Weak mixing angle at low energies: an update.

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Several upcoming experiments!

1. Qweak at JLab has measured the weak charge of the proton, $Q_W(p) \sim 1 - 4\sin^2\theta_W$. In polarized $e^- H \to e^- H$ at $Q^2 \approx 0.026\ GeV^2$.
2. P2 experiment, same observable, at $Q^2 \approx 0.0045\ GeV^2$, in Mainz, Germany.
3. MOLLER at JLab $Q_W(e)$, in polarized Moller scattering at $Q^2 \approx 0.0056\ GeV^2$.
4. The PVDIS Collaboration at the 6 GeV CEBAF complex at JLab $e^- D \to e^- X$.
5. SoLID Collaboration will increase the PVDIS precision and a correspondingly higher and broader Q^2 range.

Definitions of the weak mixing angle.

We can define the weak mixing angle in terms of the $\mathrm{SU}\left(2\right)_{\mathrm{L}}$ and $\mathrm{U}\left(1\right)_{\mathrm{Y}}$ gauge couplings or in terms of the fine structure constant:

$$\sin^2\theta_W = \frac{g^{\prime 2}}{g^2 + g^{\prime 2}} \quad \alpha = \frac{e^2}{4\pi} = \frac{g^2 \sin^2\theta_W}{4\pi} \tag{1}$$

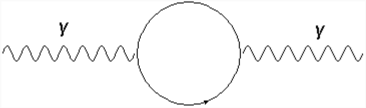
There is also a relation coming from the Electroweak Symmetry Breaking

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \tag{2}$$

the first two definitions are modified by radiative corrections. In the On-Shell Scheme one uses the equation 2 to all orders in perturbation theory.

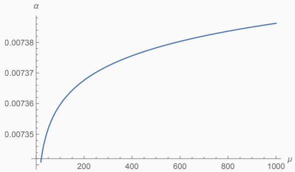
Renormalization group equation

Loops produce corrections to the coupling coefficients! for example the diagram



leads to an RGE for $\hat{\alpha}$ of the form

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{1}{3}\frac{\hat{\alpha}^2}{\pi}$$



the same type of running applies to the weak mixing angle.

RGE

The vector coupling is related to the weak mixing angle by

$$v_i = T_i - 2Q_i \sin^2 \hat{\theta} \tag{3}$$

the running of this vector coupling can be obtained from the mixed vacuum self energy diagrams

Solution to the RGE.

After some lengthy algebra they get the following result for the weak mixing angle

$$\hat{s}_{\mu}^{2} = \hat{s}_{\mu_0}^{2} \frac{\hat{\alpha}_{\mu}}{\hat{\alpha}_{\mu_0}} + \lambda_1 \left[1 - \frac{\hat{\alpha}_{\mu}}{\hat{\alpha}_{\mu_0}} \right] + \frac{\hat{\alpha}_{\mu}}{\pi} \left(\frac{\lambda_2}{3} \ln \left(\frac{\mu^2}{\mu_0^2} \right) + \frac{3\lambda_3}{4} ln \frac{\hat{\alpha}_{\mu}}{\hat{\alpha}_{\mu_0}} + \bar{\sigma}_{\mu_0} - \bar{\sigma}_{\mu} \right) \tag{4}$$

where the lambdas λ´s are numerical constants that depend on the number of particles, and

$$\bar{\sigma} = \lambda_4 \frac{5}{36} \frac{11 - 24\varsigma(3)}{33 - 2n_q} \frac{\hat{\alpha_s}^2}{\pi} + \frac{\hat{\alpha}_s^3}{\pi^3} \tilde{C}$$

is the contribution of the OZI diagrams.

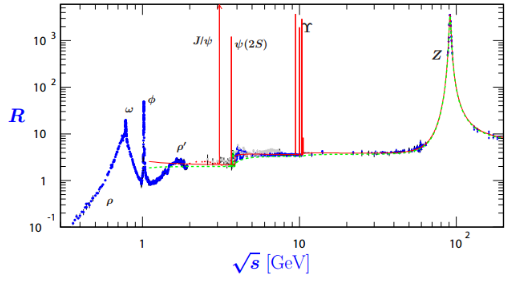
Experimental input .

In the $\overline{\mathrm{MS}}$ scheme we have the relation

$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds \frac{R(s)}{s-i\epsilon} + 4\pi I^{(3)} \tag{5}$$

where

$$R(s) = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad \hat{\alpha}(\mu) = \frac{\alpha_0}{1+\Delta\alpha_{had}+\Delta\alpha_{lep}} \tag{6}$$



Experimental input .

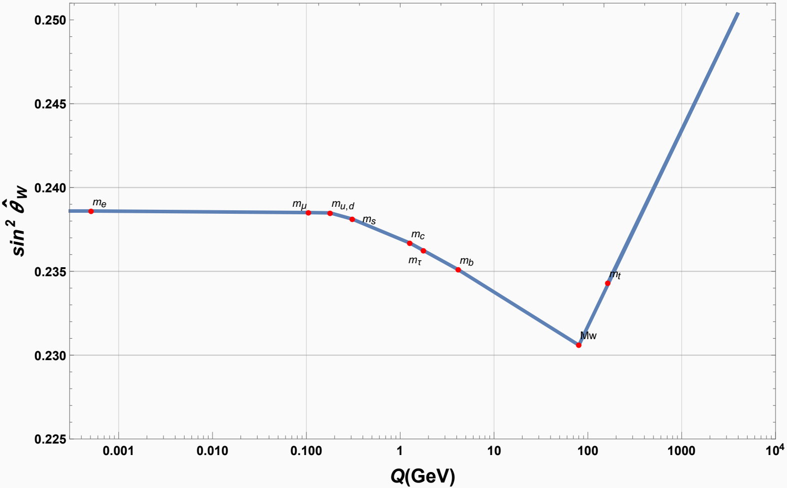
We extended the value of $I^{(3)}$ to include terms of order α_s^3

$$4\pi I^{(3)} = 2\alpha \int_0^{2\pi} d\theta \hat{\Pi}\left(\mu^2 e^{i\theta}\right) \tag{7}$$

Using the high energy expansion for $\hat{\Pi}$ and the relation between the pole and running $\overline{\mathrm{MS}}$ mass we get

$$4\pi I^{(3)} = \frac{2\alpha}{3\pi}\left[\frac{5}{3} - (0.2201)\frac{\hat{\alpha}_s}{\pi} - 2.849\left(\frac{\hat{\alpha}_s}{\pi}\right)^2 - 13.13\left(\frac{\hat{\alpha}_s}{\pi}\right)^3\right] \tag{8}$$

Running of the weak mixing angle



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Zweig (OZI) rule deviations $\delta_{OZI} \sin^2 \hat{\theta}_W\left(0\right)$.

Threshold masses.

We define the threeshold masses $\bar{m}_s$ $\bar{m}_u$ $\bar{m}_d$, with matching conditions trivial $\alpha_i^+ = \alpha_i^-$. In the perturbative case

$$\hat{\Pi}(0,\mu) + \delta\hat{\Pi}(0,\mu) = 0$$

the higher order corrections imply a correction of the form

$$\bar{m}^{(3)} = \bar{m}^{(2)}(1+\delta) \tag{9}$$

where using physical inputs give us for example

$$\bar{m}_c^{(3)} = 1.184\,\mathrm{GeV} \quad \delta = 0.008 \tag{10}$$

but what about the non perturbative case?

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We put bounds on the, for example the heavy limit

$$\xi_q = \frac{2\bar{m}_q}{M_{1S}} \qquad \rightarrow \bar{m}_s < \frac{\xi_c M_\phi}{2} = 390\,\mathrm{Mev} \quad \Delta_s \hat{\alpha}\left(\bar{m}_c\right) > 6.76{\times}10^{-4} \tag{11}$$

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while the SU(3) limit

$$\triangle \hat{\alpha}\,(\bar{\mu})^{(3)} = \frac{2\hat{\alpha}^2}{\pi}\left[\left(Q_u^2 + Q_d^2\right) K_{QCD}^u ln\left(\frac{2\bar{\mu}}{\xi_{u,d} M_\omega}\right) + Q_s^2 K_{QCD}^c ln\left(\frac{2\bar{\mu}}{\xi_s M_\phi}\right)\right] \tag{12}$$

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$$\xi_s > \frac{2\bar{\mu}}{M_\omega^{\frac{5}{6}} M_\phi^{\frac{1}{6}}} e^{-\frac{3\pi}{4}\frac{\Delta\hat{\alpha}^{(3)}}{\alpha K}} \qquad \rightarrow \Delta_s\hat{\alpha}\left(\bar{m}_c\right) < 9.4\times 10^{-4} \tag{13}$$

The Implications.

Plugging in these results into the equation for the weak mixing angle (the one we obtained using the α RGE) we see that the contribution to the uncertainty given by $\Delta\hat{\alpha}^{(3)}(\bar{m}_c)$ and $\Delta\hat{\alpha}^{(2)}(\bar{m}_c)$ are

$$\delta_\alpha \sin^2\hat{\theta}_W(0) < \pm 2.6 \times 10^{-5} \tag{14}$$

$$\delta_s \sin^2\hat{\theta}_W(0) \simeq \pm 4 \times 10^{-5} \tag{15}$$

Using a lattice result (T.Blum 2016)

$$\delta_{OZI}\hat{s} = 7 \times 10^{-6} \tag{16}$$

and for the SU(2) breaking

$$\delta_{CVC} \sin^2\hat{\theta}_W(0) =^{+}_{-7.1} \times 10^{-6} \tag{17}$$

Theoretical uncertainty.

Adding the uncertainties in quadrature give us the final theoretical uncertainty

$$\delta_{theory}\hat{s}^2 = 5 \times 10^{-5} \tag{18}$$

with a central value given by

$$\sin^2 \hat{\theta}_W (0) = 0.23862 \pm 5 \times 10^{-5} \tag{19}$$

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Upcoming ... use of τ decay to constraint more $\Delta_s \hat{\alpha}\,(\bar{m}_c)$.