UNIVERSIDAD DE COLIMA

Lepton Flavor and CPV

Alexander J. Stuart
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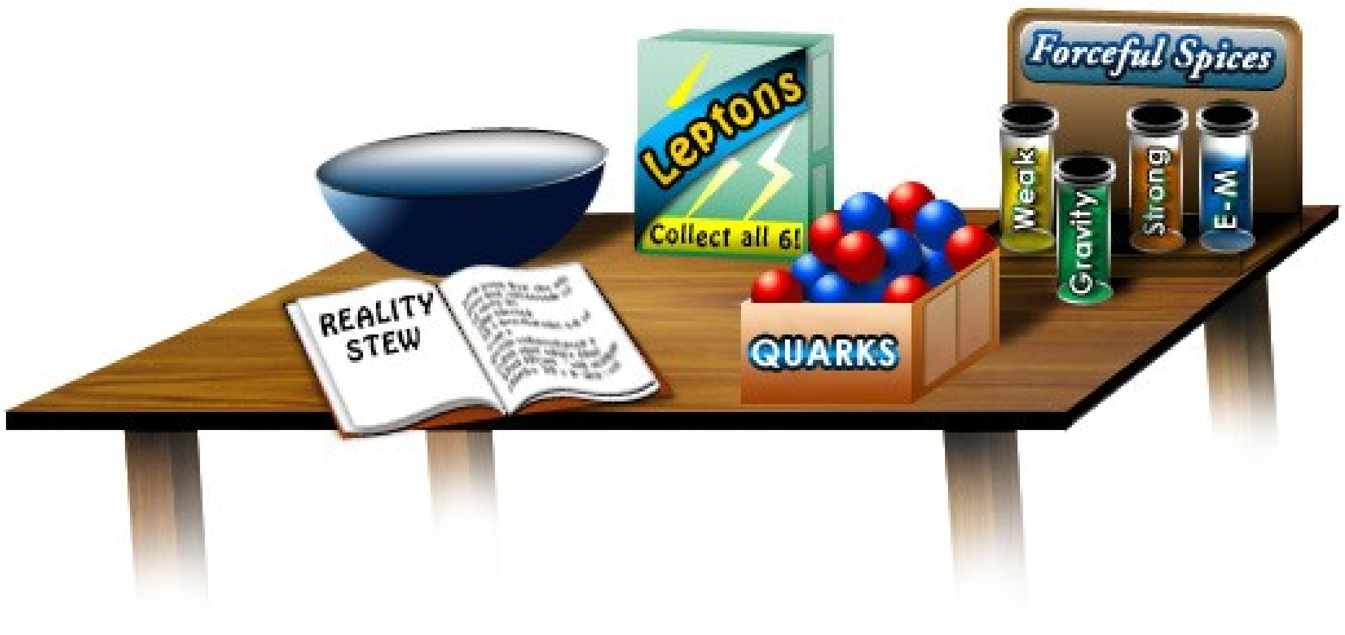
Based on: L.L. Everett, T. Garon, and AS, JHEP **1504**, 069 (2015) [arXiv:1501.04336]; L.L. Everett and AS, arXiv:1611.03020 [hep-ph].

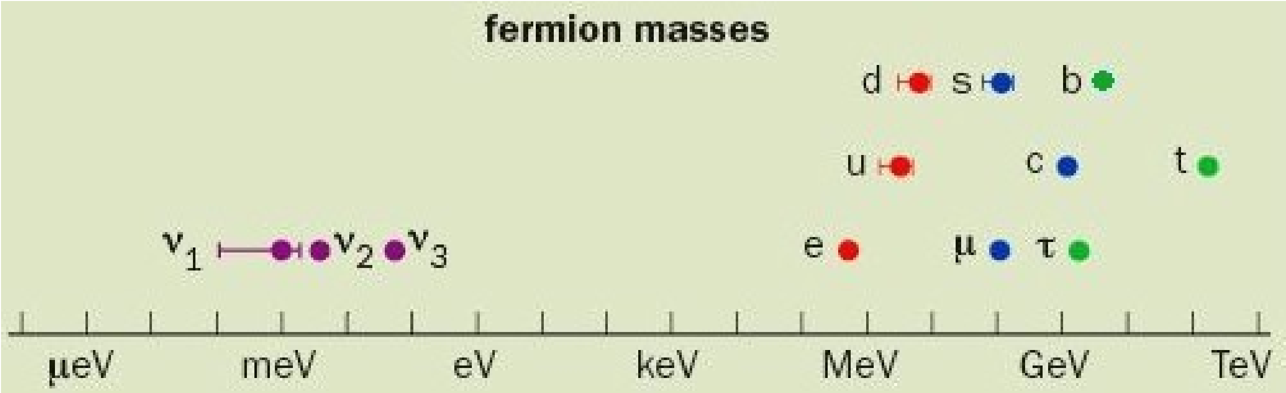
The Standard Model

Triumph of modern science, but incomplete....
Does not predict fermion masses and mixings.



http://www.particleadventure.org/standard_model.html

What We Taste



Quark Mixing

$$U_{CKM} = R_1(\theta_{23}^{CKM})R_2(\theta_{13}^{CKM}, \delta_{CKM})R_3(\theta_{12}^{CKM})$$

$$\theta_{13}^{CKM} = 0.2^\circ \pm 0.1^\circ$$

$$\theta_{23}^{CKM} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{12}^{CKM} = 13.0^\circ \pm 0.1^\circ$$

$$\delta_{CKM} = 60^\circ \pm 14^\circ$$

Lepton Mixing

$$U_{PMNS} = R_1(\theta_{23})R_2(\theta_{13}, \delta_{CP})R_3(\theta_{12})P$$

$$\theta_{13}^{MNSP} = (8.46^\circ)^{+0.15^\circ}_{-0.15^\circ}$$

$$\theta_{23}^{MNSP} = (41.6^\circ)^{+1.5^\circ}_{-1.2^\circ}$$

$$\theta_{12}^{MNSP} = (33.56^\circ)^{+0.77^\circ}_{-0.75^\circ}$$

$$\delta_{CP} = (261^\circ)^{+51^\circ}_{-59^\circ}$$

M.C. Gonzalez-Garcia et al: 1611.01514

http://lbne.fnal.gov/how-work.shtml

Quarks look like perturbations away from Identity.
Where does this enter the SM?

Fermion Masses in the Standard Model

The masses and mixings of the fermions in the SM have their origin in Yukawa interactions with the Higgs condensate:

$$\mathcal{L}_Y = -Y^d_{ij}\overline{Q_i}Hd_j - Y^u_{ij}\overline{Q_i}\epsilon H^* u_j - Y^e_{ij}\overline{L_i}He_j + h.c.$$

Q_i are left-handed quark doublets, L_i are left-handed lepton doublets, u_j are right-handed up-type quark singlets, d_j are right-handed down-type quark singlets, e_j are right-handed electron-type singlets, *H* is the Higgs field, *i,j* are fermion generation/flavor labels running from 1 to 3. As such $Y^{u,d,e}$ are 3 x 3 complex matrices.

When *H* acquires a vacuum expectation value: $< H >= v \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

the above operators yield mass terms for the quarks and charged leptons. The masses can be obtained by diagonalizing $M^{u,d,e} = vY^{u,d,e}$ as:

$$M^{Diag}_{u,d,e} = U^\dagger_{u,d,e}(vY^{u,d,e})V_{u,d,e}$$

Since there are no right-handed neutrino singlets in the SM, neutrinos are massless. However, neutrinos **oscillate**. Therefore, neutrinos have mass. How can we accommodate this theoretically?

Seesaw Mechanism (Type-I)

(P. Minkowski (1977); M. Gell-Mann, P. Ramond, R. Slansky (1979); T. Yanagida (1980); R. Mohapatra, G.Senjanovic (1980)....)

Could just add right handed neutrino fields to Standard Model.... But then neutrino Yukawa coupling is $\sim 10^{-12}$! Very small, even by comparison with electron, $y_e \sim 10^{-5\text{-}6}$.

Instead add three heavy right-handed Majorana neutrinos with mass around GUT scale ($M_N \sim 10^{16}$ GeV) generating both Dirac and Majorana mass term for neutrinos:

$$\mathcal{L}_\nu = -Y^\nu_{ij}\overline{L}_i \epsilon H^* N_j - \frac{1}{2}(M_N)_{ij} N_i N_j + h.c.$$

Then the 6x6 neutrino mass matrix is: $M = \begin{pmatrix} 0 & M_\nu^D \\ (M_\nu^D)^T & M_N \end{pmatrix}$ $M_\nu^D = vY^\nu$

With fair assumption that scale of Dirac mass terms is less than scale of Majorana mass terms ($M_\nu^D << M_N$), we can obtain light neutrino masses by integrating out heavy right-handed states and diagonalizing resulting matrix:

$$M_\nu = -M_\nu^D M_N^{-1} (M_\nu^D)^T$$

Heavy mass states can be found by diagonalizing M_N.

How do we explain the smallness of other fermion masses?

Froggatt-Nielsen Mechanism

(C. Froggatt and H. Nielsen (1979))

Propose a flavor symmetry (originally U(1)) that is spontaneously broken by a set of additional scalar fields (flavons). Couple these fields to Yukawa terms rendering them non-renormalizable, e.g:

$$\frac{Y_{ij}\phi\overline{Q_i}Hd_j}{\Lambda} = Y'_{ij}\overline{Q_i}Hd_j$$

After the the flavor symmetry is spontaneously broken by the flavons acquiring a vev:

$$\frac{Y_{ij} < \phi > \overline{Q_i}Hd_j}{\Lambda} = Y_{ij}\lambda\overline{Q_i}Hd_j \qquad \lambda = \frac{< \phi >}{\Lambda} \approx .22$$

Notice that 'Bare' Yukawa coupling is O(1), but effective Yukawa coupling is smaller:

$$Y'_{ij} = Y_{ij}\lambda$$

Smaller Yukawa couplings can be generated by couplings to more flavons.

So this more or less takes care of the fermion mass hierarchy problem. But what about the different mixing angles?

The Cabibbo-Kobayashi-Maskawa (CKM) Mixing Matrix

The CKM matrix results from looking at charged-current interactions which couple the *W* bosons to the physical left-handed up- and down-type quarks with couplings given as:

$$-g(\overline{u_L}, \overline{c_L}, \overline{t_L})\gamma^\mu W_\mu^+ U_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

where

$$U_{CKM} = U_u^\dagger U_d = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}$$

3 x 3 unitary matrix can be parametrized by 3 mixing angles and a phase (CP-violating):

$$U_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad \begin{aligned} s_{ij} &= \sin\theta_{ij} \\ c_{ij} &= \cos\theta_{ij} \end{aligned}$$

It is convenient to express this matrix in another parametrization......

Wolfenstein Parametrization

(L. Wolfenstein (1983))

Define $s_{12} = \lambda,\ s_{23} = A\lambda^2,\ s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$

Then,

$$U_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22537 \pm 0.00061\,, \qquad A = 0.814^{+0.023}_{-0.024}\,,$$

$$\bar{\rho} = 0.117 \pm 0.021\,, \qquad \bar{\eta} = 0.353 \pm 0.013$$

$$\bar{\rho} = \rho(1 - \lambda^2/2 + \ldots)$$

$$If\ \lambda \to 0,\ then\ U_{CKM} \to 1.$$

Is this a meaningful limit? Can we do a similar thing in the lepton sector?

The Cabibbo Haze

(A. Datta, L. Everett, P. Ramond (2005); L. Everett (2006); L. Everett, P. Ramond (2006))

If quarks and leptons are to be unified at some scale and the quark mixing matrix receives Cabibbo-sized corrections about an initial starting point, then we should expect the same behavior in the lepton sector, i.e., an initial lepton mixing matrix which receives Cabibbo-sized corrections to it. Hence the original starting point has been hidden from us by a haze of Cabibbo-sized corrections/effects.

By shifting the different mixing angles in simple ways, the authors find that in some of these cases (A. Datta, L. Everett, P. Ramond (2005)):

$$\theta_{13} \simeq a\lambda \sin\eta_{\oplus} \quad \text{and} \quad \theta_{13} \simeq a\lambda\cos\eta_{\oplus}$$

Notice that if a~1, the initial atmospheric mixing angle is maximal, and the initial reactor mixing angle is zero, then corrected reactor mixing angle is approximately 8.9° at first order in the expansion.

Maybe there is something to this idea.. Actually if you look at more flavor models you will see that most of them can be viewed within this framework... What are popular starting points for these large lepton angles?

$$r = (1+\sqrt{5})/2$$

Popular Starting Points

$$\tilde{U}_\nu = R_{23}\left(\theta_{23}^\nu\right) R_{12}\left(\theta_{12}^\nu\right) = \begin{pmatrix} \cos\theta_{12}^\nu & \sin\theta_{12}^\nu & 0 \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Marzocca, et al. (2013)
Petcov (2014)
Girardi, Petcov, Titov (2015)

TriBiMaximal (TBM) Mixing: $\sin^2\theta_{12}^\nu = 1/3$ (P. Harrison, D. Perkins, W. Scott (2002) ; Z. Xing (2002); X. He, A. Zee (2003))

BiMaximal (BM) Mixing: $\sin^2\theta_{12}^\nu = 1/2$ (F. Vissani (1997); V. Barger, S. Pakvasa, T. Weiler, K. Whisnant, (1998); A. Baltz, A. Goldhaber, M. Goldhaber (1998))

Golden Ratio A (GRA) Mixing: $\sin^2\theta_{12}^\nu = (2+r)^{-1} \cong 0.276$ (A. Datta, F. Ling, P. Ramond (2003))

Golden Ratio B (GRB) Mixing: $\sin^2\theta_{12}^\nu = (3-r)/4 \cong 0.345$ (W. Rodejohann (2009))

HexaGonal (HG) Mixing: $\sin^2\theta_{12}^\nu = 1/4$ (C. Albright, A. Dueck and W. Rodejohann (2010); J. E. Kimand M. Seo (2011))

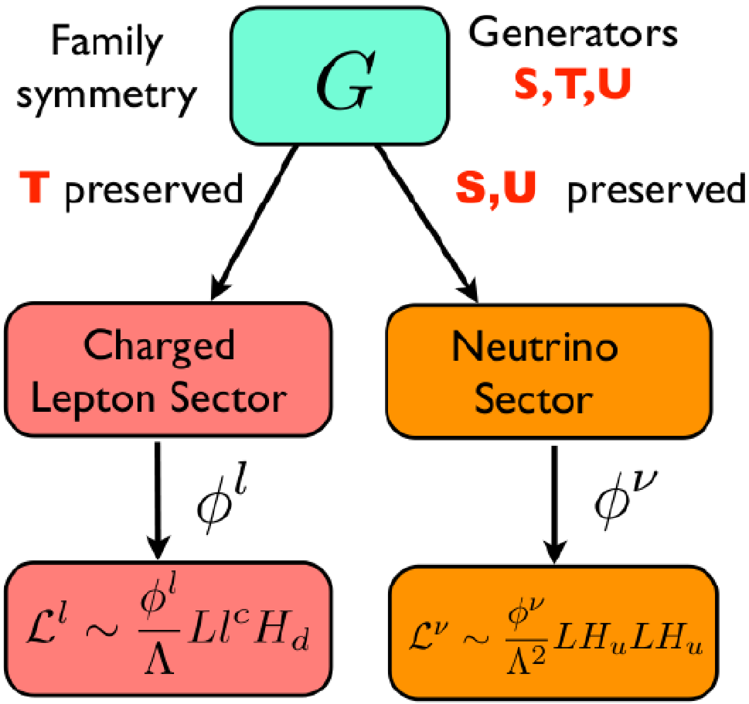
How do we get these initial predictions?

Motivated by Symmetry

Introduce set of flavon fields (e.g. ϕ^ν and ϕ^ℓ) whose vevs break G_f to G_ν in the neutrino sector and G_e in the charged lepton sector.

$$T\langle\phi^\ell\rangle \approx \langle\phi^\ell\rangle \qquad S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle$$

Non-renormalizable couplings of flavons to mass terms can be used to explain the smallness of Yukawa Couplings.



S. King, C. Luhn (2013)

Now that we better understand the framework, what can these symmetries be?

Residual Charged Lepton Symmetry

Since charged leptons are Dirac particles, consider $M_e = m_e m_e^\dagger$.
When **diagonal**, this combination is left invariant by a phase matrix

$$Q_e = \mathrm{Diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

Because $Q_e^\dagger M_e Q_e = M_e$

$$\mathrm{Det}(Q_e) = +1 \implies \beta_1 = -\beta_2 - \beta_3$$

Assume $\beta_{2,3} = 2\pi k_{2,3}/n_{2,3}$ with $k_{2,3} = 0, \ldots, n_{2,3} - 1$

Supposed we keep all $T = Q_e$, then

$$G_e \cong Z_{n_2} \times Z_{n_3} = Z_n \times Z_m$$

Can apply same logic to neutrino sector if neutrinos are Dirac particles, but what if they are Majorana?

Residual Neutrino Flavor Symmetry

Key: Assume neutrinos are Majorana particles

$$U_\nu^T M_\nu U_\nu = M_\nu^{\text{Diag}} = \text{Diag}(m_1, m_2, m_3) = \text{Diag}(|m_1|e^{-i\alpha_1}, |m_2|e^{-i\alpha_2}, |m_3|e^{-i\alpha_3})$$

Notice $U_\nu \to U_\nu Q_\nu$ with $Q_\nu = \text{Diag}(\pm 1, \pm 1, \pm 1)$ also diagonalizes the neutrino mass matrix. Restrict to $\text{Det}(Q_\nu) = 1$ and define $G_0^{\text{Diag}} = 1$

$$G_1^{\text{Diag}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad G_2^{\text{Diag}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, G_3^{\text{Diag}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Observe non-trivial relations:

$$(G_i^{\text{Diag}})^2 = 1, \text{ for } i\text{=1, 2, and 3,}$$
$$G_i^{\text{Diag}} G_j^{\text{Diag}} = G_k^{\text{Diag}}, \text{ for } i \neq j \neq k$$

Sometimes called *SU, S,* and *U*

Therefore, these form a $Z_2 \times Z_2$ residual Klein symmetry!

In non-diagonal basis: $M_\nu = G_i^T M_\nu G_i$ with $G_i = U_\nu G_i^{\text{Diag}} U_\nu^\dagger$

(L. Everett, T. Garon, AS (2015))

Let's be a little bit more clear...

Relating Diagonal to Nondiagonal

(L. Everett, AS (2016))

Nondiagonal mass matrix basis flavor symmetry invariance conditions are

$$S_\nu^T M_\nu S_\nu = M_\nu \qquad T_e^\dagger M_e T_e = M_e$$

These (with diagonalization relationships) imply

$$S_\nu = U_\nu Q_\nu U_\nu^\dagger \qquad T_e = U_e Q_e U_e^\dagger$$

Notice: The above relationships are similarity transformations!
Furthermore, the transformation

$$U_\nu \to U_\nu Q_\nu \qquad U_e \to U_e Q_e$$

leaves the similarity relationships unchanged.

What is an example of what these Non-diagonal elements look like, when all mixing comes from neutrino sector?

Non-Diagonal Klein Elements

$$G_i = U_\nu G_i^{\mathrm{Diag}} U_\nu^\dagger$$

$$G_1 = \begin{pmatrix} (G_1)_{11} & (G_1)_{12} & (G_1)_{13} \\ (G_1)_{12}^* & (G_1)_{22} & (G_1)_{23} \\ (G_1)_{13}^* & (G_1)_{23}^* & (G_1)_{33} \end{pmatrix} \quad G_2 = \begin{pmatrix} (G_2)_{11} & (G_2)_{12} & (G_2)_{13} \\ (G_2)_{12}^* & (G_2)_{22} & (G_2)_{23} \\ (G_2)_{13}^* & (G_2)_{23}^* & (G_2)_{33} \end{pmatrix}$$

$$G_3 = \begin{pmatrix} -c'_{13} & e^{-i\delta} s_{23} s'_{13} & -e^{-i\delta} c_{23} s'_{13} \\ e^{i\delta} s_{23} s'_{13} & s_{23}^2 c'_{13} - c_{23}^2 & -c_{13}^2 s'_{23} \\ -e^{i\delta} c_{23} s'_{13} & -c_{13}^2 s'_{23} & c_{23}^2 c'_{13} - s_{23}^2 \end{pmatrix}$$

$$s_{ij} = \sin(\theta_{ij}) \quad c_{ij} = \cos(\theta_{ij}) \quad s'_{ij} = \sin(2\theta_{ij}) \quad c'_{ij} = \cos(2\theta_{ij})$$

L. Everett, T. Garon, AS (2015)

Notice that in general the Klein elements are complex and Hermitian!
Cannot depend on Majorana phases because
$U_\nu \to U_\nu P_{\mathrm{Maj}}$ leaves transformation invariant.

Non-Diagonal Klein Elements (II)

$$(G_1)_{11} = c_{13}^2 c'_{12} - s_{13}^2 \quad (G_1)_{12} = -2c_{12}c_{13}\left(c_{23}s_{12} + e^{-i\delta}c_{12}s_{13}s_{23}\right)$$

$$(G_1)_{13} = 2c_{12}c_{13}\left(e^{-i\delta}c_{12}c_{23}s_{13} - s_{12}s_{23}\right)$$

$$(G_1)_{22} = -c_{23}^2 c'_{12} + s_{23}^2\left(s_{13}^2 c'_{12} - c_{13}^2\right) + \cos(\delta)s_{13}s'_{12}s'_{23}$$

$$(G_1)_{23} = c_{23}s_{23}c_{13}^2 + s_{13}\left(i\sin(\delta) - \cos(\delta)c'_{23}\right)s'_{12} + \frac{1}{4}c'_{12}\left(c'_{13} - 3\right)s'_{23}$$

$$(G_1)_{33} = \left(s_{13}^2 c'_{12} - c_{13}^2\right)c_{23}^2 - s_{23}^2 c'_{12} - \cos(\delta)s_{13}s'_{12}s'_{23}$$

$$(G_2)_{11} = -c'_{12}c_{13}^2 - s_{13}^2 \quad (G_2)_{12} = 2c_{13}s_{12}\left(c_{12}c_{23} - e^{-i\delta}s_{12}s_{13}s_{23}\right)$$

$$(G_2)_{13} = 2c_{13}s_{12}\left(e^{-i\delta}c_{23}s_{12}s_{13} + c_{12}s_{23}\right)$$

(L. Everett, T. Garon, AS (2015))

$$(G_2)_{22} = c'_{12}c_{23}^2 - s_{23}^2\left(c_{13}^2 + s_{13}^2 c'_{12}\right) - \cos(\delta)s_{13}s'_{12}s'_{23}$$

$$(G_2)_{23} = e^{-i\delta}s_{13}s'_{12}c_{23}^2 + \frac{1}{4}s'_{23}\left(2c_{13}^2 - c'_{12}\left(c'_{13} - 3\right)\right) - e^{i\delta}s'_{12}s_{13}s_{23}^2$$

$$(G_2)_{33} = -c_{23}^2\left(c_{13}^2 + s_{13}^2 c'_{12}\right) + s_{23}^2 c'_{12} + \cos(\delta)s_{13}s'_{12}s'_{23}$$

There is a Klein symmetry for each choice of mixing angle and CP-violating phase, implying a mass matrix left invariant for each choice.

Invariant Mass Matrix

$$M_\nu = U_\nu^* M_\nu^{\mathrm{Diag}} U_\nu^\dagger$$

$$(M_\nu)_{11} = c_{13}^2 m_2 s_{12}^2 + c_{12}^2 c_{13}^2 m_1 + e^{2i\delta} m_3 s_{13}^2$$

$$(M_\nu)_{12} = c_{13}(c_{12} m_1(-c_{23} s_{12} - c_{12} e^{-i\delta} s_{13} s_{23}) + m_2 s_{12}(c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23}) + \\ + e^{i\delta} m_3 s_{13} s_{23}),$$

$$(M_\nu)_{13} = c_{13}(-c_{23} m_3 s_{13} e^{i\delta} + m_2 s_{12}(c_{12} s_{23} + c_{23} e^{-i\delta} s_{12} s_{13}) + \\ + c_{12} m_1(-s_{12} s_{23} + c_{12} c_{23} e^{-i\delta} s_{13})),$$

$$(M_\nu)_{22} = m_1(c_{23} s_{12} + c_{12} e^{-i\delta} s_{13} s_{23})^2 + m_2(c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23})^2 + c_{13}^2 m_3 s_{23}^2$$

$$(M_\nu)_{23} = m_1(s_{12} s_{23} - c_{12} c_{23} e^{-i\delta} s_{13})(c_{23} s_{12} + c_{12} e^{-i\delta} s_{13} s_{23}) + \\ + m_2(c_{12} s_{23} + c_{23} e^{-i\delta} s_{12} s_{13})(c_{12} c_{23} - e^{-i\delta} s_{12} s_{13} s_{23}) - c_{13}^2 c_{23} m_3 s_{23}$$

$$(M_\nu)_{33} = m_2(c_{12} s_{23} + c_{23} e^{-i\delta} s_{12} s_{13})^2 + m_1(-s_{12} s_{23} + c_{12} c_{23} e^{-i\delta} s_{13})^2 + c_{13}^2 c_{23}^2 m_3$$

(L. Everett, T. Garon, AS (2015))

Recall these masses are complex. How can we predict their phases?

Generalized CP Symmetries

G. Branco, L. Lavoura, M. Rebelo (1986)...

Superficially look similar to flavor symmetries:

$$X_\nu^T M_\nu X_\nu = M_\nu^* \qquad Y_e^\dagger M_e Y_e = M_e^*$$

$X_\nu = Y_e = 1$ is 'traditional' CP

Related to automorphism group of flavor symmetry (Holthausen et al. (2012))

Since they act in a similar fashion to flavor symmetries, these two symmetries should be related. (Feruglio et al (2012), Holthausen et al. (2012)):

$$X_\nu G_i^* - G_i X_\nu = 0$$

Can be used to make predictions concerning both Dirac and Majorana CP violating phases, e.g. $X_\nu = G_2$

How to understand? Proceed analogously to flavor symmetry.

Arbitrary Generalized CP Transformations

Recall from our previous discussion of generalized CP:

$$X_\nu^T M_\nu X_\nu = M_\nu^* \qquad Y_e^\dagger M_e Y_e = M_e^*$$

Undiagonalizing these reveals:

$$X_\nu = U_\nu X_\nu^{\mathrm{Diag}} U_\nu^T \qquad Y_e = U_e Y_e^{\mathrm{Diag}} U_e^T$$

(L. Everett, T. Garon, AS (2015)) (I. Girardi, S. Petcov, A. Titov, AS (2016))

Notice: In general these are not similarity transformations like the flavor symmetries. Yet with all unphysical phases included in the diagonal basis these elements are expressible as:

$$X_\nu^{\mathrm{Diag}} = \mathrm{Diag}(\pm e^{i\alpha_1'}, \pm e^{i\alpha_2'}, \pm e^{i\alpha_3'}) \quad \alpha_i' = \alpha_i + \theta_\nu$$

$$Y_e^{\mathrm{Diag}} = \mathrm{Diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3})$$

(L. Everett, AS (2016))

keeps track of unphysical phase shift.

Making Similarities More Clear

(L. Everett, AS (2016))

Split apart the neutrino generalized CP and flavor transformations in:

$$X_\nu^{\mathrm{Diag}} = \mathrm{Diag}(\pm e^{i\alpha'_1}, \pm e^{i\alpha'_2}, \pm e^{i\alpha'_3})$$

by defining:

$$X_0^{\mathrm{Diag}} = \mathrm{Diag}(e^{i\alpha'_1}, e^{i\alpha'_2}, e^{i\alpha'_3})$$ (flavor symmetry 'removed'!)

$$X_\nu^{\mathrm{Diag}} = Q_\nu \times X_0^{\mathrm{Diag}}$$

For charged leptons it is “slightly” trickier because

$$Y_e^{\mathrm{Diag}} = \mathrm{Diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3})$$

Make flavor explicit and define: $\beta'_i = \gamma_i - \beta_i$

Then, $Y_0^{\mathrm{Diag}} = \mathrm{Diag}(e^{i\beta'_1}, e^{i\beta'_2}, e^{i\beta'_3})$ $\quad Y_e^{\mathrm{Diag}} = Q_e \times Y_0^{\mathrm{Diag}}$

These 2 simple relationships are actually very useful....

Orders of Generalized CP Elements I

(L. Everett, AS (2016))

Notice the application of two CP transformations looks like:

$$X_j^\dagger X_i^T M_\nu X_i X_j^* = M_\nu$$
$$Y_l^T Y_k^\dagger M_e Y_k Y_l^* = M_e$$

Therefore,

$$G_k = X_i X_j^* \qquad T_m = Y_k Y_l^*$$
$$i,j = 0,1,2,3 \qquad m,k,l = 0,\ldots,n_2+n_3-1$$

These relationships with the relationships on the previous slide can be used to find the orders of the generalized CP symmetry transformations, i.e., the smallest integers p, q such that $X^p = Y^q = 1$.

Orders of Generalized CP Elements II

Fair to assume such p and q exist. Thus,

$$(X_\nu^{\mathrm{Diag}})^p = Q_\nu^p \times \mathrm{Diag}(e^{ip\alpha'_1}, e^{ip\alpha'_2}, e^{ip\alpha'_3}) = 1$$

$$(Y_e^{\mathrm{Diag}})^q = Q_e^q \times \mathrm{Diag}(e^{iq\beta'_1}, e^{iq\beta'_2}, e^{iq\beta'_3}) = 1$$

Therefore the order of the generalized CP symmetry elements must be integer multiples of the flavor symmetry elements! Thus, the orders of the *X*'s must be even (C.C. Nishi (2013)).

Clearly only for diagonal CP elements or when $U_\nu = U_\nu^* \text{ and } U_e = U_e^*$

For other cases, can deduce relationships:

$$\mathrm{Det}(U_e^*)^{2q}\,\mathrm{Det}(Y_e)^q = 1 \quad \mathrm{Det}(U_\nu^*)^{2p}\,\mathrm{Det}(X_\nu)^p = 1$$

Relationships invariant under: $U_\nu \to U_\nu Q_\nu \quad U_e \to U_e Q_e$

These relationships must hold for all models built in this framework.

Conclusion

- Why the different particles have the different masses and mixings that they do is still an open question in particle physics. Perhaps more importantly, 'Why 3?'
- There are mechanisms which can explain the smallness of fermion masses (Froggatt-Nielsen and Seesaw Mechanisms)
- Discrete symmetries provide an interesting framework in which to generate large mixing angles in the lepton sector as an initial starting point in which to correct to their measured values (perhaps in a Cabibbo Haze type context)
- If neutrinos are Majorana particles, the possibility exists that there is a high scale flavor symmetry spontaneously broken to a residual Klein symmetry in the neutrino sector, completely determining lepton mixing parameters (except Majorana phases).
- To predict CP-violating phases (Dirac and Majorana), recently people have begun implementing a generalized CP symmetry alongside a flavor symmetry to be prepared for possible experimental measurement of these phases (see 1501.04336 and 1611.03020 for a bottom-up approach to this method).

Back-up Slides

Hinting at the Unphysical

Recall each nontrivial Klein element has one +1 eigenvalue.

The eigenvector associated with this eigenvalue will be one column of the MNSP matrix (in the diagonal charged lepton basis).
As an example consider tribimaximal mixing:

$$U^{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

P. F. Harrison, D. H. Perkins, W. G. Scott (2002)
P. F. Harrison, W. G. Scott (2002)
Z. -z. Xing (2002)

Can be shown to originate from the preserve Klein symmetry:

$$G_1^{\mathrm{TBM}} = \frac{1}{3}\begin{pmatrix} 1 & -2 & -2 \\ -2 & -2 & 1 \\ -2 & 1 & -2 \end{pmatrix} \quad G_2^{\mathrm{TBM}} = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad G_3^{\mathrm{TBM}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Notice the eigenvectors are not in the standard MNSP parametrization.

A Caveat

If low energy parameters are not taken as inputs for generating the possible predictions for the Klein symmetry elements, it is possible to generate them by breaking a flavor group G_f *to* $Z_2 \times Z_2$ in the neutrino sector and Z_m in the charged lepton sector, while also consistently breaking H_{CP} *to* $X_{i.}$

Then predictions for parameters can become subject to charged lepton (CL) corrections, renormalization group evolution (RGE), and canonical normalization (CN) considerations.

Although, can expect these corrections to be subleading as RGE and CN effects are expected to be small in realistic models with hierarchical neutrino masses, and CL corrections are typically at most Cabibbo-sized. (J. Casa, J. Espinosa, A Ibarra, I Navarro (2000); S. Antusch, J Kersten, M. Lindner, M. Ratz (2003); S. King I. Peddie (2004); S. Antusch, S. King, M. Malinsky (2009);)