Baryon vector form factors: an overview

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Abstract

Some aspects of the leading vector form factors of baryon are discussed. The issue of flavor SU(3) symmetry breaking is analyzed in the framework of the $1/N_c$ expansion of QCD. Two independent sum rules for the ratios $f_1/f_1^{\mathrm{SU(3)}}$, where $f_1^{\mathrm{SU(3)}}$ is the SU(3) symmetric value of f_1, can be found. Implications of these rum rules are quite interesting.

Outline

1. Baryon semileptonic decays

2. The Ademollo-Gatto theorem

3. Some aspects of QCD

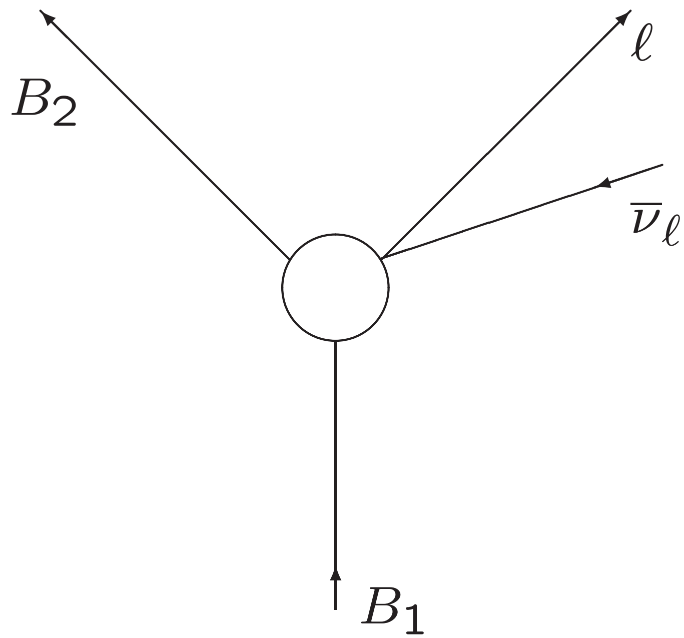
   ▷ Heavy baryon chiral perturbation theory

   ▷ The $1/N_c$ expansion of QCD

4. Sum rules

5. Concluding remarks

Baryon Semileptonic Decays



$$B_1 \rightarrow B_2 + \ell + \overline{\nu}_\ell$$

- ▷ Cabibbo[*] proposed a model for weak hadronic currents based on SU(3) symmetry: it led to detailed predictions for baryon semileptonic decays (BSD).

  The Lorentz structure of the current is $V-A$, and θ_C —the Cabibbo angle— is a parameter to be determined from experimental data.

- ▷ Kobayashi and Maskawa[†] generalized Cabibbo universality to three generations of quarks, which could accommodate CP violation.

  The matrix V is known as Cabibbo-Kobayashi-Maskawa (CKM) matrix and

$$V_{ud} \approx \cos\theta_C, \qquad V_{us} \approx \sin\theta_C$$

[*]N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963)

[†]M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)

The low-energy weak interaction Hamiltonian for HSD is given by

$$H_W = \frac{G_V}{\sqrt{2}} J_\alpha L^\alpha + \text{H.c.}$$

The leptonic current is

$$L^\alpha = \overline{\psi}_e \gamma^\alpha (1 - \gamma_5) \psi_{\nu_e} + \overline{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_{\nu_\mu}$$

The hadronic current, written in terms of the vector (V_α) and axial-vector (A_α) currents, is

$$\begin{aligned} J_\alpha &= V_\alpha - A_\alpha \,, \\ V_\alpha &= V_{ud} \overline{u} \gamma_\alpha d + V_{us} \overline{u} \gamma_\alpha s \,, \\ A_\alpha &= V_{ud} \overline{u} \gamma_\alpha \gamma_5 d + V_{us} \overline{u} \gamma_\alpha \gamma_5 s \end{aligned}$$

G_V is the weak coupling constant.

Matrix Elements of the Hadronic Current

$$
\begin{aligned}
\langle B_2|J_\alpha|B_1\rangle \; = \; & V_{\mathrm{CKM}}\,\overline{u}_{B_2}\Bigg[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_{B_1}}q_\alpha \\
& + \left(g_1(q^2)\gamma_\alpha + \frac{g_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_{B_1}}q_\alpha\right)\gamma_5\Bigg]u_{B_1}
\end{aligned}
$$

$q = p_1 - p_2$ is the momentum transfer, V_{CKM} is either V_{ud} or V_{us}, as the case may be, and

f_1	vector f.f.	g_1	axial-vector f.f.
f_2	weak magnetism f.f.	g_2	weak electricity f.f.
f_3	induced scalar f.f.	g_3	induced pseudoscalar f.f.

Flavor SU(3) Symmetry

- By neglecting the mass difference between the s and the u and d quarks, the form factors for BSD can be related to each other.

- The matrix elements of an SU(3) octet operator O_k between octet states are given in terms of two reduced matrix elements F_O and D_O

$$\langle B_n | O_k | B_m \rangle = F_O f_{knm} + D_O d_{knm},$$

  where f_{knm} are the structure constants of SU(3) and

$$\{\lambda_k, \lambda_n\} = \frac{4}{3}\delta_{kn} + 2 d_{knm} \lambda_m$$

Symmetry breaking effects in the form factors can be expanded in powers of H', the SU(3) breaking term in the hadron Hamiltonian. In the Standard Model,

$$H' = \frac{1}{\sqrt{3}} \left(m_s - \frac{m_u + m_d}{2} \right) \overline{q} \lambda^8 q$$

Low-energy consequences of QCD

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Despite the progress achieved in the understanding of the strong interactions with QCD, analytic calculations of the spectrum and properties of hadrons are not possible because the theory is strongly coupled at low energies.

Some methods to study the low-energy dynamics of QCD:

- ▷ Chiral perturbation theory
- ▷ The $1/N_c$ expansion

Large-N_c QCD

- The generalization of QCD from $N_c = 3$ to $N_c \gg 3$ colors, known as the large-N_c limit, was proposed[*] to understand the nonperturbative dynamics of hadrons.

- Large-N_c QCD is the SU(N_c) gauge theory of quarks and gluons, where N_c is a parameter of the theory.

- In the large-N_c limit the meson sector consists of a spectrum of narrow resonances and meson-meson scattering amplitudes are suppressed by powers of $1/\sqrt{N_c}$. The baryon sector is more subtle to analyze.[†]

- Physical quantities are considered in this limit, where corrections arise at relative orders $1/N_c$, $1/N_c^2, \ldots$, which leads to the $1/N_c$ expansion.

[*] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974); B **75**, 461 (1974).

[†] E. Witten, Nucl. Phys. B **160**, 57 (1979)

The $1/N_c$ Expansion of QCD

In the large-N_c limit the baryon sector possesses a contracted spin-flavor symmetry SU($2N_f$), with N_f the number of light-quark flavors.

SU($2N_f$) decomposes under SU(2) $\times$ SU(N_f) into a tower of baryon states with spins $J = \frac{1}{2}, \ldots, \frac{N_c}{2}$ in the flavor representation.*

Any physical operator $\mathcal{O}^{(m)}$ that scales as N_c^m may be written as

$$\mathcal{O}^{(m)} = N_c^m \sum_{n,p,q} c_n \left(\frac{J^i}{N_c}\right)^p \left(\frac{T^a}{N_c}\right)^q \left(\frac{G^{jb}}{N_c}\right)^{n-p-q}$$

The c_n have power series expansions in $1/N_c$ beginning at order unity.

*R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993)
J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phy. Rev. D **30**, 1795 (1989)

A QCD operator transforming according to a given $\mathrm{SU}(2) \times \mathrm{SU}(N_f)$ representation can be expanded as*

$$\mathcal{O}_{\mathrm{QCD}} = \sum_n c_n \frac{1}{N_c^{n-1}} \mathcal{O}^{(n)}$$

The spin-flavor generators J^i, T^a, and G^{ia} of $\mathrm{SU}(2N_f)$ are

$$\begin{aligned} J^i &= q^\dagger \left(\frac{\sigma^i}{2} \otimes I \right) q \,, && (1, \mathbf{1}) \\ T^a &= q^\dagger \left(I \otimes \frac{\lambda^a}{2} \right) q \,, && (0, \mathbf{8}) \\ G^{ia} &= q^\dagger \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q \,. && (1, \mathbf{8}) \end{aligned}$$

*R. F. Dashen, E. Jenkins, A.V. Manohar, Phys. Rev. D **49**, 4713 (1994)

Examples of Baryon Operators (Tree level)

- Mass

$$\mathcal{M} = \sum_{n=0}^{(N_c-1)/2} m_{2n}^{0,1} \frac{1}{N_c^{2n-1}} (J^2)^n$$

- Axial vector current

$$A^{kc} = a_1 G^{kc} + \sum_{n=2,3}^{N_c} b_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{kc} + \sum_{n=3,5}^{N_c} c_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{kc}$$

- Vector current

$$V^c = T^c$$

- Magnetic moment

$$M^{kQ} = d_1 G^{kQ} + \sum_{n=2,3}^{N_c} d_n \frac{1}{N_c^{n-1}} \mathcal{D}_n^{kQ} + \sum_{n=3,5}^{N_c} o_n \frac{1}{N_c^{n-1}} \mathcal{O}_n^{kQ}$$

Link to Physics

- Baryon mass

$$M_B = \langle B|\mathcal{M}|B\rangle, \qquad M_T = \langle T|\mathcal{M}|T\rangle$$

- Baryon axial vector coupling

$$g_A = \langle B_2|A^{kc}|B_1\rangle, \qquad g = \langle B_2|A^{kc}|T_1\rangle$$

- Baryon vector coupling

$$g_V = \langle B_2|V^c|B_1\rangle$$

- Baryon magnetic moment

$$\mu_B = \langle B|M^{kQ}|B\rangle, \qquad \mu_T = \langle T|M^{kQ}|T\rangle, \qquad \mu_{BT} = \langle B|M^{kQ}|T\rangle$$

The Ademollo-Gatto theorem

- By assuming that the vector currents and the electromagnetic current are members of the same unitary octet and that the breaking of the unitary symmetry behaves as the eighth component of an octet, M. Ademollo and R. Gatto* set up a theorem on the nonrenormalization for the $|\Delta S| = 1$ vector currents.

- The theorem asserts that all the corresponding vector form factors are protected against SU(3) symmetry breaking (SB) corrections to lowest order in $m_s - \widehat{m}$, where $\widehat{m}$ denotes the mean mass of the up and down quarks.

*M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

In the original paper...

To first order in SB, the current J^a is

$$
\begin{aligned}
\langle B|J^a|B\rangle \;=\; & a_0 \mathrm{Tr}(\overline{B}B\lambda^a) + b_0 \mathrm{Tr}(\overline{B}\lambda^a B) \\
& + a \mathrm{Tr}(\overline{B}B\{\lambda^a, \lambda_8\}) + b \mathrm{Tr}(\overline{B}\{\lambda^a, \lambda^8\}B) \\
& + c \left[\mathrm{Tr}(\overline{B}\lambda^a B\lambda^8) - \mathrm{Tr}(\overline{B}\lambda^8 B\lambda^a)\right] \\
& + g \mathrm{Tr}(\overline{B}B)\mathrm{Tr}(\lambda^a\lambda^8) \\
& + h \left[\mathrm{Tr}(\overline{B}\lambda^a B\lambda^8) + \mathrm{Tr}(\overline{B}\lambda^8 B\lambda^a)\right]
\end{aligned}
$$

where B represents the baryon matrix, λ^a denote the Gell-Mann matrices, and a_0, b_0, . . . , h are coupling constants.

The baryon electric charges are $Q_B = \langle B|J^Q|B\rangle$, with $Q = 3 + \frac{1}{\sqrt{3}}8$.

For instance,

$$Q_n = -\frac{2}{3}a_0 - \frac{2}{3}b_0 + \frac{8}{3\sqrt{3}}a - \frac{4}{3\sqrt{3}}b + \frac{2}{\sqrt{3}}c + \frac{2}{\sqrt{3}}g + \frac{\sqrt{3}}{10}h,$$

$$Q_p = -\frac{2}{3}a_0 + \frac{4}{3}b_0 + \frac{8}{3\sqrt{3}}a + \frac{8}{3\sqrt{3}}b - \frac{2}{\sqrt{3}}c + \frac{2}{\sqrt{3}}g - \frac{7\sqrt{3}}{10}h,$$

$$Q_{\Sigma^+} = -\frac{2}{3}a_0 + \frac{4}{3}b_0 - \frac{4}{3\sqrt{3}}a + \frac{8}{3\sqrt{3}}b + \frac{2}{\sqrt{3}}c + \frac{2}{\sqrt{3}}g + \frac{\sqrt{3}}{10}h,$$

Solving for the couplings,

$$a_0 = -\frac{1}{2}, \quad b_0 = \frac{1}{2}, \quad a = b = c = g = h = 0$$

SB in the framework of the $1/N_c$ expansion

SB in QCD is due to the strange quark mass m_s and transforms as a flavor octet. To linear order in SB the correction is obtained from the tensor product $(0,\mathbf{8}) \otimes (0,\mathbf{8})$ so that the $\mathrm{SU}(2) \times \mathrm{SU}(3)$ representations involved are $(0,\mathbf{1})$, $(0,\mathbf{8})$, $(0,\mathbf{10}+\overline{\mathbf{10}})$ and $(0,\mathbf{27})$. At second-order SB, the representation $(0,\mathbf{64})$ also appears.

The $1/N_c$ expansion for a $(0,\mathbf{8})$ operator, Q^c, containing first-order SB

$$\begin{aligned}
O^c + \epsilon O^c \; = \; & c_{(1)}^{\mathbf{8}} T^c + c_{(2)}^{\mathbf{8}} \frac{1}{N_c}\{J^r, G^{rc}\} \\
& + \epsilon N_c a_{(0)}^{\mathbf{1}} \delta^{c8} + \epsilon a_{(1)}^{\mathbf{8}} d^{c8e} T^e + \epsilon a_{(2)}^{\mathbf{8}} \frac{1}{N_c} d^{c8e}\{J^r, G^{re}\} \\
& + \epsilon a_{(3)}^{\mathbf{10}+\overline{\mathbf{10}}} \frac{1}{N_c^2}\left[\{T^c,\{J^r,G^{r8}\}\} - \{T^8,\{J^r,G^{rc}\}\}\right] \\
& + \epsilon a_{(2)}^{\mathbf{27}} \frac{1}{N_c}\{T^c, T^8\} \\
& + \epsilon a_{(3)}^{\mathbf{27}} \frac{1}{N_c^2}\left[\{T^c,\{J^r,G^{r8}\}\} + \{T^8,\{J^r,G^{rc}\}\}\right]
\end{aligned}$$

At second order in SB

$$
\begin{aligned}
\epsilon^2 O^c \;=\; & \epsilon^2 b_{11} N_c d^{c88} + \epsilon^2 b_{12} \delta^{c8} T^8 + \epsilon^2 b_{13} d^{c8e} d^{8eg} T^g \\
& + \epsilon^2 b_{28} \frac{1}{N_c} \delta^{c8} \{J^r, G^{r8}\} \\
& + \epsilon^2 b_{38} \frac{1}{N_c} d^{c8e} d^{8eg} \{J^r, G^{rg}\} \\
& + \epsilon^2 b_{410} \frac{1}{N_c^2} d^{c8e} \left[\{T^e, \{J^r, G^{r8}\}\} - \{T^8, \{J^r, G^{re}\}\} \right] \\
& + \epsilon^2 b_{527} \frac{1}{N_c} d^{c8e} \{T^e, T^8\} \\
& + \epsilon^2 b_{627} \frac{1}{N_c^2} d^{c8e} \left[\{T^e, \{J^r, G^{r8}\}\} + \{T^8, \{J^r, G^{re}\}\} \right] \\
& + \epsilon^2 b_{627} \frac{1}{N_c^2} \{T^c, \{T^8, T^8\}\}
\end{aligned}
$$

Sum rules for vector form factors

In the limit of axact SU(3) symmetry and neglecting isospin breaking:

$$\left[f_1^{\mathrm{SU(3)}}\right]_{\Xi^-\Sigma^0}+\sqrt{3}\left[f_1^{\mathrm{SU(3)}}\right]_{\Xi^-\Lambda}+\frac{1}{\sqrt{2}}\left[f_1^{\mathrm{SU(3)}}\right]_{\Sigma^-n}+\sqrt{3}\left[f_1^{\mathrm{SU(3)}}\right]_{\Lambda p}=0$$

and

$$\sqrt{3}\left[f_1^{\mathrm{SU(3)}}\right]_{\Xi^-\Sigma^0}-\left[f_1^{\mathrm{SU(3)}}\right]_{\Xi^-\Lambda}-\sqrt{\frac{3}{2}}\left[f_1^{\mathrm{SU(3)}}\right]_{\Sigma^-n}+\left[f_1^{\mathrm{SU(3)}}\right]_{\Lambda p}=0,$$

where $f_1^{\mathrm{SU(3)}}$ stands for the SU(3) symmetric value of f_1.

Violations to the above expressions can be written as

$$[f_1]_{\Xi^-\Sigma^0}+\sqrt{3}\,[f_1]_{\Xi^-\Lambda}+\frac{1}{\sqrt{2}}\,[f_1]_{\Sigma^-n}+\sqrt{3}\,[f_1]_{\Lambda p}=\delta^{\mathbf{27}}$$

and

$$\sqrt{3}\,[f_1]_{\Xi^-\Sigma^0}-[f_1]_{\Xi^-\Lambda}-\sqrt{\frac{3}{2}}\,[f_1]_{\Sigma^-n}+[f_1]_{\Lambda p}=\delta^{\mathbf{10}+\overline{\mathbf{10}}},$$

where δ^{rep} arise from SB.

Concluding remarks

- ▷ The understanding of flavor SU(3) SB in the baryon sector still presents some challenges. The outcome could be rewarding.