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Monochromatization for Direct Higgs Production in Future Circular e+e- Colliders

M. Alan Valdivia G. valdivia@fisica.ugto.mx RADPyC'17, CINVESTAV, Mexico City, 250517

Outline

- Motivation
- Objectives
- Introduction: Accelerator Physics
- Standard Monochromatization
- Radiation Effects In FCC
- Optimized Monochromatization
- Conclusion
- Discussion

Motivation

- Questions still unanswered about Higgs boson properties
- Post-LHC high energy resolution requirements
- Upcoming radiation Effects
- FCCe+e- proposal opportunity

Objectives

- Characterize the radiation effects at the FCCe+e-
- Quantify the IP parameters modification due to the radiation effects.
- Define the IP parameters to produce monochromatization
- Develop an optimized monochromatization scheme for the FCCe+e-

Colliding Particles A Qualitative Description



- Particles are injected into the vacuum chamber
- Stored particles loss energy by radiation, compensated by the RFS
- Particles are focused by guiding fields toward a designed orbit
- Accelerating field collects particles into circulating bunches
- Amplitud **damping** occurs due to energy loss by radiation
- Amplitude excitation occurs as a quantum effect
- Between damping and excitation a **balance** is reached
- Collision are produced at defined interaction points



A proposal Future Circular e+e- Collider



- Luminosity is expected of order 10e36
- Energy range per beam should be from 40 to 175 GeV
- Circumference would be of 100 km
- Energy Resolution requires an improvement for direct Higgs production

Hill Differential Equation

$$x'' + K(s)x = 0 \qquad y'' - K(s)y = 0, \qquad K(s) \equiv \frac{1}{(B\rho)} \left(\frac{\partial B_y}{\partial x}\right)(s)$$
$$K(s+C) = K(s)$$

Ansatz
$$x(s) = A\omega(s)\cos(\phi(x) + \phi_0)$$

Courant-Snyder Parameters

$$\beta(s) \equiv rac{\omega^2(s)}{k} \qquad \alpha(s) \equiv -rac{1}{2} eta\prime(s) \qquad \gamma(s) \equiv rac{1+lpha^2(s)}{eta(s)}$$

$$\begin{array}{c} & & & \\ & & \\ \hline & & \\ &$$

Emittance

$$\epsilon = \pi \mathcal{W} = \pi [\gamma(s)x^{2}(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^{2}(s)]$$

Size of the Distribution

$$\sigma_{RMS} = \sqrt{\epsilon \beta(s)}$$

Lattice Design Dispersion



General Contribution

 $x'' + K(s)x = \frac{\delta}{\rho(s)}$

Dispersion Contribution

Dispersion Function $D(s) = S(s) \int_0^s \frac{C(\tau)}{\rho(\tau)} d\tau - C(s) \int_0^s \frac{S(\tau)}{\rho(\tau)} d\tau$ $x(s) = C(s)x_0 + S(s)x'_0 + D(s)\delta_0$

Off-Energy Particles $E = E_0 + \epsilon$ Off-Energy Contribution $x = x_\beta + x_\epsilon$ Ultrarelativistic Limit $E \gg mc^2$ $p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} \approx \frac{E}{c}$ $\sum_{i=1}^{2} \frac{1}{\sqrt{1 - \beta^2}}$ $\Delta p = \frac{\Delta E}{E_0}$ $\delta = \frac{\epsilon}{E_0}$ $\sum_{i=1}^{2} \frac{1}{\sqrt{1 - \beta^2}}$ Off-Energy Contribution $x_\epsilon = D(s)\epsilon/E_0$ β

Equation of Motion for Off-Energy Particles

 $x'' = -K_x x + G\left(\frac{\epsilon}{E_0}\right)$

Radius of Curvature

 $G(s) = \rho^{-1}(s)$

Dispersion Equation

$$D''(s) = -K_x D(s) + G(s)$$

Beam Parameters Distributions

Transverse Displacement $x = x_{\beta} + D_x \epsilon_0$, $y = y_{\beta} + D_y \epsilon_0$ $\epsilon_0 = \frac{\epsilon}{E_0}$

Phase Space Distribution

$$f^{\pm}(x,y,\epsilon) = \frac{f^{\pm}(p_x,p_y,z)}{\sqrt{8\pi^3\beta_x^*\epsilon_{xc}\beta_y^*\epsilon_{yc}\sigma_\epsilon^2}} \exp\left\{-\frac{(x_\beta + D_x\epsilon_0)^2}{2\beta_x^*\epsilon_{xc}} - \frac{(y_\beta + D_y\epsilon_0)^2}{2\beta_y^*\epsilon_{yc}} - \frac{\epsilon^2}{2\sigma_\epsilon^2}\right\}$$

Average at the IP
$$\langle \mathcal{A} \rangle^{\pm} = \int f^{\pm}(X^{\pm})\mathcal{A}(X^{+}, X^{-})dX^{\pm} \qquad \langle \mathcal{A} \rangle^{*} = \left\langle \langle \mathcal{A} \rangle^{+} \right\rangle^{-} = \left\langle \langle \mathcal{A} \rangle^{-} \right\rangle^{+}$$

Average over distribution

$$\sigma_x^* = \sqrt{\langle x^2 \rangle^{\pm}} = \sqrt{\beta_x^* \epsilon_{xc} + D_x^{*2} \sigma_\epsilon^2} \qquad \langle x \epsilon_0 \rangle^{\pm} = D_x^{*\pm} \sigma_\epsilon^2 \qquad \sigma_\epsilon^* = \sqrt{\langle \epsilon_0^2 \rangle^{\pm}}$$
$$\sigma_y^* = \sqrt{\langle y^2 \rangle^{\pm}} = \sqrt{\beta_y^* \epsilon_{yc} + D_y^{*2} \sigma_\epsilon^2} \qquad \langle y \epsilon_0 \rangle^{\pm} = D_y^{*\pm} \sigma_\epsilon^2$$

Notes

- **Relative Momentum Deviation** of particles differs from ideality
- **Dispersion** quantifies the effects on displacement due to off-momentum particles
- **Emittance** is a measure of how far our distribution is from ideal orbit
- **Beta Function** is related to the physical transverse size of bunches

Standard Monochromatization

Monochromatization Colliding Beams

Collision Energy Spread $\sigma_w = \sqrt{2}E_0\sigma_\epsilon$ Beam Energy Spread $\sigma_\epsilon^2 \propto \frac{55\hbar cE_0^2}{32\sqrt{3(mc^2)^3}}\frac{I_3}{I_2}\frac{1}{J_\epsilon}$ Function of radius and J_ϵ $\sigma_w \propto (\rho J_\epsilon)^{-1/2}$. $J_\epsilon \in [0.5, 2.5]$

Typical Options
$$\begin{pmatrix} \rho >> \rho_0 \\ J_{\epsilon} > J_{0\epsilon} \end{pmatrix} \Rightarrow \sigma_w < \sigma_{0w}$$

Monochromatization Colliding Beams

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Typical Options
$$\begin{pmatrix} \rho >> \rho_0 \\ J_{\epsilon} > J_{0\epsilon} \end{pmatrix} \Rightarrow \sigma_w < \sigma_{0w}$$

Principle of Monochromatization

Monochromatization Factor

$$\lambda \equiv rac{\mathcal{L}_0}{\mathcal{L}}$$

Baseline Scheme
$$D_{x^+}^* = -D_{x^-}^* = 0$$
 $\mathcal{L} = \mathcal{L}_0$

Monochromatization principle A "simple explanation"

Standard collision Dispersion has the same sign in the IP

$$e^{-} \xrightarrow{E + \Delta E} \overleftarrow{E} \overleftarrow{E} \overleftarrow{E} e^{+}$$

 $\xrightarrow{E - \Delta E} \overleftarrow{E} \overleftarrow{E} e^{+}$
 $w = 2(E_0 + \epsilon)$

Monochromatization principle A "simple explanation"

Standard collision Dispersion has the same sign in the IP

$$e^{-} \xrightarrow{E + \Delta E} \overleftarrow{E} + \Delta E} \overrightarrow{E} e^{-}$$

 $\xrightarrow{E - \Delta E} \overleftarrow{E} - \Delta E} e^{-}$
 $w = 2(E_0 + \epsilon)$

Monochromatization Dispersion has opposite sign in the IP

$$e^{-} \xrightarrow{E + \Delta E} \stackrel{E}{\longleftrightarrow} \stackrel{E - \Delta E}{\xleftarrow{E}} e^{+}$$

 $w = 2E_0 + 0(\epsilon)^2$

Notes

- **Baseline Parameters** fail to produce the required center-of-mass energy resolution for direct Higgs production at 125 GeV
- Higgs Width of ~4.2 MeV requires an improvement in resolution by at least a factor of 10 since baseline produced a ~40 MeV width distribution
- **Double Ring System** of the FCCee may produced dispersion at the interaction point as required for monochromatization.
- **Radiation Effects** could compromise the performance of a monochromatization scheme

Radiation Effects

Beamstrahlung Sokolov-Ternov Theory



Beamstrahlung Sokolov-Ternov Theory

Relative Energy Loss
$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 N V_b^2}{\sigma_z (\sigma_x + \sigma_y)^2} \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 N V_b^2}{\sigma_z \sigma_x^2}$$
Average Emitted Photons
$$n_\gamma \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x + \sigma_y} \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x}$$

$$\int_{0.95} \frac{e^4 (u^2)}{(u^2)} \frac{1}{(u^2)} \approx \frac{25 \times 11}{64} (u)^2$$
Classical Formula
$$\langle u^2 \rangle \approx \frac{25 \times 11}{64} (u)^2$$

$$\int_{0.8} \frac{e^2 N V_b^2}{(u^2)} = \frac{\delta_B}{10^6} \approx \frac{2\sqrt{3}}{10^2} \frac{r_e^2 N V_b^2}{\alpha \sigma_z \sigma_x}$$
Excitation Term
$$n_\gamma \langle u^2 \rangle \approx 1.4 \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 (\sigma_x + \sigma_y)^3} \approx 192 \frac{r_e^5 N_b^3 \gamma^2}{\sigma_z^2 \sigma_x^3}$$

Beamstrahlung Self-Consistant Energy Spread

Total Energy Spread
$$\sigma_{\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \sigma_{\delta,\text{BS}}^2$$
$$\sigma_{\delta,\text{tot}}^2 - \sigma_{\delta,\text{SR}}^2 = A \left(\frac{\sigma_{\delta,\text{SR}}}{\sigma_{\delta,\text{tot}}} \frac{1}{\sigma_{z,\text{SR}}} \right)^2 \qquad \sigma_{\delta,\text{tot}} = \left[\frac{1}{2} \sigma_{\delta,\text{SR}}^2 + \left(\frac{1}{4} \sigma_{\delta,\text{SR}}^4 + A \frac{\sigma_{\delta,\text{SR}}^2}{\sigma_{z,\text{SR}}^2} \right)^{1/2} \right]^{1/2}$$

Bunch Length

 $\sigma_{z,\mathrm{tot}} = rac{lpha_{\mathrm{C}} C}{2\pi Q_s} \sigma_{\delta,\mathrm{tot}}$

Dispersion Invariant

$$\mathcal{H}_x^* \equiv \frac{\left(\beta_x^* {D'_x}^* + \alpha_x^* D_x^*\right)^2 + {D_x^*}^2}{\beta_x^*}$$

Self-Consistent Equations

$$\epsilon_{x,\text{tot}} = \epsilon_{x,\text{SR}} + \frac{\tau_x n_{\text{IP}}}{4T_{\text{rev}}} \{ n_\gamma < u^2 \} \mathcal{H}_x^*$$

$$\sigma_{\delta,\text{tot}}^2 = \sigma_{\delta,\text{SR}}^2 + \frac{n_{\text{IP}}\tau_{E,\text{SR}}}{4T_{\text{rev}}} \{n_{\gamma} < u^2 > \}$$

Optimized Monochromatization

Baseline monochromatization

$E_e [{ m GeV}]$	62.5
scheme	m.c.
	basel.
I_b [mA]	408.3
$N_b [10^{10}]$	3.3
$n_b [1]$	25760
n_{IP} [1]	2
β_x^* [m]	1.0
$\beta_y^* \; [\mathrm{mm}]$	2
D^*_x [m]	0.22
$\epsilon_{x,\mathrm{SR}} \; [\mathrm{nm}]$	0.17
$\epsilon_{x,\mathrm{tot}}$ [nm]	0.21
$\epsilon_{y,\mathrm{SR}} \mathrm{[pm]}$	1
$\sigma_{x,\mathrm{SR}} \; [\mathrm{\mu m}]$	132
$\sigma_{x,\mathrm{tot}}~[\mathrm{\mu m}]$	144
$\sigma_y \mathrm{[nm]}$	45
$\sigma_{z,\mathrm{SR}} \; [\mathrm{mm}]$	1.8
$\sigma_{z,{ m tot}} [{ m mm}]$	1.8
$\sigma_{\delta,\mathrm{SR}}$ [%]	0.06
$\sigma_{\delta,\mathrm{tot}}$ [%]	0.06
$\theta_c \; [\mathrm{mrad}]$	0
circ. C [km]	100
$\alpha_{\rm C} \ [10^{-6}]$	7
$f_{ m rf}~[m MHz]$	400
$V_{\rm rf} [{ m GV}]$	0.4
$U_{0,\mathrm{SR}}$ [GeV]	0.12
$U_{0,\mathrm{BS}} \mathrm{[MeV]}$	0.01
$ au_E/T_{ m rev}$	509
Q_s	0.030
$\Upsilon_{ m max}$ $[10^{-4}]$	0.3
$\Upsilon_{\rm ave} [10^{-4}]$	0.1
$\theta_c \; [\mathrm{mrad}]$	0
$\xi_x [10^{-2}]$	1
$\xi_y [10^{-2}]$	4
λ [1]	9.2
$L [10^{35}]$	1.0
$cm^{-2}s^{-1}$]	
$\sigma_w [{ m MeV}]$	5.8

Width of standard model Higgs 4-5 MeV requires $\lambda \ge 10$ $N_b = 3.3 \times 10^{10} n_b = 25760 \beta_y * = 2 \text{ mm}$



 \star : λ = 9.2, L = 1 x 10³⁵ cm⁻²s⁻¹

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$E_e [{ m GeV}]$	62.5
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$\Upsilon_{\rm max}$ [10 ⁻⁴]	0.3
$\Upsilon_{\rm ave} [10^{-4}]$	0.1
$\theta_c \; [\text{mrad}]$	0
$\xi_x [10^{-2}]$	1
$\xi_y [10^{-2}]$	4
λ [1]	9.2
$L [10^{30} - 2^{-1}]$	1.0
cm^2s^1	
$\sigma_w \; [{ m MeV}]$	5.8

Width of standard model Higgs 4-5 MeV requires $\lambda \ge 10$ $N_b = 3.3 \times 10^{10} n_b = 25760 \beta_y^* = 2 mm$ Luminosity contours -00e+35 4.5 3.5 1.36e+35 2.08e+35 1.90e+35 1.54e+35 1.18e+35 1.72e+35 2.26e+35 44e+35 2e+35 <u>ב</u> י 13035 × 2.5 β 1.358138.59e+35 1.5 0.5 0.05 0.1 0.15 0.2 0.25 D _ * [m]

 \star : λ = 9.2, L = 1 x 10³⁵ cm⁻²s⁻¹

Optimized Monochromatization Luminosity_max and λ

S = [0.1, 3], T = [0.1, 3] $\beta_{0x} = 1.0 \text{ m}, \beta_{0y} = 1.0 \text{ mm}$ $D_{0x} = 0.22 \text{ m}$ $N_{0b} = 3.3 \times 10^{10}, n_{0b} = 25760$ $\beta_x = \beta_{0x} * S^2$, $D_x * = D_{0x} * S$ $N_b = N_{0b} / T$, $n_b = n_{0b} * T$

Monochromatization Factor





Optimized Monochromatization Luminosity max and λ

S = [0.1, 3], T = [0.1, 3] $\beta_{0x} = 1.0 \text{ m}, \beta_{0y} = 1.0 \text{ mm}$ $D_{0x} = 0.22 \text{ m}$ $N_{0b} = 3.3 \times 10^{10}, n_{0b} = 25760$ $\beta_x = \beta_{0x} * S^2, D_x * = D_{0x} * S$



$$N_b = N_{0b} / T$$
, $n_b = n_{0b} * T$

 \bigstar : λ₀ = 10.17321, L ~ 1 x 10³⁶ cm⁻²s⁻¹

Optimized Monochromatization Luminosity_max and λ



 \bigstar : λ₀ = 10.17321, L ~ 1 x 10³⁶ cm⁻²s⁻¹

S = [0.1, 3], T = [0.1, 3] $\beta_{0x} = 1.0 \text{ m}, \beta_{0y} = 1.0 \text{ mm}$ $D_{0x} = 0.22 \text{ m}$ $N_{0b} = 3.3 \times 10^{10}, n_{0b} = 25760$ $\beta_x = \beta_{0x} * S^2, D_x * = D_{0x} * S$

 $N_b = N_{0b} / T$, $n_b = n_{0b} * T$

Monochromatization Factor



 \star : λ = 5.07, β = 1.96 m, D_x = 0.308 m, L = 3.736 x 10³⁵ cm⁻²s⁻¹

Conclusions

 \star : λ = 5.07, β = 1.96 m, D_x = 0.308 m, L = 3.736 x 10³⁵ cm⁻²s⁻¹

E_e [GeV]	45.6	62.5	62.5	62.5	80
scheme	CW	ho.	m.c.	m.c.	CW
			basel.	opt'd	
I_b [mA]	1450.3	408.3	408.3	408.3	151.5
N_b [10 ¹⁰]	3.3	1.05	3.3	11.1	6.0
n_b [1]	91500	80960	25760	7728	5260
n_{IP} [1]	2	2	2	2	2
β_x^* [m]	1	1.0	1.0	1.96	1
β_y^* [mm]	2	2	2	1	2
$\check{D_x^*}$ [m]	0	0	0.22	0.308	0
$\epsilon_{x,\mathrm{SR}}$ [nm]	0.09	0.17	0.17	0.17	0.26
$\epsilon_{x,\mathrm{tot}}$ [nm]	0.09	0.17	0.21	0.70	0.26
$\epsilon_{y,\mathrm{SR}} \; \mathrm{[pm]}$	1	1	1	1	1
$\sigma_{x,\mathrm{SR}} \; [\mu\mathrm{m}]$	9.5	9.2	132	185.7	16
$\sigma_{x,\mathrm{tot}} \; [\mathrm{\mu m}]$	9.5	9.2	144	188.5	16
σ_y [nm]	45	45	45	32	45
$\sigma_{z,\mathrm{SR}}$ [mm]	1.6	1.8	1.8	1.8	2.0
$\sigma_{z,\mathrm{tot}} \mathrm{[mm]}$	3.8	1.8	1.8	1.8	3.1
$\sigma_{\delta,\mathrm{SR}}$ [%]	0.04	0.06	0.06	0.06	0.07
$\sigma_{\delta,\mathrm{tot}}$ [%]	0.09	0.06	0.06	0.06	0.10
$\theta_c \text{ [mrad]}$	30	0	0	0	30
circ. C [km]	100	100	100	100	100
$\alpha_{\rm C} [10^{-6}]$	7	7	7	7	7
$f_{\rm rf} [{ m MHz}]$	400	400	400	400	400
$V_{\rm rf} [{ m GV}]$	0.2	0.4	0.4	0.4	0.8
$U_{0,\mathrm{SR}} \; [\mathrm{GeV}]$	0.03	0.12	0.12	0.12	0.33
$U_{0,\mathrm{BS}} \; [\mathrm{MeV}]$	0.5	0.05	0.01	0.01	0.21
$ au_E/T_{ m rev}$	1320	509	509	509	243
Q_s	0.025	0.030	0.030	0.030	0.037
$\Upsilon_{ m max}$ $[10^{-4}]$	1.7	0.8	0.3	0.85	4.0
$\Upsilon_{\rm ave} \ [10^{-4}]$	0.7	0.3	0.1	0.35	1.7
$\theta_c \text{ [mrad]}$	30	0	0	0	30
$\xi_x [10^{-2}]$	5	12	1	2.22	7
$\xi_{y} [10^{-2}]$	13	15	4	6.76	16
λ [1]	1	1	9.2	5.08	1
$L [10^{35}]$	9.0	2.2	1.0	3.74	1.9
$cm^{-2}s^{-1}$]					
$\sigma_w [{ m MeV}]$	58	53	5.8	10.44	113

- Monochromatization scheme can be implemented
- Beamstrahlung effects may be controlled
- Simulation supports predictions
- Lattice designed is still in progress and the required

modification should be possible

• Theory confirmation could be achieved at the FCCe+e-



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M. Alan Valdivia G. valdivia@fisica.ugto.mx RADPyC'17, CINVESTAV, Mexico City, 250517