Forward physics and the glue at small $x$

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May 24, 2017
Outline

A short introduction

High energy effective action (canceled)

BFKL & exclusive Vector Mesons

3 parton production in the presence of high gluon densities

Conclusion
Low $x$ physics .... a small community

- yellow report on LHC forward physics (184 authors) [arXiv:1611.05079]
- typical workshop size: 89 participants (DIFFRACTION 2016, International Workshop on Diffraction in High-Energy Physics), 44 talks in Low x and Diffraction working group at DIS 2017

- overlap with various (QCD) communities
  - heavy ion/high multiplicity physics
  - transverse momentum dependence in parton distribution functions
  - physics of a future Electron Ion Collider
  - higher order corrections & precision physics (resummation!)
  - new physics searches in ‘strange’ hadronic processes

- forward physics in e.g. $pp$: rapidities close to the beam line probe 2nd proton at low $x$ ...
interested in strong interactions, in particular the dynamics of the gluon in the high energy limit
Important QCD property: asymptotic freedom

Strong coupling strong at large distances $\equiv$ low energy scales

Weak at small distances $\equiv$ hard energy scales

$\alpha_s = \alpha_s(Q)$ running coupling

$\Rightarrow$ process with hard scale accessible to perturbative treatment
A short introduction

**Hadron structure from Deep Inelastic Scattering (DIS) of electrons on protons**

- knowledge of scattering energy + nucleon mass
- + measure scattered electron ➔ control kinematics

\[ k \cdot p \]

\[ k' \]

\[ q \]

\[ p \]

\[ X \]

**Photon virtuality**

\[ Q^2 = -q^2 \]

**Bjorken**

\[ x = \frac{Q^2}{2p \cdot q} \]

**Resolution**

\[ \lambda \sim \frac{1}{Q} \]

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Forward physics & small \( x \) gluon  
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Interpretation of observation (MIT-SLAC ’67)

Bjorken: parametrize cross-section in terms of proton structure functions $F_1$ and $F_2$ ($F_L = F_2 - 2xF_1$)

$$\frac{d^2\sigma^{eh\to eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[(1 + (1 - y)^2 F_2(x, Q^2) - y^2 F_L(x, Q^2)\right]$$

Feynman [Feynman, August 1968]:

- proton composed of generic point-like free constituents, called “partons” – later identified as quarks & gluons

**proton structure function**

$$F_2(x, Q^2) = \sum_{\text{partons}} e_{\text{parton}}^2 \cdot f_{\text{parton}}(x)$$

**parton distribution function**

$f_{\text{parton}}(x)$ probability to hit a parton with proton momentum fraction $x$
- fit $x$-dependence of pdf's at initial scale $Q_0^2 \sim 2$ GeV$^2$

- QCD (DGLAP equation): evolution from $Q_0^2$ to $Q^2$

$$\partial \ln \mu_f^2 f_k(x, \mu_f) = \sum_{l=q,g} P_{kl} \otimes f_l$$
The proton at high center of mass energies

At small $x$: proton dominated by gluons (and sea-quarks)

At small $x$: Parton fluctuations time dilated on strong interaction times scales

... long lived gluons radiate further small $x$ gluons

power-like rise of gluon and sea-quark distribution – probability distribution!

....

H1 and ZEUS

$Q^2 = 10 \text{ GeV}^2$
The proton at high energies: saturation

theory considerations:

- effective finite size $1/Q$ of partons at finite $Q^2$
- at some $x \ll 1$, partons ‘overlap’ = recombination effects
- turning it around: system is characterized by saturation scale $Q_s$
- grows with energy $Q_s \sim x^{-\Delta}$, $\Delta > 0$ & can reach in principle perturbative values $Q_s > 1\text{GeV}$
Can attempt description using models, ... ultimate understanding: QCD perturbation theory

requires:

- expansion of perturbative amplitudes in $x \ll 1$
- naïve expansion breaks down due to large logs $\alpha \ln \frac{1}{x} \sim 1$
- resummation of enhanced terms $(\alpha_s(Q^2) \ln \frac{1}{x})^n \sim 1$ to all orders in $\alpha_s$

$\rightarrow$ BFKL equation

**LL:** [Fadin, Kuraev, Lipatov, PLB 60 (1975) 50]
[Balitsky, Lipatov, SJNP (1978 822)]

**NLL:** [Fadin, Lipatov; PLB 429 (1998) 127];
[Ciafaloni, Camici; PLB 430 (1998) 349]

**+RG:** [Salam; hep-ph/9806482], [many others ...],
[MH, Salas, Sabio Vera; 1209.1353]

resummed/collinear improved NLO BFKL kernel
linear vs. non-linear evolution

\[ \partial \ln \frac{1}{x} G(x, k) = K \otimes G \quad \text{BFKL = linear/low density evolution} \]

roughly speaking: powerlike rise \( G \sim x^{-\lambda} \) of gluon

assuming dense gluonic field \( A_\mu \sim \frac{1}{g} \): non-linear extension of BFKL

\[ \partial \ln \frac{1}{x} G(x, k) = K \otimes G - G \otimes G \quad \text{BK-JIMWLK = non-linear evolution} \]


fixed point at \( x = 0 \): gluon density constant for \( s \to \infty \)
Phenomenology at the LHC

- events close in rapidity to one of the rescattering protons \( \equiv \) forward events \( \Rightarrow \) probe second proton at low values of

\[
x = \frac{p_t}{\sqrt{s}} e^{-\eta},
\]

- want to find:

- evidence for BFKL evolution & fix the region of its applicability
- determine the region where saturation effects become relevant
- needed to arrive at a proper understanding of saturated matter \( \rightarrow \) determination of the initial state of heavy ion collisions
- want to understand which picture is correct (low density/DGLAP) or (low density/BFKL) or (high density/JIMWLK/BK)
photo-production of $J/\Psi$ and $\Upsilon$: explore proton at ultra-small $x$
[Bautista, Ferandez-Tellez, MH; 1607.05203]

- measured at HERA ($ep$) and LHC ($pp$, ultra-peripheral $pPb$)
- charm and bottom mass provide hard scale → pQCD
- exclusive process, but allows to relate to inclusive gluon

reach values down to $x = 4 \times 10^{-6}$ → (unique ?) opportunity to explore the low $x$ gluon
BFKL & exclusive Vector Mesons

DGLAP vs. saturation (I)

- log($xg(x)$)
- LHC UPC data
- HERA fit
- rcBK, sat. models
- evolve, extrapolate
- larger $W^2$
- log($1/x$)

$g \left( x = \frac{M_p^2}{W^2} \right)$

- describes data or not $\rightarrow$ re-fit
- if yes: do we really see saturation effects?

i.e. BK type evolution

$$\frac{d}{d \ln 1/x} G(x) = K \otimes G(x) - \underbrace{G \otimes G}_{\text{present, relevant?}}$$
DGLAP vs. saturation (II)

\[
\log(xg(x))
\]

evolve to higher scales e.g. \(M_\Upsilon\)

fit HERA + LHC data

at \(Q_0 \approx M_{J/\Psi}\)

\(g\left(x = \frac{M_\Upsilon^2}{W^2}\right)\)

\[
\log\left(\frac{1}{x}\right)
\]

\(J/\Psi \rightarrow \Upsilon \rightleftharpoons\) evolution 2.4 GeV\(^2\) \(\rightarrow\) 22.4 GeV\(^2\)

- high density effects die away in collinear limit
- DGLAP unstable at ultra-small \(x\) and small scales ...
- convinced: pdf studies highly valuable \(\rightarrow\) constrain pdf’s at ultra-small \(x\)
- useful benchmark for saturation searches (?)
Will argue in the following ...

- a far better dilute (!) benchmark might (?) be given by BFKL evolution (→ applies for UPCs@LHC, might be different for a future (US-)EIC ... → phase space!)

- why? .... BFKL ≡ low $x$ evolution without high density/saturation effects

- available up to NLO [Fadin, Lipatov; PLB 429 (1998) 127]; [Ciafaloni, Camici; PLB 430 (1998) 349], resummation schemes for coll. logs exist & to some degree well explored [Salam; hep-ph/9806482], ...

- not only explored in $n = 0$ sector → additional constrains from e.g. angular decorrelation studies of jets → see talks on Wednesday
The underlying NLO BFKL fit to DIS data

\[ F_2(x, Q^2) = \int_0^\infty dk^2 \int_0^\infty dq^2 \frac{q^2}{Q^2} \Phi_2 \left( \frac{k^2}{Q^2} \right) \mathcal{F}^{\text{DIS}}_{\text{BFKL}}(x, k^2, q^2) \Phi_p \left( \frac{q^2}{Q_0^2} \right) \]

virtual photon: quarks mass-less, \( n_f = 4 \) fixed

proton impact factor: \( \Phi_p \left( \frac{q^2}{Q_0^2}, \delta \right) = \frac{C}{\pi \Gamma(\delta)} \left( \frac{q^2}{Q_0^2} \right)^\delta e^{-\frac{q^2}{Q_0^2}} \)

free parameters of proton impact factor from fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

allows for definition of unintegrated gluon density [Chachamis, Deak, MH, Rodrigo, Sabio Vera; 1507.05778]

<table>
<thead>
<tr>
<th>virt. photon impact factor</th>
<th>( Q_0 / \text{GeV} )</th>
<th>( \delta )</th>
<th>( C )</th>
<th>( \Lambda_{\text{QCD}} / \text{GeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit 1 leading order (LO)</td>
<td>0.28</td>
<td>8.4</td>
<td>1.50</td>
<td>0.21</td>
</tr>
<tr>
<td>fit 2 LO with kinematic improvements</td>
<td>0.28</td>
<td>6.5</td>
<td>2.35</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Good description of combined HERA [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
Solve BFKL equation in conjugate ($\gamma$) Mellin space

$$G(x, k^2, M) = \frac{1}{k^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \hat{g} \left(x, \frac{M^2}{Q_0^2}, \frac{M^2}{M^2}, \gamma\right) \left(\frac{k^2}{Q_0^2}\right)^\gamma$$

re-introduce two scales: hard scale of process ($M$) and scale of running coupling ($\overline{M}$)

$\hat{g}$: operator in $\gamma$ space!

$$\hat{g} \left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, \gamma\right) = \frac{C \cdot \Gamma(\delta - \gamma)}{\pi \Gamma(\delta)} \cdot \left(\frac{1}{x}\right)^\chi\left(\gamma, \frac{M^2}{\overline{M}^2}\right).$$

$$\left\{ 1 + \frac{\bar{\alpha}_s^2/\beta_0 \chi_0 (\gamma)}{8N_c} \log \left(\frac{1}{x}\right) \left[ -\psi(\delta - \gamma) + \log \frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\},$$

resummed NLO BFKL eigenvalue with optimal scale setting ($\rightarrow$ modifies $\chi_1(\gamma)$):

$$\chi \left(\gamma, \frac{\overline{M}^2}{M^2}\right) = \bar{\alpha}_s \chi_0 (\gamma) + \bar{\alpha}_s^2 \tilde{\chi}_1 (\gamma) - \frac{1}{2} \bar{\alpha}_s^2 \chi'_0 (\gamma) \chi_0 (\gamma)$$

$$+ \chi_{RG}(\bar{\alpha}_s, \gamma, \tilde{a}, \tilde{b}) - \frac{\bar{\alpha}_s^2/\beta_0}{8N_c} \chi_0 (\gamma) \log \frac{\overline{M}^2}{M^2}.$$
provide study for two hard scales:
photoproduction scale: $M_{pp} = M_V/2$
impact factor motivated: $M_{if}^2 = 8 R_V^{-2}$
fix normalization by low energy H1 data point $\rightarrow$ K-factor; no further adjustments
**BFKL & exclusive Vector Mesons**

**comparison to data: \( J/\Psi \) production**

- NEW (wrt. [Bautista, Fernando Tellez, MH; 1607.05203]): 13 TeV LHCb data
- fix normalization by low energy ALICE data point \( \to \) K-factor
  - believe: related to HERA fit (massless, \( n_f = 4, (C_1/C_2)^2 = 2.45 \))
- often included (not here): GPD motivated factor ("\( x' \neq x \" \)); known for collinear [Shuvaev, Golec-Biernat, Martin, Ryskin, hep-ph/9902410]
  - to be calculated for \( k_T \) factorized BFKL impact factor
  - \( \sim \) kinematic improvements for \( \gamma \to V \)
Concluding remarks for Vector mesons

BFKL fits [MH, Salas, Sabio Vera; 1301.5283] somehow approach its limits, but so far works very well → keep in mind: most simple combination of existing NLO BFKL fit & existing VM impact factor

first suggestion: no need for saturation effects, linear NLO evolution sufficient

why so hard to manifest saturation? .... two possible reasons:

a) BFKL simply appropriate framework,... saturation effects not (yet) present

b) observable \sim N_{\text{dipole}} \Leftrightarrow G^{\text{BFKL}}_{\text{ugd}} \Rightarrow high density effects (if at all present) only through evolution

but evolve not even an order of magnitude w.r.t. HERA data;
A possible way out ...
observables with higher order correlators of Wilson lines
→ inclusive observables (no gap) + resolved final states
(e.g. inclusive di- & tri-hadrons/jets)

\[ \mathcal{N} \sim 1 - \frac{1}{N_c} \text{tr} \left[ V(x)V^\dagger(y) \right] \leftrightarrow G^{\text{BFKL}}(x, k) \]

\[ Q^{(4)} \sim 1 - \frac{1}{N_c} \text{tr} \left[ V(x)V^\dagger(y)V(y')V^\dagger(x') \right] \leftrightarrow G + #G^2 + #G^4 + ... \]
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prelim. study using bCGC model

... work in progress, no conclusion to be drawn yet
A new process: 3 particle production at high densities

[Ayala, MH, Jalilian-Marian, Tejeda-Yeomans; 1604.08526, 1701.07143]

► assumption: high gluon densities exists ⇒ higher order correlators ≡ new dynamics w.r.t. BFKL
► access to $Q^{(4)} \sim 1 - \frac{1}{N_c} \text{tr} \left[ V(x) V^{\dagger}(y) V(y') V^{\dagger}(x') \right] \leftrightarrow G + \#G^2 + \#G^4 + \ldots$ and more complicated stuff
► experimental difficult observable: relate to more inclusive quantity (energy loss etc.)
Theory: Propagators in background field

use light-cone gauge, with $k = n \cdot k$, $(n)^2 = 0$, $n \sim$ target momentum

\[ S_F^{(0)}(p) = \frac{i p^\mu + m}{p^2 - m^2 + i 0} \quad \tilde{G}_\mu^\nu(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i 0} \]

\[ d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu n_\nu}{n^- \cdot p} \]

interaction with the background field:

\[ V(z) \equiv V_{ij}(z) \equiv P \exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, z) t^c \]

\[ U(z) \equiv U^{ab}(z) \equiv P \exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, z) T^c \]

strong background field resummed into path ordered exponentials (Wilson lines)

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...
Two-fold interest: phenomenology & further development of theory

- spinor helicity formalism $\equiv$ expand elements of amplitudes (propagators, spinors, polarization vectors, ...) in terms of mass-less spinors with helicity $\lambda = \pm$

$$|k_i^\pm\rangle \equiv \frac{1 \pm \gamma_5}{2} u(k_i), \quad \epsilon_\mu^\pm(k, n) = \pm \frac{\langle n^\mp|\gamma_\mu|k^\mp\rangle}{\sqrt{2}\langle n^\mp|k^\pm\rangle}$$

- important relations $\langle i^\pm|j^\pm\rangle = 0$ and $\langle i^\pm|i^\mp\rangle = 0$

- successfully employed in multi-loop & leg calculations
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- QCD in the high-energy limit: helicity conserved during interaction with high-energy gluon + use of axial gauge \(A \cdot n = 0\)

- expansion in spinors reflects these symmetries in a (perfect?) way \(\rightarrow\) very compact expressions at amplitude level [Ayala, MH, Jalilian-Marian, Tejeda-Yeomans; arXiv:1701.07143]
First phenomenology on partonic level: angular decorrelation

- here: density effects through large nucleus (Au)
- fix one (transverse) angle at \( \theta_{qg} = \frac{2\pi}{3} \), vary \( \theta_{qg} \)

- no \( p_T \) from the nucleus/proton \( Q_S \rightarrow 0 \)

\[ \Rightarrow \text{Mercedes-Benz-star configuration dominant} \]

- higher correlators: Gaussian-approximation expanded to quadratic order in 2-point correlator (model)
Conclusions

- low $x$/forward physics: QFT under extreme conditions
- need to resum high energy logarithms and possibly high density effects
- leading order description often insufficient for precise quantitative understanding $\Rightarrow$ higher order corrections
- require the development of new calculational techniques
- at the same time need to identify discriminative observables
- lots to be done, but exciting times!