Electromagnetic response of topological insulators

A. Martín-Ruiz (ICN-UNAM)

in collaboration with M. Cambiaso (UNAB) and L. Urrutia (ICN)

Mexico City. November 10, 2016

Plan of the talk

- Basics of topological Insulators
- Effective field theory describing the electromagnetic response of TIs
- The topological magnetoelectric effect
- The gravitational θ term

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Basics of Topological Insulators

The search for new states of matter

- The search for new elements led to a golden age of chemistry.
- The search for new particles led the golden age of particle physics.
- In condensed matter physics, we ask what are the fundamental states of matter?

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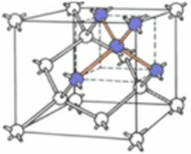
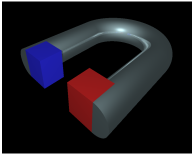
- Most of these states are differentiated by the broken symmetries:



Crystal- Translational



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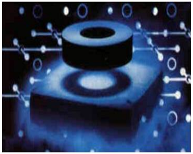

Superconductor- Gauge

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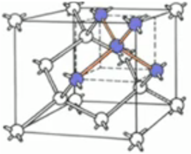

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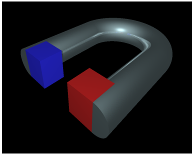

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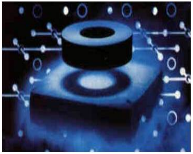

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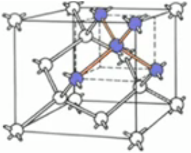

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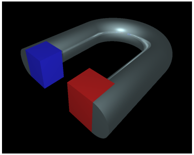

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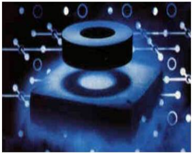

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- The pattern of symmetry breaking leads to a unique order parameter.
- EFT $\rightarrow$ Landau-Ginzburg theory $\rightarrow$ quantum states of matter
- In 1980, a new quantum state was discovered which does not fit into this paradigm.

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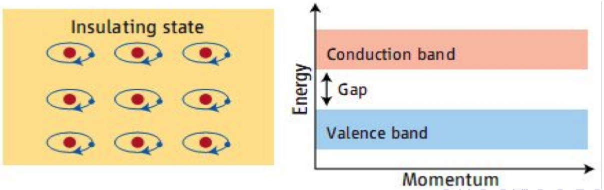
The quantum Hall state

The insulating state

- Electrons bound to atoms in closed shells
- Electrically inert (finite energy gap to dislodge electrons)
- Occupied valence and empty conduction bands
- Classify states by momentum **k**
- Equivalent to Dirac's vacuum (energy gap for pair production)



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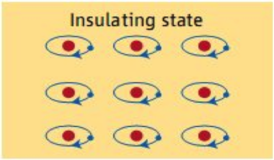

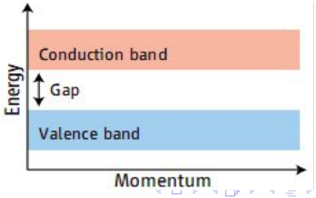

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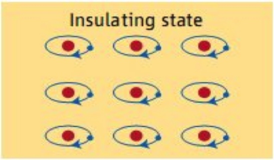

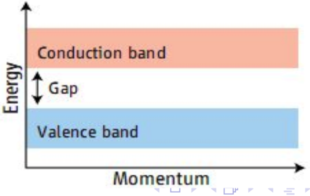

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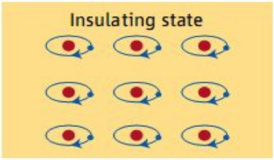

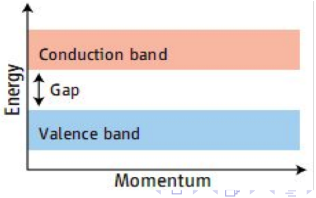

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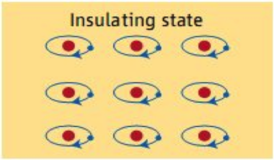

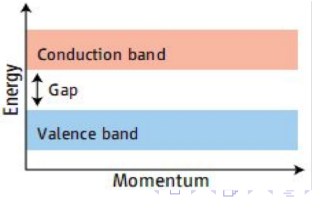

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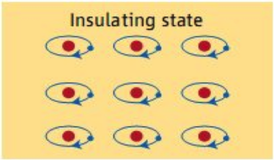

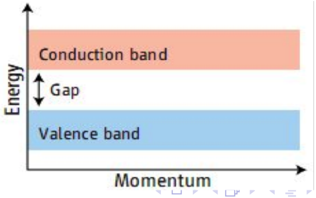

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- Electrons in 2D / strong **B** field
- Quantized Landau levels (band structure)
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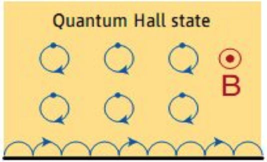
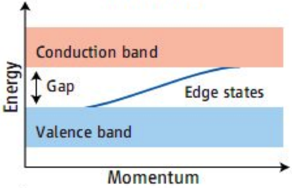
- An **E** field causes cyclotron orbits to drift (diff to insulator)
- Quantizaed Hall conductivity $\sigma_{xy} = ne^2/h$.





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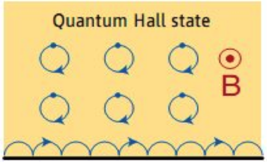

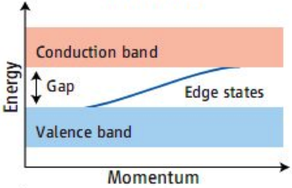

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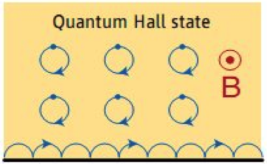

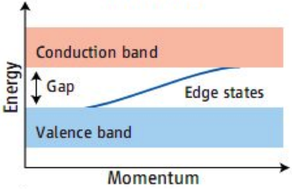

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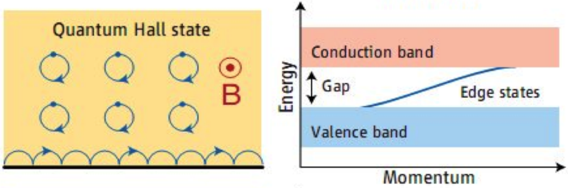

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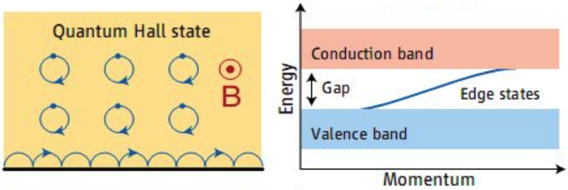

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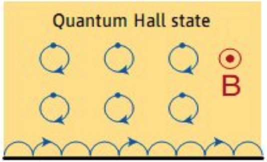
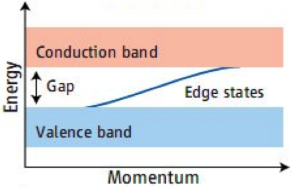




Topological quantization?

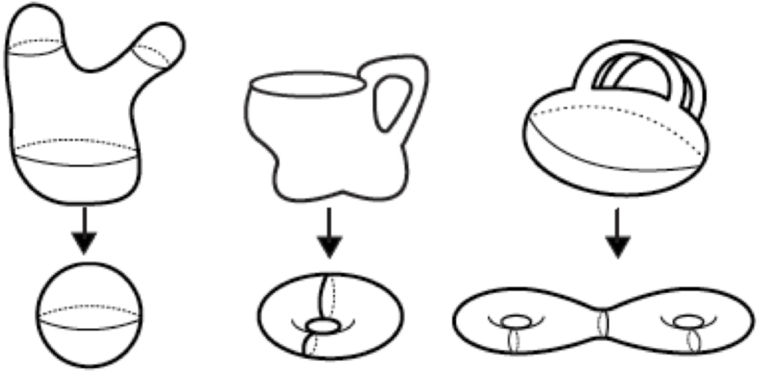
What is the difference between a quantum Hall state and an ordinary insulator?

The answer is a matter of **topology!**

Topological quantization?

Topologically equivalent classes

Shape and Genus (holes)

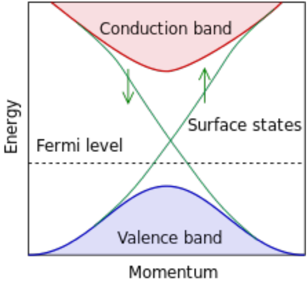


Topological quantization?

Topologically equivalent classes in physics

Hamiltonian (of many particle systems) with an energy gap (separating the ground state from the excited states)

$\Rightarrow$ Insulating states are disconnected with the Hall states! (or Hamiltonians)



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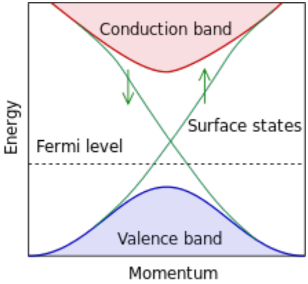

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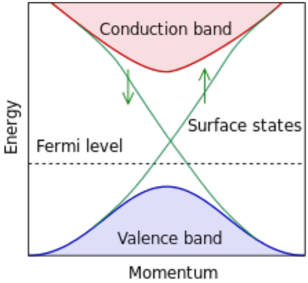

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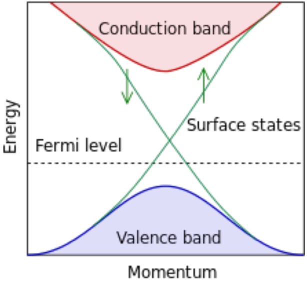


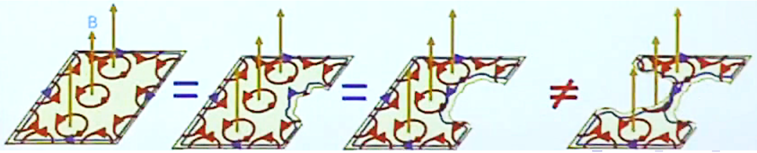
Topological quantization?

Topological invariance of the Hall conductance

HC remains unchanged by small changes in the sample.

Genus (TI) $\Rightarrow$ integral over the local curvature
HC (TI) $\Rightarrow$ integral over the frequency momentum space (Berry curvature)

$$n_m = (i/2\pi) \int d^2\mathbf{k} \nabla \times \langle u_m | \nabla_\mathbf{k} | u_m \rangle \quad (1)$$



Electromagnetic response of Topological Insulators

Effective action of 3D TIs

In the presence of an electromagnetic background, the fermionic action is given by

$$\mathcal{S}[\overline{\psi}, \psi, A_\mu] = \int d^4x \overline{\psi} \left(i\gamma^\mu \partial_\mu - eA_\mu \gamma^\mu - m \right) \psi$$

The effective electromagnetic action $\mathcal{S}_\theta[A_\mu]$ for the electromagnetic field is obtained from the fermionic path integral

$$e^{i\mathcal{S}_\theta[A_\mu]} = \int \mathcal{D}\overline{\psi} \mathcal{D}\psi e^{i\mathcal{S}[m, \overline{\psi}, \psi, e]}$$

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Effective action of 3D TIs

We can flip the sign of mass continuously by the chiral rotation $\psi \to e^{i\gamma^5\theta/2}\psi'$, under which

$$\overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi = \overline{\psi}'\left(i\gamma^\mu\partial_\mu - m'(\theta)\right)\psi'$$

where $m'(\theta) = me^{i\theta\gamma^5}$, so that $m'(0) = m$ and $m'(\pi) = -m$.

The chiral transformation that rotates m continuously cost the Jacobian $\mathcal{J}$ of the path integral measure $\mathcal{D}\overline{\psi}\mathcal{D}\psi = \mathcal{J}\mathcal{D}\overline{\psi}'\mathcal{D}\psi'$.

$$S_\theta[A_\mu] = -\ln\mathcal{J} = \frac{\theta e^2}{32\pi^2}\int d^4x\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

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TR invariant 3D TIs

Topological term?

The θ term is a total derivative:

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{E}\cdot\mathbf{B} = 2\epsilon^{\mu\nu\alpha\beta}\partial_\mu\left(A_\nu\partial_\alpha A_\beta\right)$$

TR invariant TIs

- Time-reversal invariance requires $\theta = 0$ (trivial) or $\theta = \pi(\mathrm{mod}2\pi)$ (topological).
- θ is determined by the nature of the TR breaking perturbation (controlled experimentally with a thin magnetic layer)

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Topological magnetoelectric effect

Electromagnetic response of TRI 3D TIs

$$\mathcal{S} = \int d^4x \mathcal{L} = \int d^4x \left(\mathcal{L}_0 + \mathcal{L}_\theta \right)$$

$$\mathcal{L}_0 = \frac{1}{8\pi}[\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2] \quad , \quad \mathcal{L}_\theta = \frac{\alpha}{4\pi}\theta \mathbf{E} \cdot \mathbf{B}$$

Axion electrodynamics?

- In particle physics the Axion is a dynamical field...
- On a TR breaking boundary, θ suddenly changes! (domain wall)

Topological magnetoelectric effect

Electromagnetic response of TRI 3D TIs

$$\mathcal{S} = \int d^4x \mathcal{L} = \int d^4x \left(\mathcal{L}_0 + \mathcal{L}_\theta \right)$$

$$\mathcal{L}_0 = \frac{1}{8\pi}[\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2] \quad , \quad \mathcal{L}_\theta = \frac{\alpha}{4\pi}\theta \mathbf{E}\cdot\mathbf{B}$$

Axion electrodynamics?

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- On a TR breaking boundary, θ suddenly changes! (domain wall)

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Field equations

Functional variation produces the Maxwell's equations in matter

$$\nabla \cdot \mathbf{D} = 4\pi\rho \quad , \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J},$$

$$\nabla \cdot \mathbf{B} = 0 \quad , \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

with the constitutive relations

$$\mathbf{D} = 4\pi \frac{\delta \mathcal{L}}{\delta \mathbf{E}} = \varepsilon \mathbf{E} + \frac{\alpha}{\pi}\theta \mathbf{B} \quad , \quad \mathbf{H} = -4\pi \frac{\delta \mathcal{L}}{\delta \mathbf{B}} = \frac{\mathbf{B}}{\mu} - \frac{\alpha}{\pi}\theta \mathbf{E}.$$

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Boundary conditions at the interface

$$\left[\varepsilon \mathbf{E}\right]_{\Sigma} \cdot \mathbf{n} = \tilde{\theta}(\mathbf{B}\cdot\mathbf{n})\Big|_{\Sigma} \quad , \quad \left[\tfrac{1}{\mu}\mathbf{B}\right]_{\Sigma} \times \mathbf{n} = -\tilde{\theta}(\mathbf{E}\times\mathbf{n})\Big|_{\Sigma},$$

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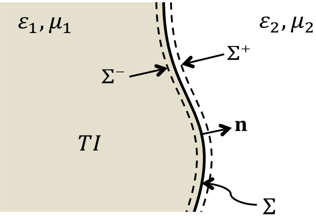

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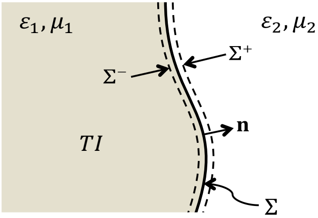

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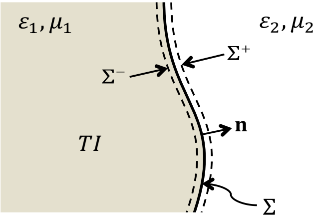

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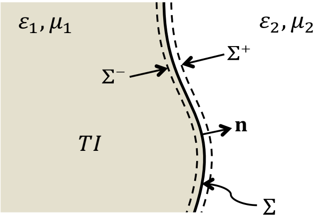

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- Maxwell's equations remain unaltered in the bulk.
- The θ term modifies the EM field only at the surface.
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Quantized topological magnetoelectric effect:

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A Green's functions approach to the TME

Motivations

- The TME has not yet been observed experimentally (the θ and Maxwell terms in (3+1)D are equally important at low energies).
- Knowledge of Green's function allows one to compute the electromagnetic fields for an arbitrary distribution of sources.
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In the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$), the electromagnetic potentials satisfy

$$-\varepsilon(\mathbf{r})\nabla^2\phi - \nabla\varepsilon(\mathbf{r}) \cdot \nabla\phi + \frac{\alpha}{\pi}\nabla\theta(\mathbf{r}) \cdot \nabla \times \mathbf{A} = 4\pi\rho,$$

$$-\tilde{\mu}(\mathbf{r})\nabla^2\mathbf{A} + \frac{\alpha}{\pi}\nabla\theta(\mathbf{r}) \times \nabla\phi + \nabla\tilde{\mu}(\mathbf{r}) \times (\nabla \times \mathbf{A}) = 4\pi\mathbf{J}.$$

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We introduce the Green's function matrix $G^{\mu}{}_{\nu}(\mathbf{x},\mathbf{x}')$ which satisfies

$$\left[\mathcal{O}^{\mu}{}_{\nu}\right]_{\mathbf{x}} G^{\nu}{}_{\sigma}(\mathbf{x},\mathbf{x}') = 4\pi\eta^{\mu}{}_{\sigma}\delta(\mathbf{x}-\mathbf{x}'),$$

such that

$$A^{\mu}(\mathbf{x}) = \int G^{\mu}{}_{\nu}(\mathbf{x},\mathbf{x}')J^{\nu}(\mathbf{x}')d^3\mathbf{x}'.$$

We introduce the Fourier transform in the direction parallel to the plane Σ, and define

$$G^{\mu}{}_{\nu}(\mathbf{x},\mathbf{x}') = 4\pi \int \frac{d^2\mathbf{p}}{(2\pi)^2} e^{i\mathbf{p}\cdot\mathbf{R}} g^{\mu}{}_{\nu}(z,z').$$

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One can further solve for the reduced GF. After Fourier transforming we obtain

$$G^0{}_0(\mathbf{x},\mathbf{x}') = \frac{1}{\varepsilon(z')}\left[\frac{1}{|\mathbf{x}-\mathbf{x}'|} - \frac{\text{sgn}(z')(\varepsilon-1)(\frac{1}{\mu}+1)+\tilde{\theta}^2}{(\varepsilon+1)(\frac{1}{\mu}+1)+\tilde{\theta}^2}\frac{1}{|\mathbf{x}-\mathbf{x}''|}\right],$$

$$G^i{}_0(\mathbf{x},\mathbf{x}') = \frac{i\tilde{\theta}}{(\varepsilon+1)(\frac{1}{\mu}+1)+\tilde{\theta}^2}\epsilon^{0ij3}I_j(\mathbf{x},\mathbf{x}'),$$

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where

$$\mathbf{I}(\mathbf{x},\mathbf{x}') = 2i\frac{\mathbf{R}}{R^2}\left(1-\frac{Z}{|\mathbf{x}-\mathbf{x}''|}\right) \quad , \quad \mathbf{K}(\mathbf{x},\mathbf{x}') \; = |\mathbf{x}-\mathbf{x}''|\mathbf{I}(\mathbf{x},\mathbf{x}'),$$

with $Z = |z| + |z'|$, $\mathbf{R} = (x-x', y-y')$ and $R = |\mathbf{R}|$

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Topological Kerr and Faraday rotation

$$\tan\theta_K = \frac{4\alpha P_3 \sqrt{\varepsilon_1/\mu_1}}{\varepsilon_2/\mu_2 - \varepsilon_1/\mu_1 + 4\alpha^2 P_3^2}$$

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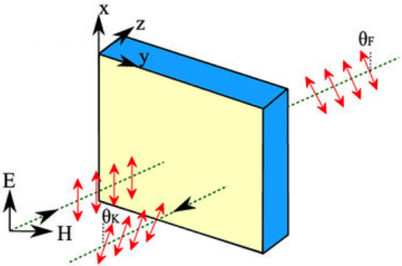

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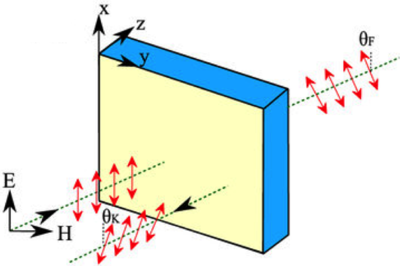

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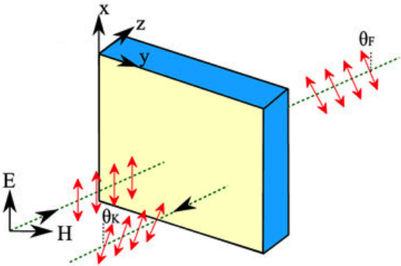

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$$q'' = q\frac{(1-\varepsilon)(1+1/\mu) - \tilde{\theta}^2}{(1+\varepsilon)(1+1/\mu) + \tilde{\theta}^2}$$

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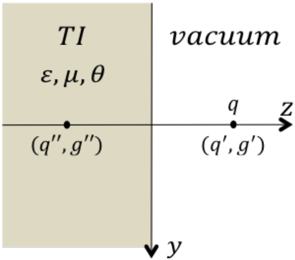

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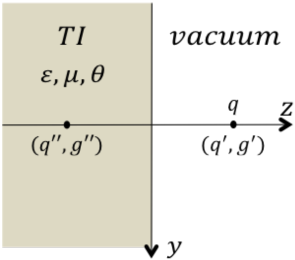

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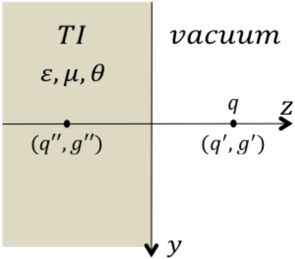

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Maxwell's law $\nabla \cdot \mathbf{B} = 0$ remains unaltered due to $U(1)$ gauge invariance of the θ term!!!!

Zero magnetic flux at the TI !!!

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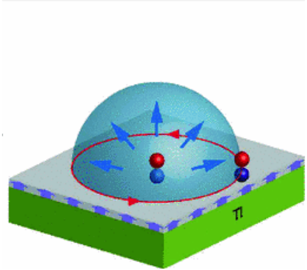

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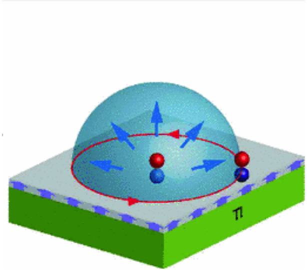

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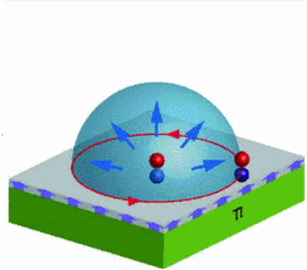

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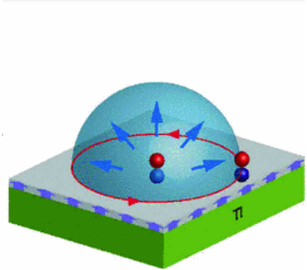

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Magnetic monopoles induced by topological surface states

Hall current

Induced current density at the surface:

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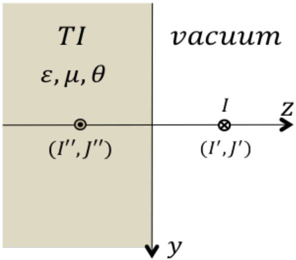

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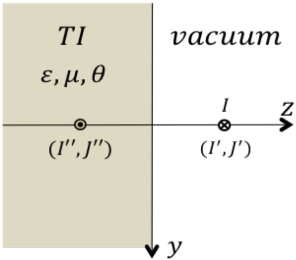

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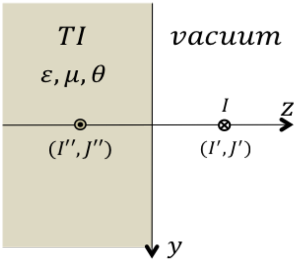

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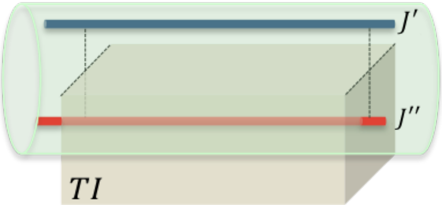

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The physical origin of magnetic current density

Maxwell's law $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ remains unaltered due to $U(1)$ gauge invariance of the θ term!!!!

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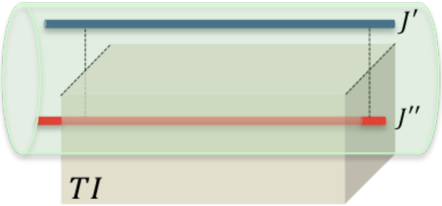

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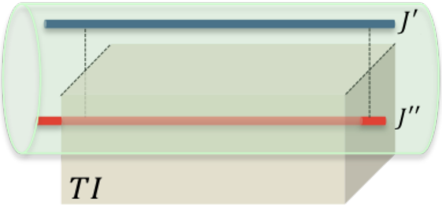

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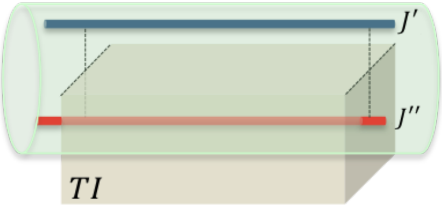

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Magnetic current induced by topological surface states

Induced current density at the surface:

$$\rho = \tilde{\theta}\mathbf{B} \cdot \mathbf{n}|_{\Sigma} = \frac{1}{2\pi}\frac{y\tilde{\theta}I'}{y^2 + b^2}$$

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Casimir force with TR-invariant TIs

Tunable Casimir repulsion between TR invariant 3D TIs

$$\mathcal{E}_c = \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d\mathbf{p}}{(2\pi)^2} \log\det\left[1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2k_3 d}\right]$$

where $k_3 = \sqrt{\mathbf{p}^2 + \xi^2}$ and the reflexion matrix

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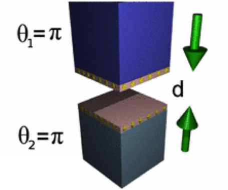

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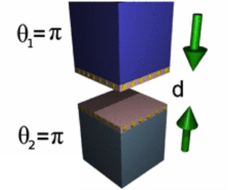

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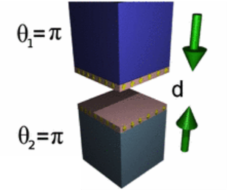

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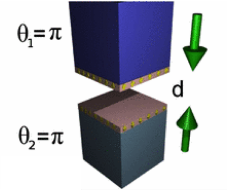

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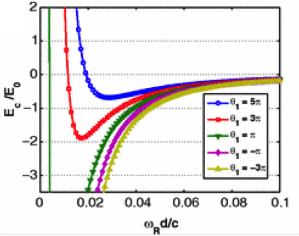

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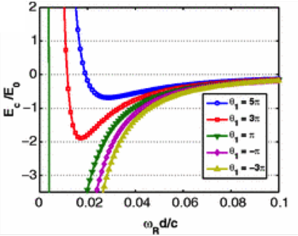

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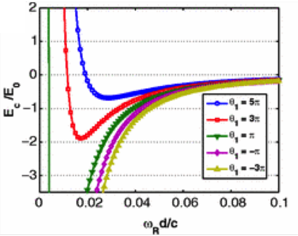

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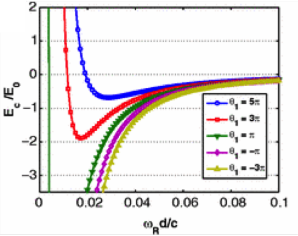

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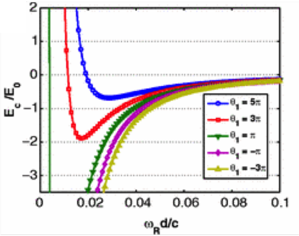

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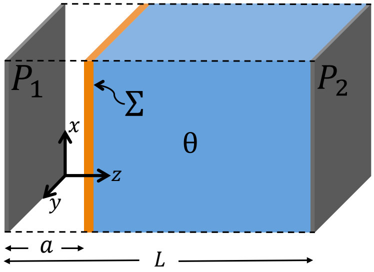
A Green's function approach

Renormalized vacuum stress-energy tensor:

$$\langle T_\theta^{\mu\nu} \rangle = \mathcal{E}_L \left(\eta^{\mu\nu} + 4n^\mu n^\nu \right) \left[u_\theta(\chi) H_{vac} + u_\theta(1-\chi) H_{TI} \right],$$

where $\mathcal{E}_L = -\pi^2/720L^4$ and $\chi = a/L$.



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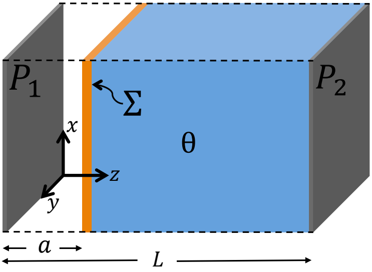

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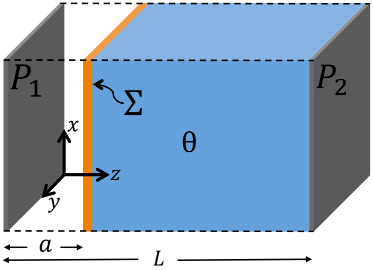

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Energy density stored between the plates:

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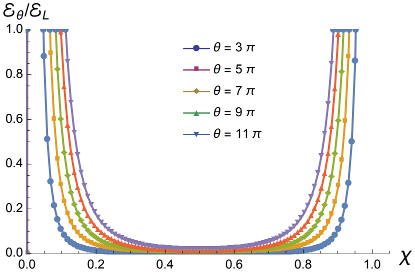
The Casimir stress upon Σ is $F_\theta = -d\mathcal{E}_\theta/da$.



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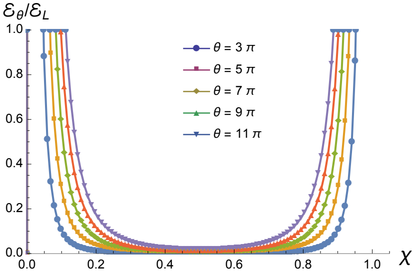

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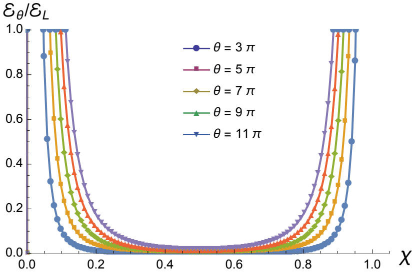

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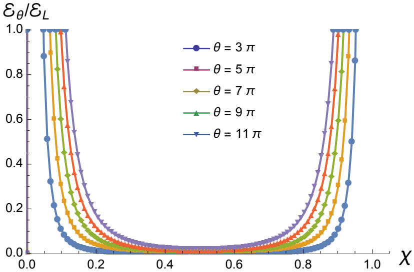

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θ	$\pm 7\pi$	$\pm 15\pi$	$\pm 23\pi$	$\pm 31\pi$	$\pm 39\pi$
f_θ	0.0005	0.0025	0.0060	0.0109	0.0172

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Gravitational θ term

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- In the presence of a gravitational background, the fermionic action is

$$S[\overline{\psi},\psi,e^{\mu}{}_{a}]=\int d^4x\sqrt{g}\,\overline{\psi}\left[e_a{}^{\mu}i\gamma^a\left(\partial_\mu-\frac{i}{2}\omega_\mu{}^{ab}\Sigma_{ab}\right)-m\right]\psi,$$

where $e^{\mu}{}_{a}$ is the vielbein, $\omega_\mu{}^{ab}$ the spin connection, and $\Sigma_{ab}=\frac{1}{4i}[\gamma_a,\gamma_b]$.

Topological Superconductor

$e^{\mu}{}_{a}$ can be interpreted as the internal order parameter of the ^{3}He-B phase.

Volkovik, Physica B **162**, 222 (1990)

Gravitational θ term

- Integration of (gapped Majorana) fermions yields the effective action

$$S_\theta = -\ln \mathcal{J} = -\frac{\theta}{1536\pi 2} \int d^4x \epsilon^{\mu\nu\rho\sigma} R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\sigma},$$

where $\mathcal{J}$ is the chiral Jacobian.

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The gravito-electromagnetic θ term

- By direct analogy (**incorrect!**)

$$S_\theta^g = \int d^4x\theta \mathbf{E}_g \cdot \mathbf{B}_g \tag{2}$$

where $\mathbf{E}_g$ and $\mathbf{B}_g$ are the gravitoelectric and gravitomagnetic fields.

- (First order) Maxwell-like equations!
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Modified field equations

- Variation of the full action $S_{EH} + S_\theta$ yields

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = C^{\mu\nu},$$

where

$$C^{\mu\nu} = \frac{-1}{2\sqrt{-g}} \left[\upsilon_\lambda \epsilon^{\lambda\mu\alpha\beta} \nabla_\alpha R^\nu{}_\beta + \upsilon_{\lambda\sigma} {}^* R^{\sigma\mu\lambda\nu} + (\mu \leftrightarrow \nu) \right]$$

is a 4D Cotton like-tensor. Here $\upsilon_\lambda = \partial_\lambda \theta$ and $\upsilon_{\lambda\sigma} = \nabla_\sigma \upsilon_\lambda$.

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Domain wall and linearized field equations

- We first consider a (planar) domain wall (similar to the surfaces of TIs)

$$\theta(z) = \theta H(z)$$

- The field equations at linear order become

$$\partial^2 h^{\mu\nu} = \frac{\Delta\theta}{2}\epsilon^{3\alpha\beta\mu}\left[\delta(z)\,\partial^2 + \delta'(z)\,\partial_3\right]\partial_\alpha h^{\nu}{}_{\beta} + (\mu \leftrightarrow \nu) \tag{3}$$

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GWs propagating across the θ interface

- For plane GW (circularly polarized) $h_{R/L}$ the equations of motion (in the TT gauge) are decoupled

$$\left(-\frac{d^2}{dz^2} \mp \Delta\theta\omega\cos\alpha\frac{d}{dz}\delta(z)\frac{d}{dz} \mp \Delta\theta\omega^3\cos^3\alpha\delta(z) - \omega^2\cos^2\alpha \right) h_{R/L} = 0,$$

where α is the incidence angle.

Gravitational point interactions!

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CS term as self-adjoint extension

- The additional operator is a self-adjoint extension of the D'Alambert operator $\Rightarrow$ **Discontinuous metric!**
- Using the theory of distributions the boundary conditions at the interface are

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Threshold anomaly

- Using the standard ansatz

$$h^{(1)}_{R/L} = e^{ik_\mu x^\mu} + \mathcal{R}_{R/L} e^{i\tilde{k}_\mu x^\mu}, \quad h^{(2)}_{R/L} = \mathcal{T}_{R/L} e^{ik_\mu x^\mu},$$

we find

$$\mathcal{T}_{R/L}(\xi) = \frac{4-\xi^2}{4+\xi^2} \quad , \quad \mathcal{R}_{R/L}(\xi) = \pm i \frac{4\xi}{4+\xi^2}.$$

Threshold anomaly

- Using the standard ansatz

$$h^{(1)}_{R/L} = e^{ik_\mu x^\mu} + \mathcal{R}_{R/L} e^{i\tilde{k}_\mu x^\mu}, \quad h^{(2)}_{R/L} = \mathcal{T}_{R/L} e^{ik_\mu x^\mu},$$

we find

$$\mathcal{T}_{R/L}(\xi) = \frac{4-\xi^2}{4+\xi^2} \quad , \quad \mathcal{R}_{R/L}(\xi) = \pm i \frac{4\xi}{4+\xi^2}.$$

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- Our solutions agree with those in the literature and allow to applications to other configurations not tackled yet.
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