Contribution of the sea quark pairs to the electromagnetic decay of S-wave baryons problem

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Índice

Motivation

Electromagnetic decay widhts

Transition (keV)	CQM	Exp	Reference
$\Gamma_{\Delta^+ \to p\gamma}$	399	660±60	PDG (2014)
$\Gamma_{\Sigma^{*+} \to \Sigma^+\gamma}$	110	250±56	CLAS, PRD 85 052004 (2012)
$\Gamma_{\Sigma^{*0} \to \Lambda^0\gamma}$	258	445±80	CLAS, PRD 83 072004(2011)

Is there important differences between the CQM predictions and the recently collected experimental data for this baryon decay widths.

EM decay widths

$$\Gamma(B_{10} \to B_8\gamma)_{exp} \approx 2\Gamma(B_{10} \to B_8\gamma)_{CQM}$$

CQM under predict these values (we can't understand the experiment in the CQM frame)

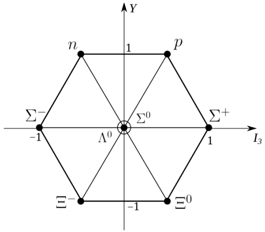
We can study this in any quark model using the following relation

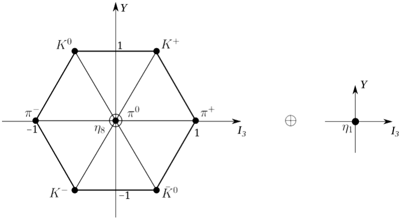
$$\Gamma(B_{10} \to B_8\gamma) = 2_{pol} 2\pi \left| \langle \Psi_{A_8}\gamma | \hat{H}_{int} | \Psi_{A_{10}} \rangle \right|^2 4\pi \frac{E_{A_8}}{m_{A_{10}}} p_\gamma^2 .$$

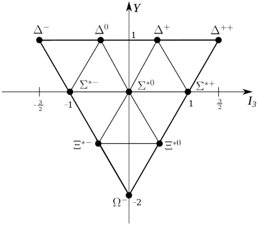
Model dependent ?
The quark model dependece of this expression lie in
specifying the baryon states (p. ej. $|\Psi_A\rangle_{CQM}$, $|\Psi_A\rangle_{UQM}$, ...)

Constituent Quark Model

$$|\Psi\rangle_{total} = |\psi_r\rangle_{orb} \otimes |\phi\rangle_{flavor} \otimes |\chi\rangle_{spin} \otimes |\psi_c\rangle_{color}.$$

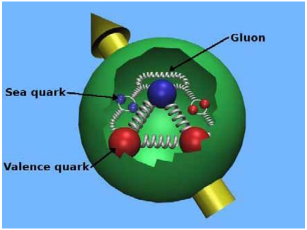






Baryons $(q^3) \rightarrow qqq$
Mesons $\rightarrow q\bar{q}$.
The interesed transitions are between the S-wave decuplet baryons and the S-wave octet baryons.

Unquenched Quark Model



Exotic degrees of freedom

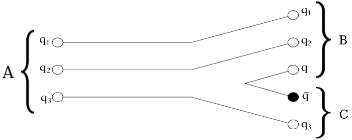
-Quark-Antiquark sea pairs :
Meson Cloud Model (Speth & Weise, 1998).
Chiral Quark Model (Eichten et al, 1992).
Unquenched Quark Model (Geiger & Isgur, 1997), (Törnqvist & Zenczykowski, 1984) (Bijker & Santopinto, 2009).
-Higher Fock states included in the wave function.

$$\psi = \mathcal{N}\left[\psi(q^3) + \alpha\,\psi(q^3 - q\bar{q})\right]$$

$$|\psi_A\rangle = \mathcal{N}_A \left[|A\rangle + \sum_{BClJ} \int d\vec{K} k^2 dk |BC\vec{K}klJ\rangle \frac{\langle BC\vec{K}klJ|T^\dagger|A\rangle}{m_A - E_B(k) - E_C(k)} \right]$$

$$\begin{aligned} T^\dagger &= T^\dagger(^3P_0) \\ &= -3\sum_{ij}\int d\vec{p}_i d\vec{p}_j \delta(\vec{p}_i+\vec{p}_j) C_{ij} F_{ij} V(\vec{p}_i-\vec{p}_j)[\chi_{ij}\times\mathcal{Y}_1(\vec{p}_i-\vec{p}_j)]^{(0)} b_i^\dagger(\vec{p}_i) d_j^\dagger(\vec{p}_j). \end{aligned}$$

This is the quark-pair creation operator of the $^3P_0$ model which considers the quantum number of vacuum (Micu, 1969). $V(\vec{p}_i-\vec{p}_j) = \gamma e^{-r_q^2(\vec{p}_i-\vec{p}_j)^2/6}$, where γ correspond to an adimensional coupling constant between the $|A\rangle$ and intermediate states $\langle BC\rangle$. It can be determined from the asymmetry flavor in the proton.



It's considered baryons $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and pseudoscalar mesons $J^P = 0^-$. For example

$$\begin{aligned} |\Psi_{\Delta^{++}}\rangle &= \mathcal{N}_\Delta \left[\left|\Delta^{++}\right\rangle + a_{\Delta\to N\pi}|p\pi\rangle \right. \\ &+ a_{\Delta\to\Sigma K}|\Sigma K\rangle + a_{\Delta\to\Delta\pi}|\Delta\pi\rangle \\ &+ a_{\Delta\to\Delta\eta}|\Delta\eta\rangle + a_{\Delta\to\Delta\eta'}\left|\Delta\eta'\right\rangle \\ &+ \left. a_{\Delta\to\Sigma^* K}|\Sigma^* K\rangle \right] \end{aligned}$$

CQM can't explain this

Gottfried sum rule

$$S_G = \int_0^1 dx \frac{F_{2p}(x) - F_{2n}(x)}{x} = \frac{1}{3} - \frac{2}{3}\int_0^1 dx[\bar{d}(x) - \bar{u}(x)]$$

the non-nule asymmetric contribution of the sea quarks in the proton

$$S_G = 0.255 \pm 0.008,$$

i.e., $\Delta P = \int_0^1 dx[\bar{d}(x) - \bar{u}(x)] = N(\bar{d}) - N(\bar{u}) = 0.118 \pm 0.012$ (Fermilab E866 Drell-Yan experiment)

Flavor asymmetry

$$N(\bar{d}) > N(\bar{u})$$

There is an excess of $\bar{d}$ than $\bar{u}$ into the proton.

We need to study another quarks model that can consider new degrees of freedom (extension)–> Higher Fock components
UQM can explain it.

Electromagnetic decay of S-wave baryons

$$\Gamma_{i\to f} = \frac{d(probability)}{d(time)} = 2\pi \left|\langle f|\hat{H}_{int}|i\rangle\right|^2 \rho_f ,$$

$$\hat{H}_{int} = -\int d^3x \hat{j}^\mu(\vec{x})\hat{A}_\mu(\vec{x},t)$$

where

$$\hat{j}^\mu(\vec{x}) = \sum_q \hat{\bar{q}}(\vec{x}) Q_q \gamma^\mu \hat{q}(\vec{x})$$

and

$$\hat{q}(\vec{x}) = \sum_{r=1}^{2} \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{\varepsilon(\vec{p})}} \left(\hat{b}_r(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} u_r(\vec{p}) + (-1)^{r+1} \hat{d}_r^\dagger(\vec{p}) e^{i\vec{p}\cdot\vec{x}} v_r(\vec{p}) \right),$$

then

$$\hat{j}^\mu(\vec{x}) = \hat{j}_1^\mu(\vec{x}) + \hat{j}_2^\mu(\vec{x}) + \hat{j}_3^\mu(\vec{x}) + \hat{j}_4^\mu(\vec{x})$$

$$\hat{j}_1^\mu(\vec{x}) \sim \hat{b}_r^\dagger \hat{b}_s \to \text{quark transition}$$

$$\hat{j}_2^\mu(\vec{x}) \sim \hat{d}_r \hat{b}_s \to \text{pair anihilation } q\bar{q}$$

$$\hat{j}_3^\mu(\vec{x}) \sim \hat{b}_r^\dagger \hat{d}_s^\dagger \to \text{pair creation } q\bar{q}$$

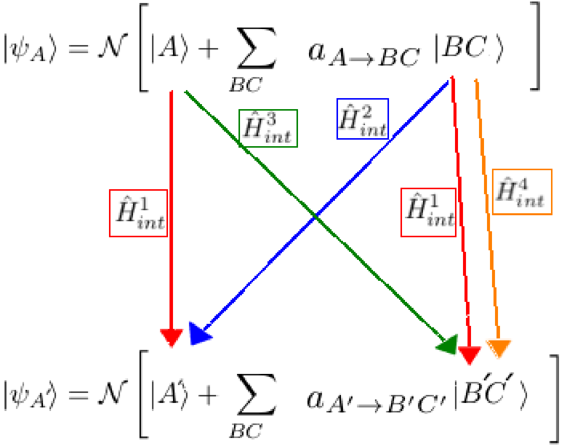
$$\hat{j}_4^\mu(\vec{x}) \sim \hat{d}_r \hat{d}_s^\dagger \to \text{antiquark transition}$$

In consecuence

$$\hat{H}_{int} = \hat{H}^1_{int} + \hat{H}^2_{int} + \hat{H}^3_{int} + \hat{H}^4_{int}$$

$$\begin{aligned}\Gamma_{A\rightarrow A'\gamma} &= 2_{pol} 2\pi \left|\langle\Psi_{A'}\gamma|\hat{H}_{int}|\Psi_A\rangle\right|^2 \rho_f \\ &= 4\pi \Big|\langle\Psi_{A'}\gamma|\hat{H}^1_{int}|\Psi_A\rangle + \langle\Psi_{A'}\gamma|\hat{H}^2_{int}|\Psi_A\rangle \\ &\quad + \langle\Psi_{A'}\gamma|\hat{H}^3_{int}|\Psi_A\rangle + \langle\Psi_{A'}\gamma|\hat{H}^4_{int}|\Psi_A\rangle\Big|^2 \rho_f\end{aligned}$$

In the particular CQM frame $\Gamma_{A\rightarrow A'\gamma} = 4\pi \left|\langle A'\gamma|\hat{H}^1_{int}|A\rangle\right|^2 \rho_f$


$|\psi_A\rangle = \mathcal{N}\left[|A\rangle + \sum_{BC} a_{A\rightarrow BC}\,|BC\rangle\right]$
$\hat{H}^3_{int}$
$\hat{H}^2_{int}$
$\hat{H}^1_{int}$
$\hat{H}^1_{int}$
$\hat{H}^4_{int}$
$|\psi_{A'}\rangle = \mathcal{N}\left[|A'\rangle + \sum_{BC} a_{A'\rightarrow B'C'}\,|B'C'\rangle\right]$

$\hat{H}^1_{int}$ contribution

$$\langle \Psi_{A'}, \gamma | \hat{H}^1_{int} | \Psi_A \rangle = i \sqrt{\frac{2\pi}{p_\gamma V}} \langle \Psi_{A'} | \vec{\mu}_S | \Psi_A \rangle \times \vec{p}_\gamma \cdot \vec{\epsilon}^{*\beta}.$$

CQM frame

$$\Gamma^1_{A \to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2} \mu_S^2 (A/A')$$

UQM frame

$$\Gamma^1_{A \to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2} \mu_S^2 (\Psi_A/\Psi_{A'})$$

$$\mu(\Psi_A/\Psi_{A'}) = \sqrt{\frac{2 m_N^2 \Gamma_{A \to A'\gamma}}{\alpha p_\gamma^3}}$$ (D. Keller, H. Hicks, 2011)

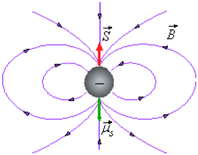
Magnetic moments

-Orbital angular momentum

$$\hat{\mu}_l = \frac{e}{2m}\hat{l}$$

-Spin

$$\mu_s = \frac{e_q \hbar}{2m_q} 2\hat{S}$$



$$\vec{\mu} = \sum_i 2\mu_i \vec{s}_i + \sum_i \mu_i \vec{l}_i = \vec{\mu}_{spin} + \vec{\mu}_{orbital}$$

matrix elements

$$\langle \Psi_{A'} | \sum_i \mu_i (2\vec{s}_i + \vec{l}_i) | \Psi_A \rangle \quad (1)$$

Experimental information for CQM mm

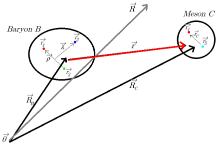
magnetic moments of Baryons
For example :
$\mu_p = 2.7928473508 \pm 0.0000000085(\mu_N)$
$\mu_n = -1.91304273 \pm 0.00000045(\mu_N)$
$\mu_\Lambda = -0.613 \pm 0.004(\mu_N)$

$$\left.\begin{array}{l}\mu_p \\ \\ \mu_n \\ \\ \mu_\Lambda\end{array}\right\} \qquad \begin{array}{l} CQM\mu_u \\ \\ CQM\mu_d \\ \\ CQM\mu_s\end{array}$$

Experimental information for UQM mm

$$\langle \hat{O} \rangle_{UQM} = \mathcal{N}^2 \left[\langle \hat{O} \rangle_{CQM} + \sum_{B,C,l} a^2_{A\to BC} \langle BC; l | \hat{O} | BC; l \rangle + ... \right]$$

$$a^2_{A\to B\eta} = (6\gamma\varepsilon')^2 \int_0^\infty dk \frac{k^4 e^{-2F^2k^2}}{[m_A - E_B(k) - E_\eta(k)]^2} \left(\theta_{A\to B\eta_8} \cos\theta_P - \theta_{A\to B\eta_1} \sin\theta_P\right)^2 .$$



Harmonic oscillator

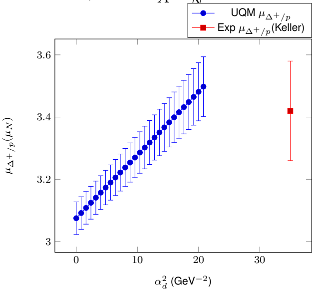
$$\psi_{baryon} \backsim f_b(\vec{p}) \times e^{-\alpha_b^2 \vec{p}/2}$$
$$\psi_{meson} \backsim f_c(\vec{p}) \times e^{-\alpha_c^2 \vec{p}/2}$$
$$\psi_{q\bar{q}} \backsim f_{q\bar{q}}(\vec{p}) \times e^{-\alpha_d^2 \vec{p}/2}$$
$$r_d = \sqrt{2\alpha_d^2 \ln(2)} = 4 \pm 1$$

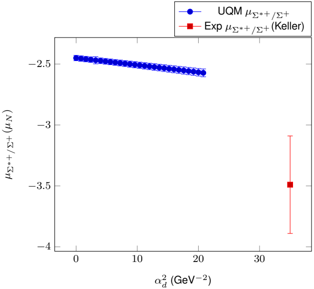
μ_p
μ_n
μ_Λ
$hadron \quad masses$
θ_P
$\Delta P(asymmetry)$
$\langle r^2 \rangle_{baryon}$
$\langle r^2 \rangle_{meson}$
$\langle r^2 \rangle_{q\bar{q}}$

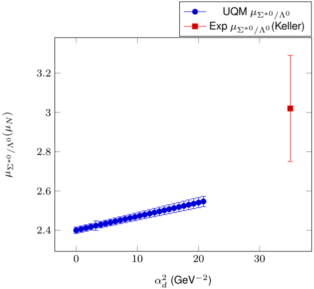
$\Bigg\}$

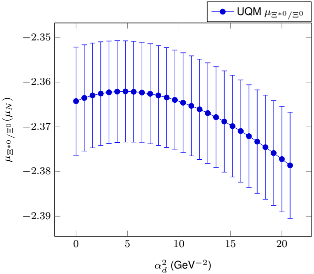
$UQM\mu_u$
$UQM\mu_d$
$UQM\mu_s$
$\gamma^2(\Delta P, \theta_P, m, \langle r^2 \rangle)$
$\backsim 200$
$F^2(\langle r^2 \rangle) = 4.0 \pm 0.3 \quad GeV$

$$\Gamma^{1}_{A\to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2}\,\mu_S^2(\Psi_A/\Psi_{A'})$$



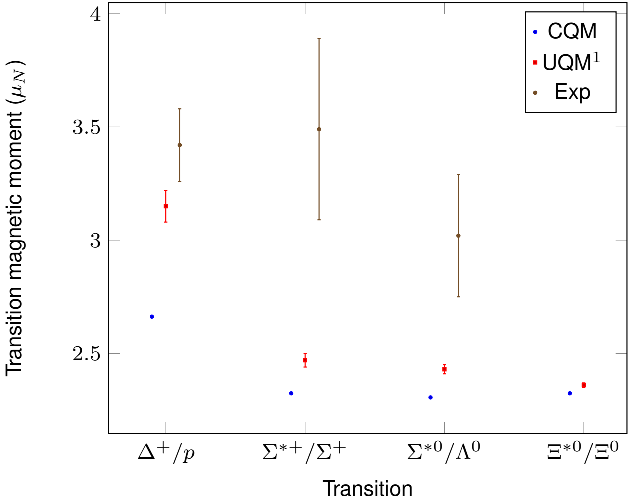






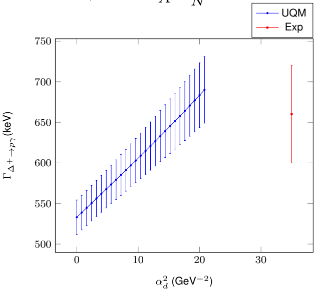
Values for μ in the UQM fited to the experimental data : CODATA ep

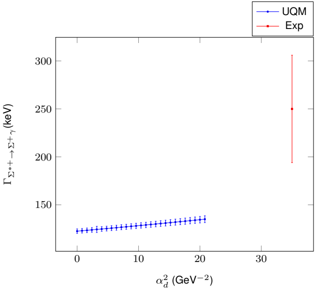
$$\Gamma^{1}_{A\to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2} \mu_S^2(\Psi_A/\Psi_{A'})$$

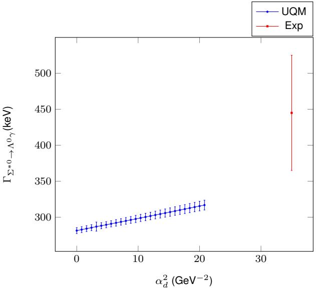


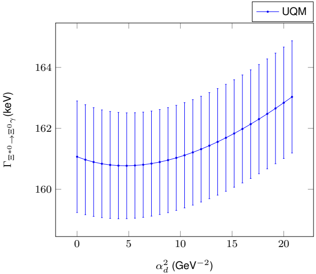
Exp data (CLAS 2011, 2012), Transición UQM (Guerrero-Navarro, unpublished)

$$\Gamma^{1}_{A\to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2} \mu_S^2 (\Psi_A/\Psi_{A'})$$



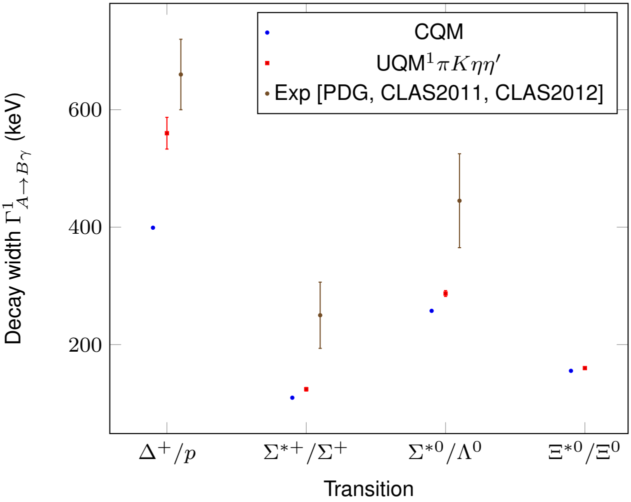






Values for μ in the UQM fited to the experimental data : CODATA ep

$$\Gamma^{1}_{A\to A'\gamma} = \frac{\alpha E_{A'} p_\gamma^3}{2 m_A m_N^2} \mu_S^2 (\Psi_A/\Psi_{A'})$$



How can we understand the difference by using an Unquenching ?

$$\begin{aligned}\Gamma_{A\to A'\gamma} &= 2_{pol} 2\pi \left| \langle \Psi_{A'}\gamma | \hat{H}_{int} | \Psi_A \rangle \right|^2 \rho_f \\ &= 4\pi \Big| \langle \Psi_{A'}\gamma | \hat{H}^1_{int} | \Psi_A \rangle + \langle \Psi_{A'}\gamma | \hat{H}^2_{int} | \Psi_A \rangle \\ &\quad + \langle \Psi_{A'}\gamma | \hat{H}^3_{int} | \Psi_A \rangle + \langle \Psi_{A'}\gamma | \hat{H}^4_{int} | \Psi_A \rangle \Big|^2 \rho_f\end{aligned}$$

If we consider only the first term $\hat{H}^1_{int}$, it would be valid into the CQM expressions. Doing this gives us non-consistent expressions into the UQM. Then

$$\begin{aligned}\Gamma_{A\to A'\gamma} &= 4\pi \rho_f \mathcal{N}^2_{A'} \mathcal{N}^2_A \\ &\times \Big| i\sqrt{\frac{2\pi}{p_\gamma V}} \Big[\langle A' | \vec{\mu}_S | A \rangle + \sum_{BB'C} a_{A'\to B'C}\, a_{A\to BC} \langle B'C | \vec{\mu}_S | BC \rangle \Big] \times \vec{p}_\gamma \\ &+ \sum_{BC} a_{A\to BC} \langle A', \gamma | \hat{H}^2_{int} | BC \rangle \\ &+ \sum_{B'C'} a_{A'\to B'C'} \langle B'C', \gamma | \hat{H}^3_{int} | A \rangle \\ &+ \sum_{BCB'C'} a_{A'\to B'C'}\, a_{A\to BC} \langle B'C', \gamma | \hat{H}^4_{int} | BC \rangle \Big|^2\end{aligned}$$

Conclusions

$$\begin{array}{ccccc} exp & & & \color{red}{model} & \\ \sigma_{exp} & \color{blue}{\leftrightsquigarrow} & \Gamma_{A\to A'\gamma} & \color{blue}{\leftrightsquigarrow} & \mu(\Psi_A/\Psi_{A'}) \\ & \color{red}{?} & & \color{red}{!} & \end{array}$$

- In the UQM all the "parameters" are well defined into this model and fited with the experimental information.
- The dependence of the magnetic moment is given strongly by the baryon and pair size.
- Due to the additional terms, there is still a lack in the expression of the decay width in the UQM, for that the effective contribution is not so clear even.
- Is there clear differences with the CQM.
- It could be more appropiated a comparison using another expression related to direct experimental data.

Thank you

	$\Gamma_{\Delta\to N\gamma}$	$\Gamma_{\Sigma^{*0}\to\Lambda^0\gamma}$	$\Gamma_{\Sigma^{*+}\to\Sigma^+\gamma}$
U-spin [Keller, 2011, 2012]		423±38	250±23
HBχPT [Butler]	670-790	252-540	70-220
Algebraic model [Bijker,Franco]	342-344	221.3	140.7
QCD SR [Wang]	887	409	150
Large N_c [Lebed]	669±42	336±81	149±36
Spectator [Ramalho]	648	399	154
NRQM [Koniuk]		273	104
RCQM [Rollnick]		267	
χCQM [Wagner]		265	105
MIT Bag [Soyeur]		152	117
Soliton [Scoccola]		243	91
Skyrme [Weigel]		157-209	47
UQM[1] $\pi K\eta\eta'$	560± 27	287± 5	124± 3
Exp	660±60 [PDG, 2014]	445±80 [CLAS, 2011]	250±56 [CLAS, 2012]

TABLE : EM decay widths $A \to A'\gamma$ (keV) corresponding to distinct models (includying this) and the exp. data.

Transición (μ_N)	CQM	Large N_c[Jenkins]	Large $N_c \chi$PT [F. Mendieta]	UQM $\pi K \eta \eta'$	Exp
Δ^+/p	2.66249	3.51*	3.51	3.03954	3.42 $\pm$0.16
Σ^{*+}/Σ^+	-2.32402	2.96	3.17	-2.45244	3.49 $\pm$0.40
Σ^{*0}/Λ^0	2.30579	2.96	2.73	2.5014	3.02 $\pm$0.27
Ξ^{*0}/Ξ^0	-2.32402	2.96	3.14	-2.44828	—

TABLE : Resultados de los momentos magnéticos de transición suponiendo la relación de Keller-Hicks.

Octete

Barión	CQM (μ_N)	UQM (μ_N)	$\mu_{exp}(\mu_N)$
p	2.793	2.793*	2.793
n	-1.913	-1.913*	-1.913
Σ^+	2.673	2.589	2.458$\pm$0.010
Σ^0	0.791	0.783	-
Σ^-	-1.091	-1.023	-1.160$\pm$0.025
Λ^0	-0.613	-0.613*	-0.613$\pm$0.004
Ξ^0	-1.435	-1.359	-1.250$\pm$0.014
Ξ^-	-0.493	-0.530	-0.651$\pm$0.003
Σ^0/Λ^0	1.630	1.640	1.610$\pm$0.08

TABLE : Momentos magnéticos de los bariones del octete

Decuplete

Barión	$\mu(\vec{s})$ val (μ_N)	$\mu(\vec{s})$ mar (μ_N)	$\mu(\vec{s})$ (μ_N)	$\mu(\vec{l})$ (μ_N)	$\mu(\vec{s},\vec{l})$ (μ_N)
Δ^{++}	2.954	2.022	4.977	0.334	5.312
Δ^{+}	1.453	0.907	2.361	0.122	2.483
Δ^{0}	-0.049	-0.207	-0.256	-0.090	-0.346
Δ^{-}	-1.551	-1.322	-2.873	-0.303	-3.175
Σ^{*+}	1.911	0.615	2.526	0.264	2.789
Σ^{*0}	0.165	-0.1310	0.034	0.003	0.037
Σ^{*-}	-1.580	-0.877	-2.458	-0.259	-2.716
Ξ^{*0}	0.473	-0.291	0.182	0.159	0.340
Ξ^{*-}	-1.661	-0.422	-2.083	-0.168	-2.251
Ω^{-}	-0.929	-0.755	-1.6848	-0.173	-1.858

TABLE : Resultados de los momentos magnéticos de los bariones del decuplete en el UQM para la contribución del espín, $\mu(\vec{s})$, del momento angular relativo, $\mu(\vec{l})$ y el total, $\mu(\vec{s},\vec{l})$.

Barión	CQM (μ_N)	UQM (μ_N)	Exp (μ_N)
Δ^{++}	5.556	5.31165	3.7 a 7.5
Δ^{+}	2.7318	2.48262	-
Δ^{0}	-0.092	-0.346408	-
Δ^{-}	-2.916	-3.17544	-
Σ^{*+}	3.091	2.78921	-
Σ^{*0}	0.267	0.036555	-
Σ^{*-}	-2.557	-2.71611	-
Ξ^{*0}	0.626	0.340423	-
Ξ^{*-}	-2.198	-2.25133	-
Ω^{-}	-1.839	-1.85787	-2.02 $\pm$ 0.05

TABLE : Comparación de los momentos magnéticos con los resultados del CQM y los resultados experimentales

Momentos magnéticos de transición

Transición (μ_N)	$\mu_s \pi$	$\mu_l \pi$	$\mu_T \pi$
Δ^+/p	2.68091	0.210584	2.89
Σ^{*+}/Σ^+	-2.1453	-0.0779084	-2.22
Σ^{*0}/Λ^0	2.13222	0.174976	2.31
$\frac{2}{\sqrt{3}}\Sigma^{*0}/\Lambda^0$	2.46207	0.202045	2.66
Ξ^{*0}/Ξ^0	-2.00569	-0.0749409	-2.08

TABLE : Resultados de los momentos magnéticos de transición en el UQM para la contribución de espín, μ_s, del momento angular relativo, μ_l y el total, μ_T, considerando la contribución del meson π.

Transition (μ_N)	$\mu_s \pi K \eta_1 \eta_8$	$\mu_l \pi K \eta_1 \eta_8$	$\mu_T \pi K \eta_1 \eta_8$	$\mu_s \pi K \eta \eta'$	$\mu_l \pi K \eta \eta'$	$\mu_T \pi K \eta \eta'$
Δ^+/p	2.75012	0.301182	3.0513	2.74089	0.298646	3.03954
Σ^{*+}/Σ^+	-2.29202	-0.158617	-2.45063	-2.29381	-0.158625	-2.45244
Σ^{*0}/Λ^0	2.27782	0.233697	2.51152	2.26705	0.234352	2.5014
$\frac{2}{\sqrt{3}}\Sigma^{*0}/\Lambda^0$	2.6302	0.26985	2.90005	2.61777	0.270607	2.88837
Ξ^{*0}/Ξ^0	-2.2728	-0.183334	-2.45614	-2.2625	-0.185787	-2.44828

TABLE : Resultados de los momentos magnéticos de transición en el UQM para la contribución de espín, μ_s, del momento angular relativo, μ_l y el total, μ_T, considerando la contribución de los mesones $\pi K \eta_1 \eta_8$ y en la mezcla $\pi K \eta \eta'$.

Transition (μ_N)	$\mu_s \pi$	$\mu_l \pi$	$\mu_T \pi$
Δ^+/p	2.68091	0.210584	2.8915
Σ^{*+}/Σ^+	-2.1453	-0.0779084	-2.22324
Σ^{*0}/Λ^0	2.13222	0.174976	2.30719
$\frac{2}{\sqrt{3}}\Sigma^{*0}/\Lambda^0$	2.46207	0.202045	2.66412
Ξ^{*0}/Ξ^0	-2.00569	-0.0749409	-2.08063

TABLE : Resultados de los momentos magnéticos de transición en el UQM para la contribución de espín, μ_s, del momento angular relativo, μ_l y el total, μ_T, considerando la contribución del meson π.

Cálculo del parámetro γ^2

$$\bar{d} - \bar{u} = \int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right] = 0.118 \pm 0.012,$$

$$\begin{aligned} \bar{d} &= \mathcal{N}_N^2 \left(\frac{1}{6}a^2_{N\to N\pi} + \frac{1}{6}a^2_{N\to N\eta} + \frac{4}{6}a^2_{N\to\Delta\pi} + \frac{2}{6}a_{N\to N\pi}a_{N\to N\eta}\right) \\ \bar{u} &= \mathcal{N}_N^2 \left(\frac{5}{6}a^2_{N\to N\pi} + \frac{1}{6}a^2_{N\to N\eta} + \frac{2}{6}a^2_{N\to\Delta\pi} - \frac{2}{6}a_{N\to N\pi}a_{N\to N\eta}\right). \end{aligned}$$

$$\Delta P = \bar{d} - \bar{u} = 0.118 = \mathcal{N}_N^2 \left(\frac{2}{3}a^2_{N\to N\pi} - \frac{1}{3}a^2_{N\to\Delta\pi} - \frac{2}{3}a_{N\to N\pi}a_{N\to N\eta}\right),$$

$$\begin{aligned} \gamma^2 = & \frac{-3\Delta P}{\alpha^2_{N\to\Delta\pi}(3\Delta P + 1) + 2\alpha_{N\to N\pi}\alpha_{N\to N\eta}} \\ \times & \frac{1}{\alpha^2_{N\to N\pi}(3\Delta P - 2) + 3\Delta P(\alpha^2_{N\to N\eta} + \alpha^2_{N\to N\eta} + \alpha^2_{N\to\Sigma K} + \alpha^2_{N\to\Lambda K} + \alpha^2_{N\to\Sigma^*}} \end{aligned}$$

Cálculo de las ampitudes de probabilidad de estados $|BC\rangle$

$$a^2_{A\to BC} = \left(6\gamma\theta_{A\to BC}\varepsilon'\right)^2 \int_0^\infty dk_0 \frac{k_0^4 e^{-2F^2k_0^2}}{\left[m_A - \sqrt{m_B^2 + k_0^2} - \sqrt{m_C^2 + k_0^2}\right]^2}.$$

$$a_{A\to B_8 C} a_{A\to B_{10} C} = \left(6\gamma\varepsilon'\right)^2 \theta_{A\to B_{10}C}\theta_{A\to B_8 C} \times$$

$$\int_0^\infty dk_0 \frac{k_0^4 e^{-2F^2k_0^2}}{\left[m_A - \sqrt{m_{B_8}^2 + k_0^2} - \sqrt{m_C^2 + k_0^2}\right]\left[m_A - \sqrt{m_{B_{10}}^2 + k_0^2} - \sqrt{m_C^2 + k_0^2}\right]}.$$

$$a^2_{A\to B\eta} = (6\gamma\varepsilon')^2 \int_0^\infty dk \frac{k^4 e^{-2F^2k^2}}{[m_A - E_B(k) - E_\eta(k)]^2} \left(\theta_{A\to B\eta_8}\cos\theta_P - \theta_{A\to B\eta_1}\sin\theta_P\right)^2.$$

$$a^2_{A\to B\eta'} = (6\gamma\varepsilon')^2 \int_0^\infty dk \frac{k^4 e^{-2F^2k^2}}{[m_A - E_B(k) - E_{\eta'}(k)]^2} \left(\theta_{A\to B\eta_8}\sin\theta_P + \theta_{A\to B\eta_1}\cos\theta_P\right)^2.$$

$$a_{A_1 \to B_1 \eta} a_{A_2 \to B_2 \eta} =$$

$$(6\gamma\varepsilon')^2 \int_0^\infty dk \frac{k^4 e^{-2F^2 k^2}}{[m_{A_1} - E_{B_1}(k) - E_\eta(k)][m_{A_2} - E_{B_2}(k) - E_\eta(k)]}$$
$$\times (\theta_{A_1 \to B_1 \eta_8} \cos\theta_P - \theta_{A_1 \to B_1 \eta_1} \sin\theta_P)(\theta_{A_2 \to B_2 \eta_8} \cos\theta_P - \theta_{A_2 \to B_2 \eta_1} \sin\theta_P).$$

$$a_{A_1 \to B_1 \eta'} a_{A_2 \to B_2 \eta'} =$$

$$(6\gamma\varepsilon')^2 \int_0^\infty dk \frac{k^4 e^{-2F^2 k^2}}{[m_{A_1} - E_{B_1}(k) - E_{\eta'}(k)][m_{A_2} - E_{B_2}(k) - E_{\eta'}(k)]}$$
$$\times (\theta_{A_1 \to B_1 \eta_8} \sin\theta_P + \theta_{A_1 \to B_1 \eta_1} \cos\theta_P)(\theta_{A_2 \to B_2 \eta_8} \sin\theta_P + \theta_{A_2 \to B_2 \eta_1} \cos\theta_P).$$