Use of a non-relativistic basis for describing the low energy meson spectrum

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in collaboration with

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XXXI Reunión Anual de la División de Partículas y Campos

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Justification

- Description of QCD in the non-perturbative regime.
- MIT bag model, Lattice QCD, Dyson-Schwinger, etc.
- The present model pretends to offer an alternative:
  - That requieres less computational power.
  - That requieres as few parameters as possible.
  - That offers a more intuitive physical interpretation.
  - That is realistic enough, ie, that contains the important ingredients of the theory.
  - Non-perturbative!

Characteristics of this model

Which is our approach to the NP-QCD problem?

- One considers an effective QCD Hamiltonian inspired in the Coulomb Gauge formalism.
- The fermionic fields are expanded in the harmonic oscillator basis in the coordinate space.
- Traslational invariance of the center of mass is to be recovered with the help of the Talmi-Moshinsky transformations.
- Many-body methods such as TDA and RPA are used to diagonalize the effective Hamiltonian.
- Flavor simmetry breaking is set by introducing an *a posteriori* flavor mixing procedure.

The election of a Gauge

- Why is it a good idea to work in the Coulomb Gauge?
  - Elimination of nondynamical degrees of freedom creates an instantaneous confining interaction.
  - This potential, even though is NR, introduces the effect of gluons in an effective manner.
  - Retardation effects are minimized for heavy quarks, making this a natural framework for studying NR bound states.
  - It is also relevant for light flavors once the constituent quarks are identified with quasiparticle excitations which saturate at $\sim$200 MeV as the bare mass is reduced.
  - The appearance of a quark-antiquark vacuum condensate is typically associated with the confining potential.

[A. Szczepaniak, E. Swanson, Phys. Rev. D, **55**, 1578, (1997)]

[S. L. Adler, A. C. Davis, Nucl. Phys. B **244**, 469 (1984)] .

The election of a basis for the fields

- Why is it a good idea to use the harmonic oscillator basis?
  - One expects the fields to be restricted to a finite volume.
  - Talmi-Moshinsky transformations allow to recover the traslational invariance of the Center of Mass

$$\left[\Psi_{n_a l_a}(\mathbf{x}) \otimes \Psi_{n_b l_b}(\mathbf{y})\right]^L_M = \sum_{n_r l_r n_R l_R} \langle n_r l_r, n_R l_R; L | n_a l_a n_b l_b; L \rangle \left[\Psi_{n_r l_r}(\mathbf{r}) \otimes \Psi_{n_R l_R}(\mathbf{R})\right]^L_M$$

$$\mathbf{r} = \frac{1}{\sqrt{2}}(\mathbf{x} - \mathbf{y}) \quad , \qquad \mathbf{R} = \frac{1}{\sqrt{2}}(\mathbf{x} + \mathbf{y})$$

  - Only one integral is to be solved in this frame, with analytical results!

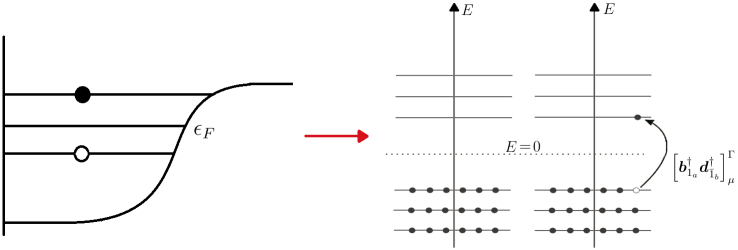
$$I(n_i, l_i, L) = \int R^2 dR \int r^2 dr R_{n_1 l_1}(r) R_{n_2 l_2}(r) \left(-\frac{\alpha}{r} + \beta r\right) R_{n_3 l_3}(R) R_{n_4 l_4}(R)$$

  - The integrals are long-range and short-range well-behaved
  - Free-propagation solutions may not be the best approach to a strong interacting theory

[G.P. Kamuntavicius, R.K. Kalinauskas, Nuclear Physics A 695 (2001) 191-201.]

The election of a diagonalization method

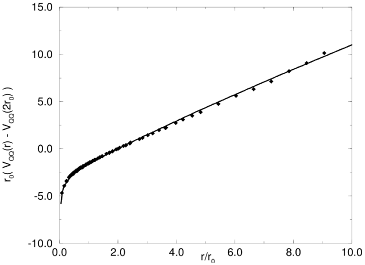
- Why is it a good idea to use many-body methods?
  - In analogy to the particle-hole excitations well known in nuclear physics, colorless quark-antiquark pairs are used to model mesons.
  - As said, Coulomb gauge gives rise to a vacuum with structure of a quark-antiquark condensate.



The full Hamiltonian

A Cornell potential simulates the quark-antiquark interaction due to gluons.

$$\langle a, \mathbf{r}| \frac{1}{\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}}(-\boldsymbol{\nabla}^2)\frac{1}{\boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}}|a'\mathbf{r}'\rangle \longrightarrow V(|\mathbf{r}-\mathbf{r}'|) = \frac{-\alpha}{|\mathbf{r}-\mathbf{r}'|} + \beta|\mathbf{r}-\mathbf{r}'|$$



$$H \approx \underbrace{\int \boldsymbol{\psi}^\dagger(\mathbf{r})[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}]\boldsymbol{\psi}(\mathbf{r})d\mathbf{r}}_{\mathbf{H}_{Kq}} + \underbrace{\int \boldsymbol{\psi}^\dagger(\mathbf{r})[\beta m]\boldsymbol{\psi}(\mathbf{r})d\mathbf{r}}_{\mathbf{H}_{mq}} + \underbrace{\frac{1}{2}g^2\delta_{a'a}\int \rho_a^{(q)}(\mathbf{r})V(|\mathbf{r}-\mathbf{r}'|)\rho^{(q)a}(\mathbf{r}')d\mathbf{r}d\mathbf{r}'}_{\mathbf{V}} ,$$

Mass and kinetic terms for quark sector

The quark fields are expanded in the harmonic oscillator basis

$$\psi^{\dagger}_{1,2}(\mathbf{r},\sigma,c,f) = \sum_{Nlm\sigma cf} \mathbf{b}^{\dagger}_{\pm\frac{1}{2},Nlm,\sigma cf} R^{*}_{Nl}(r) Y^{*}_{lm}(\hat{r}) \chi^{\dagger}_{\sigma} ,$$

The kinetic term is not diagonal in this basis! So a unitary transformation is introduced to diagonalize the free Hamiltonian

$$\begin{pmatrix} \mathbf{b}^{\dagger}_{\frac{1}{2}Nljmcf} \\ \mathbf{b}^{\dagger}_{-\frac{1}{2}Nljmcf} \end{pmatrix} = \sum_k \begin{pmatrix} \gamma^{j}_{Nl,k} & -\beta^{j}_{Nl,k} \\ \beta^{j}_{Nl,k} & \gamma^{j}_{Nl,k} \end{pmatrix} \begin{pmatrix} \mathbf{b}^{\dagger}_{kjmcf} \\ \mathbf{d}_{kjmfc} \end{pmatrix} ,$$

with this, the mass and kinetic terms turn to be

$$\mathbf{K} = \mathbf{H}_{K_q} + \mathbf{H}_{m_q} = \sum_{kj} \tilde{\epsilon}_{kj} \left[\left(\mathbf{b}^{\dagger}_{kj} \cdot \mathbf{b}^{kj} - \mathbf{d}_{kj} \cdot \mathbf{d}^{\dagger kj} \right) \right] .$$

where we introduce different masses for different quarks.

$$m_{\frac{1}{3}\frac{1}{2}} = 0.008 \text{ GeV}, \quad \mathrm{m}_{-\frac{2}{3}0} = 0.092 \text{ GeV}. \tag{1}$$

[A. Amor-Quiroz, P. O. Hess, O. Civitarese, T. Yépez-Martínez, J. Phys.: Conf. Ser. **639** 011001, (2015).]

The effective quark interaction

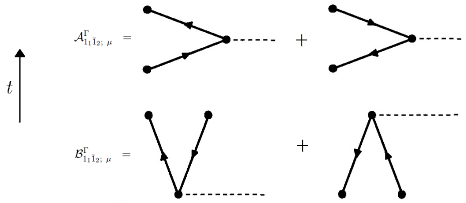
By defining the short-hand notation

$$\mathcal{A}^{\Gamma}_{1_1\bar{1}_2;\ \mu} = \left[\mathbf{b}^{\dagger}_{1_1}\mathbf{b}_{\bar{1}_2}\right]^{\Gamma}_{\mu} - \left[\mathbf{d}_{1_1}\mathbf{d}^{\dagger}_{\bar{1}_2}\right]^{\Gamma}_{\mu}$$

$$\mathcal{B}^{\Gamma}_{1_1\bar{1}_2;\ \mu} = \left[\mathbf{b}^{\dagger}_{1_1}\mathbf{d}^{\dagger}_{\bar{1}_2}\right]^{\Gamma}_{\mu} + \left[\mathbf{d}_{1_1}\mathbf{b}_{\bar{1}_2}\right]^{\Gamma}_{\mu},$$

the final structure of an arbitrary local potential turns to be

$$\mathbf{V} = \sum_{L_0}\sum_{\{k,j\}}\left(E^{L_0}_{\{k,j\}}\left[\mathcal{A}_{12}\mathcal{A}_{34}\right]^0_0 + F^{L_0}_{\{k,j\}}\left[\mathcal{A}_{12}\mathcal{B}_{34}\right]^0_0 + F'^{L_0}_{\{k,j\}}\left[\mathcal{B}_{12}\mathcal{A}_{34}\right]^0_0 + G^{L_0}_{\{k,j\}}\left[\mathcal{B}_{12}\mathcal{B}_{34}\right]^0_0\right),$$



$$E, F, F', G \sim \int R^2 dR \int r^2 dr R_{n_1 l_1}(r) R_{n_2 l_2}(r)\left(-\frac{\alpha}{r} + \beta r\right) R_{n_3 l_3}(R) R_{n_4 l_4}(R)$$

Exploring the use of Many-body methods

The Hamiltonian to diagonalize is then given by

$$
\begin{aligned}
H = \textstyle\sum_{kj} 3\hat{j} C_{kj} \left(\left[\boldsymbol{b}^{\dagger}_{kj} \boldsymbol{b}_{\bar{k}\bar{j}} \right]^{0}_{0} + \left[\boldsymbol{d}_{kj} \boldsymbol{d}^{\dagger}_{\bar{k}\bar{j}} \right]^{0}_{0} \right) \\
+ \textstyle\sum_{L_0} \sum_{\{k,j\}} \left(E^{L_0}_{\{k,j\}} \left[\mathcal{A}_{12} \mathcal{A}_{34} \right]^{0}_{0} + \cancel{F^{L_0}_{\{k,j\}} \left[\mathcal{A}_{12} \mathcal{B}_{34} \right]^{0}_{0}} + \cancel{F'^{L_0}_{\{k,j\}} \left[\mathcal{B}_{12} \mathcal{A}_{34} \right]^{0}_{0}} + G^{L_0}_{\{k,j\}} \left[\mathcal{B}_{12} \mathcal{B}_{34} \right]^{0}_{0} \right)
\end{aligned}
$$

To apply the Many-body methods, the mesonic states are build up as quark-antiquark pairs coupled to a singlet in color (physical states); and by defining some transformation laws

$$
\begin{aligned}
\gamma^{\dagger}_{1_a \bar{1}_b; \Gamma\mu} &\equiv \left[\mathbf{b}^{\dagger}_{1_a} \mathbf{d}^{\dagger}_{\bar{1}_b} \right]^{\Gamma}_{\mu} , \\
\gamma^{\bar{1}_b 1_a;\, \Gamma\mu} &\equiv \left(\gamma^{\dagger}_{1_a \bar{1}_b; \Gamma\mu} \right)^{\dagger} , \\
\gamma^{\dagger 1_b \bar{1}_a;\, \bar{\Gamma}\bar{\mu}} &\equiv (-1)^{\phi\mu} \gamma^{\dagger}_{1_a \bar{1}_b; \Gamma\mu} ,
\end{aligned}
$$

Tamm-Dankoff Method (TDA)

The TDA phonon is defined as a combination of quark-antiquark pairs $\gamma^\dagger$

$$\Gamma^\dagger_{\Gamma\mu;\alpha} \equiv \sum_{ab} X^\alpha_{ab;\Gamma} \gamma^\dagger_{ab;\Gamma\mu} .$$

The method consists in mapping a non-diagonal Hamiltonian into an effective harmonic oscillator Hamiltonian in TDA pairs basis.

$$H \to \sum_{\Gamma\mu} \sum_\alpha \hbar\Omega^\Gamma_\alpha \Gamma^\dagger_{\Gamma\mu;\alpha} \Gamma^{\Gamma\mu;\alpha} .$$

The *forward matrix* is defined as

$$\mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \equiv \left[\gamma^{b'a';\Gamma'\mu'}, \left[H, \gamma^{\dagger\Gamma}_{ab;\mu}\right]\right] = \left(\epsilon^\Gamma_{ab}\delta_{a'a}\delta_{b'b} + V^\Gamma_{(a'b')(ab)}\right)\delta_{\Gamma'\Gamma}\delta_{\mu'\mu} .$$

By doing so, one gets the following eigenvalues equation

$$\sum_{ab} \mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} X^\alpha_{ab;\Gamma} = \hbar\Omega^\Gamma_\alpha X^\alpha_{a'b';\Gamma}\delta_{\Gamma'\Gamma}\delta_{\mu'\mu} ,$$

But the TDA vacuum is the same as the free-theory vacuum!

[P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980)]

Random Phase Approximation (RPA)

The RPA phonon is defined as

$$\Gamma^{\dagger}_{\Gamma\mu;\alpha} \equiv \sum_{ab} \left[X^{\alpha}_{ab;\Gamma} \gamma^{\dagger}_{ab;\Gamma\mu} - Y^{\alpha}_{ab;\Gamma} \gamma_{ab;\Gamma\mu} \right] .$$

This time the vacuum does change: it is correlated!

By defining the *Backward Matrix* $\mathbb{B}$ as

$$\mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \equiv - \left[\gamma^{\dagger b'a';\Gamma'\mu'}, \left[H, \gamma^{\dagger}_{ab;\Gamma\mu} \right] \right] = \frac{1}{2} \left(W^{\Gamma}_{(a'b')(ab)} + W^{\Gamma}_{(ab)(a'b')} \right) \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} ,$$

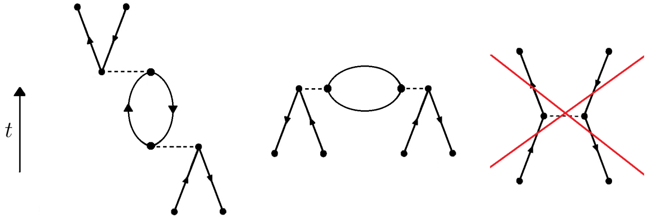
One obtains an eigenvalues equation of the form

$$\sum_{ab} \begin{pmatrix} \mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} & \mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \\ -\mathbb{B}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} & -\mathbb{A}^{(\Gamma\mu)(\Gamma'\mu')}_{(ab)(a'b')} \end{pmatrix} \begin{pmatrix} X^{\alpha}_{ab;\Gamma} \\ Y^{\alpha}_{ab;\Gamma} \end{pmatrix} = \hbar\Omega^{\Gamma}_{\alpha} \begin{pmatrix} X^{\alpha}_{a'b';\Gamma} \\ Y^{\alpha}_{a'b';\Gamma} \end{pmatrix} \delta_{\Gamma'\Gamma} \delta_{\mu'\mu} ,$$

[P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980)]

Random Phase Approximation (RPA)

This means that diagonalizing the above matrix equation allows to find the energy spectrum for the RPA phonon and the coefficients X and Y from the linear combination.
These coefficients may also allow us to construct the effective propagators and obtain transition amplitudes



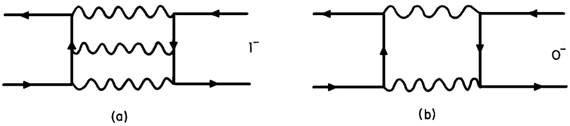
[P. Ring, P. Schuck, **The Nuclear Many-Body Problem**, Springer (1980)]

A posteriori flavor mixing

After applying the many-body methods, it is possible to identify η and η'-like mesons, as pure $q\bar{q}$ and $s\bar{s}$ states, which can be mixed by introducing an *a posteriori* interaction via

$$\begin{pmatrix} M_{q\bar{q}} + 2H_{FM} & \sqrt{2}\,H_{FM} \\ \sqrt{2}\,H_{FM} & M_{s\bar{s}} + H_{FM} \end{pmatrix} .$$

$$\Delta E_{\pm} = \frac{1}{2}\left(M_{q\bar{q}} + M_{s\bar{s}} + 3H_{FM} \pm \sqrt{9H_{FM}^2 + 2H_{FM}(M_{q\bar{q}} - M_{s\bar{s}}) + (M_{q\bar{q}} - M_{s\bar{s}})^2} \right) .$$

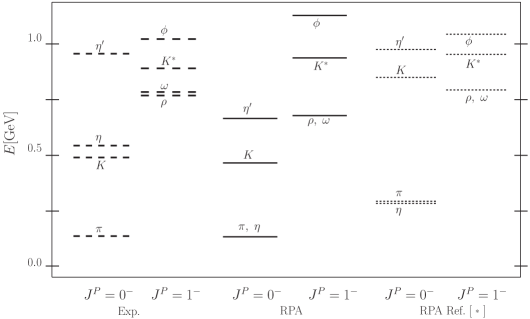


Virtual annihilation of neutral mesons into gluons for a) $J^P = 1^-$ and b) $J^P = 0^-$

[A. de Rújula, H. Georgi, S. L. Glashow, **Hadron masses in a gauge theory**, Phys Rev. D (1975)]

Results

• Without a flavor mixing interaction, the pion-like states are degenerated with the η-like state.

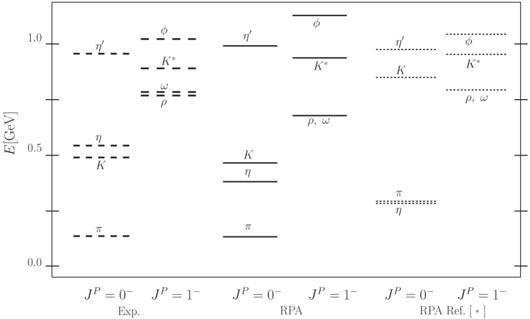


$m_{u,d}[GeV]$	$m_s[GeV]$	α	$\beta[GeV^2]$
0.05	0.31	0.16	0.40

[Phys Rev Lett, **84**, 6, Llanes-Estrada, Cotanch (2000)]

Results

• With a flavor mixing interaction, the η meson is still too low, but in better agreement.

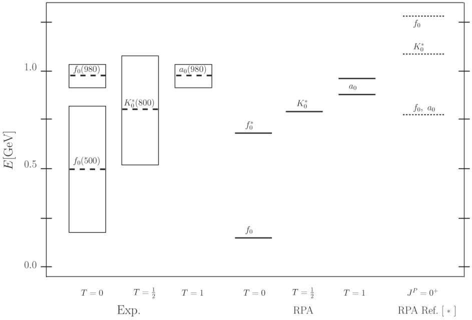


$m_{u,d}$[GeV]	m_s[GeV]	α	β[GeV^2]	H_{FM}[GeV]
0.05	0.31	0.16	0.40	0.188

[Phys Rev Lett, **84**, 6, Llanes-Estrada, Cotanch (2000)]

Results

• The understanding of the scalar states requires the calculation of their widths, and may also improve with a flavor mixing interaction



$m_{u,d}[GeV]$	$m_s[GeV]$	α	$\beta[GeV^2]$
0.05	0.31	0.16	0.40

[Phys Rev Lett, **84**, 6, Llanes-Estrada, Cotanch (2000)]

Conclusions

Actual status: Not ready yet!

- Recover traslational invariance.
- Introduce a dynamical flavor mixing.

Advantages:

- A simple yet semi-realistic model.
- The harmonic oscillator basis is confining and allows analytic integration.
- Rapid convergence of the solutions respect to the number of quanta considered.
- Does not require too many computational power.
- The coordinate space does not contain any divergences.
- The structure of a correlated vacuum simulates the virtual pairs creation.

Thank you!


This is Proton.
Proton has so many problems
But Proton still stays positive
Proton is Good
Be Like Proton