Heavy quarks within electroweak multiplet

Jaime Besprosvany

En colaboración: Ricardo Romero

Instituto de Física
Universidad Nacional Autónoma de México

Instituto de Ciencias Nucleares, UNAM, 15 de marzo de 2017

J. Besprosvany y R. Romero “Representation of quantum field theory in an extended spin space and fermion mass hierarchy " *Int. J. Mod. Phys. A* **29**, No. 29 1450144 (17 pp.) (2014), arXiv:1408.4066[hep-th].

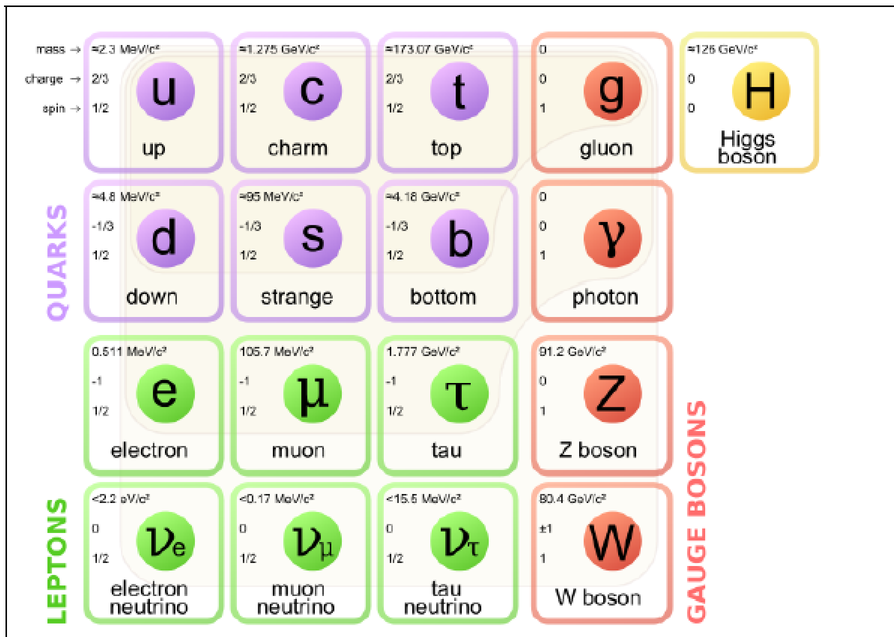
Ricardo Romero and Jaime Besprosvany, “Quark horizontal flavor hierarchy and two-Higgs- doublet model in a (7+1)-dimensional extended spin space ", arXiv:1611.07446[hep-ph]

Jaime Besprosvany and Ricardo Romero, “Heavy quarks within electroweak multiplet", arXiv:11701.01191[hep-ph]

Contents

- Motivation: puzzles in the standard model, multiplet structure
- Spin-extended model, interpretation, and formulation
- (7+1)-dimensional space: states and operators; conventional and spin-extended bases: Lagrangian equivalence
- Electroweak scalar-vector symmetry, and 3 Lagrangian representations
- Scalar-field uniqueness: scalar-vector and scalar-fermion terms comparison
- Quark-mass relations and physical interpretation
- Argument summary

Modelo estándar



- Muchos parámetros libres
- Sin información experimental
- Extensiones populares sin avance
- Necesidad de alternativas

Motivation: multiplet structure

Puzzles in the standard model:

- Fermion-mass parameters; Yukawa sector independent of scalar-vector.
- Origin of electroweak symmetry breaking (Higgs mechanism).

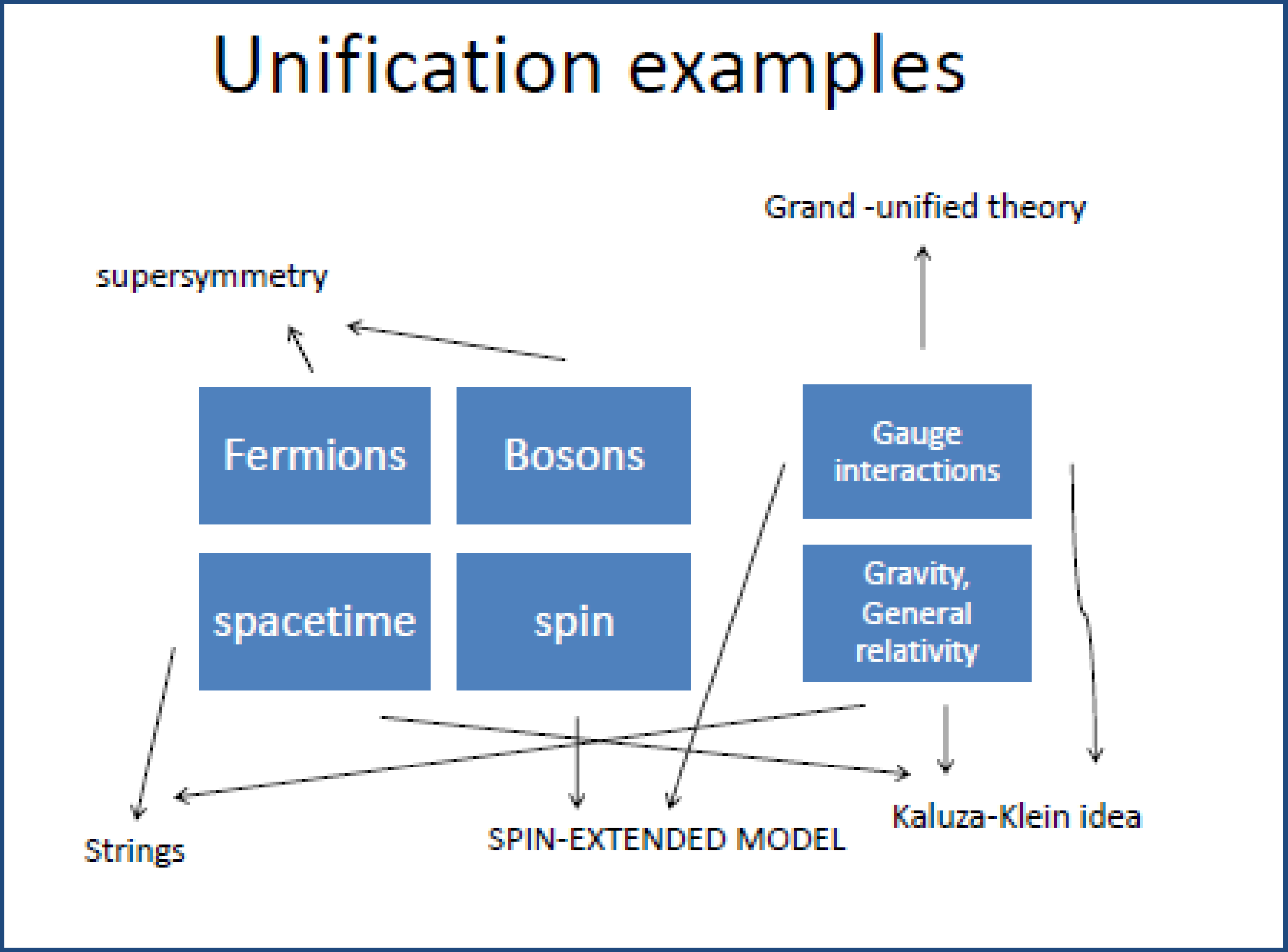
	Masses (GeV)	Spin	Weak I^2	Hypercharge Y
• $W^{+/-}$	80.4	1	1	0
• Z	91.2	1	0	0
• H	126	0	½	1
• t	173	½	½ ,0	1/3, 4/3
• b	4	½	½ ,0	1/3, -2/3

Composite multiplet structure suggested

Physics revealed by new basis

- Landau’s quasiparticles: effective mass.
- Superconductivy: Cooper pairs.
- Application to field theory in Nambu Jona-Lasinio model: composite bound particle states.
- Constituent quarks.
- Interactive boson model in nuclear physics.

Spin-extended model within standard-model extensions



Two physical interpretations

- Kaluza-Klein type of framework, for in higher than (3+1)-dimensions, only the spin component in

$$\mathcal{P}_P\left[\frac{1}{2}\sigma_{\mu\nu} + i(x_\mu\partial_\nu - x_\nu\partial_\mu)\right]$$

μ=5,…,N, ν=5,…,N

  remains as symmetry operator; thus, spatial components are frozen.

- Elementary discrete degree-of-freedom matrix construction: q-bits

To generalize to higher-dimensional spaces, we continue with the Lorentz-group case as paradigm. The most general spinor construction containing l spinors ξ^{a_i}, k spinors $\xi^{\dot{a}_i}$, n spinors ξ_{a_i}, and m spinors $\xi_{\dot{a}_i}$, has the form

$$= \xi^{a_1}\ldots\xi^{a_l}\xi^{\dot{a}_1}\ldots\xi^{\dot{a}_k}\xi_{a_1}\ldots\xi_{a_l}\xi_{\dot{a}_n}\ldots\xi_{\dot{a}_m} \qquad (1)$$

Use of conventional and spin bases

spin basis → conventional basis

- Constrain representations and interactions at given dimension.
- Finite number of possible partitions.

spin basis ← conventional basis

Reinterpretation of fields:

- SV: scalar operator acting over vectors
- SF: scalar operator acting over fermions
- Standard-model projection.

LORENTZ AND MAXIMAL SCALAR SYMMETRY AT D DIMENSION

$$\underbrace{\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3} \qquad \underbrace{\gamma_4, \ldots \gamma_{D-1}}$$

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \quad \mu,\nu = 0,..,3 \qquad \gamma_a \quad a = 4,..,D-1$$

4-D Lorentz symmetry $\otimes$ Scalar symmetry

unitary: $U(2^{(D-4)/2})$

$$[\sigma_{\mu\nu}, \gamma_a] = 0$$

$$[\tilde{\gamma}_5, \gamma_a] = 0 \qquad \tilde{\gamma}_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$[H, \gamma_a] = 0 \qquad H = i\gamma_0 \bar{\nabla}\cdot\bar{\gamma}$$

maximal scalar symmetry

$$U_R \otimes U_L \qquad U_R = \frac{1}{2}(1+\tilde{\gamma}_5)\, U(2^{(D-4)/2})$$

$$U_L = \frac{1}{2}(1-\tilde{\gamma}_5)\, U(2^{(D-4)/2})$$

Coleman-Mandula OK

Propiedades de los generadores escalares

Conjunto de generadores escalares

- $\mathscr{S}_{N-4} = \frac{1}{2}(I+\tilde{\gamma}_5)\mathrm{U}\left(2^{(N-4)/2}\right) \oplus \frac{1}{2}(I-\tilde{\gamma}_5)\mathrm{U}\left(2^{(N-4)/2}\right)$
- Matriz quiral $\tilde{\gamma}_5 \equiv -i\gamma_0\gamma_1\gamma_2\gamma_3$

Teorema de Coleman-Mandula

$\left[\mathscr{S}_{N-4}, \sigma_{\mu\nu}\right] = 0$

- Simetrías continuas (globales y locales)

Propiedades del modelo de espín extendido

Representaciones asociadas a partículas en $\mathscr{C}_4 \otimes \mathscr{S}_{N-4}$

- $(\text{elementos del espacio } 3{+}1\ ) \times \left(\begin{array}{c} \text{combinación de productos} \\ \text{de elementos de } \mathscr{S}_{N-4} \end{array} \right)$

Transformaciones

- $\Psi \to U \Psi U^\dagger$

Evaluación de operadores

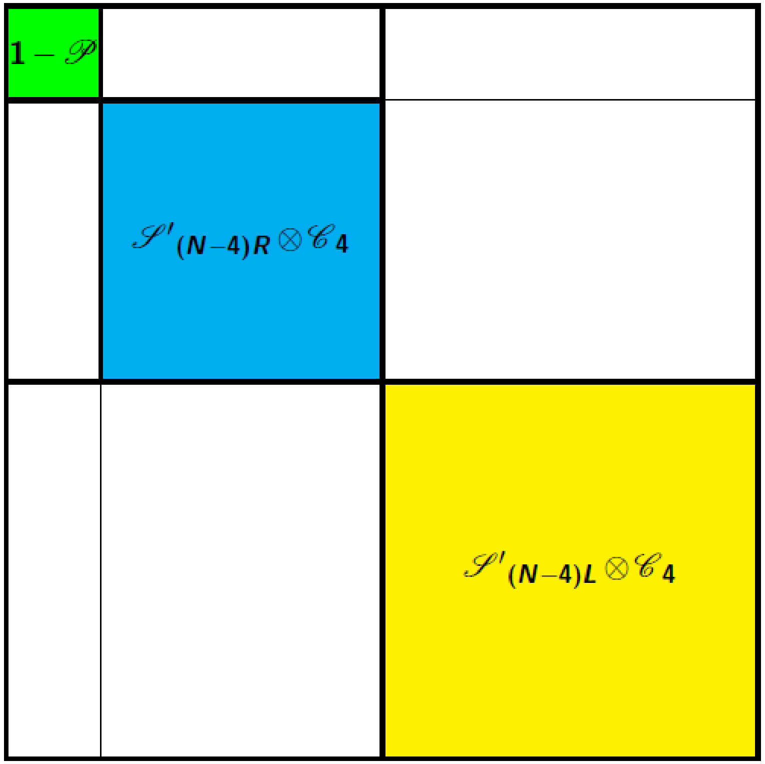
- $[\mathscr{O}, \Psi] = \lambda \Psi$

Producto interno

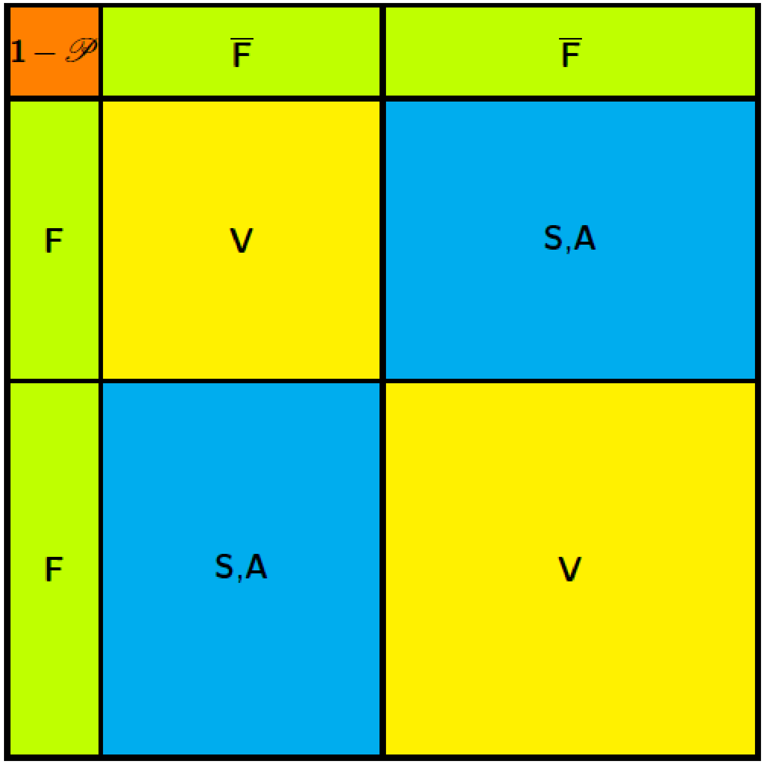
- $\langle \Phi \mid \Psi \rangle = \mathrm{Tr}\left(\Phi^\dagger \Psi\right)$

Esquema del espacio matricial

Operadores



Estados



Bases y generadores

Matrices gama

- $\gamma_0, \gamma_1, \ldots, \gamma_8$

Generadores escalares

- 4 γ_a, $a = 5, \ldots, 8$, 6 $\gamma_{ab} \equiv \gamma_a \gamma_b$, $a < b$, 4 $\gamma_{abc} \equiv \gamma_a \gamma_b \gamma_c$, $\gamma_5 \gamma_6 \gamma_7 \gamma_8$
- 32 escalares $\mathscr{S}_4 = P_+ \mathrm{U}(4) \oplus P_- \mathrm{U}(4)$

Base de Cartan

- $1, \quad \widetilde{\gamma}_5, \quad \gamma_5\gamma_6, \quad \gamma_7\gamma_8, \quad \gamma_5\gamma_6\gamma_7\gamma_8, \quad \gamma_5\gamma_6\widetilde{\gamma}_5, \quad \gamma_7\gamma_8\widetilde{\gamma}_5, \quad \gamma_5\gamma_6\gamma_7\gamma_8\widetilde{\gamma}_5.$

Representación matricial base de Cartan

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	P_{R1}																1
2		P_{R1}															2
3			P_{R2}														3
4				P_{R2}													4
5					P_{R3}												5
6						P_{R3}											6
7							P_{R4}										7
8								P_{R4}									8
9									P_{L1}								9
10										P_{L1}							10
11											P_{L2}						11
12												P_{L2}					12
13													P_{L3}				13
14														P_{L3}			14
15															P_{L4}		15
16																P_{L4}	16
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	

Operadores de norma y número bariónico

- $\mathscr{P} = B = \frac{1}{6}(1 - i\gamma_5\gamma_6)$.
- $Y = \frac{1}{6}\left(1 - i\gamma_5\gamma_6\right)\left(1 + i\frac{3}{2}(1 + \tilde{\gamma}_5)\gamma_7\gamma_8\right)$.
- $I^1 = \frac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma_7\gamma_8$.
- $I^2 = \dfrac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma^7$.
- $I^3 = \frac{i}{8}(1 - \tilde{\gamma}_5)(1 - i\gamma_5\gamma_6)\gamma^8$

Esquema matricial de quarks y escalares

<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td></td></tr>
<tr><td>1</td><td>f_3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>1</td></tr>
<tr><td>2</td><td></td><td>f_3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>2</td></tr>
<tr><td>3</td><td></td><td></td><td>f_3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>3</td></tr>
<tr><td>4</td><td></td><td></td><td></td><td>f_3</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>4</td></tr>
<tr><td>5</td><td colspan="2" rowspan="2">U_R^1</td><td colspan="2" rowspan="2">U_R^2</td><td>Y_R</td><td></td><td></td><td></td><td colspan="2" rowspan="2">U_R^3</td><td colspan="2" rowspan="2">U_R^4</td><td colspan="2" rowspan="2">$\varphi^0_{R,1,2}$</td><td colspan="2" rowspan="2">$\varphi^+_{R,1,2}$</td><td>5</td></tr>
<tr><td>6</td><td></td><td>Y_R</td><td></td><td></td><td>6</td></tr>
<tr><td>7</td><td colspan="2" rowspan="2">D_R^1</td><td colspan="2" rowspan="2">D_R^2</td><td></td><td></td><td>Y_R</td><td></td><td colspan="2" rowspan="2">D_R^3</td><td colspan="2" rowspan="2">D_R^4</td><td colspan="2" rowspan="2">$\varphi^-_{R,1,2}$</td><td colspan="2" rowspan="2">$\varphi^{0*}_{R,1,2}$</td><td>7</td></tr>
<tr><td>8</td><td></td><td></td><td></td><td>Y_R</td><td>8</td></tr>
<tr><td>9</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>$\hat{f}_3$</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>9</td></tr>
<tr><td>10</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>$\hat{f}_3$</td><td></td><td></td><td></td><td></td><td></td><td></td><td>10</td></tr>
<tr><td>11</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>$\hat{f}_3$</td><td></td><td></td><td></td><td></td><td></td><td>11</td></tr>
<tr><td>12</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>$\hat{f}_3$</td><td></td><td></td><td></td><td></td><td>12</td></tr>
<tr><td>13</td><td colspan="2" rowspan="2">U_L^1</td><td colspan="2" rowspan="2">U_L^2</td><td colspan="2" rowspan="2">$\varphi^{0*}_{L,1,2}$</td><td colspan="2" rowspan="2">$\varphi^+_{L,1,2}$</td><td colspan="2" rowspan="2">U_L^3</td><td colspan="2" rowspan="2">U_L^4</td><td>I^3</td><td></td><td></td><td></td><td>13</td></tr>
<tr><td>14</td><td></td><td>I^3</td><td></td><td></td><td>14</td></tr>
<tr><td>15</td><td colspan="2" rowspan="2">D_L^1</td><td colspan="2" rowspan="2">D_L^2</td><td colspan="2" rowspan="2">$\varphi^-_{L,1,2}$</td><td colspan="2" rowspan="2">$\varphi^0_{L,1,2}$</td><td colspan="2" rowspan="2">D_L^3</td><td colspan="2" rowspan="2">D_L^4</td><td></td><td></td><td>I^3</td><td></td><td>15</td></tr>
<tr><td>16</td><td></td><td></td><td></td><td>I^3</td><td>16</td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td></td></tr>
</table>

(7+1)-d fermions

hypercharge 1/3 left-handed doublet	I^3	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$\begin{pmatrix} T_L^1 \\ B_L^1 \end{pmatrix} = \begin{pmatrix} \frac{1}{16}(1-\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(\gamma^7 + i\gamma^8)(\gamma^0+\gamma^3) \\ \frac{1}{16}(1-\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(1 - i\gamma^7\gamma^8)(\gamma^0+\gamma^3) \end{pmatrix}$	$1/2$ $-1/2$	$2/3$ $-1/3$	$1/2$ $1/2$

(a)

$I^3 = 0$ right-handed singlet	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$T_R^1 = \frac{1}{16}(1+\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(\gamma^7 + i\gamma^8)\gamma^0(\gamma^0+\gamma^3)$	$4/3$	$2/3$	$1/2$
$B_R^1 = \frac{1}{16}(1+\tilde{\gamma}_5)(\gamma^5 - i\gamma^6)(1 - i\gamma^7\gamma^8)\gamma^0(\gamma^0+\gamma^3)$	$-2/3$	$-1/3$	$1/2$

(b)

(7+1)-d scalars

Table 4. Scalar Higgs-like pairs.

0 baryon-number scalar	I_3	Y	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}(1-i\gamma^5\gamma^6)(\gamma^7+i\gamma^8)\gamma^0 \\ \frac{1}{8}(1-i\gamma^5\gamma^6)(1+i\gamma^7\gamma^8\tilde{\gamma}_5)\gamma^0 \end{pmatrix}$	$1/2$ $-1/2$	1	1 0	0
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{8}(1-i\gamma^5\gamma^6)(\gamma^7+i\gamma^8)\tilde{\gamma}_5\gamma^0 \\ \frac{i}{8}(1-i\gamma^5\gamma^6)(1+i\gamma^7\gamma^8\tilde{\gamma}_5)\gamma^7\gamma^8\gamma^0 \end{pmatrix}$	$1/2$ $-1/2$	1	1 0	0

Conventional and spin-extended bases, Lagrangian equivalence: fermion-vector

conventional base

spin-extended base

Field formulation: $A_\mu(x) = g_\mu{}^\nu A_\nu(x)$

$A_\mu(x)\gamma_0\gamma^\mu$

$$\mathcal{L}_{FV} = \bar{\mathrm{q}}_L(x)[i\partial_\mu + \frac{1}{2}g\tau^a W_\mu^a(x) + \frac{1}{6}g' B_\mu(x)]\gamma^\mu \mathrm{q}_L(x) +$$

$$\bar{t}_R(x)[i\partial_\mu + \frac{2}{3}g' B_\mu(x)]\gamma^\mu t_R(x) + \bar{b}_R(x)[i\partial_\mu - \frac{1}{3}g' B_\mu(x)]\gamma^\mu b_R(x)$$

$$\mathrm{q}_L(x) = \begin{pmatrix} t_L(x) \\ b_L(x) \end{pmatrix}$$

$$t_L(x) = \begin{pmatrix} \psi_{tL}^1(x) \\ \psi_{tL}^2(x) \end{pmatrix}$$

$$\mathcal{L}_{FV} = \mathrm{tr}\{\Psi_{qL}^\dagger(x)[i\partial_\mu + gI^a W_\mu^a(x) + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{qL}(x) +$$

$$\Psi_{tR}^\dagger(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{tR}(x) + \Psi_{bR}^\dagger(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{bR}(x)\}P_f$$

$$\Psi_{qL}(x) = \sum_\alpha \psi_{tL}^\alpha(x) T_L^\alpha + \psi_{bL}^\alpha(x) B_L^\alpha$$

Conjugate SU(2) property

- For a set of generators $G_{i,}$ conjugate $-G_i^*$ satisfy the same Lie algebra.
- SU(2) property: conjugate representation obtained by similarity transformation:

$$\sigma_2\, \sigma_i\, \sigma_2 = -\sigma_i^* \qquad\qquad \sigma_i : \quad \text{Pauli matrices}$$

$$\sigma_2\, \psi^* \qquad \text{transforms as } \psi$$

Scalar-vector Lagrangian representations

Single scalar representation

$$\mathbf{F}_{\mu}(x) = i\partial_{\mu} + \tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_{\mu}(x) + \tfrac{1}{2}g'B_{\mu}(x)$$

$$\mathbf{W}_{\mu}(x) = (W_{\mu}^{1}(x), W_{\mu}^{2}(x), W_{\mu}^{3}(x))$$

$$\mathbf{H}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \eta_3(x) + i\eta_4(x) \end{pmatrix}$$

$$\mathcal{L}_{SV} = \mathbf{H}^{\dagger}(x)\mathbf{F}^{\mu\dagger}(x)\mathbf{F}_{\mu}(x)\mathbf{H}(x)$$

Scalar and conjugate representation

$$\mathbf{F'}_{\mu}\bar{\mathbf{H}}_{\chi_t\chi_b}(x) = (i\partial_{\mu} + \tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_{\mu}(x))\bar{\mathbf{H}}_{\chi_t\chi_b}(x) + g'\bar{\mathbf{H}}_{\chi_t\chi_b}(x)B_{\mu}(x)\tau_3$$

Sikivie et al. [80], Chivukula [98] $\bar{\mathbf{H}}_{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}(x)$

$$\bar{\mathbf{H}}_{\chi_t,\chi_b}(x) = (\chi_t\mathbf{H}(x), \chi_b\tilde{\mathbf{H}}(x))$$

$$\mathcal{L}_{SV} = \mathrm{tr}[\mathbf{F'}_{\mu}\bar{\mathbf{H}}_{\chi_t\chi_b}(x)]^{\dagger}\mathbf{F'}^{\mu}\bar{\mathbf{H}}_{\chi_t\chi_b}(x) \quad \alpha \quad |\chi_t|^2 + |\chi_b|^2$$

Scalar-field normalization requires $|\chi_t|^2 + |\chi_b|^2 = 1$

SV Lagrangian and scalar t-b spin representation

Scalar correspondence

$$\mathbf{H}(\mathbf{x}) \to \phi_1(x) - \phi_2(x)$$

$$\tilde{\mathbf{H}}^\dagger(x) \to \phi_1(x) + \phi_2(x).$$

$$\mathbf{H}_t(x) = \phi_1(x) + \phi_2(x), \; \mathbf{H}_b(x) = \phi_1(x) - \phi_2(x).$$

$$R_5 = \tfrac{1}{2}(1+\tilde{\gamma}_5), \text{ e. g., } R_5\mathbf{H}_t(x)L_5 = \mathbf{H}_t(x)$$

$$L_5\mathbf{H}_t(x)R_5 = 0, \; R_5\mathbf{H}_b(x)L_5 = 0$$

$$\mathbf{H}_{af}(x) = a\phi_1(x) + f\phi_2(x).$$

$$\mathbf{H}_{af}(x) = \frac{1}{\sqrt{2}}(\chi_t\mathbf{H}_t(x) + \chi_b\mathbf{H}_b(x))$$

$$\chi_t = \frac{1}{\sqrt{2}}(a+f), \; \chi_b = \frac{1}{\sqrt{2}}(a-f)$$

SV spin representation

$$\mathbf{F}''(x) = [i\partial_\mu + gW^i_\mu(x)I^i + \tfrac{1}{2}g'B_\mu(x)Y_o]\gamma_0\gamma^\mu$$

$$\mathcal{L}_{SV} = \mathrm{tr}\{[\mathbf{F}''(x), \mathbf{H}_{af}(x)]^\dagger_\pm [\mathbf{F}''(x), \mathbf{H}_{af}(x)]_\pm\}_{\mathrm{sym}}$$

Lagrangian correspondence in two bases for Z-mass term:

$$\mathbf{H}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1^r(x) e^{ip_{t1} + ip_{\eta 1}(x)} \\ \eta_0^r(x) e^{ip_{t0} + ip_{\eta 0}(x)} \end{pmatrix}$$

$$\tilde{\mathbf{H}}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\eta_1^r(x) e^{ip_{b1} - ip_{\eta 0}(x)} \\ i\eta_0^r(x) e^{ip_{b0} - ip_{\eta 1}(x)} \end{pmatrix}$$

$$\bar{\mathbf{H}}_{\chi_t, \chi_b}(x) = (\chi_t \mathbf{H}(x), \chi_b \tilde{\mathbf{H}}(x))$$

$$\begin{aligned} \mathbf{H}_{ab}^{tot\lambda}(x) = \frac{1}{\sqrt{(1+\lambda^2}} & \Big[\chi_t \left[\eta_1^r(x) e^{i\phi_1^t + ip_{\eta 1}(x)} (\phi_1^+ + \phi_2^+) + \eta_0^r(x) e^{i\phi_0^t + ip_{\eta 0}(x)} (\phi_1^0 + \phi_2^0)\right] \\ & + \chi_b \left[\eta_1^r(x) e^{i\phi_1^b - ip_{\eta 1}(x)} (\phi_1^+ - \phi_2^+)^\dagger + \eta_0^r(x) e^{i\phi_0^b - ip_{\eta 0}(x)} (\phi_1^0 - \phi_2^0)^\dagger + \right] \\ & + \lambda\chi_t \left[\eta_1^r(x) e^{i\phi_1^{\lambda t} - ip_{\eta 1}(x)} (\phi_1^+ + \phi_2^+)^\dagger + \eta_0^r(x) e^{i\phi_0^{\lambda t} - ip_{\eta 0}(x)} (\phi_1^0 + \phi_2^0)^\dagger\right] \\ & + \lambda\chi_b \left[\eta_1^r(x) e^{i\phi_1^{\lambda b} + ip_{\eta 1}(x)} (\phi_1^+ - \phi_2^+) \eta_0^r(x) e^{i\phi_0^{\lambda b} + ip_{\eta 0}(x)} (\phi_1^0 - \phi_2^0)\right]\Big], \end{aligned}$$

Square Z. $\mathrm{tr}\bar{\mathbf{H}}^\dagger(x)\frac{1}{2}g'B_\mu(x)\frac{1}{2}g'B^\mu(x)\bar{\mathbf{H}}(x) \leftrightarrow$
$\frac{1}{2}\mathrm{tr}\{[\frac{1}{2}g'B_0(x)Y_o\gamma_0\gamma^0, \mathbf{H}_{ab}^{tot\lambda}(x)]^\dagger[\frac{1}{2}g'B_0(x)Y_o\gamma_0\gamma^0, \mathbf{H}_{ab}^{tot\lambda}(x)] +$
$\{\frac{1}{2}g'B_j(x)Y_o\gamma_0\gamma^j, \mathbf{H}_{ab}^{tot\lambda}(x)\}^\dagger\{\frac{1}{2}g'B_k(x)Y_o\gamma_0\gamma^k, \mathbf{H}_{ab}^{tot\lambda}(x)\}\}$

$$\frac{1}{8}g'^2\left(\chi_t^2 + \chi_b^2\right)\left(\eta_0^r(x)^2 + \eta_1^r(x)^2\right)B_\mu(x)B^\mu(x) \tag{28}$$

Spin-space: connection between scalar-vector and Yukawa terms

$$\mathcal{L}_{SV} = \mathrm{tr}\{[\mathbf{F}''(x), \mathbf{H}_{af}(x)]_{\pm}^{\dagger}[\mathbf{F}''(x), \mathbf{H}_{af}(x)]_{\pm}\}_{\mathrm{sym}}$$

$$\mathbf{H}_{af}(x) = \tfrac{1}{\sqrt{2}}(\chi_t \mathbf{H}_t(x) + \chi_b \mathbf{H}_b(x))$$

$$-\mathcal{L}_{SF} = \mathrm{tr}\frac{\sqrt{2}}{v}[m_t \Psi_{tR}^{\dagger}(x)\mathbf{H}_t^{\dagger}(x)\boldsymbol{\Psi}_{qL}(x) + m_b \boldsymbol{\Psi}_{qL}^{\dagger}(x)\mathbf{H}_b^{\dagger}(x)\Psi_{bR}(x)] + \{hc\}$$

$$\mathbf{H}_m(x) = \tfrac{\sqrt{2}}{v}(m_t \mathbf{H}_t(x) + m_b \mathbf{H}_b(x))$$

Scalar-vector scalar-fermion comparison

$$\langle \eta_3(x) \rangle = v, \ \langle \mathbf{H}(x) \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Z-vector mass

Higgs mechanism

$$\langle \mathbf{H}_{af}(x) \rangle = H_n = \frac{v}{2}(\chi_t H_t^0 + \chi_b H_b^0),$$

$$\begin{aligned} \mathcal{L}_{SZm0} &= \mathrm{tr}[H_n, W_0^3(x) g I^3 + B_0(x)\frac{1}{2}g' Y_o]^\dagger [H_n, W_0^3(x) g I^3 + B_0(x)\frac{1}{2}g' Y_o] \qquad (11) \\ &= Z_0^2(x)\frac{1}{g^2+g'^2}\mathrm{tr}[H_n, g^2 I^3 - \frac{1}{2}g'^2 Y_o]^\dagger [H_n, g^2 I^3 - \frac{1}{2}g'^2 Y_o] = \frac{1}{2}Z_0^2(x) m_Z^2, \end{aligned}$$

Top-quark mass

Higgs mechanism

$$H_m = \langle \mathbf{H}_m(x) \rangle = m_t H_t^0 + m_b H_b^0.$$

$$H_m^h T_M^1 = m_t T_M^1, \quad H_m^h T_M^{c1} = -m_t T_M^{c1},$$

$$H_m^h B_M^1 = m_b B_M^1, \quad H_m^h B_M^{c1} = -m_b B_M^{c1}, \qquad (13)$$

where $H_m^h = H_m + H_m^\dagger$, and T_M^{c1}, B_M^{c1} correspond to negative-energy solution states

Spin-space connection: vector and fermion masses

vector

$$m_Z = v\sqrt{g^2 + g'^2}/2$$

fermion

massive quarks	H_m^h	Q	$\frac{3i}{2}B\gamma^1\gamma^2$
$T_M^1 = \frac{1}{\sqrt{2}}(T_L^1 + T_R^1)$	m_t	$2/3$	$1/2$
$B_M^1 = \frac{1}{\sqrt{2}}(B_L^1 - B_R^1)$	m_b	$-1/3$	$1/2$
$T_M^{c1} = \frac{1}{\sqrt{2}}(T_L^1 - T_R^1)$	$-m_t$	$2/3$	$1/2$
$B_M^{c1} = \frac{1}{\sqrt{2}}(B_L^1 + B_R^1)$	$-m_b$	$-1/3$	$1/2$

Table 3: Massive quark eigenstates of H_m^h

$$\sqrt{2}H_n = H_m$$

Quark-mass relation

The "punchline:" $|\langle Z|\sqrt{2}H_n|Z\rangle|^2 = m_Z^2$ and $\langle t|H_m + H_m^\dagger|t\rangle = m_t$

Higgs mechanism

$$\langle \mathbf{H}_{af}^\dagger(x)\mathbf{H}_{af}(x)\rangle = (|a|^2 + |f|^2)v^2/2 = (|\chi_t|^2 + |\chi_b|^2)v^2/2 = v^2/2$$

$$(|a|^2 + |f|^2)v^2/2 = |m_t|^2 + |m_b|^2 = v^2/2$$

$$m_t = \sqrt{v^2/2 - m_b^2} \simeq 173.90 \text{ GeV}$$

$v = 246$ GeV

$m_b = 4$ GeV

Top-quark mass from hierarchy argument

$$\mathbf{H}_{af}(x) = a\phi_1(x) + f\phi_2(x),$$

$$\chi_t = \frac{1}{\sqrt{2}}(a+f),\ \chi_b = \frac{1}{\sqrt{2}}(a-f)$$

$$\mathbf{H}_{af}(x) = \frac{1}{\sqrt{2}}(\chi_t \mathbf{H}_t(x) + \chi_b \mathbf{H}_b(x))$$

$$(|a|^2 + |f|^2)v^2/2 = |m_t|^2 + |m_b|^2 = v^2/2$$

$\Rightarrow$ $\mathrm{O}(a) \simeq \mathrm{O}(f)$, $(m_b \ll m_t,)$ we get $\frac{1}{\sqrt{2}}v \simeq 173.95$, for $v = 246$ GeV

Geometric assumption (weaker argument)

$$\mathbf{H}_m(x) = \frac{\sqrt{2}}{v}(m_t \mathbf{H}_t(x) + m_b \mathbf{H}_b(x))$$

$\Rightarrow$ $\chi_t = m_t / \frac{v}{\sqrt{2}},\ \chi_b = m_b / \frac{v}{\sqrt{2}}$ $\qquad |\chi_t|^2 + |\chi_b|^2 = 1$

Correspondence's physical interpretation

- Dynamical: action of scalar on fermion and vector share the same effect: common Hamiltonian H.

$$[H+H^\dagger,F] \quad \text{vs} \quad [H,V]^\dagger\,[H,V]$$

- Symmetry: e. g., $SU(2)_L xU(1)$ fundamental-adjoint representation connection.

- Compositeness: Standard-model gauge structure. No information on whether this a formal or physical feature.

from gauge invariance. Formally, a boson field $B_o(x)$ expansion may be obtained using $B_o(x) = \sum_{lk} F_l(x)F_k(x) + [B_o(x) - \sum_{lk} F_l(x)F_k(x)]$, where $F_l(x)$, $F_k(x)$ are fermion fields reproducing $B_o(x)$'s quantum numbers, and the last two terms give corrections.

Argument summary

- Electroweak conventional fields and their Lagrangian can be written in a spin-extended space.
- Scalar-vector term, invariant under conjugate scalar parametrization.
- *Same* scalar field within SV and SF terms connects V and F; after the Higgs mechanism, it constrains quark masses.
- Yukawa constants are reinterpreted as geometrical.
- Multiplet structure suggested for heavy standard-model particles.

Composite models

- 1961 Nambu Jona-Lasinio. Superconductivity model in which four-fermion interaction generates both fermion and boson masses.
- 1989 Nambu. Higgs from top quark condensate.
- Bardeen, Hill, Lindner, use fixed point in renormalization.
- Technicolor: Higgs composed of fermions alleviates fine-tuning problem.
- Spin extended model.

Can standard-model bosons be constructed in terms of fermions?

Higgs

$$\Phi \propto \frac{1}{2}\begin{pmatrix} \overline{b^a}(1+\gamma_5)t^a \\ -\overline{t^a}(1+\gamma_5)t^a \end{pmatrix} = i\tau_2(\overline{t}_R \Psi_L)^{\dagger T},$$

$$\tilde{\Phi} \equiv i\tau_2 \Phi^{\dagger T} \propto \overline{t}_R \Psi_L .$$

$Y = -1$ $I_3 = 1/2$ states: $H_t^0 = t_L^\dagger \bar{t}_L^\dagger + \bar{t}_R t_R$

W

$W_\downarrow^+ = t_L^\dagger \bar{b}_R^\dagger$

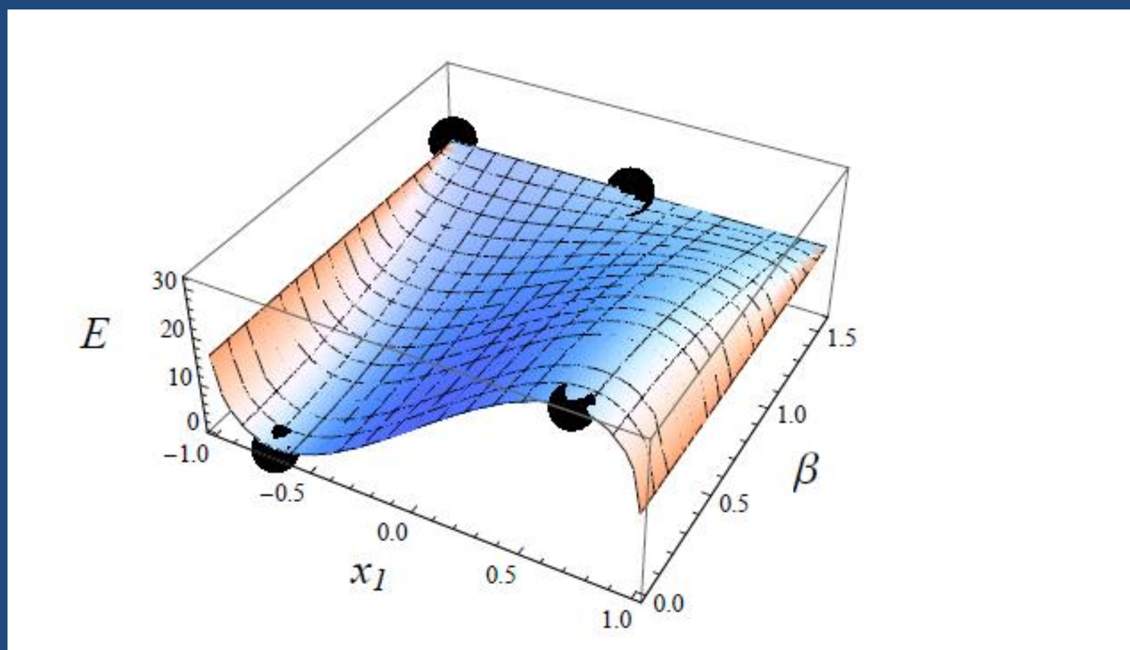
Z

left-handed helicity -1 hypercharge carrier is described by $A_L = \frac{2}{3}(t_L^\dagger \bar{t}_R^\dagger + b_L^\dagger \bar{b}_R^\dagger)$

Hamiltonian model: known mesons

- Fermion Hamiltonian of the form:

H=H(□$_t$, □$_{b,}$$m_t$, m_b); variational calculation



toponium and bottomium: masses $2m_t$ $2m_b$

Spin-extended model equivalent Lagrangian terms

Fermion-vector

$$\frac{1}{N_f}\mathrm{tr}\,\Psi^\dagger\{[i\partial_\mu I_{\mathrm{den}} + gA^a_\mu(x)I_a]\gamma_0\gamma^\mu - M\gamma_0\}\Psi P_f\,,$$

Projection operator

$$P_f = \frac{1}{\sqrt{2}}(\tilde{\gamma}^5 - \gamma^0\gamma^1)$$

INTERACTIVE THEORY FOR FIELDS IN AN EXTENDED SPIN SPACE

Keep

Polarization basis for fields

vector $A^{4a}\gamma_4 G_a$

scalar $\phi^a G'_a$

transformation rule

$\bar{\Psi} \rightarrow u\bar{\Psi}u^{+}$

drop

free-field generalized Dirac equation

There is an equivalence between a field theory and its formulation in an extended spin space