γγ* Transition form factors of pseudoscalar mesons

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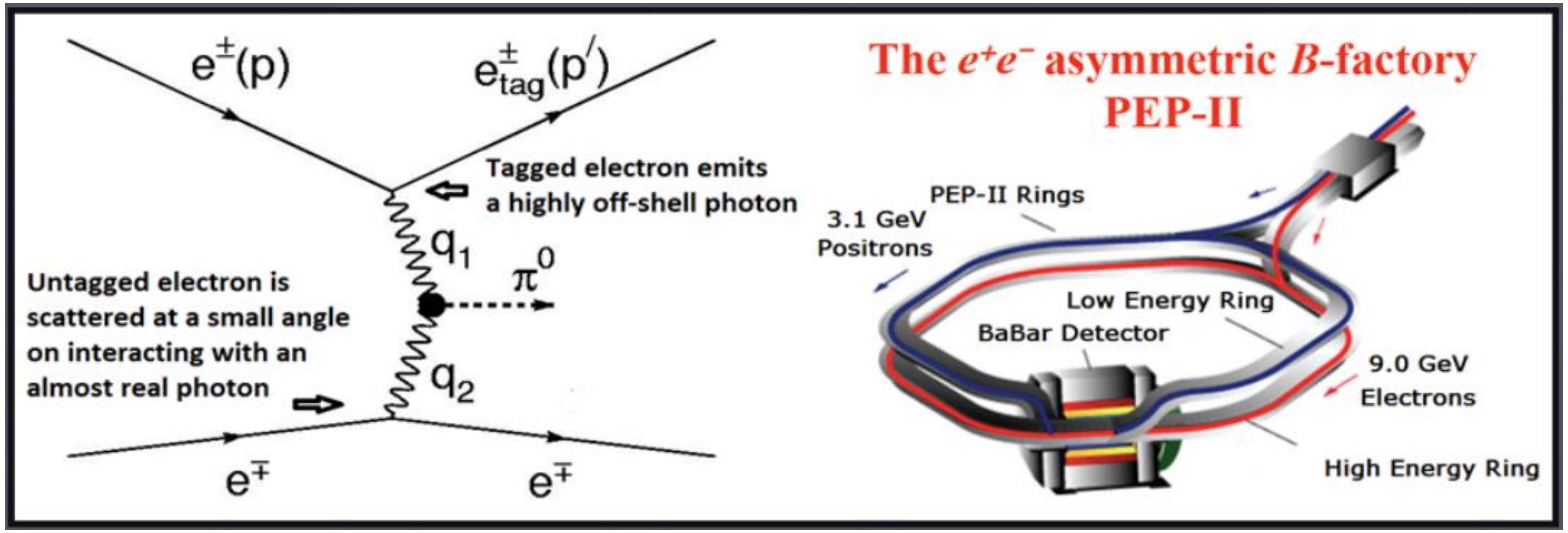
Outline

- Transition form factor: Generalities
- Pion transition form factor.
- Eta-c, eta-b transition form factors.
- Conclusions and scope.

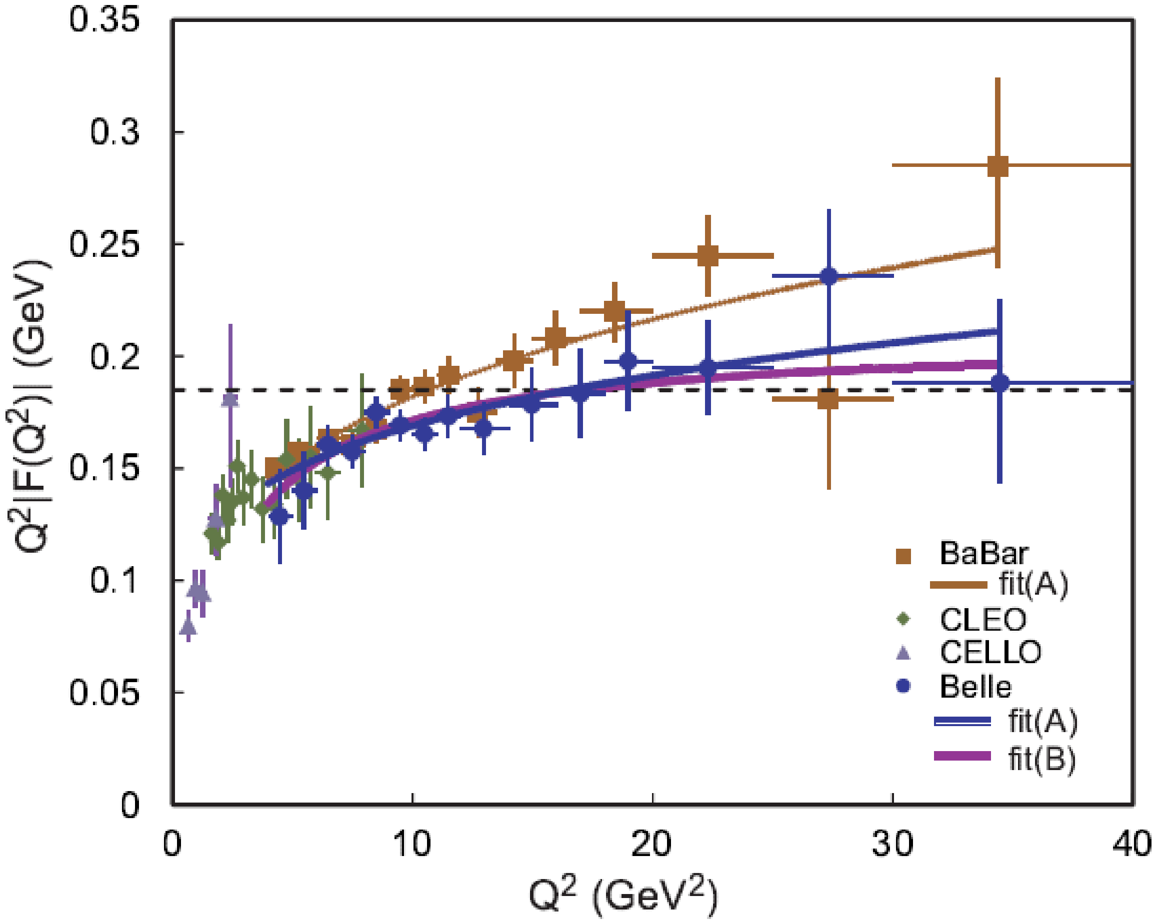
$\gamma\gamma^* \rightarrow$PS transition form factors

- Pseudoscalar meson production via photon fusion is typically studied through electron-positron scattering.



- One of the outgoing fermions is detected after a large-angle scattering. The other is scattered through a small angle and it is not detected. The detected fermion emits a highly off-shell photon; the undetected fermion, a soft photon.

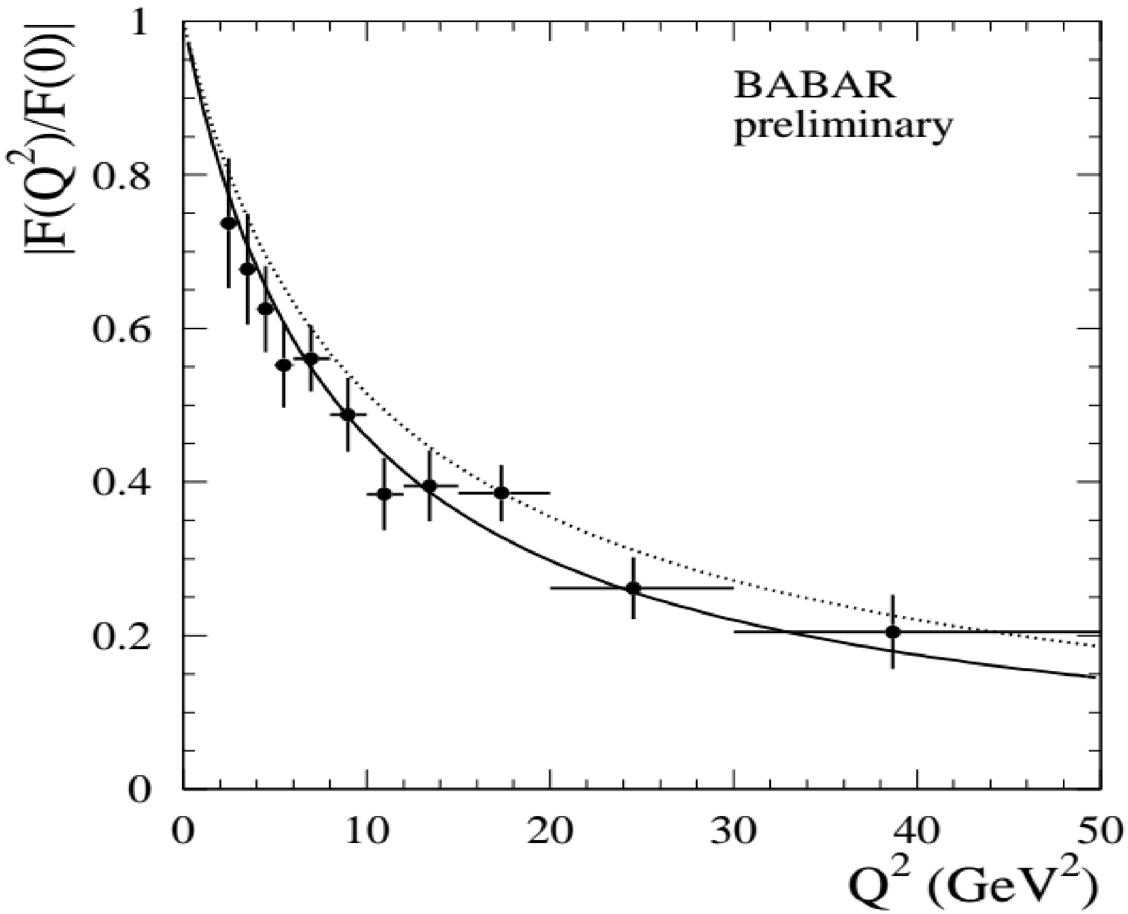
Pion transition form factor - $\gamma\gamma^* \to \pi^0$



- Many experiments have been done so far; but, at large Q^2, there is no agreement between the only available data (Babar[1] and Belle[2]).
- This needs to be explained, as well as how the conformal or BL limit[3], $2f_\pi$, is reached.
- Also, its study demands an explanation of the Abelian anomaly[4], which determines the value of the form factor at $Q^2=0$.

1. Phys. Rev. D80, 052002 (2009)
2. Phys. Rev. D86, 092007 (2012)
3. Phys. Rev. D22, 2157 (1980)
4. Phys. Rev. 177, 2426 (1969)

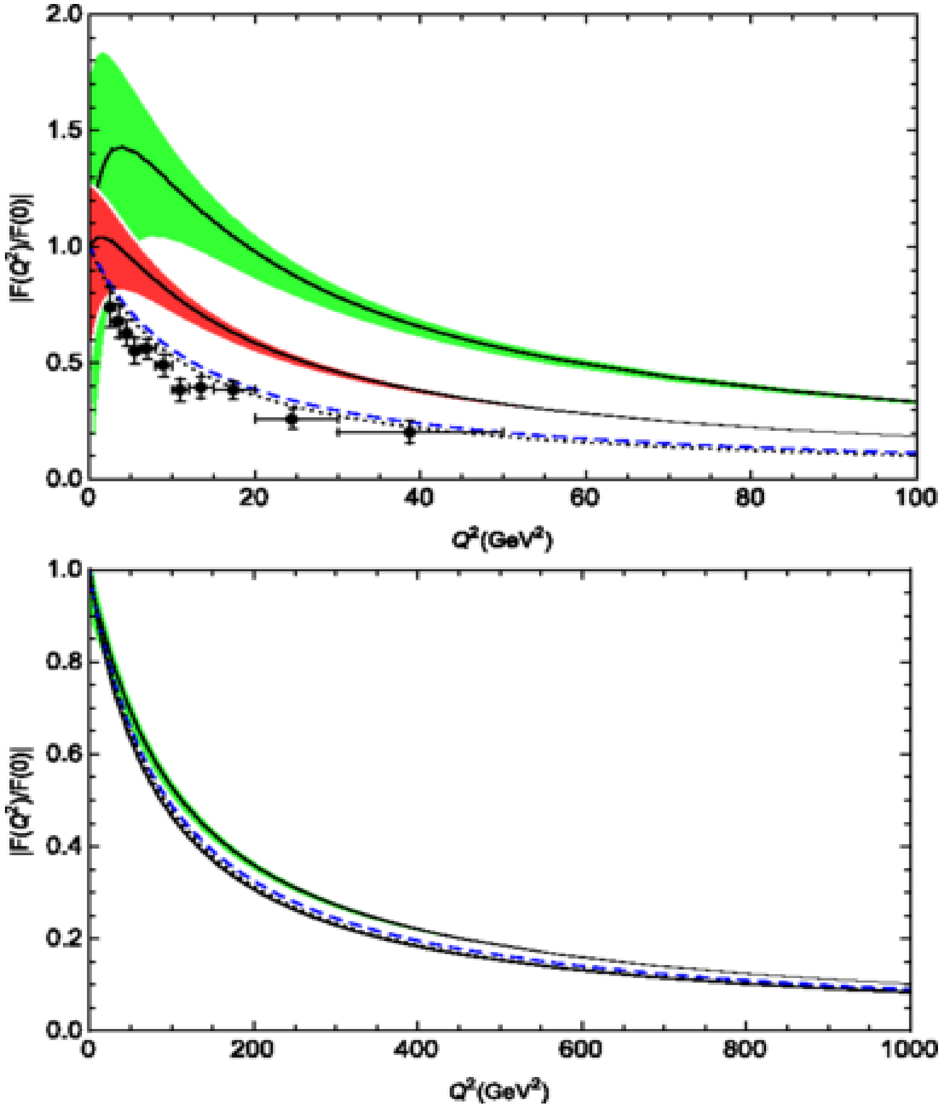
eta-c transition form factor - γγ*→eta-c



- eta-c transition form factor has been measured in Babar[5] at photon virtualities of 50 GeV^2.
- Such information can be used to refine effective field theories developed for application to systems involving heavy quarks[6].
- At first sight, It seems to be agreement between experimental data and asymptotic QCD.

5. Phys. Rev. D81, 052010 (2010)
6. Eur. Phys. J. C9, 459 (1999)

eta-b transition form factor - $\gamma\gamma^* \rightarrow$ eta-b



- It has been seen[7] that LO and NLO corrections of nrQCD, explain well eta-c experimental data. However, NNLO corrections are in a very notorious disagreement.
- One should therefore, also question if related predictions of $\gamma\gamma^*$ -> eta-b TFF are reliable.
- We present a consolidated explanation of all three transition form factors within a single theoretical approach.

7. Phys. Rev. Lett. 115, 222001 (2015)

Transition form factor

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- The pseudoscalar meson transition form factor (TFF) is computed from[8]:

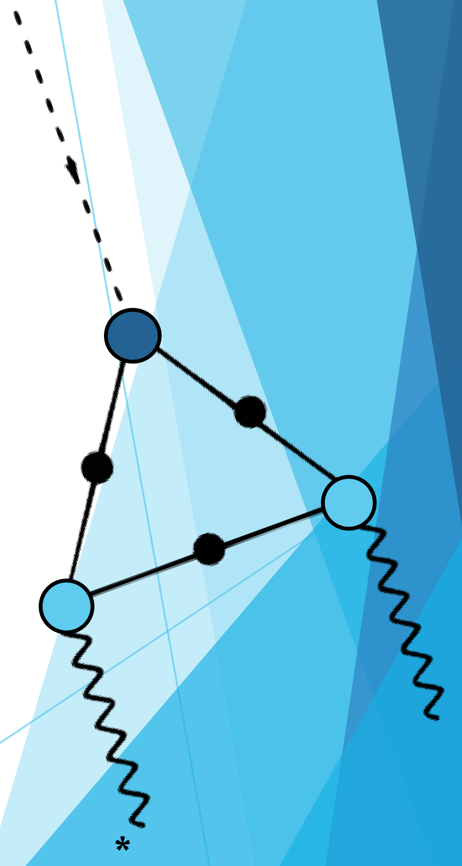
$$T_{\mu\nu}(k_1,k_2) = T_{\mu\nu}(k_1,k_2) + T_{\nu\mu}(k_2,k_1)\ ,$$

$$T_{\mu\nu}(k_1,k_2) = \frac{\alpha_{em}}{\pi}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G(k_1^2,k_2^2,k_1\cdot k_2)$$

- Where k_1, k_2 are the photon momenta and $P=k_1+k_2$ is the meson's momentum.
- At leading order, Rainbow-Ladder (RL), in the systematic and symmetry-preserving DSE:

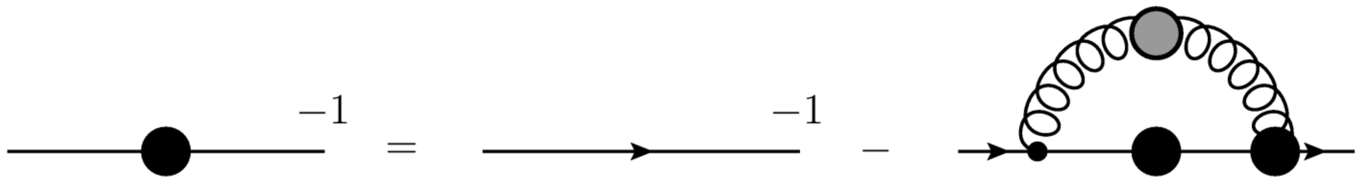
$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int \frac{d^4l}{(2\pi)^4} i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_M(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l)\ .$$

- We will gather the ingredients of the right-hand side of the above equation.

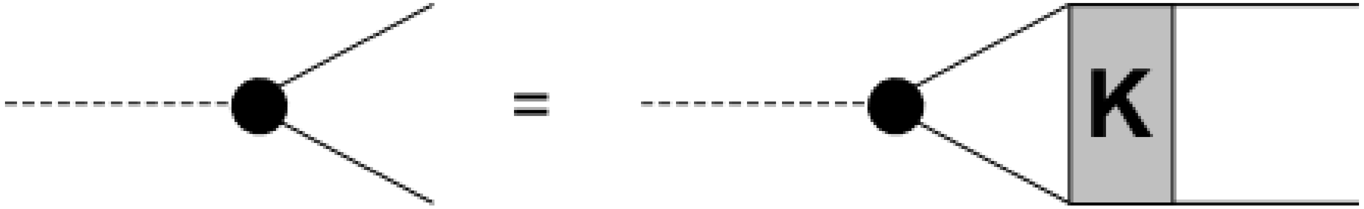


The DSE-BSE approach

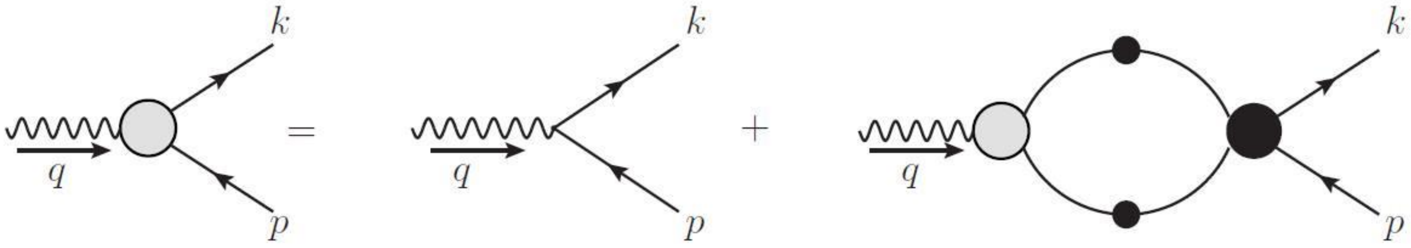
1. Quark Propagator

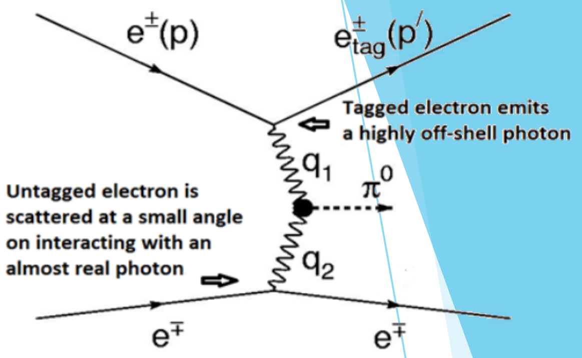


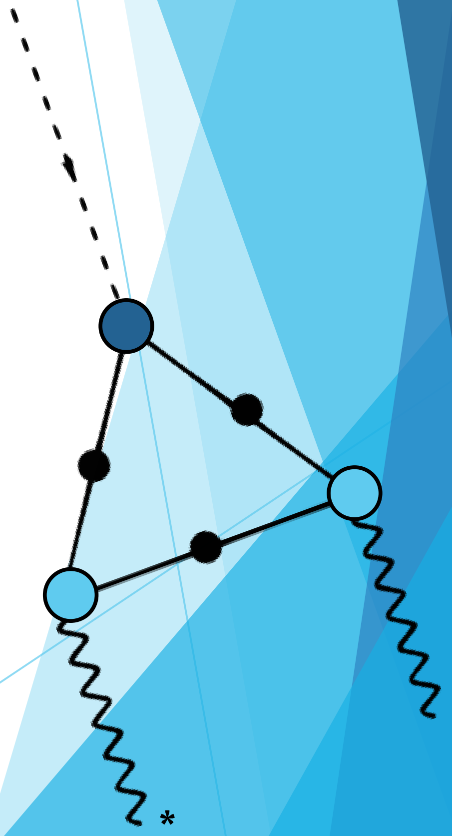
2. Bethe-Salpeter Amplitudes



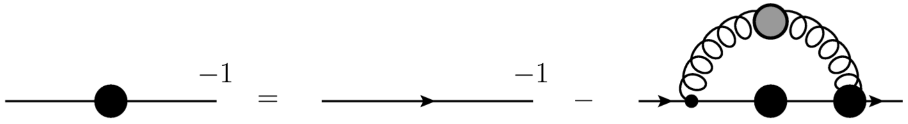
3. Quark-Photon Vertex

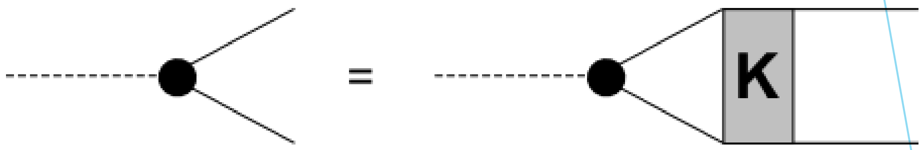






The DSE-BSE approach





- The renormalized DSE for the quark propagator is written as:

$$S^{-1}(p,\mu) = \mathcal{Z}_{2F}(i\gamma\cdot p) + \mathcal{Z}_4 m(\mu) + \mathcal{Z}_{1F}\int_q^\Lambda g^2 D_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\Gamma^a_\nu(p,q,\mu)\ .$$

- The corresponding Bethe-Salpeter equation and amplitude:

$$\Gamma_M^{ab}(p;P) = \int_q^\Lambda K(p,q;P)S^a(q+\eta P)\Gamma_M^{ab}(q;P)S^b(q-(1-\eta)P)$$

$$\Gamma_\pi^{qq}(p;P) = i\gamma_5\, E_\pi(p;P) + \gamma_5\gamma\cdot P\ F_\pi(p;P)$$
$$+\gamma_5(\gamma\cdot p)(p\cdot P)\ G_\pi(p;P) + \gamma_5\, p_\alpha\sigma_{\alpha\beta}P_\beta\ H_\pi(q;P)$$

- We employ RL[9] (for pion) and DB[10] (for eta-c, eta-b) truncation schemes.

9. Phys. Rev. C84, 042202 (2011)
10. Phys. Rev. Lett. 106, 072001 (2011)

The tools: N-ccp parametrization

- The quark propagator is written as:

$$S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)\;.$$

- It can be written in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N}\left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}}\right)\;,\;\; \sigma_s(q) = \sum_{k=1}^{N}\left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}}\right)\;.$$

- Constrained to the UV conditions of the free propagator form.

- N=2 is accurate enough for our purposes.

The tools: Nakanishi representation

- We parametrize the BSA using a Nakanishi-like representation[11]. Which consists in splitting the BSA into IR and UV parts and writing them as follows:

$$A(q,P) = \int_{-1}^{1} dz \int_{0}^{\infty} d\Lambda \left[\frac{\rho^{i}(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{m+n}} + \frac{\rho^{u}(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{n}} \right] .$$

- Where the spectral density is written as:

$$\rho^{i,u}(z,\Lambda) = \rho_1(z)\delta(\Lambda - \Lambda^{i,u}) + \cdots$$

- Following ref. 12 (Chang et al.), first we choose:

$$\rho_1(z) = \rho_\nu(z) \sim (1 - z^2)^\nu .$$

11. Phys. Rev. 130 1230-1235 (1963)
12. Phys. Rev. Lett. 110, 072001 (2013)

The tools: Nakanishi representation

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- Then we choose the following representation from refs. 8,12-13:

$$A^i(k,P) = c_A^i \int_{-1}^{1} dz \rho_{\nu_A^i}(z)[b_A \hat{\Delta}^4_{\Lambda_A^i}(k_z^2) + \bar{b}_A \hat{\Delta}^5_{\Lambda_A^i}(k_z^2)] \ . \quad E^u(k;P) = c_E^u \int_{-1}^{1} dz\ \rho_{\nu_E^u}(z) \hat{\Delta}^{1+\alpha}_{\Lambda_E^u}(k_z^2)$$

$$F^u(k,P) = c_F^u \int_{-1}^{1} dz \rho_{\nu_F^u}(z) k^2 \Lambda_F^u \Delta^{2+\alpha}_{\Lambda_F^u}(k_z^2) \qquad G^u(k,P) = c_G^u \int_{-1}^{1} dz \rho_{\nu_G^u}(z) \Lambda_G^u \Delta^{2+\alpha}_{\Lambda_G^u}(k_z^2)$$

- *A* stands for amplitude (E,F,G); *i*, *u* for IR and UV; Λ, ν, c, b are parameters fitted to the numerical data. With the following definitions:

$$\hat{\Delta}_\Lambda(s) = \Lambda\ \Delta_\Lambda(s)\ ,\ \Delta_\Lambda(s) = (s+\Lambda^2)^{-1}\ ,\ k_z^2 = k^2 + z\ k \cdot P\ .$$

- H(k,P) is negligible for pion and eta-c; G(k,P) and H(k,P) are negligible for eta-b.

12. Phys. Rev. Lett. 111, 141802 (2013)
13. Phys. Rev. Lett. 111, 141802 (2013)

Quark-Photon vertex.

- We employ unamputated vertex ansatz:

$$S\Gamma_\mu S \to \chi(k_f, k_i) = \sum_{i=1}^{3} T_{\mu i} X_i(k_f, k_i)$$

- Where the tensor structures are:

$$T_{1\mu} = \gamma_\mu$$
$$T_{2\mu} = \beta\gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{\beta}\gamma \cdot k_i \gamma_\mu \gamma \cdot k_f$$
$$T_{3\mu} = i\,\beta(\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + i\bar{\beta}(\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f)$$

$$\beta = 1 + s(Q^2)$$
$$\bar{\beta} = 1 - \beta$$
$$s(Q^2) = s_0 \mathrm{Exp}[-\mathcal{E}/M_E]$$

- And, the dressing functions:

$$X_1(k_f, k_i) = \Delta_{k^2\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_2(k_f, k_i) = \Delta_{\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_3(k_f, k_i) = \Delta_{\sigma_S}(k_f^2, k_i^2)\,.$$

$$\Delta_F(k_f, k_i) = \frac{F(k_f) - F(k_i)}{k_f - k_i}$$

Quark-Photon vertex.

$$s(Q^2) = s_0 \mathrm{Exp}[-\mathcal{E}/M_E]$$

- The vertex is constructed through the gauge technique[14], satisfies the longitudinal Ward-Green-Takahashi identity, is free of kinematic singularities, reduces to the bare vertex in the free-field limit, and has the same Poincaré transformation properties as the bare vertex.
- We have introduced transverse terms proportional to s0, where:

$$\mathcal{E} = \sqrt{Q^2/4 + m^2} - m, \; M_E = \{p|p^2 = M^2(p^2), \; p^2 > 0\} \; ,$$

are the kinetic energy in Breit frame (m ~ 0 for pion) and Euclidian constituent quark mass, respectively.

- Owing to the Abelian anomaly, it is impossible to simultaneously conserve the vector and axial vector currents. We have thus included a momentum redistribution factor s_0.

$$2f_\pi G(Q^2 = 0) = 1 \Rightarrow s_0 = 1.9.$$

14. J. Phys. A10, 1049 (1977)

Quark-Photon vertex.

$$s(Q^2) = s_0 \mathrm{Exp}[-\mathcal{E}/M_E]$$

- In the case of eta-c and eta-b, $G(Q^2=0)$ is fixed by the decay width:

$$\Gamma[M \to \gamma\gamma] = \tfrac{1}{4}\pi\alpha_{\rm em}^2 m_M^2 \, |G_M \, (Q^2=0)|^2$$

$$= \frac{8\pi\alpha_{\rm em}^2 e_{M^q}^4 f_M^2}{m_M^2} \begin{cases} \overset{\eta_c}{=} 6.1\,\mathrm{keV} \\ \overset{\eta_b}{=} 0.52\,\mathrm{keV} \end{cases}, \quad \Rightarrow \quad \begin{matrix} s_0 = 0.89 \\ s_0 = 0.23 \end{matrix}$$

$[e_{M^q} = (2/3), (-1/3)$ for $\eta_{c,b}$, respectively$]$

- The values of decay constants are taken from ref. 15 (Ding et al.):

$$f_{\eta_c} = 0.262 \text{ GeV}, \; f_{\eta_b} = 0.543 \text{ GeV} .$$

- Fixing $G(Q^2=0)$ to $\Gamma[\eta_c \to \gamma\gamma] = 5.1 \ keV$ instead, yields to an identical result.

15. Phys. Lett. B753, 330-335 (2016)

Transition form factor

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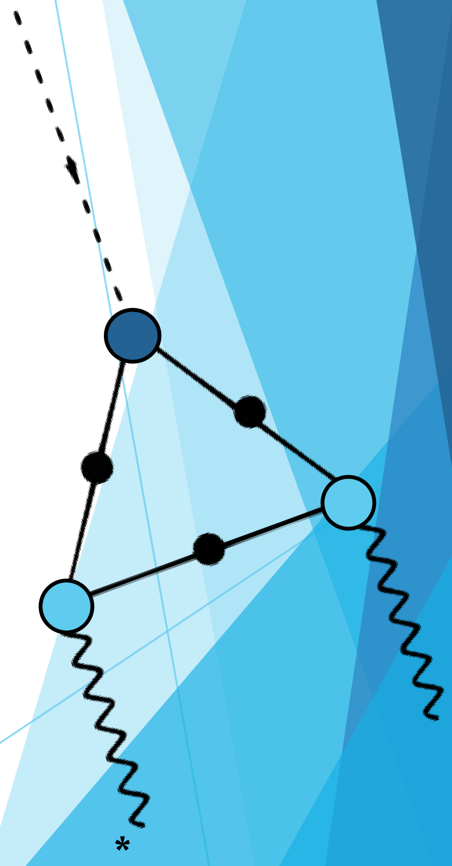
- Let's recall the expression for a pseudoscalar meson transition form factor (TFF):

$$\mathcal{T}_{\mu\nu}(k_1,k_2) = T_{\mu\nu}(k_1,k_2) + T_{\nu\mu}(k_2,k_1) \ ,$$

$$T_{\mu\nu}(k_1,k_2) = \frac{\alpha_{em}}{\pi}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G(k_1^2,k_2^2,k_1\cdot k_2)$$

$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int\frac{d^4l}{(2\pi)^4}i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_M(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l) \ .$$



- The parametrizations of quark propagator and BSA allow us to solve analytically the integrations over momentum after Feynman Parametrization. After calculation of the four-momentum integration, evaluation of the individual term is complete after one computes a finite number of simple integrals (over Feynman parameters and the spectral integral).

- Complete result comes after summing the series.

TFF: Asymptotic limit

- The asymptotic behavior of the TFF is written as:

$$Q^2 G_M(Q^2) \to 2 f_M \mathcal{Q}_M^2 \int_0^1 \frac{\phi(x;\zeta^2 = Q^2)}{x}\, , M = \pi_0,\ \eta_c,\ \eta_b,$$

where, $\mathcal{Q}_M^2 = \{1/3, 4/9, 1/9\}$ for pion, eta-c and eta-b respectively. It depends on the quark composition (quark charges) of the mesons.

- For asymptotic QCD, we have: $\phi^{asy}(x;\zeta^2 \to \infty) = 6x(1-x)$.

Therefore, we arrive at the *Conformal limit*[3]:

$$Q^2 G_{\pi,\eta_c,\eta_b}(Q^2) \to 2f_\pi, (8/3) f_{\eta_c}, (2/3) f_{\eta_b} \ .$$

Parton Distribution Amplitudes

- To fully understand mesons, one must extract information from its PDA, the projection of the BSA onto the lightcone[11]. For a pseudoscalar meson:

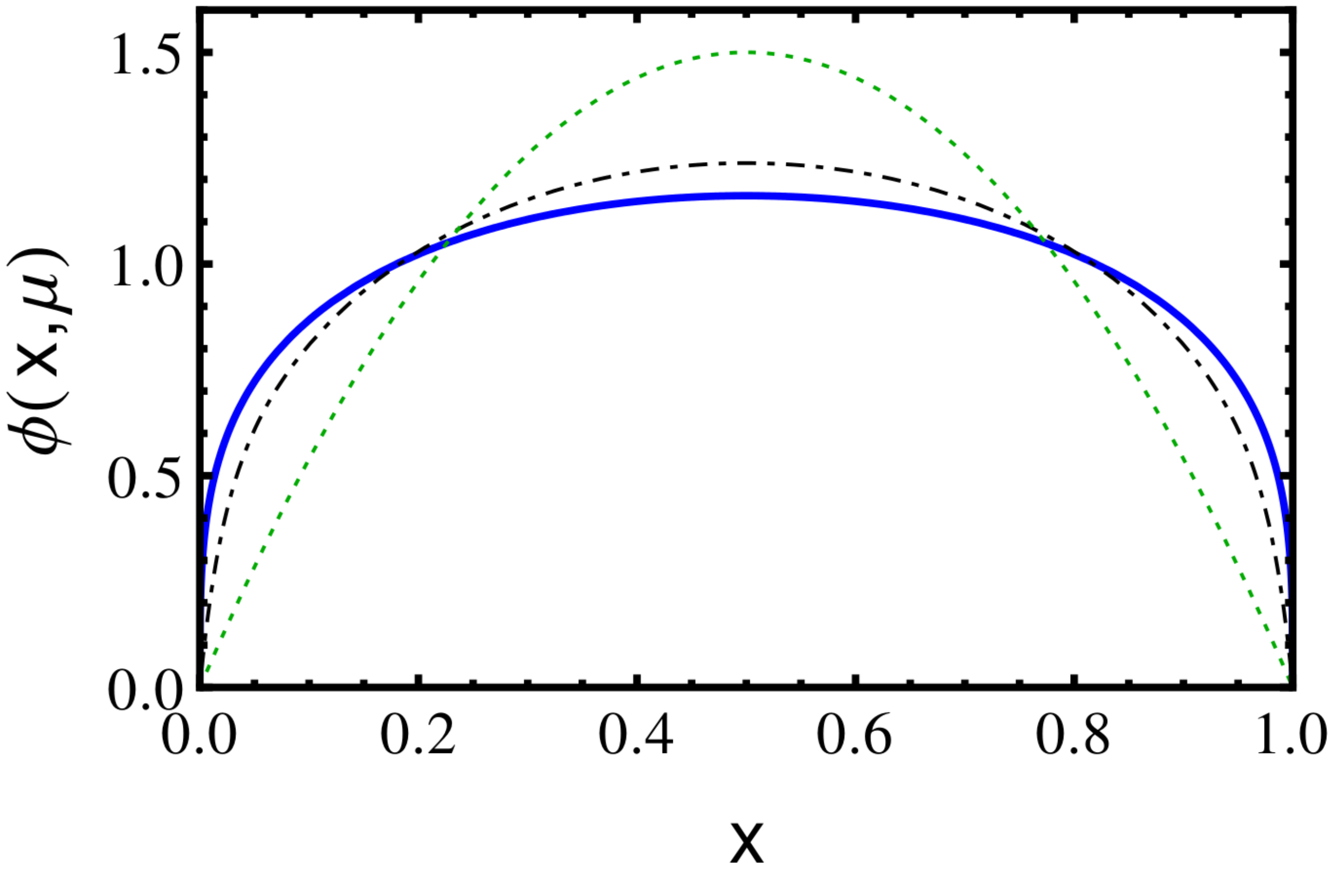
$$f_M \phi_M(x;\zeta) = Z_2(\zeta,\Lambda) \int_q^\Lambda \delta(n\cdot q^+ - x n\cdot P)\gamma_5 \gamma\cdot n \chi_M(q;P)\ .$$

- PDA should evolve with the resolution scale zeta^2=Q^2 through the ERBL evolution equations[16].
- Evolution enables the dressed-quark and -antiquark degrees-of-freedom, to split into less well-dressed partons via the addition of gluons and sea quarks in the manner prescribed by QCD dynamics. This can be read from the leading twist expansion:

$$G(Q^2) \sim f_M \int_0^1 T_H(x, Q^2;\zeta)\phi_M(x;\zeta)\ .$$

16. Phys. Rev. Lett. 11, 092001 (2013)

Pion Distribution Amplitude



- **PDA at different scales:** [Green, dashed] Asymptotic PDA[3], 6x(1-x). [Blue, solid] PDA at μ=2 GeV[12]. [Black, dot-dashed] Evolution from μ=2 GeV to μ=10 GeV.

Pion TFF

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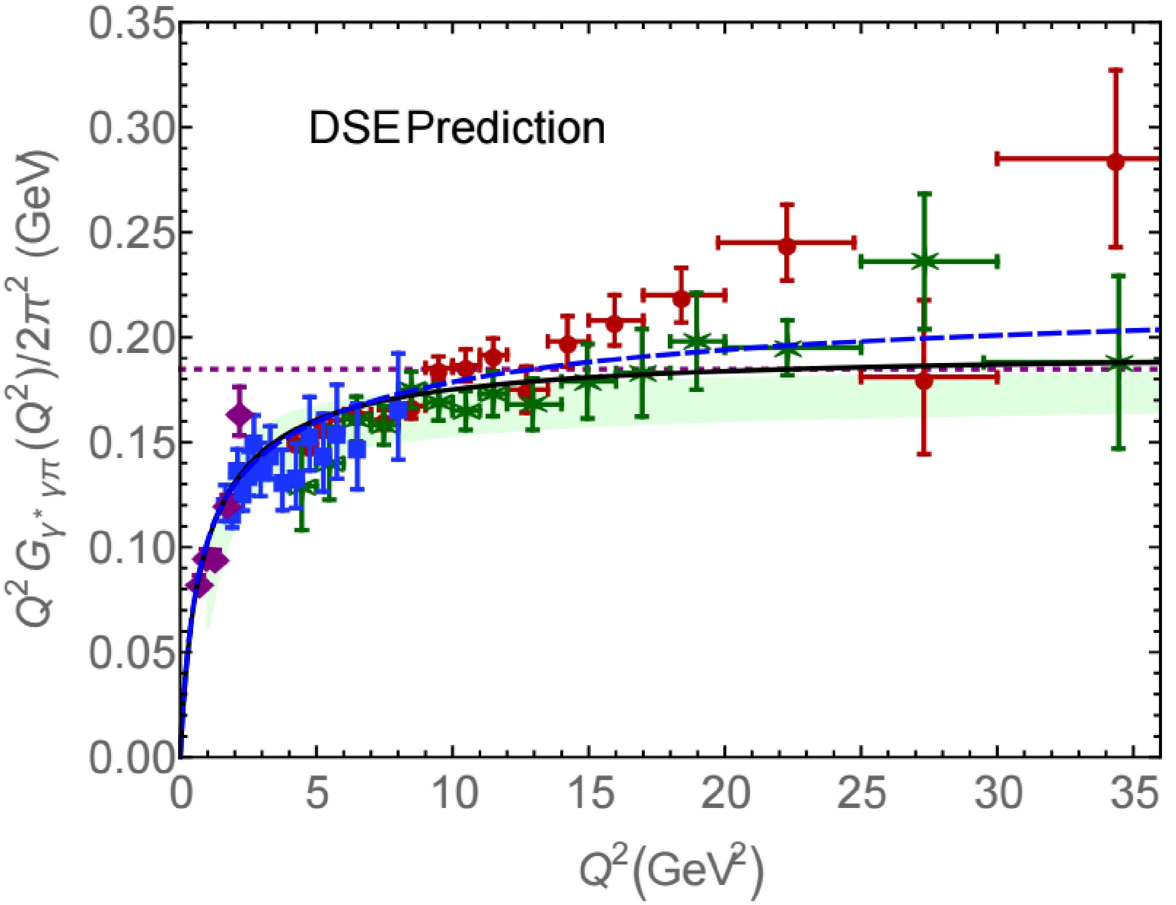
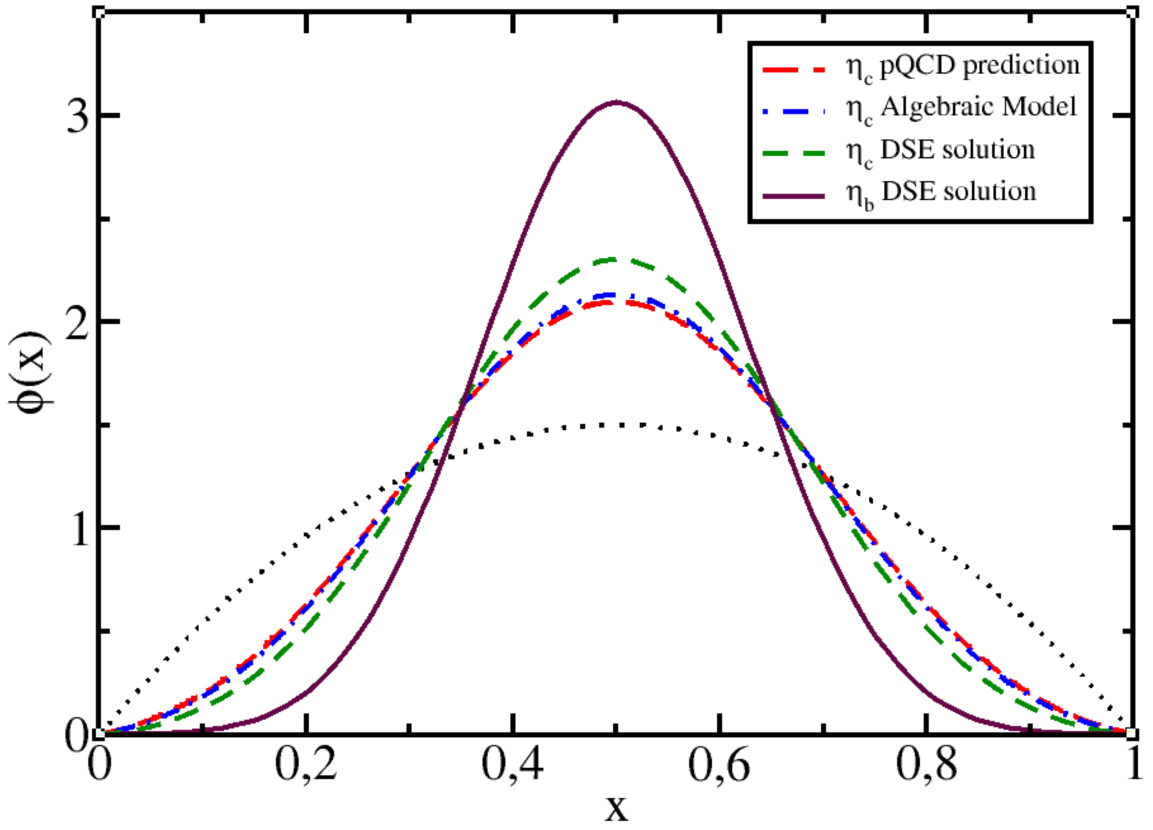


- **Pion TFF:** [**Black**, solid] DSE-RL Prediction (G(0)=1/(2fpi), r=0.68 fm)[8]. [Blue, dashed] *Frozen* DSE Prediction (μ = 2 GeV). [Band] BMS model[17].

17. Phys. Rev. D84, 034015 (2011)

PDA: eta-c and eta-b

Phys.Rev. D83 (2016) no.9, 094025

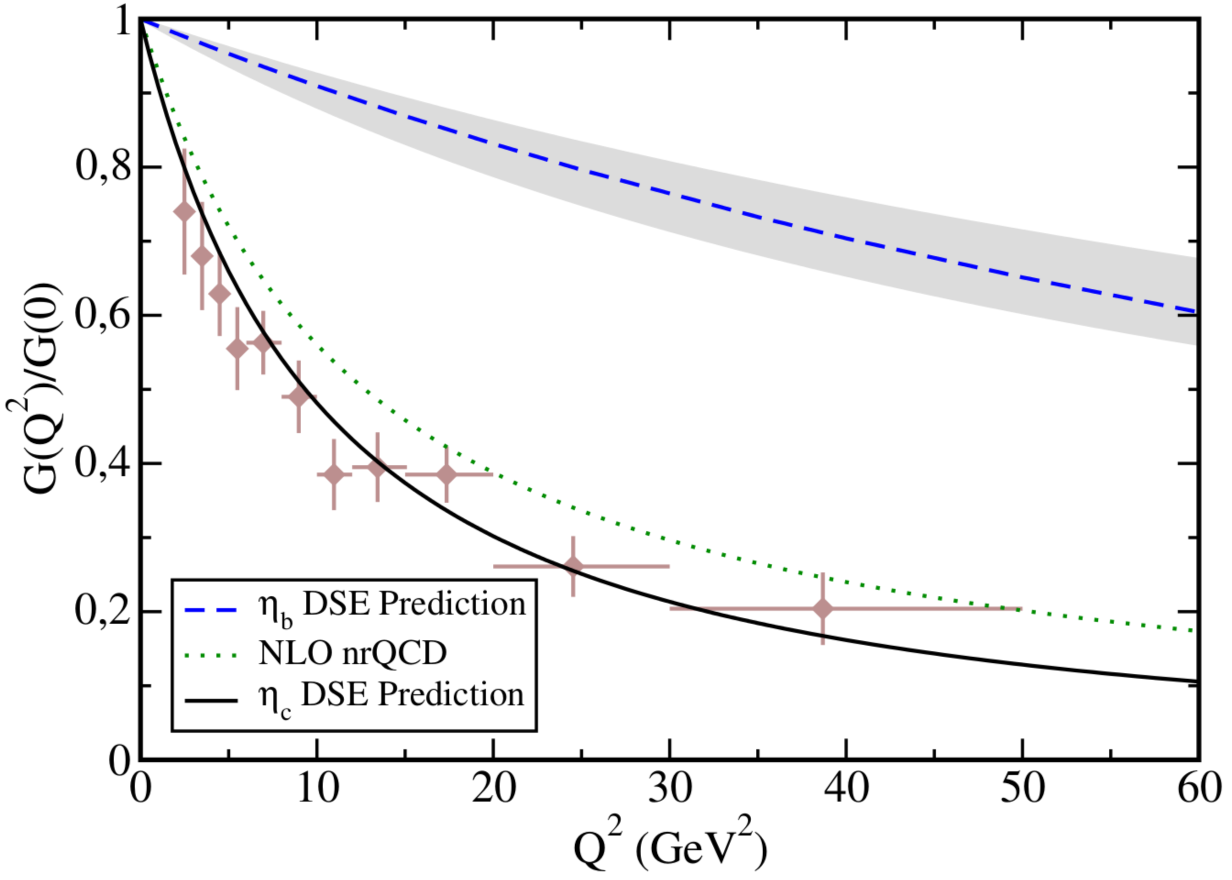


- From the figure above, we see that heavy mesons PDAs have a convex-concave-convex form. **DSE solutions:** Ding et al., ref. 15. **pQCD:** Kroll et al., ref. 18. **Algebraic Model:** Bedolla et al., ref. 19.

18. Phys. Lett. B413, 410 (1997)
19. Phys. Rev. D93, 094025 (2016)

eta-c and eta-b TFF

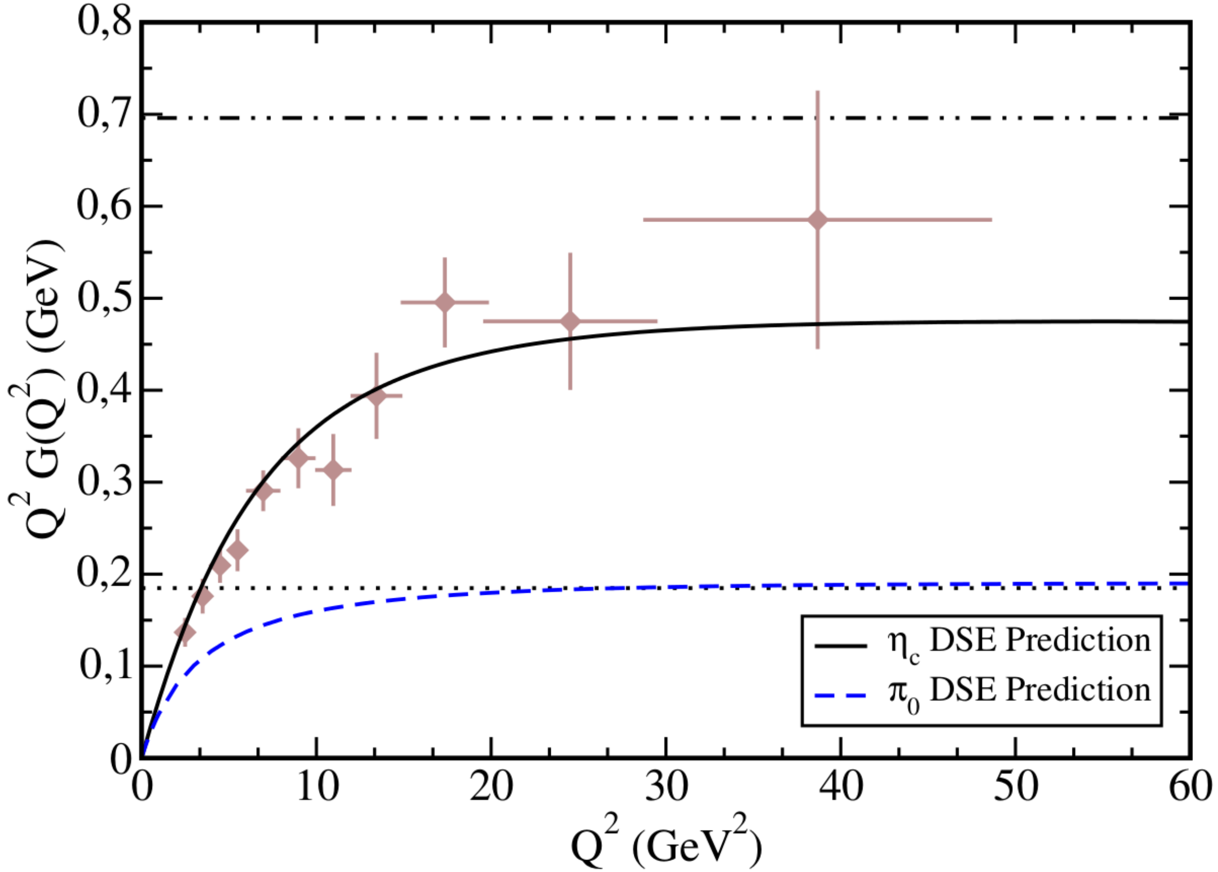
arXiv:1610.06575 [nucl-th]



- **TFF eta-c:** [**Black**, solid] DSE-DB Prediction (Width=6.1keV, r=0.16fm, s_0=0.78). [Green, dotted] NLO nrQCD result of ref. 7. [Data] Babar measurements.
- **TFF eta-b:** [Blue, solid] DSE-DB Prediction (Width=0.52keV, r=0.04fm, s_0=0.23). [Band] NNLO result of ref. 7.

Conformal limits

[arXiv:1610.06575 [nucl-th]]



- **Conformal limit:** From the figure, we see that for pion, the corresponding CL limit [blue] is reached around 30 GeV². This is far from being true for eta-c [**black**], which magnitude at 60 GeV² is less than 68% of its CL.

Conclusions and scope

- We described a computation of pion, eta-c and eta-b transition form factors, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations. Our results are relevant for BaBar and Belle experiments.
- We have unified the description with that of the valence-quark distribution amplitudes, masses, decay constants (refs. 11,13,15), and also with the charged pion electromagnetic form factor in ref. 12.
- The novel analysis techniques we employed made it possible to compute $G(Q^2)$, on the entire domain of space-like momenta, for the first time in a framework with a direct connection to QCD.

Phys.Rev. D93 (2016) no.7, 074017

arXiv:1610.06575 [nucl-th]

Conclusions and scope

- Those techniques could be easily adapted to other hadrons and other non perturbative objects, such as parton distribution functions and generalised parton distributions[20].
- Under development:
    - Mesons:
    1. eta and eta' transition form factors.
    2. eta-c Generalised Parton Distribution.
    - Baryons:
    1. N-N*(1535) transition form factor.

20. Phys. Lett. B741, 190-196 (2015)

References

1. Phys. Rev. D80, 052002 (2009)
2. Phys. Rev. D86, 092007 (2012)
3. Phys. Rev. D22, 2157 (1980)
4. Phys. Rev. 177, 2426 (1969)
5. Phys. Rev. D81, 052010 (2010)
6. Eur. Phys. J. C9, 459 (1999)
7. Phys. Rev. Lett. 115, 222001 (2015)
8. Phys.Rev. D93, 074017 (2016)
9. Phys. Rev. C84, 042202 (2011)
10. Phys. Rev. Lett. 106, 072001 (2011)
11. Phys. Rev. 130 1230-1235 (1963)
12. Phys. Rev. Lett. 110, 072001 (2013)
13. Phys. Rev. Lett. 111, 141802 (2013)
14. J. Phys. A10, 1049 (1977)
15. Phys. Lett. B753, 330-335 (2016)
16. Phys. Rev. Lett. 11, 092001 (2013)
17. Phys. Rev. D84, 034015 (2011)
18. Phys. Lett. B413, 410 (1997)
19. Phys. Rev. D93, 094025 (2016)
20. Phys. Lett. B741, 190-196 (2015)