the number of particles involved, e.g. for gluon scattering at *tree level* in QCD one have

$$gg \to gg, \ \text{4 diagrams}$$
$$gg \to ggg, \ \text{25 diagrams}$$
$$gg \to gggg, \ \text{220 diagrams}$$
$$gg \to gggggg, \ \text{more than 1 million of diagrams}$$

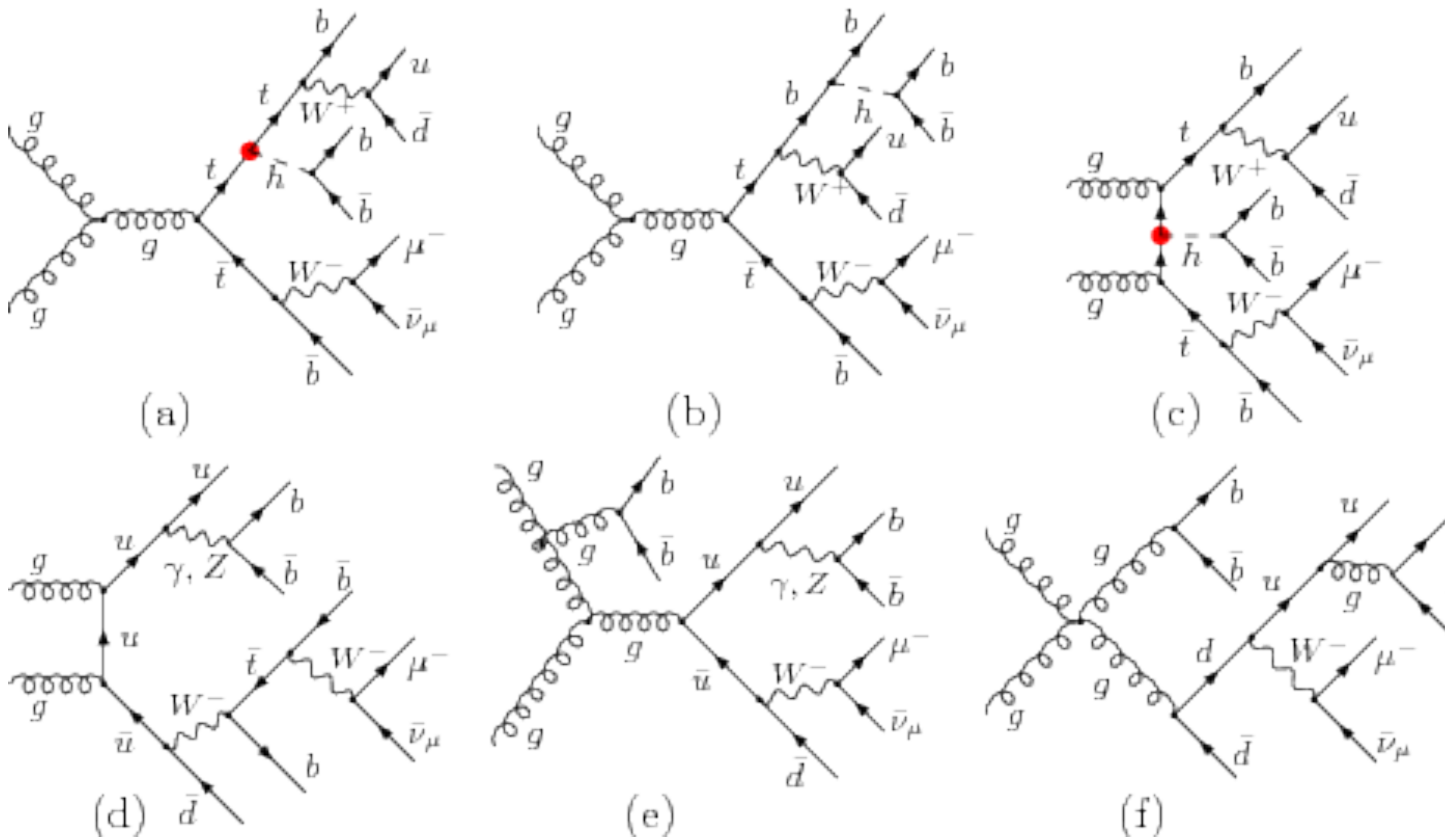
Mangano & Parke, 1991

If one compute $gg \to ggg$, part of the result is

$$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$$

From Z. Bern talk, ICTP-SAIFR, 15

“When the number of external particles grow, the mathematical expression for each diagram becomes significantly more complicated”.



“When the number of external particles grow, the mathematical expression for each diagram becomes significantly more complicated”.

grad student

“When the number of external particles grow, the mathematical expression for each diagram becomes significantly more complicated”.

grad student

me!!!!!, The Spinor Helicity Formalism

The Spinor Helicity Formalism

The key of the SHF is to express the 4-momentum of each external particle in terms of 2-component numerical spinor $p_{a\dot{a}} = -\phi_a \phi^*_{\dot{a}}$, and consider these spinors as the fundamental blocks of the amplitude.

$$\bar{u}_+(\vec{p})u_-(\vec{k}) = \phi^a \kappa_a = [pk] = -[kp]$$

$$\bar{u}_-(\vec{p})u_+(\vec{k}) = \phi^*_{\dot{a}} \kappa^{*\dot{a}} = \langle pk \rangle = -\langle kp \rangle$$

$$\bar{u}_+(\vec{p})u_+(\vec{k}) = [pk\rangle = 0$$

$$\bar{u}_-(\vec{p})u_-(\vec{k}) = \langle pk] = 0$$

Massless case

The SHF is implemented to massive particles as well.

$$u_- = |r] + \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad u_+ = \frac{m}{[rq]}|q] + |r\rangle;$$
$$v_+ = |r] - \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad v_- = -\frac{m}{[rq]}|q] + |r\rangle;$$
$$\bar{u}_- = \frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{u}_+ = [r| + \frac{m}{\langle qr\rangle}\langle q|;$$
$$\bar{v}_+ = -\frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{v}_- = [r| - \frac{m}{\langle qr\rangle}\langle q|.$$

$$\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|r]}{\sqrt{2}\langle rq\rangle},$$
$$\epsilon^\mu_- = \frac{\langle r|\gamma^\mu|q]}{\sqrt{2}[qr]},$$

$$\epsilon^\mu_0 = \frac{1}{2m}\left(\langle r|\gamma^\mu|r] - \alpha\langle q|\gamma^\mu|q]\right)$$
$$= \frac{1}{m}r^\mu + \frac{m}{2p\cdot q}q^\mu.$$

With these ingredients it is possible in principle to compute processes and reactions in the SM, we only need to know the rules of these 2-component spinors.

Some of the most important formulas that are needed to compute scattering amplitudes :

$$\boxed{[ij] = -[ji],}$$

$$\boxed{\langle ij\rangle = [ji]^*,}$$

$$\boxed{\langle ij\rangle[ji] = \langle ij\rangle\langle ij\rangle^* = |\langle ij\rangle|^2,}$$

$$\boxed{\langle ij\rangle[ji] = -2k_i \cdot k_j = s_{ij},}$$

$$\boxed{\langle i|\gamma_\mu|j] = [j|\gamma_\mu|i\rangle,}$$

$$\boxed{\langle i|\gamma_\mu|j]\langle k|\gamma^\mu|l] = 2\langle ik\rangle[lj],}$$

$$\boxed{\langle ab\rangle\langle cd\rangle = \langle ac\rangle\langle bd\rangle + \langle ad\rangle\langle cb\rangle,}$$

$$\boxed{\sum_{k=1}^{n}\langle ik\rangle[kj] = 0,}$$

Returning to the 5 gluons amplitude in QCD and using the SHF, it is possible to find the following helicity amplitude

$$A_5(1^{\pm}, 2^{+}, 3^{+}, 4^{+}, 5^{+}) = 0$$

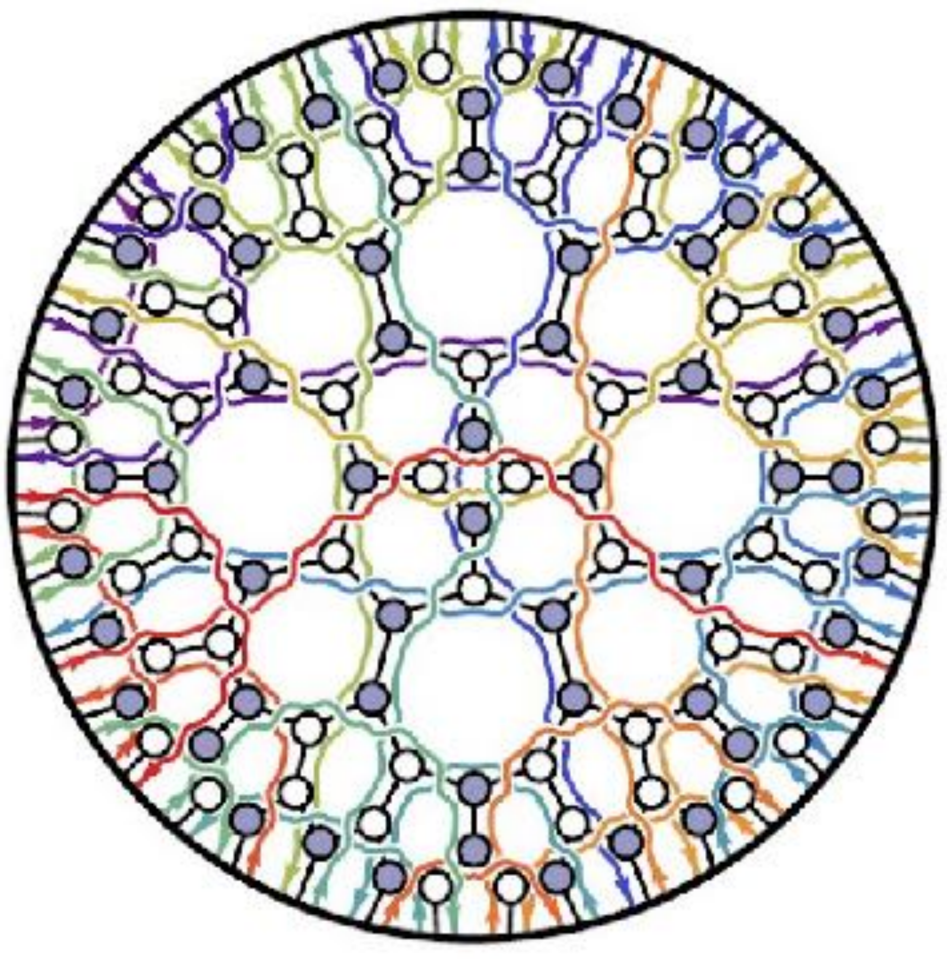
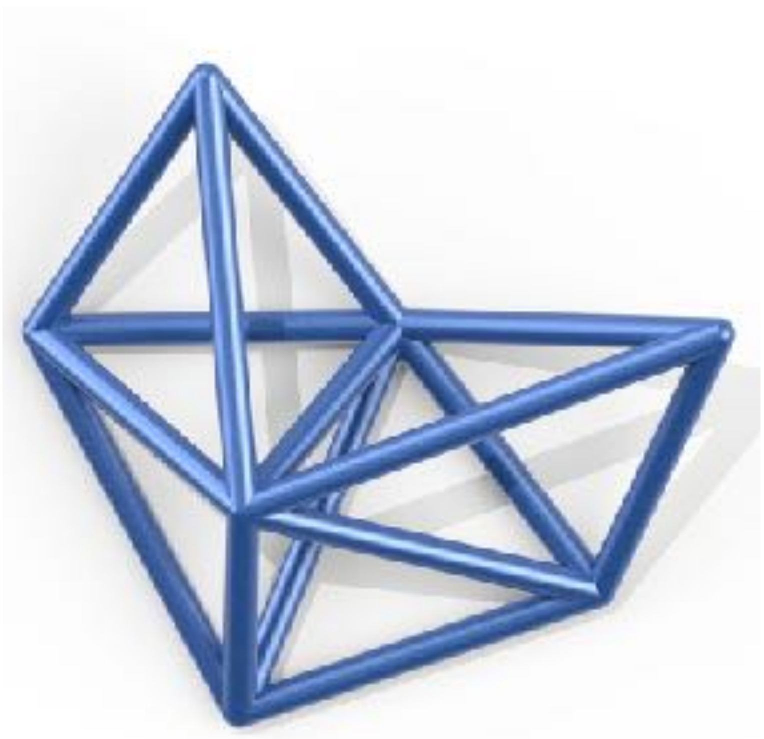
$$A_5(1^{-}, 2^{-}, 3^{+}, 4^{+}, 5^{+}) = i\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

Parke & Taylor, 1986

Just to remind you.......

$$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$$

Frontier in Scattering Amplitudes

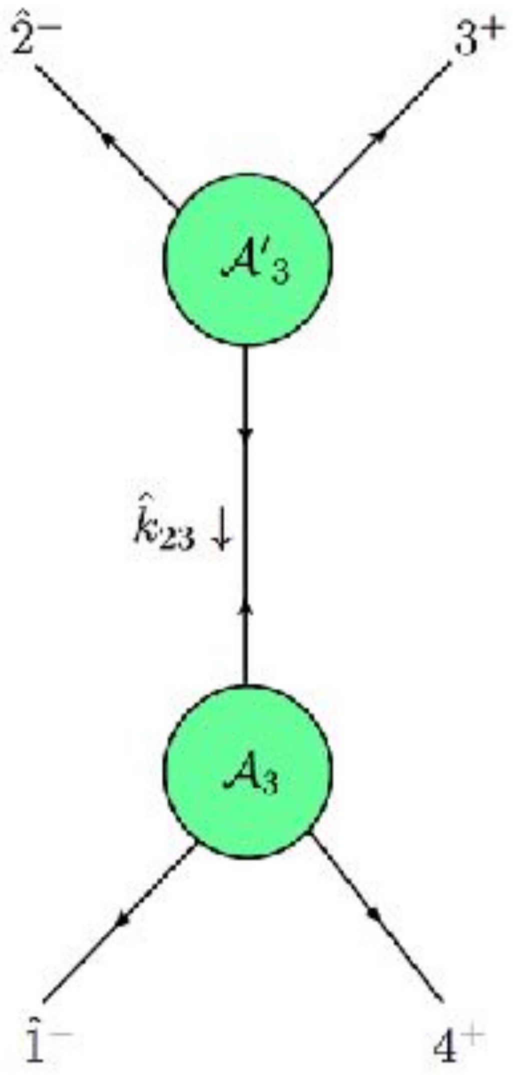




During the last decade, a lot of progress has been done to understand the mathematical structure of the *scattering amplitudes*, I would like to point out some of the most important

- on-shell recursion relations

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations ✦BCFW



$$\mathcal{A}_n = \sum_{diagramas\, I} \hat{A}_L(z_I)\frac{1}{\hat{P}_I^2}\hat{A}_R(z_I)$$

Britto, Cachazo, Feng and Witten, 2005

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations ✦KLT

This amazing result that came from *String Theory* relates SA of gravity with SA of Yang-Mills theory (tree level)

$$\boxed{\mathcal{A}_4(\text{Gravity}) = \mathcal{A}_4(\text{YM})^2}$$

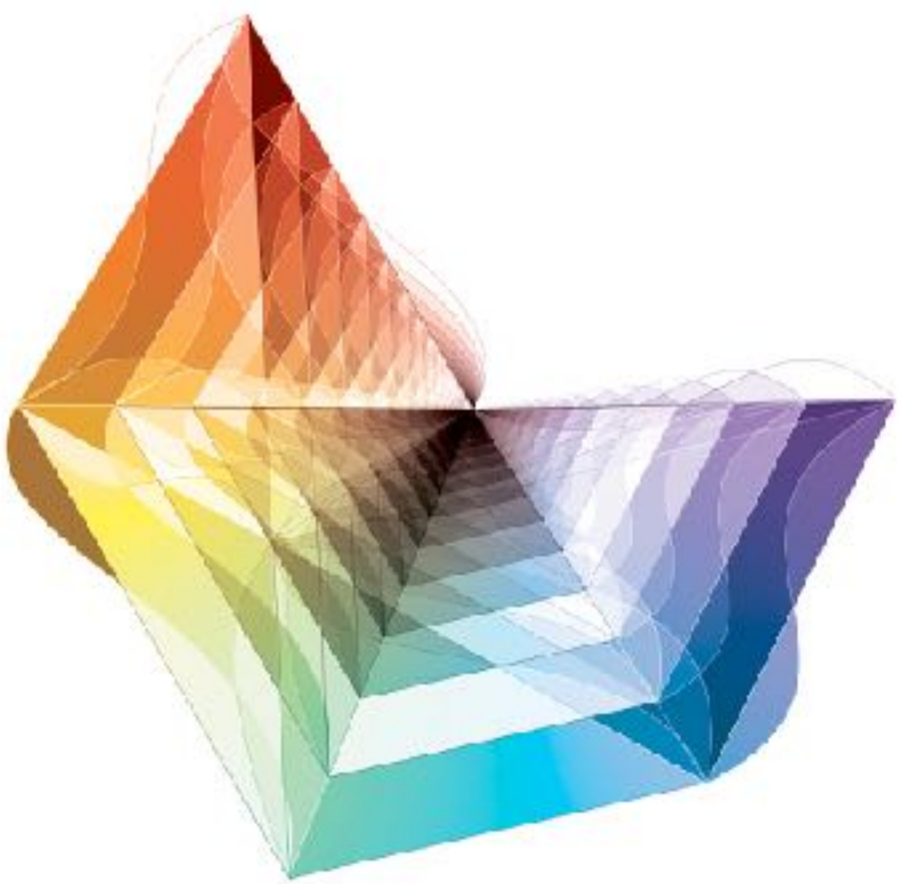
Kaway, Lewellen, Tye, 1986

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations
- Geometrization of SA

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations
- Geometrization of SA ✦ The Amplituhedron



Applying BCFW and complex analysis in several variables it is possible to express (in some toy model theories) the SA as the volume of a polyhedron.

N.Arkani-Hamed, et.al.

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations
- Geometrization of SA
- SA from first principles

During the last decade, a lot of progress has been done to understand the mathematical structure of the scattering amplitudes, I would like to point out some of the most important

- on-shell recursion relations
- Geometrization of SA
- SA from first principles

Just applying momentum conservation and gauge invariance it is possible to express some tree level amplitudes for gluons and gravitons.

R. Medina, et.al.
N. Arkani-Hamed, et.al.

Gravitino Phenomenology with SHF

In SUSY theories with gravity, the spin-3/2 gravitino is the superpartner of the graviton and it is considerate one candidate for DM, when this is the LSP.
In order to investigate the nature of the gravitino and the NLSP (Cosmology and Collider Physics), it is necessary to compute scattering amplitudes that involve gravitinos in the final state. Using the traditional Feynman approach (Trace technology) result extremely laborious to compute observables.

Some progress has been done in order to express the 4 gravitino states in terms of spinor variables

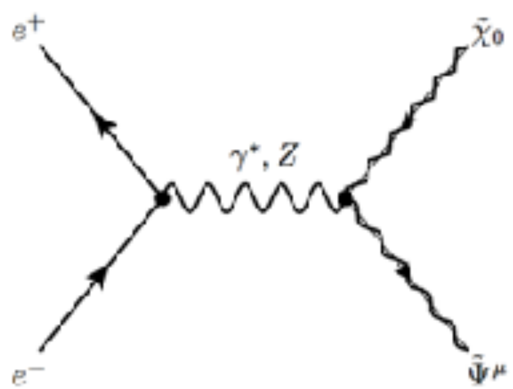
$$\tilde{\Psi}^{\mu}_{++}(p) = \beta^{\mu}_{1}|r\rangle + \tilde{m}\beta^{\mu}_{2}|q],$$
$$\tilde{\Psi}^{\mu}_{--}(p) = -\beta^{*\mu}_{1}|r] + \tilde{m}\beta^{*\mu}_{2}|q\rangle,$$
$$\tilde{\Psi}^{\mu}_{-}(p) = \beta^{\mu}_{3}|r] + \tilde{m}(\beta^{\mu}_{4}|q\rangle + \beta^{\mu}_{5}|r\rangle) + \tilde{m}^{2}(\beta^{\mu}_{6}|r] + \beta^{\mu}_{7}|q]) + \tilde{m}^{3}\beta^{\mu}_{8}|q\rangle,$$
$$\tilde{\Psi}^{\mu}_{+}(p) = \beta^{*\mu}_{3}|r\rangle - \tilde{m}(\beta^{*\mu}_{4}|q] + \beta^{*\mu}_{5}|r]) + \tilde{m}^{2}(\beta^{*\mu}_{6}|r\rangle + \beta^{*\mu}_{7}|q\rangle) - \tilde{m}^{3}\beta^{*\mu}_{8}|q],$$

L. Diaz-Cruz, BL, 2017

With these states at hand it has been possible to evaluate several processes and reactions considering the full (massive) gravitino and also with the goldstino approximation.

Some calculations with gravitino

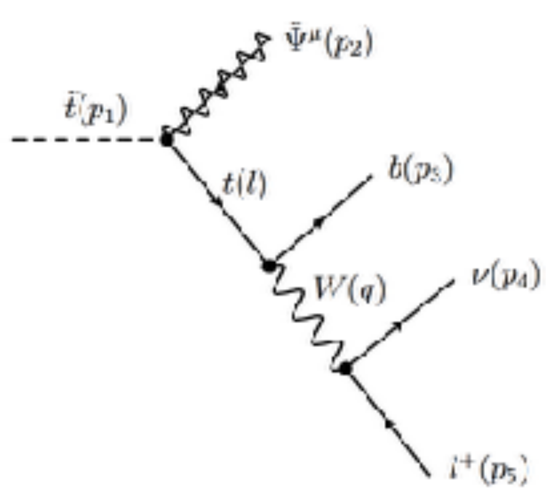
- Associated production of $e^+e^- \to \tilde{\Psi}^\mu \tilde{\chi}_0$



$\lambda_1\lambda_2\lambda_3\lambda_4$	$\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}$
$-,+;+,-$	$-\frac{2e\eta(2s_{12}-M_Z^2)(A_{\tilde{G}}^2 m_{\tilde{\chi}_0} s_{qr}+A_{\tilde{\chi}_0}^2 \tilde{m}^3)}{s_{12}(s_{12}-M_Z^2)M A_{\tilde{G}}\langle qr\rangle}\langle 2r\rangle[1r]$
$-,+;+,+$	$\frac{e\eta(2s_{12}-M_Z^2)(A_{\tilde{G}} s_{qr}^2+A_{\tilde{\chi}_0}\tilde{m}^3 m_{\tilde{\chi}_0})}{s_{12}(s_{12}-M_Z^2)s_{qr}M}\langle 2q\rangle[1r]$
$-,+;-,-$	$\frac{e\eta(2s_{12}-M_Z^2)(A_{\tilde{G}} s_{qr}^2+A_{\tilde{\chi}_0}\tilde{m}^3 m_{\tilde{\chi}_0})}{s_{12}(s_{12}-M_Z^2)s_{qr}M}\langle 2r\rangle[1q]$
$-,+;-,+$	$-\frac{2e\eta(2s_{12}-M_Z^2)(A_{\tilde{G}}^2 m_{\tilde{\chi}_0} s_{qr}+A_{\tilde{\chi}_0}^2 \tilde{m}^3)}{s_{12}(s_{12}-M_Z^2)M A_{\tilde{G}}[qr]}\langle 2r\rangle[1r]$
$-,+,++,-$	$-\frac{2e(2s_{12}-M_Z^2)(A_{\tilde{\chi}_0}\tilde{m}+A_{\tilde{G}} m_{\tilde{\chi}_0})}{\sqrt{2}s_{12}(s_{12}-M_Z^2)s_{qr}M}[rq]^2[1r]\langle 2q\rangle$
$-,+,--,+$	$-\frac{2e(2s_{12}-M_Z^2)(A_{\tilde{\chi}_0}\tilde{m}+A_{\tilde{G}} m_{\tilde{\chi}_0})}{\sqrt{2}s_{12}(s_{12}-M_Z^2)s_{qr}M}\langle qr\rangle^2[1q]\langle 2r\rangle$

- 4-Body stop decay $\tilde{t} \to \tilde{\Psi}^\mu \, b \, l^+ \, \nu_l$

L.Diaz-Cruz, BL, 2017



$\lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5$	$\mathcal{T}_{\lambda_2,\lambda_3,\lambda_4\lambda_5}$
$-;-,-,+$	$-2\langle 43\rangle[r_5q_1](A_{\tilde{t}}\tilde{m}\cos\theta_{\tilde{t}} - m_t\sin\theta_{\tilde{t}})$
$+;-,-,+$	$\frac{2\langle 43\rangle[r_5r_1]}{[q_1r_1]}(A_{\tilde{\Psi}} s_{q_1r_1}\sin\theta_{\tilde{t}} + m_t\tilde{m}\cos\theta_{\tilde{t}})$

L.Diaz-Cruz, BL, 2017

Final Comments

- It will be interesting if in the future the *scattering amplitudes* involved in realistic theories are computed from first principles or with a geometric approach.
- Our next step (wish) is to implement recursion relations to SA with gravitinos in the final state.
- Apply our results to Cosmology.

Thank you