The AdS/CFT correspondence and the Quark-Gluon Plasma

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Introduction

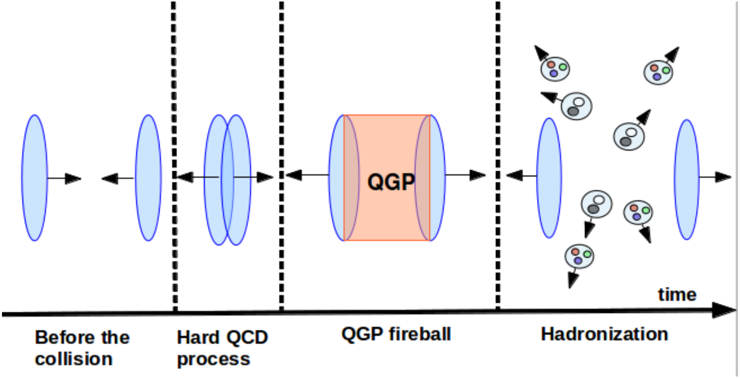
Goal

To use the AdS/CFT correspondence to study the Quark-Gluon Plasma (QGP) produced in heavy-ion collisions at RHIC and LHC.

Motivation

- In principle, the QGP is described by QCD.
- At $T \sim T_c$, the QGP is strongly coupled, what makes problematic the use of perturbative QCD;
- Lattice QCD is poorly suited for the computation of quantities in the real-time formalism, like transport coefficients or spectral functions.
- The AdS/CFT correspondence can be used the study strongly coupled systems similar to the QGP.

Heavy-Ion Collisions



Probes of the QGP:

- thermal probes: photons and dileptons
- heavy quarks
- quarkonium mesons

QCD phase diagram

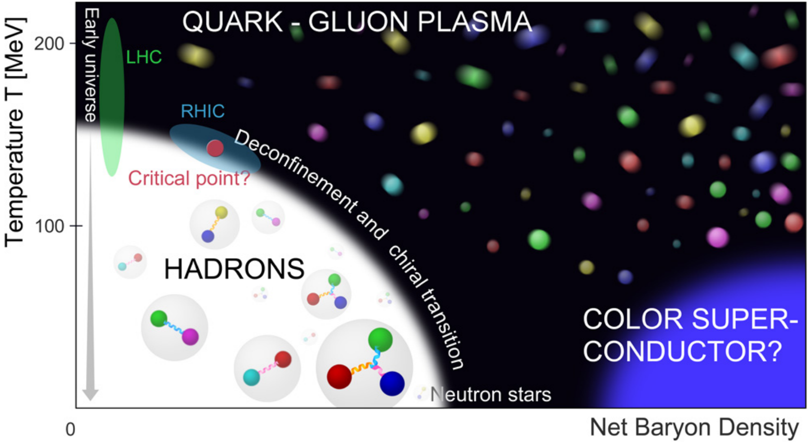
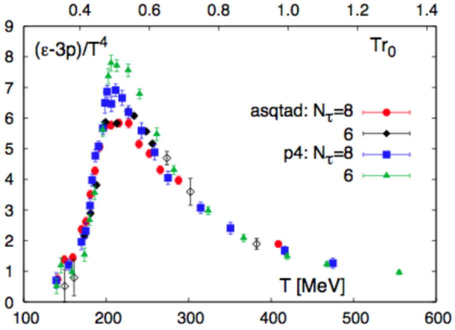


Figure from: http://www.jicfus.jp/en/wp-content/uploads/2012/12/QGPT.jpg

Conformal Anomaly - Lattice QCD

For an ideal gas: $\varepsilon = 3P$ and $\varepsilon \sim T^4$, $P \sim T^4$.
The *conformal anomaly* $\Delta = (\varepsilon - 3P)/T^4$ measures the strength of interactions in non-conformal theories.



Bazavov, Bhattacharya, Cheng, Christ, DeTar, Ejiri, Gotlieb, Gubta *et al* 2009

The results of lattice QCD for Δ show that the QGP behaves as a strongly coupled fluid for $T \approx T_c$.

The AdS/CFT correspondence Maldacena 1997

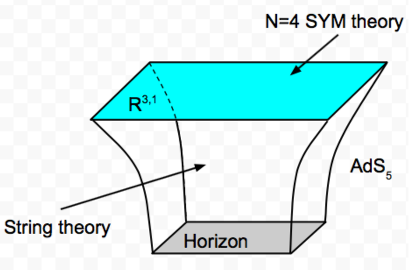
Holographic Principle:
gravity theory in M_{d+1} = gauge theory in ∂M_{d+1}.

AdS_{d+1}
gravity theory in Anti-de Sitter space in $d+1$ dimensions.

$\leftrightarrow$

CFT_d
gauge theory in Minkowski space in d dimensions.

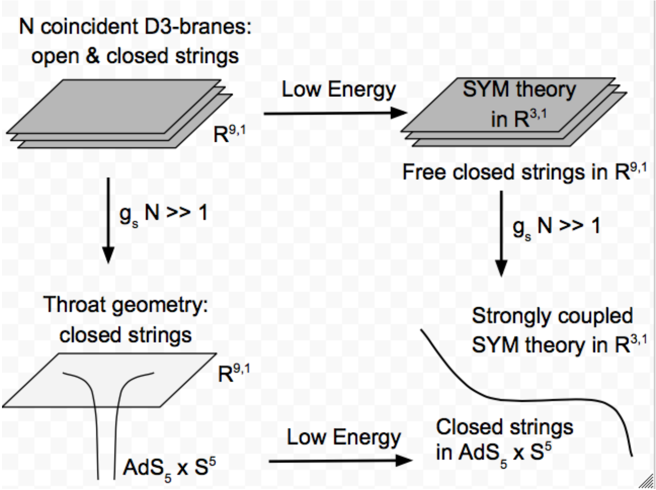
Most known example:
type IIB superstrings in $AdS_5 \times S^5 \leftrightarrow \mathcal{N} = 4$ $SU(N_c)$ SYM in $\mathbb{R}^{3,1}$



"Derivation" of the AdS/CFT correspondence

Gravitational effect of N D-branes: $g_s N = g_{YM}^2 N = \lambda$

$g_s N << 1$



AdS/CFT dictionary Gubser, Klebanov, Polyakov 1998, Witten 1998

Fundamental Equation:

$$\boxed{\left\langle \exp \int d^4x\, \phi_{\partial \mathrm{AdS}}(x)\mathcal{O}(x) \right\rangle_{\mathrm{CFTs}} = Z_{\mathrm{string}}[\phi_{\partial \mathrm{AdS}}]}$$

Field/Operator Correspondence

- For each gauge field operator $\mathcal{O}(x)$ there is a corresponding field $\phi(x, r)$ in the gravity theory;
- The value of the field at the boundary of AdS, $\phi_{\partial \mathrm{AdS}}$, acts as the source of the operator $\mathcal{O}(x)$.

Parameters of the correspondence:

$\mathcal{N} = 4$ SYM: g_{YM}, N.

type IIB strings: g_s, $R/\sqrt{\alpha'}$

$$\boxed{g_{\mathrm{YM}}^2 = 4\pi g_s}$$

$$\boxed{R^4/\alpha'^2 = N\, g_{\mathrm{YM}}^2 \equiv \lambda}$$

AdS/CFT

Quantum Corrections: $\boxed{\ell_p/R = \pi^4/(2N^2)}$

Higher Derivative Corrections: $\boxed{\alpha'/R^2 = 1/\sqrt{\lambda}}$

Limits $N \to \infty$, $\lambda \to \infty$

type IIB string theory$\to$ Classical type IIB Supergravity

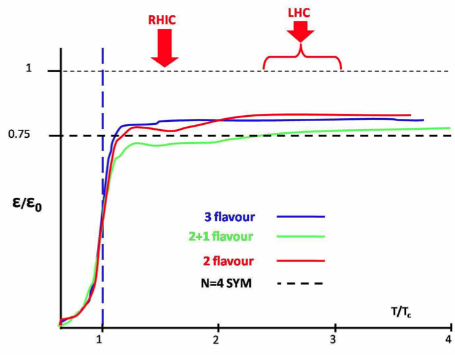
$$Z_{\text{string}}[\phi_{\partial AdS}] \to \exp\left(-S^{\text{on-shell}}_{\text{SUGRA}}\right)$$

$$\boxed{\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\ldots\mathcal{O}(x_n)\rangle = -\frac{\delta^n S^{\text{on-shell}}_{\text{SUGRA}}[\phi_{\partial \text{AdS}} = J]}{\delta J(x_1)\delta J(x_1)\ldots\delta J(x_n)}}$$

Calculations at finite temperature $\to$ Black Brane in AdS

Strongly Coupled Plasma

$SU(N_c)$ $\mathcal{N}=4$ SYM theory
at finite temperature and with $\lambda >> 1$ $\rightarrow$ model for the QGP



Myers, Vázquez 2008

QCD and $\mathcal{N}=4$ SYM are not so different at $T_c \leq T \leq 5T_c$.

$\mathcal{N} = 4$ SYM plasma

$\mathcal{N} = 4$ SYM plasma - unrealistic features

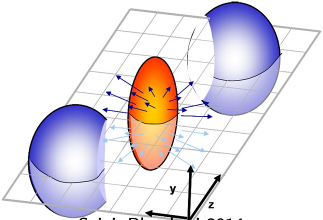
- static
- isotropic
- $\lambda = \infty$ (fixed)
- $N = \infty$
- only have adjoint fields, etc

In this work we investigate the effects of

- **anisotropy**
- **higher curvature corrections**

Sources of Anisotropy in Heavy-Ion Collisions

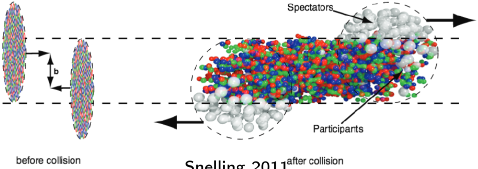
There are at least two sources of anisotropy



Saini, Bhardwaj 2014

Spatial Anisotropy $\rightarrow$ Elliptic Flow

Occurs only in non-central collisions



Snelling 2011

Anisotropy related to the rapid expansion of the plasma along the beam axis

- $P_z < P_{xy}$
- Occurs even in central collisions!

Holographic Model for an Anisotropic Plasma Mateos, Trancanelli 2011

Gauge Theory - Deformation of the $\mathcal{N}=4$ theory by a θ-term

$S = S_{\mathcal{N}=4} + \int \theta(z)\,\mathrm{Tr}\,F \wedge F, \quad \theta(z) \propto z.$

Gravity Theory - solution of type IIB SUGRA field equations with D7-branes dissolved in the geometry

$$S = \frac{1}{2\kappa^2}\int_{\mathcal{M}} \sqrt{-g}\left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial\chi)^2\right) + \text{boundary term}$$

$$ds^2 = \frac{e^{-\phi/2}}{u^2}\left(-F(u)B(u)dt^2 + dx^2 + dy^2 + H(u)dz^2 + \frac{du^2}{F(u)}\right)$$

$$\chi = a\,z, \qquad \phi = \phi(u).$$

u is AdS radial coordinate

Boundary at $u = 0$ and Horizon at $u = u_H$

a $=$ parameter of **anisotropy** $\boxed{P_z < P_{xy}}$

Higher Curvature Corrections

The limit $\lambda = \infty$ in the gauge theory suppress higher curvature corrections in the gravity theory.

- In the gauge theory the finite-λ corrections appears as powers of $1/\sqrt{\lambda}$;
- In the gravity side this corrections appears as higher curvature terms scaled by powers of α'/R^2;
- In type IIB superstring the leading corrections arise as terms with the schematic form $\alpha'^3 R^4$;

Lovelock Theory and AdS/CFT Edelstein 2013

- they are generalizations of Einstein-Hilbert action;
- these theories contain higher curvatures corrections, but the equations of motion are stilll of second order;
- they admit a large class of asymptotically AdS black holes.

Simplest example:
Gauss-Bonnet Theory of Gravity (in 5 dimensions)

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \frac{L^2}{2}\lambda_{\rm GB}\mathcal{L}_{\rm GB}\right)$$

where $\mathcal{L}_{\rm GB} = R^2 - 4R_{mn}R^{mn} + R_{mnrs}R^{mnrs}$.

In the gauge side: CFT with two independent central charges
Unitarity, Causality and Positive of Energy Fluxes:
$\rightarrow -7/36 \leq \lambda_{\rm GB} \leq 9/100$ Hofman 2009, Buchel, Myers 2009

Anisotropic Plasma with Higher Curvature Corrections

Jahnke, Misobuchi, Trancanelli 2014

Gravity Theory - anisotropic solution with a GB term

$$S_{a,\mathrm{GB}} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial\chi)^2 + \frac{\lambda_{\mathrm{GB}}}{2}\mathcal{L}_{\mathrm{GB}}\right)$$

$$\mathcal{L}_{\mathrm{GB}} = R^2 - 4R_{mn}R^{mn} + R_{mnrs}R^{mnrs},$$

Analitic solution up to $\mathcal{O}(a^2)$ and for any λ_{GB}

$$ds^2 = \frac{e^{-\phi/2}}{u^2}\left(-F(u)B(u)dt^2 + dx^2 + dy^2 + H(u)dz^2 + \frac{du^2}{F(u)}\right)$$

$$\chi = a\,z, \qquad \phi = \phi(u).$$

- Holographic Renormalization $\to$ $T_{\mu\nu}$ up to $\mathcal{O}(a^2, \lambda_{\mathrm{GB}})$
- DC conductivities: $\sigma_{||}$ and $\sigma_{\perp}$
- Ratios $\eta_{||}/s$ and $\eta_{\perp}/s$ $\to$ violation of the KKS bound $\eta/s \geq 1/(4\pi)$

Probing strongly coupled anisotropic plasmas from higher curvature gravity Jahnke, Misobuchi 2015

Model: anisotropic plasma with $\lambda_{\text{GB}} \neq 0$

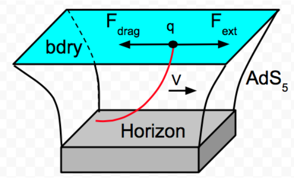
$$S = S_{a,\text{GB}} + S_{U(1)}$$

Observables

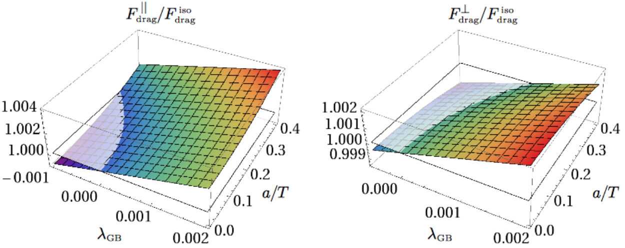
- drag force
- jet quenching parameter
- quarkonium static potential
- photon production rate

Drag Force on a heavy quark Gubser 2006, Holzhey, Karch, Kovtun, Kozcaz, Yaffe, 2006

Trailing string



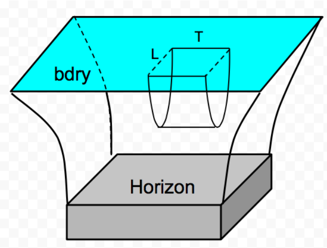
$F_{\text{drag}} = \text{momentum flux: bdry} \to \text{horizon}$



Left: motion along the anisotropic direction. Right: motion along the transverse plane. We have fixed $v = 0.3$.

Quarkonium Static Potential $V_{Q\bar{Q}}$

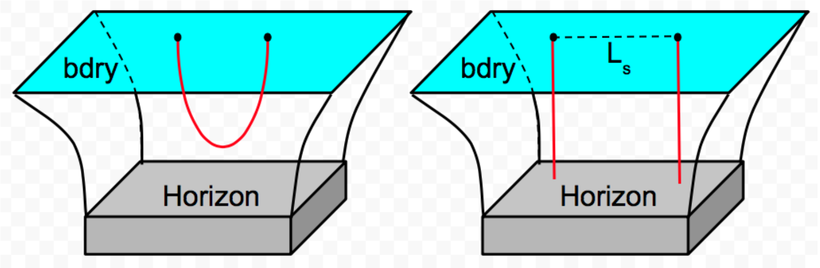
$S_{\text{NG}}^{\text{on-shell}} \sim$ word-sheet area



Retangular Wilson loop with sizes T and L:

$\lim_{T\to\infty} \langle W(C) \rangle \approx e^{iT(V_{Q\bar{Q}}+2M_Q)} = e^{iS_{NG}^{\text{on-shell}}}$

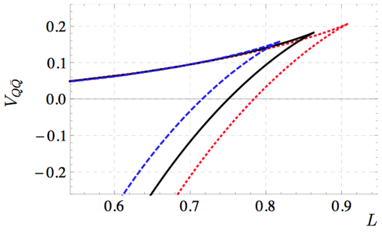
Screening Length L_s = property of the plasma

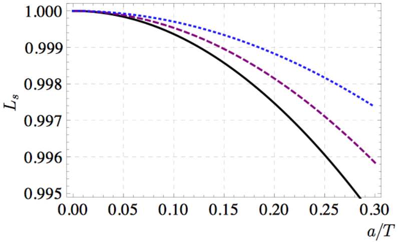


Quarkonium Static Potential $V_{Q\bar{Q}}$ - Results

$$V_{Q\bar{Q}} = S_{NG}^{\text{on-shell}}/T - 2M_Q$$

$L_s \to L$ such that $V_{Q\bar{Q}}(L) = 0$

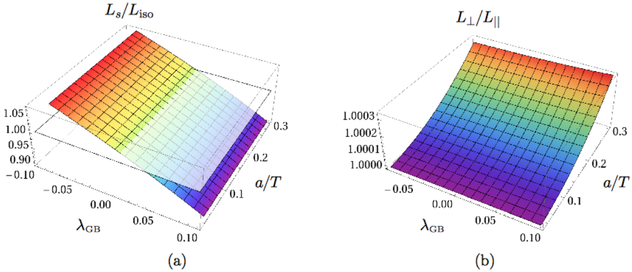




Left: $\lambda_{GB} = -0.1$ (red), $\lambda_{GB} = 0$ (black), $\lambda_{GB} = 0.1$ (blue),
We have fixed: $a/T \approx 0.3$, $\theta = \pi/4$.

Right: $\theta = 0$ (black), $\theta = \pi/4$ (purple), $\theta = \pi/2$ (blue), $\lambda_{GB} = 0$

Quarkonium Static Potential $V_{Q\bar{Q}}$ - Results



(a) Screening length $L_s(a, \lambda_{\mathrm{GB}})$ normalized with respect to the isotropic result $L_{\mathrm{iso}} = L_s(0,0)$.
(b) Ratio $L_\perp/L_{||}$, where $L_\perp$ is calculated for $\theta = \pi/2$ and $L_{||}$ is calculated for $\theta = 0$.

Discussion - Effects of $(a, \lambda_{\text{GB}}) \neq (0, 0)$

Effects of $\lambda_{\text{GB}} \neq 0$

	η/s	Drag force	Jet quenching	Screening length	Photon production
$\lambda_{\text{GB}} > 0$	decrease	increase	increase	decrease	increase
$\lambda_{\text{GB}} < 0$	increase	decrease	decrease	increase	decrease
$\alpha'^3 R^4$	increase	increase	decrease	decrease	increase

Effects of Anisotropy

- Shear Viscosity: $\eta_\perp > \eta_{||} \Longrightarrow \ell^\perp_{\text{mfp}} > \ell^{||}_{\text{mfp}}$
- Drag Force: $F^\perp_{\text{drag}} < F^{||}_{\text{drag}}$
- Jet Quenching Parameter: $\hat{q}_\perp < \hat{q}_{||}$
- Screening Length: $L_\perp > L_{||}$
- Photon Production Rate: $\Gamma^\perp_{\text{photon}} < \Gamma^{||}_{\text{photon}}$

At weak coupling: $\boxed{\eta/s \sim \ell_{\text{mfp}}}$

Conclusion

- the AdS/CFT correspondence can be used to understand strongly coupled plasmas similar to the QGP;
- we were able to understand the effects of the anisotropy and higher curvature corrections in some physical observables of the plasma;
- this information might be useful in the construction of phenomenological models.

THE END