Invariancia local de Lorentz y teoría de campos efectiva

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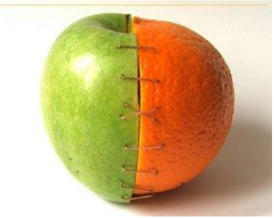
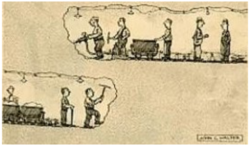
Theoretical physics status

- Fundamental physics = GR + QM.
- Accurate empirical description.
- Theoretically inconsistent.
- Need a new theory (QG).



Top down *vs.* bottom up

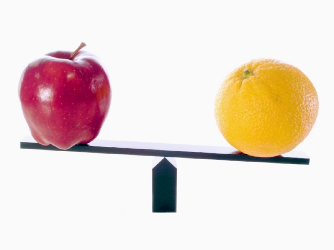
- Towards QG: top down *vs.* bottom up.
- This talk: phenomenological approach.
- Goal: find empirical evidence of new physics.
- We focus on testing Lorentz invariance.
- Typical expectation: need to reach the Planck scale.
- Effects could appear in low-energy sensitive experiments.



Lorentz invariance

- Lorentz invariance states that *all* inertial frames are equivalent.
- Inertial frames = free-falling and nonrotating (linked by Lorentz transformations).
- Equivalent = same experiments will give same results.
- Test: perform the same experiment in different frames.
- No preferred (nondynamical) spacetime directions.

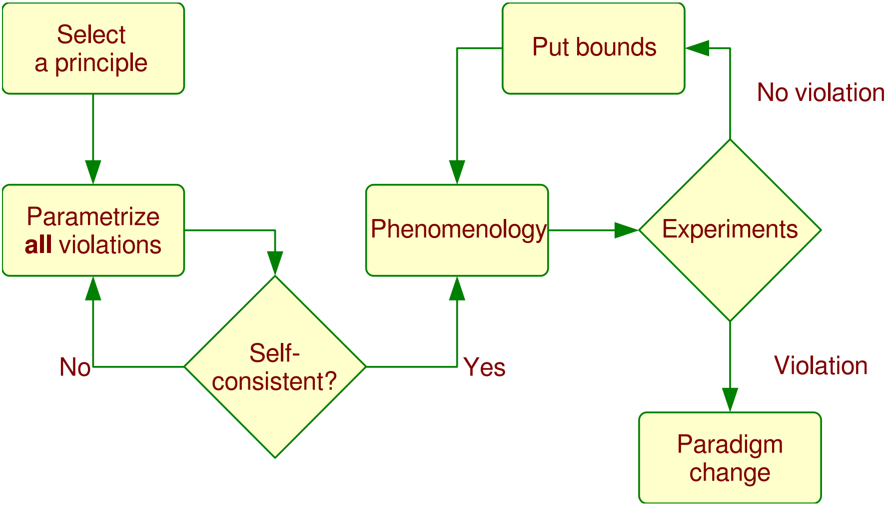
Motivation for LV



- LI is fundamental for both GR and QFT.
- LV includes CPT violation[1].
- Accommodated by most QG candidates (*e.g.*, ST, LQG).
- Possible discovery of new interactions.
- Clear phenomenology.

[1] Greenberg PRL 2002

(Idealized) phenomenologists' workflow



- Often, steps 2 and 3 not considered.
- Parametrization serves as a guide for theory and experiment.

Effective field theory

- EFT is useful when the fundamental d.o.f. are unknown.
- Requires knowing the field content and symmetries.
- Field content = standard physics;
  symmetries = standard physics without LI.
- Result: Lagrange density[1]

$$\mathcal{L} = \mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{LV}}.$$

  where $\mathcal{L}_{\mathrm{LV}}$ contains *all* possible LV additions to SM + GR.
- Naive expectation: $\mathcal{L}_{\mathrm{LV}}$ is suppressed by $E_{\mathrm{EW}}/E_{\mathrm{P}} \sim 10^{-17}$.
- Terms of every dimensionality (higher dimensions are more suppressed).

[1] "Standard Model Extension": Colladay+Kostelecký PRD 1997; PRD 1998; Kostelecký PRD 2004;...

Example: Free Dirac spinor minimal sector in flat spacetime

- Minimal = operators of renormalizable dimension:

$$\begin{aligned}
\mathcal{L} &= \frac{i}{2}\bar{\psi}\Gamma^\mu\partial_\mu\psi - \frac{i}{2}(\partial_\mu\bar{\psi})\Gamma^\mu\psi - \bar{\psi}M\psi, \\
\Gamma^\mu &= \gamma^\mu - \eta^{\mu\nu}c_{\rho\nu}\gamma^\rho - \eta^{\mu\nu}d_{\rho\nu}\gamma_5\gamma^\rho - \eta^{\mu\nu}e_\nu \\
&\quad - i\eta^{\mu\nu}f_\nu\gamma_5 - \frac{1}{2}\eta^{\mu\nu}g_{\rho\sigma\nu}\sigma^{\rho\sigma}, \\
M &= m + im_5\gamma_5 + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}.
\end{aligned}$$

- Γ^μ and M are the most general matrices (*e.g.*, m_5).
- SME coefficients: $a_\mu, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\mu, f_\mu, g_{\mu\nu\rho}, H_{\mu\nu}$.

Experiments and bounds

Experiments (partial list)

- Accelerator/collider.
- Astrophysical observations.
- Birefringence/dispersion.
- Clock-comparison.
- CMB polarization.
- Laboratory gravity tests.
- Matter interferometry.
- Neutrino oscillations.
- Particle *vs.* antiparticle.
- Resonant cavities and lasers.
- Sidereal/annual variations.
- Spin-polarized matter.

No evidence of LV $\Rightarrow$ bounds:

"Data Tables for Lorentz and CPT Violation"
Kostelecký+Russell RMP (2011),
('16 version: arXiv:0801.0287v9)

- $>$ 150 experimental results.
- Best bounds:
  matter $\sim 10^{-34}$ GeV,
  photons $\sim 10^{-43}$ GeV

Selfconsistency

- Are there theoretical restrictions to rule out LV terms?
- In flat spacetime, few interesting tests.
    - Field redefinitions: Only some linear combinations of the coefficient's components are observable.
- Strong evidence that spacetime is not flat.
- Curved spacetime tests:
    - Field redefinitions.
    - Gravitational d.o.f.
    - Spacetime boundaries.
    - Diffeomorphism invariance.
    - Dirac algorithm and Cauchy problem.

Field redefinitions in curved spacetime

- $\psi \to e^{i a_\mu x^\mu} \psi$ shows that $a_\mu \bar{\psi} \gamma^\mu \psi$ is unphysical.
- In flat spacetime it is possible to map the (minkowskian) coordinates of any point to a vector x^μ.
- This cannot be done covariantly in curved spacetime (no global minkowskian coordinates).
- Less field redefinitions $\Rightarrow$ access more coefficients[1].
- No need for curvature, only nonminkowskian coordinates[2].
- The metric can be redefined $\Rightarrow$ alternative constraints[3].

[1] Kostelecký+Tasson PRD 2011
[2] Bonder PRD 2013
[3] Bonder PRD 2015

Gravitational degrees of freedom

- In GR, possible to take the metric and the connection as *a priori* dynamically independent variables.
- The action variation w.r.t. the connection yields $\nabla_\mu g_{\nu\rho} = 0$.
- This is known as the Palatini formulation of GR.
- Minimal gravitational LV: $\mathcal{L}_{\rm LV} = \sqrt{-g} k^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$.
- For the minimal gravitational LV, the standard and Palatini approaches are equivalent[1].
- More general field redefinitions, no practical applications!
- For nonminimal LV, these approaches are inequivalent.
- Lesson: decide the gravitational d.o.f. beforehand.

[1]Bonder PRD 2015

Spacetime boundaries

- In GR, the metric variation (fixed at the boundaries) modifies Einstein equations.
- This action can be 'corrected' with a Gibbons-Hawking term:

$$\Delta S = 2\int_{\mathrm{boundary}} d^3x\sqrt{|h|}K_{\mu\nu}h^{\mu\nu},$$

  where $h_{\mu\nu} = g_{\mu\nu} \pm n_\mu n_\nu$ and $K_{\mu\nu}$ are the induced metric and extrinsic curvature of the boundary, respectively.
- In the phenomenological applications of LV, spacetime is conformally flat, which has boundaries.
- For the minimal gravitational action, add[1]

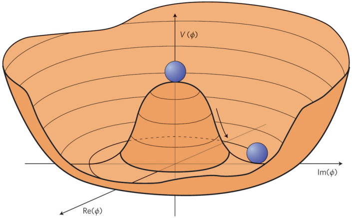
$$\Delta S_{\mathrm{LV}} = \pm 2\int_{\mathrm{boundary}} d^3x\sqrt{|h|}n_\mu n_\sigma k^{\mu\nu\rho\sigma}K_{\nu\rho}.$$

- There is no ΔS_{LV} for the nonminimal part!

[1] Bonder PRD 2015

Diffeomorphism invariance

- Nondynamical fields break (active) diffeomorphism invariance.
- Here, need to assume the standard gravitational sector.
- Well-known: diffeomorphism invariance $\Leftrightarrow \nabla_\mu T^{\mu\nu} = 0$.
- Thus, $\nabla_\mu T^{\mu\nu} \neq 0$, which implies $\nabla_\mu G^{\mu\nu} \neq 0$.
- But that goes against the Bianchi identities!
- Conclusion: LV can only be broken spontaneously[1].



[1]Kostelecký PRD 2004

Dirac algorithm and Cauchy problem

- Dirac algorithm: Is there a Hamilton density for which the evolution respects the constraints?
- Cauchy problem:
  - Is the evolution uniquely determined by proper initial data?
  - Is the evolution continuous under changes of initial data.
  - Are the effects of modifying the initial data in agreement with spacetime causal structure?

- These conditions are difficult to verify without specifying the coefficients dynamics.

Cauchy problem: concrete model

- Focus on a concrete model[1]:

$$\mathcal{L} = \frac{1}{2} D_\mu \phi D^\mu \phi^* - \frac{m^2}{2} \phi\phi^* - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\kappa}{4} (B_\mu B^\mu - b^2)^2$$

  - Flat spacetime, complex scalar field ϕ (matter), real vector field B^μ.
  - $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $D_\mu \phi = \partial_\mu \phi - ie B_\mu \phi$
    $\Rightarrow \mathcal{L}_{\mathrm{LV}} = -B^\mu J_\mu$ and no gauge freedom.
  - Generalization of the Mexican hat potential, its VEV is timelike.
  - e, κ, and b are real positive constants.

- Canonical momenta:

$$\pi^0 = \frac{\delta \mathcal{L}}{\delta \partial_0 B_0} = 0, \quad \pi^i = \frac{\delta \mathcal{L}}{\delta \partial_0 B_i} = B^{i0},$$

$$p = \frac{\delta \mathcal{L}}{\delta \partial_0 \phi} = \frac{1}{2} (\partial_0 \phi^* + ie B_0 \phi^*) = (p^*)^*.$$

[1] Bonder+Escobar PRD 2016

Cauchy problem: concrete model

$$\mathcal{L} = \frac{1}{2} D_\mu \phi D^\mu \phi^* - \frac{m^2}{2} \phi\phi^* - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\kappa}{4} (B_\mu B^\mu - b^2)^2$$

- Two second-class constraints:

$$\begin{aligned} \chi_1 &= \pi^0, \\ \chi_2 &= \partial_i \pi^i - \kappa B_0 (B_\mu B^\mu - b^2) + 2e\mathrm{Im}(\phi p). \end{aligned}$$

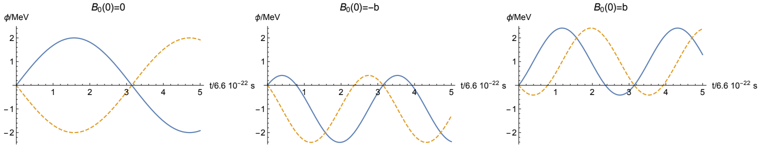
- The Dirac algorithm exhausted without inconsistencies.
- The well-known theorems don't apply for these e.o.m.
- D.o.f.: B_i, π^i, ϕ, and p (only this initial data needed) $\Rightarrow$ the initial B_0 obtained through the constraints.
- No unique initial B_0 $\Rightarrow$ ill-posed Cauchy problem!

Cauchy problem: concrete model

- Example: initially $B_i = 0$, $\pi^i = 0$, $\phi = 0$, and $p = a \in \mathbb{C}$.

$$\chi_2 = \left[B_0(0)^2 - b^2\right] B_0(0) = 0 \quad \Rightarrow \quad B_0(0) = b, 0, -b.$$

- Numerically ($\kappa = b/\mathrm{MeV}^2 = e = m/\mathrm{MeV} = \mathrm{Re}(a)/\mathrm{MeV} = \mathrm{Im}(a)/\mathrm{MeV} = 1$):



where the blue (yellow-dotted) line is for $\mathrm{Re}\phi$ ($\mathrm{Im}\phi$).

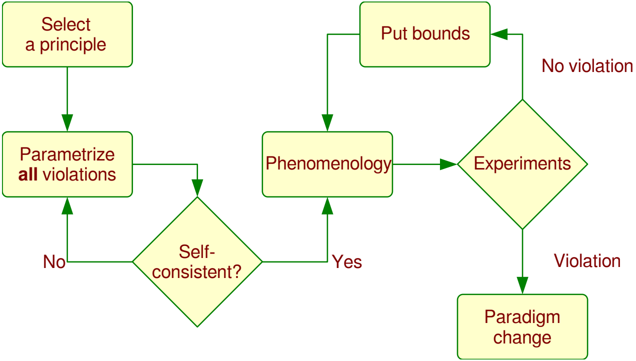
- ϕ represents matter $\Rightarrow$ physical consequences!

Cauchy problem: concrete model

- Easy fix: change the kinetic term for B_μ.
- Other options dependent on the connection and tend to damage the Cauchy problem for the metric.
- Alternatives:
  - "Only one measurement".... per experiment (unlike a fundamental constant).
  - Consider B_0 as a standard d.o.f. (*i.e.*, naive application of Lagrange's formalism $\Rightarrow$ inequivalent quantizations?, discrete number of d.o.f.).
  - Construct a criteria to choose a special B_0 (*e.g.*, initial energy, but there are degeneracies).
- Longterm goal: study if we can rule out spontaneous LV.

Conclusions

- LV promising way to empirically search for new physics.
- Effective field theory is a robust framework to parametrize LV.
- There are theoretical obstacles, mainly in curved spacetime:
  - Explicit LV is inconsistent.
  - The Cauchy problem could restrict spontaneous LV.



Gibbons-Hawking term

- In the minimal gravitational LV action-variation:

$$\begin{aligned}\delta S &\supset \frac{1}{2\kappa}\int_M d^4x\sqrt{-g}(g^{\rho\mu}\delta^\nu_\sigma + k^{\mu\nu\rho}{}_\sigma)\delta R_{\mu\nu\rho}{}^\sigma \\ &= \frac{1}{\kappa}\int_M d^4x\sqrt{-g}(\nabla_\rho\nabla_\sigma k^{\rho\mu\nu\sigma})\delta g_{\mu\nu} \\ &\quad + \frac{1}{\kappa}\int_{\partial M} d^3x\sqrt{|h|}n_\rho(2g^{\mu[\rho}g^{\nu]\sigma} + k^{\rho\mu\nu\sigma})\nabla_\sigma\delta g_{\mu\nu}\end{aligned}$$

- In ∂M: $\delta g_{\mu\nu} = 0$ (and $\delta h_{\mu\nu} = \delta n^\mu = 0$) but $n^\rho\nabla_\rho\delta g_{\mu\nu} \neq 0$.
- $K_{\mu\nu} = h^\rho_\mu\nabla_\rho n_\nu \Rightarrow \delta K_{\mu\nu} = -h^\rho_\mu n_\sigma\delta C_{\rho\nu}{}^\sigma = \frac{1}{2}h^\rho_\mu n^\sigma\nabla_\sigma\delta g_{\nu\rho} \Rightarrow$

  $n_\rho(2g^{\mu[\rho}g^{\nu]\sigma}+k^{\rho\mu\nu\sigma})\nabla_\sigma\delta g_{\mu\nu} = -\delta[(2h^{\mu\nu} \pm 2n_\rho n_\sigma k^{\rho\mu\nu\sigma})\,K_{\mu\nu}],$

- To cancel the problematic term:

$$\Delta S = \frac{1}{\kappa}\int_{\partial M} d^3x\sqrt{|h|}\,(2h^{\nu\rho} \pm 2n_\mu n_\sigma k^{\mu\nu\rho\sigma})\,K_{\nu\rho}.$$

Variation under diffeomorphisms

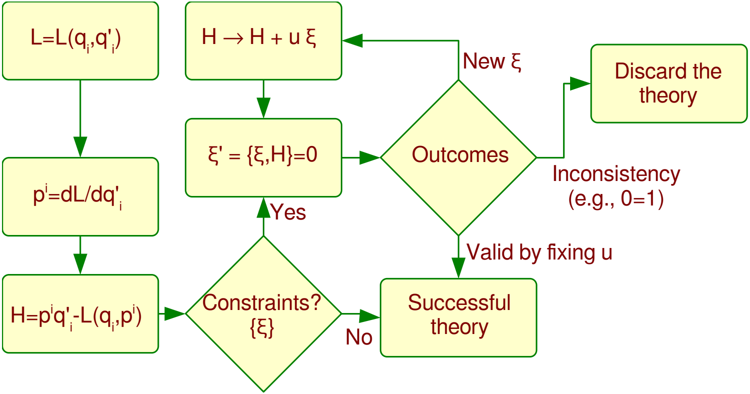
- Nongravitational LV = $S = \int d^4x\sqrt{-g}R + 2\kappa S_{\mathrm{m}}(g, \phi; k)$.
- Under a diffeo. assoc. with any w^μ (of compact sup.):

$$\begin{aligned}
\delta S &= \int d^4x \left(\frac{\delta\sqrt{-g}R}{\delta g^{\mu\nu}}\delta g^{\mu\nu} + 2\kappa\frac{\delta\mathcal{L}_{\mathrm{m}}}{\delta g^{\mu\nu}}\delta g^{\mu\nu} + 2\kappa\frac{\delta\mathcal{L}_{\mathrm{EH}}}{\delta\phi}\delta\phi \right) \\
&= \int d^4x \left(-G_{\mu\nu} + \kappa T_{\mu\nu}\right)\left(-2\nabla^{(\mu}w^{\nu)}\right) \\
&= 2\int d^4x \left(-\nabla^\mu G_{\mu\nu} + \kappa\nabla^\mu T_{\mu\nu}\right) w^\nu \\
&= 2\kappa\int d^4x w^\nu \nabla^\mu T_{\mu\nu}
\end{aligned}$$

  where use that the fields ϕ satisfy their e.o.m., $\delta g^{\mu\nu} = \mathcal{L}_w g^{\mu\nu} = -2\nabla^{(\mu}w^{\nu)}$, and the Bianchi identity.
- Hence, $\delta S = 0$ if and only if $\nabla_\mu T^{\mu\nu} = 0$.

Dirac method

- Dirac's algorithm: method to construct *the* Hamiltonian.



- May reveal inconsistencies (example: $L(q, \dot{q}) = q$).

Cauchy theorems

- Cauchy-Kowalewski requires analytic initial data, which damages causality.

Theorem

$(M, g_{\mu\nu})$ globally hyperbolic, ∇_μ any derivative operator. The following system of n linear equations for n unknown functions $\Psi_1, \ldots, \Psi_n$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\Psi_i + A^\mu_{ij}\nabla_\mu\Psi_j + B_{ij}\Psi_j + C_i = 0,$$

where A^μ_{ij}, B_{ij}, C_i are smooth vector/scalar fields, has a well-posed Cauchy problem.

- There are more general theorems[1].
- Most relevant: form of the second-derivative term.

[1] Wald's GR book