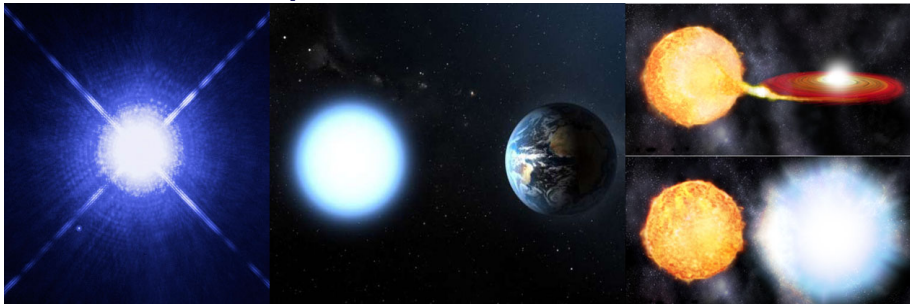


Constraints on braneworld from compact stars[†]

Daryel Manreza Paret, ICN-UNAM
Aurora Pérez Martínez, ICIMAF, Cuba
Ricardo. González Felipe, ISEL, Portugal

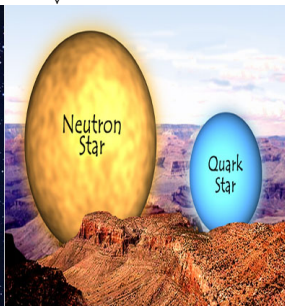
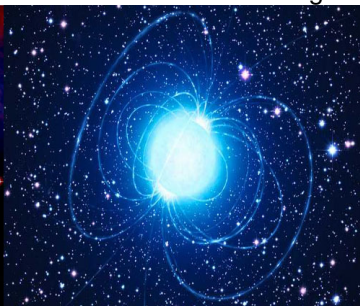
[†]R. Gonzales Felipe, D. Manreza Paret and A. Perez Martinez, Eur. Phys. J. C (2016) 76:337
(arXiv:1601.01973)

Introduction. Compact Stars.



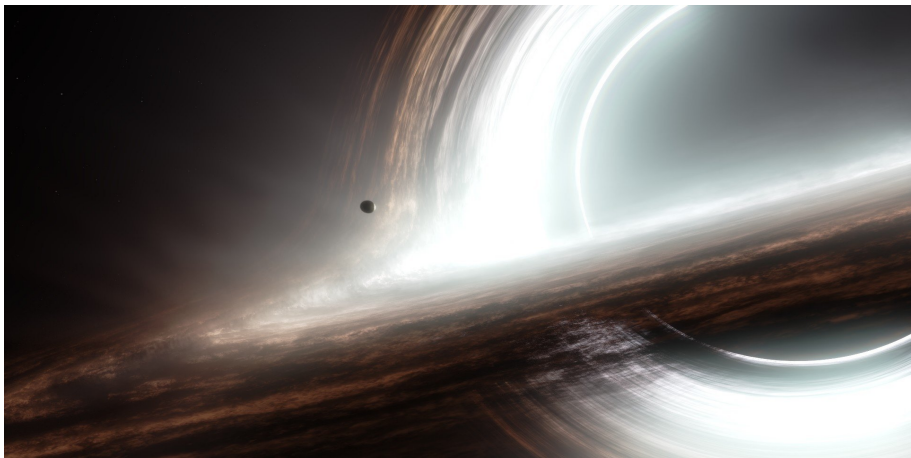
White Dwarfs↑

Neutron Stars/Strange Stars↓

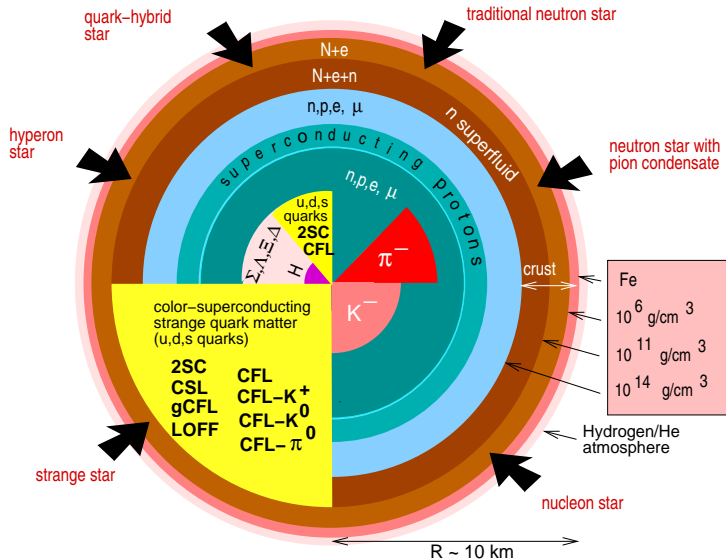


Introduction. Compact Stars.

Third type of Compact Object:
Black Holes



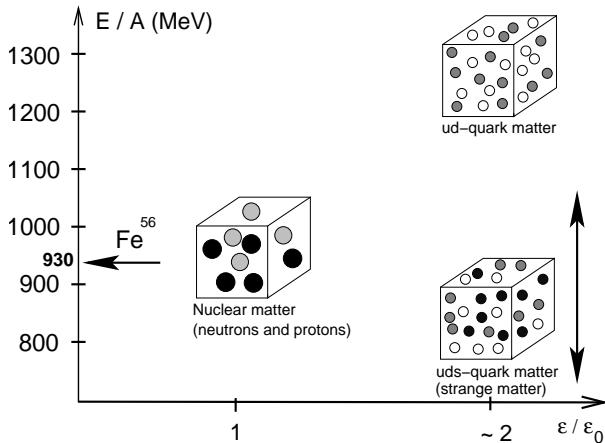
Introduction. Compact Stars.



† F. Weber. Progress in Particle and Nuclear Physics, 54:193-288 (2005).

Introduction. Compact Stars.

Quark Stars. Bodmer-Witten-Terazawa conjecture[‡]

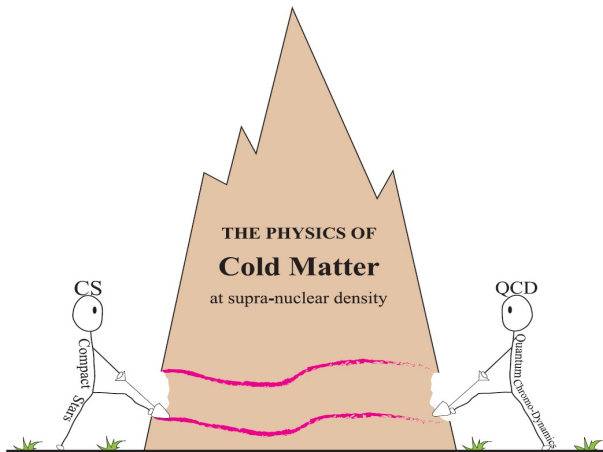


[†] Picture from: F. Weber. Progress in Particle and Nuclear Physics, 54:193-288 (2005).

[‡] A. R. Bodmer. Phys. Rev. D, (1971). E. Witten. Phys. Rev. D (1984). H. Terazawa. Journal of the Physical Society of Japan, (1989)

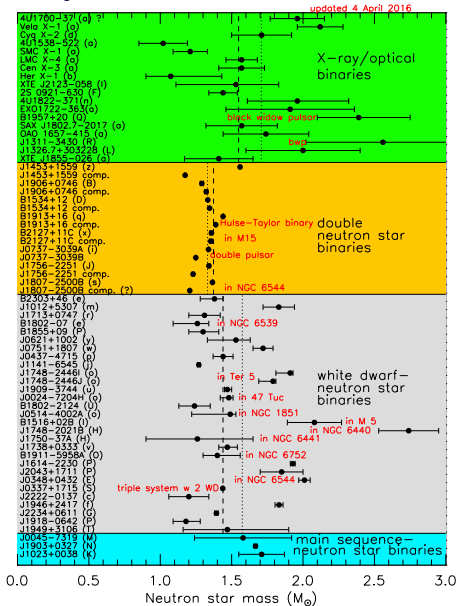
Introduction. Compact Stars.

Neutron Stars: Natural Laboratories



† Renxin Xu. J. Phys. G: Nucl. Part. Phys. 36 (2009) 064010 (9pp).
(ICN-UNAM)

Introduction. Compact Stars.



† <http://stellarcollapse.org/nsmasses>. Accedido 26-01-2017.

Tolman-Oppenheimer-Volkoff equations

The static, structure equations for a spherical symmetric relativistic star are found by solving Einstein's equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2 T_{\mu\nu}. \quad (1)$$

For a spherically symmetric star, the metric is given by

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (2)$$

and the energy momentum tensor

$$T_{\mu\nu} = \rho u_\mu u_\nu + p (g_{\mu\nu} + u_\mu u_\nu) \quad (3)$$

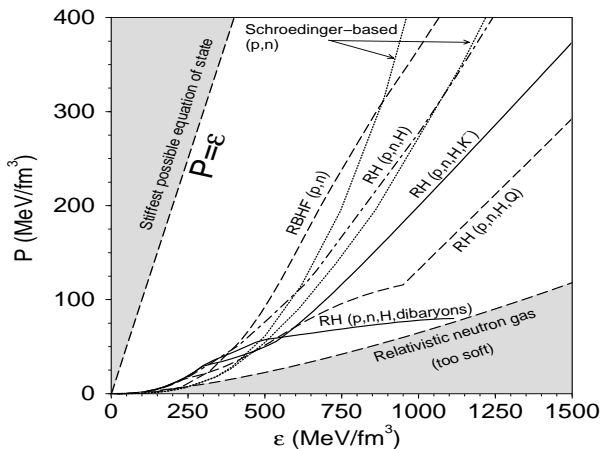
Tolman–Openheimer–Volkof (TOV):

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho, \\ \frac{dp}{dr} &= -G \frac{(\rho + p)(m + 4\pi p r^3)}{r^2 - 2G r m}, \end{aligned}$$

with initial conditions $m(0) = 0$, $p(0) = p_c$ and at stellar surface $p(R) = 0$.

Tolman-Oppenheimer-Volkoff equations

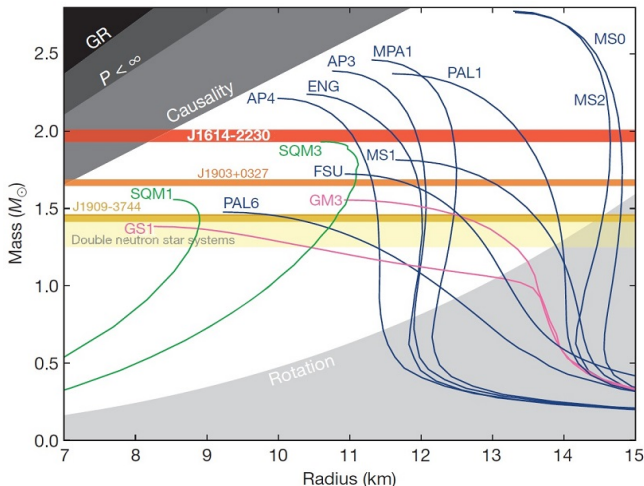
Equations of state



†F. Weber. Progress in Particle and Nuclear Physics, 54:193-288 (2005).

Tolman-Oppenheimer-Volkoff equations

Mass-Radius diagram



† P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels. *Nature*, 467:10811083 (2010).

Tolman-Oppenheimer-Volkoff equations

Theoretical constraints

- GR: $R > \frac{2GM}{c^2}$.
- $P < \infty$: $R > \frac{9}{4} \frac{GM}{c^2}$
- Causality: A sound signal cannot propagate faster than the speed of light
 $v < \sqrt{dp/d\rho} \leq c \Rightarrow R > 2.9 \frac{GM}{c^2}$
- Rotation: $R > R_{\max}$ excluded by the 716 Hz pulsar J1748-2446ad from the empirical result[†]

$$R < 10.4 \left(\frac{1000 \text{ Hz}}{\nu} \right)^{2/3} \left(\frac{M}{M_{\odot}} \right)^{1/3} \text{ km.}$$

[†]J. M. Lattimer, M. Prakash, Science 304, 536 (2004)

Tolman-Oppenheimer-Volkoff equations

Observational constraints

- For neutron star radii

$$7.6 \text{ km}^{\dagger} \leq R \leq 13.9 \text{ km}^{\ddagger}, \quad (4)$$

- For neutron star masses

$$1.08 M_{\odot}^{\S} \leq M \leq 2.05 M_{\odot}^{\dagger\dagger}, \quad (5)$$

[†]S. Guillot, M. Servillat, N.A. Webb, R.E. Rutledge, *Astrophys. J.* 772, 7 (2013)

[‡]K. Hebeler, J.M. Lattimer, C.J. Pethick, A. Schwenk, *Phys. Rev. Lett.* 105, 161102 (2010).

[§]F. Ozel, D. Psaltis, R. Narayan, A.S. Villarreal, *Astrophys. J.* 757, 55 (2012).

^{††}J. Antoniadis et al., *Science* 340, 6131 (2013).

Brane World Models

From a classical point of view brane world models can be realised via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk.

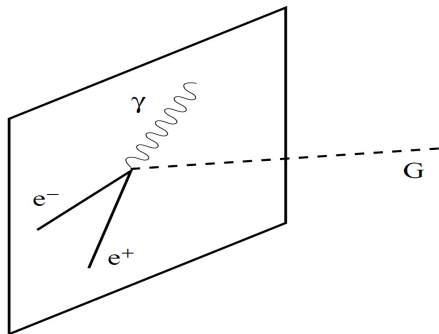


Image from: Cavaglia, M., Int. J. Mod.Phys. A, 18, 1843-1882, (2003). [hep-ph/0210296].

Brane World Models

Randall–Sundrum Brane-Worlds

- The bulk is a portion of a 5-D anti-de Sitter (AdS_5) spacetime (extra dimension is curved rather than flat).
- What prevents gravity from leaking into the extra dimension at low energies is a negative bulk cosmological constant $\Lambda_5 = -6/l^2$ where l is the curvature radius.
- The brane gravitates with self-gravity in the form of a brane tension λ .

† Randall&Sundrum PRL 1999; Maartens, PRD 2000; Shiromizu et al PRD 2000.

TOV equations on the brane

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2 T_{\mu\nu} + \frac{6\kappa^2}{\lambda} \mathcal{S}_{\mu\nu} - \mathcal{E}_{\mu\nu}, \quad (6)$$

where

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) \quad (7)$$

and

$$\mathcal{S}_{\mu\nu} = \frac{1}{12} \rho^2 u_\mu u_\nu + \frac{1}{12} \rho(\rho + 2p)(g_{\mu\nu} + u_\mu u_\nu), \quad (8)$$

where u^μ is the four-velocity of the fluid. The tensor $\mathcal{E}_{\mu\nu}$ reduces to the form

$$\mathcal{E}_{\mu\nu} = -\frac{6}{\kappa^2 \lambda} \left[\mathcal{U} u_\mu u_\nu + \mathcal{P} r_\mu r_\nu + \frac{1}{3} (\mathcal{U} - \mathcal{P}) (g_{\mu\nu} + u_\mu u_\nu) \right], \quad (9)$$

- \mathcal{U} y \mathcal{P} are dark energy and pressure respectively.
- $\mathcal{S}_{\mu\nu}$ local correction term.
- $\mathcal{E}_{\mu\nu}$ non-local correction term.
- When $\lambda \rightarrow \infty$, we recover GR.

TOV equations on the brane

$$\frac{dm}{dr} = 4\pi r^2 \rho_{\text{eff}}, \quad (10)$$

$$\frac{dp}{dr} = -(\rho + p) \frac{d\Phi}{dr}, \quad (11)$$

$$\frac{d\Phi}{dr} = \frac{2Gm + \kappa^2 r^3 [p_{\text{eff}} + (4\mathcal{P})/(\kappa^4 \lambda)]}{2r(r - 2Gm)}, \quad (12)$$

$$\frac{d\mathcal{U}}{dr} = -\frac{1}{2}\kappa^4(\rho + p) \frac{d\rho}{dr} - 2 \frac{d\mathcal{P}}{dr} - \frac{6}{r}\mathcal{P} - (2\mathcal{P} + 4\mathcal{U}) \frac{d\Phi}{dr}, \quad (13)$$

where

$$\rho_{\text{eff}} = \rho_{\text{loc}} + \frac{6}{\kappa^4 \lambda} \mathcal{U}, \quad p_{\text{eff}} = p_{\text{loc}} + \frac{2}{\kappa^4 \lambda} \mathcal{U}. \quad (14)$$

and,

$$\rho_{\text{loc}} = \rho + \frac{\rho^2}{2\lambda}, \quad p_{\text{loc}} = p + \frac{p\rho}{\lambda} + \frac{\rho^2}{2\lambda}, \quad (15)$$

We need $p(\rho)$ y $\mathcal{P}(\mathcal{U})$ and initial conditions: $m(0) = 0$, y $p(0) = p_c$. At stellar surface $p(R) = 0 \Rightarrow m(R) = M$. For the dark component \mathcal{U} , we shall assume $\mathcal{U}(0) = 0$.

TOV equations on the brane

Equations of state

- In our analysis, the non-local dark components are modelled via the simple linear proportionality relation $\mathcal{P} = w\mathcal{U}$ between the dark energy \mathcal{U} and dark pressure \mathcal{P} .
- For **dense nuclear matter**, we shall consider the analytical representation for the unified Brussels-Montreal EoS models[†], which are based on the nuclear energy-density functional theory with generalized Skyrme effective forces.
- For **quark matter**, we shall employ the simple phenomenological parametrisation[‡] which includes QCD and strange-quarkmass corrections.
- Hybrid EoS to study **hybrid stars**, i.e., stars with a hadronic outer region surrounding a quark (or mixed hadron-quark) inner core.

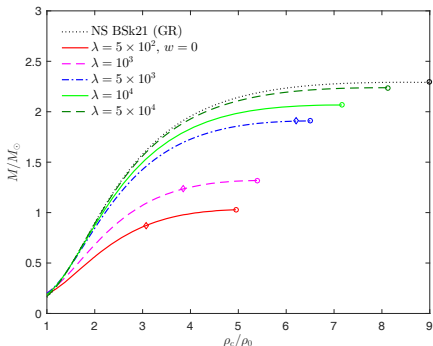
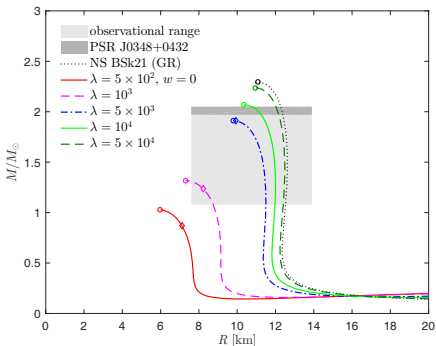
[†]A.Y. Potekhin, A.F. Fantina, N. Chamel, J.M. Pearson, S. Goriely, *Astron. Astrophys.* 560, A48 (2013).

[‡]M. Alford, M. Braby, M.W. Paris, S. Reddy, *Astrophys. J.* 629, 969 (2005).

TOV equations on the brane

Results: Neutron Stars

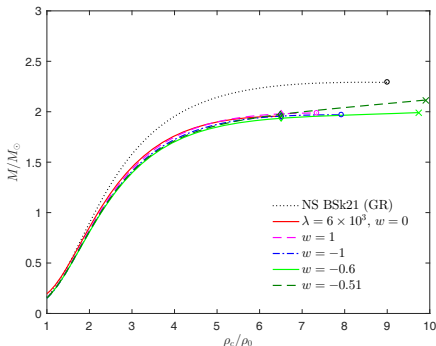
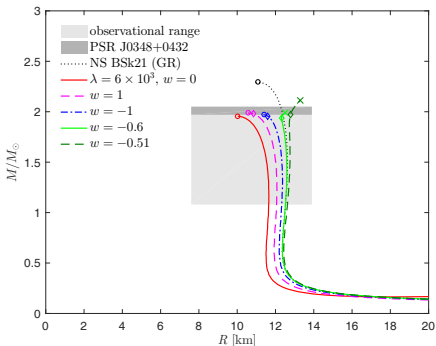
- Requiring agreement with observational constraints leads to a lower bound on the brane tension, $\lambda \gtrsim 8 \times 10^2 \text{ MeV}/\text{fm}^3$.
- The star radii lie in the range 8–13 km.



TOV equations on the brane

Results: Neutron Stars

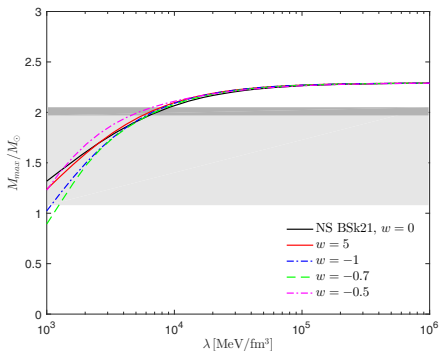
- Mass-radius curves bend clockwise for $w = -0.6$ and $w = -0.51$.
- We have indicated with crosses (\times) the mass-radius configuration at which the GR causality condition $v_s \leq 1$ is violated in such cases.



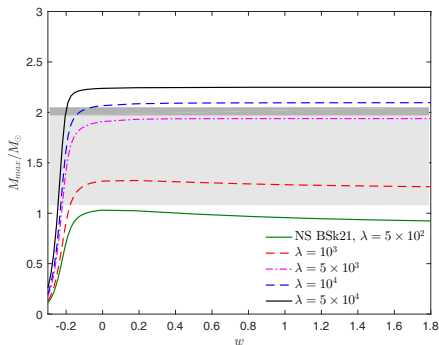
TOV equations on the brane

Results: Neutron Stars

- The maximum star mass predicted for this type of EoS is compatible with observations provided that $\lambda \gtrsim 6 \times 10^2 \text{ MeV/fm}^3$.
- For $w \gtrsim -0.1$, the value of M_{max} remains practically constant with the variation of w , depending only on the value of λ .
- For $-0.3 < w < -0.1$, the maximum mass is quite sensitive to w .



(ICN-UNAM)

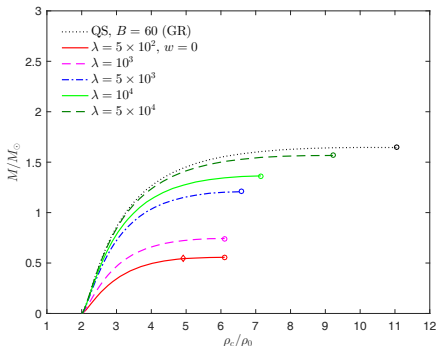
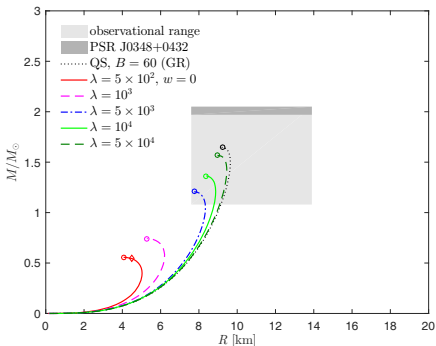


February 1, 2017 20 / 29

TOV equations on the brane

Results: Quark Stars

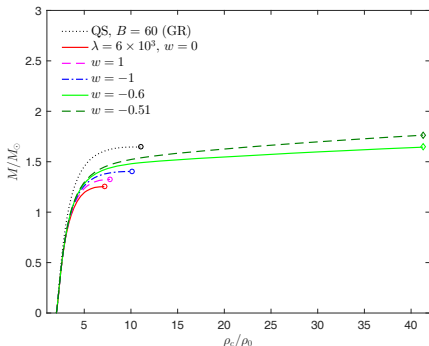
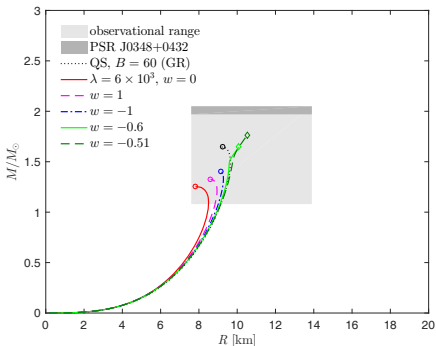
- Agreement with observational constraints imposes the lower bound $\lambda \gtrsim 4 \times 10^3 \text{ MeV/fm}^3$, for $w = 0$
- The star radii lie in the range 8–10 km.



TOV equations on the brane

Results: Quark Stars

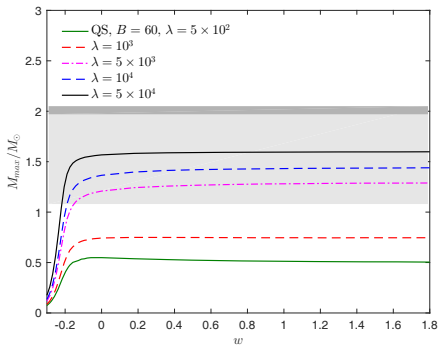
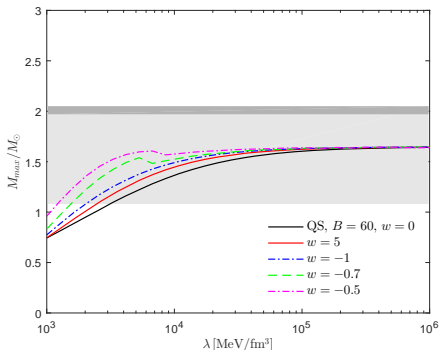
- For certain negative values of w the mass-radius curves bend clockwise, reaching the maximum mass at relatively high central densities, $\rho_c \sim 40\rho_0$, bounded by the requirement of subluminality of the EoS.



TOV equations on the brane

Results: Quark Stars

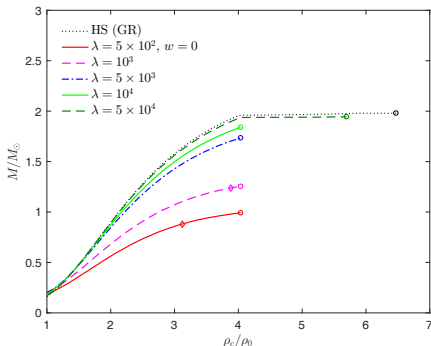
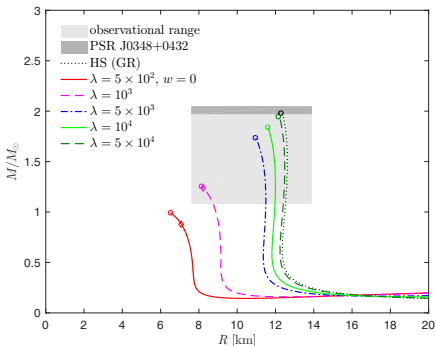
- The maximum star mass predicted for this type of EoS is compatible with observations provided that $\lambda \gtrsim 10^3 \text{ MeV/fm}^3$.
- For $w \gtrsim -0.1$, the value of M_{max} remains practically constant with the variation of w , depending only on the value of λ .
- For $-0.3 < w < -0.1$, the maximum mass is quite sensitive to w .



TOV equations on the brane

Results: Hybrid Stars

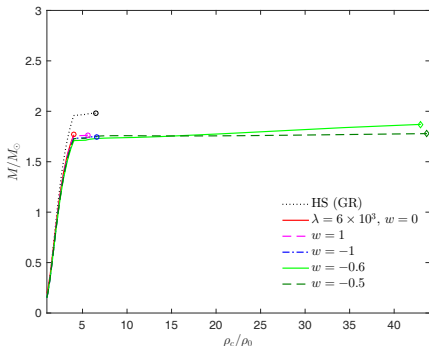
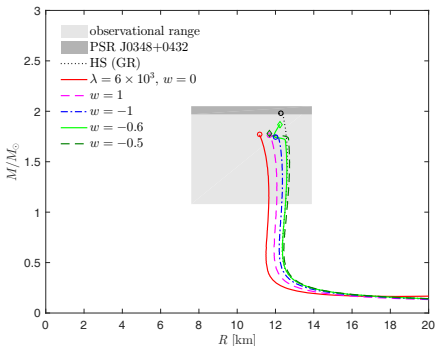
- Requiring agreement with observational constraints leads to a lower bound on the brane tension, $\lambda \gtrsim 8 \times 10^2 \text{ MeV}/\text{fm}^3$.
- The maximum mass $M \sim 1.98M_{\odot}$ is obtained for GR, and this value is consistent with the observational range of the pulsar PSR J0348+0432.



TOV equations on the brane

Results: Hybrid Stars

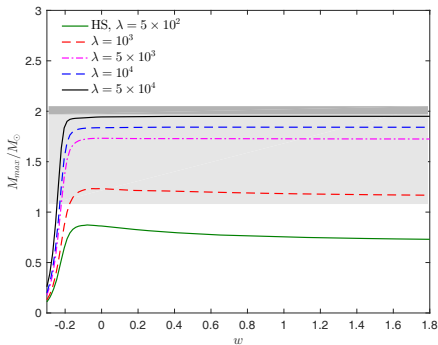
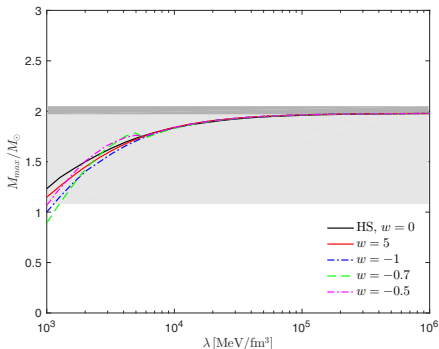
- As in the case of quark stars, we notice that the clockwise bending of the mass-radius curves persists for certain negative values of w , reaching the maximum mass at relatively high central densities, $\rho_c \sim 45\rho_0$



TOV equations on the brane

Results: Hybrid Stars

- The maximum star mass predicted for this type of EoS is compatible with observations provided that $\lambda \gtrsim 10^3 \text{ MeV/fm}^3$.
- For $w \gtrsim -0.1$, the value of M_{max} remains practically constant with the variation of w , depending only on the value of λ .
- For $-0.3 < w < -0.1$, the maximum mass is quite sensitive to w .



Conclusions

- 1 Compact Stars are natural laboratories to test new theories.
- 2 In all the three EOS cases, the maximum mass and the corresponding star radius decrease as λ decreases. Furthermore, the central energy density ρ_c required to achieve the maximum mass configuration is always less than that of GR
 - $\rho_c \lesssim 7\rho_0$ for pure neutron stars and quark stars
 - slightly lower for hybrid stars, $\rho_c \lesssim 4.5\rho_0$The star radii lie in the ranges
 - 8–12 km for pure neutron stars,
 - 8–11 km for quark stars
 - 9–14 km for hybrid stars.
- 3 The maximum star mass as a function of λ and w was also studied for the three families of stars. Requiring agreement with observational constraints leads to a lower bound on the brane tension, $\lambda \gtrsim 10^3 \text{ MeV/fm}^3$ for all three types of stars.

Muchas Gracias

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