LORENTZ SYMMETRY BREAKING AND AN INTRODUCTION TO THE QUANTUM INTEGER HALL EFFECT

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PLAN OF THE TALK

- (1) INTRODUCTION
- (2) RECENT WORK ON LSB
- (3) THE TOPOLOGICAL ORIGIN OF THE INTEGER QUANTUM HALL EFFECT

NAMBU MODELS

- **The idea here is to relate gauge particles with the Goldstone bosons arising from SLSB**

- **The simplest case is the abelian Nambu-model**

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J_\mu A^\mu, \qquad A_\mu A^\mu = M^2 n_\mu n^\mu$$

where the constraint is to be solved and substituted in the Lagrangian. This can be directly generalized to the non-abelian case.

- **The NG bosons are in the components of A_μ which are orthogonal to n_μ: 3 DOF. Completely different from ED.**

- **The Hamiltonian analysis can be made by solving one field from the constraint and dealing with the remaining 3 coordinates. In the non-abelian case, generally one encounters second class contraints and the procedure gets more involved.**

- **We have shown that the NANM model coupled to a conserved current, plus the Gauss constraints imposed as an initial condition is equivalent to the $SU(N)$ MGT, thus generalizing known results for the abelian case.**

- **Thus LSB becomes unobservable and the original Goldstone modes can be identified as the corresponding gauge particles.**

- **Also we presented a perturbative proof of the equivalence between the ANM plus initial conditions, and QED in the convenient axial gauge, paying attention to the gauge fixing procedure.**

- **Generalization to arbitrary gauge theories defined by having only first class constraints which produce non trivial gauge transformations. The ENM is then defined by an arbitray constraint upon the gauge theory coordinate variables.**

- **Work in collaboration with Carlos Escobar Ruiz. [PRD92(2015)025013, PRD92(2015)025042, preprint** *Extended Nambu models: their relation to gauge theories* **(submitted for publication).**

BREIT-WHEELER SCATERING IN THE SME

- Calculate the cross section for the process $\gamma_1 + \gamma_2 \to e^+ + e^-$, where γ_1 is a photon with very high energy (del orden de 100 TeV) and γ_2 is a low energy photon in the interstellar medium.
- Use the fermion sector of the SME to calculate the cross section.
- The cross section has to be further integrated over all the angles that the low energy photons from with the incident high energy photon: need to calculate in the reference frame where the directions of those photons form an arbitrary angle.
- Work in collaboration with S. Ramirez (Master Thesis), J. D. Vergara, Carlos Escobar. Need some clarifications.

Resultados de Ellis et al. para el CTA

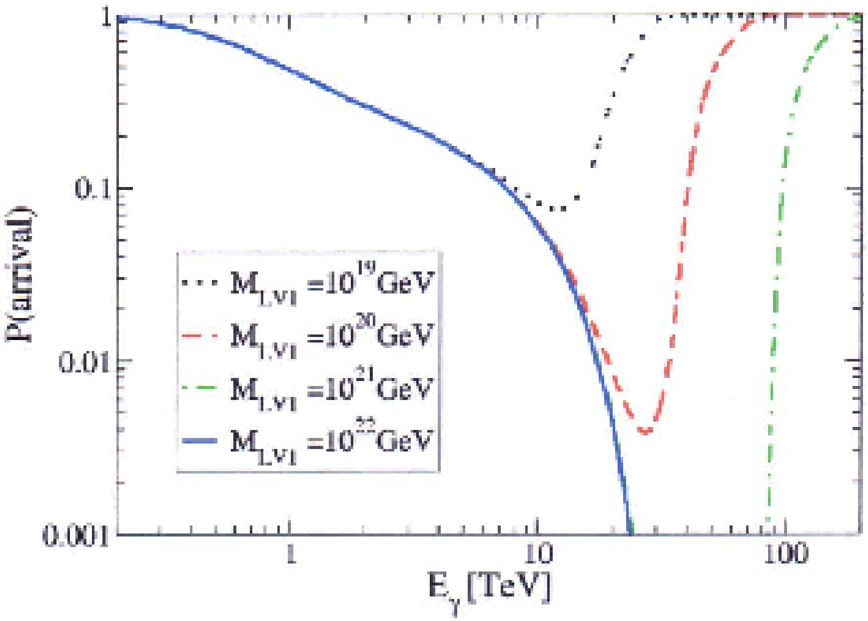

Figure 2. *The arrival probability of a photon emitted from a hypothetical source at redshift $z = 0.05$ as a function of energy. The different curves represent different values of the Lorentz-violating scale M_{LV1}. VHE photons with energies $\gtrsim 100$ TeV can travel through the CMB effectively unimpeded.*

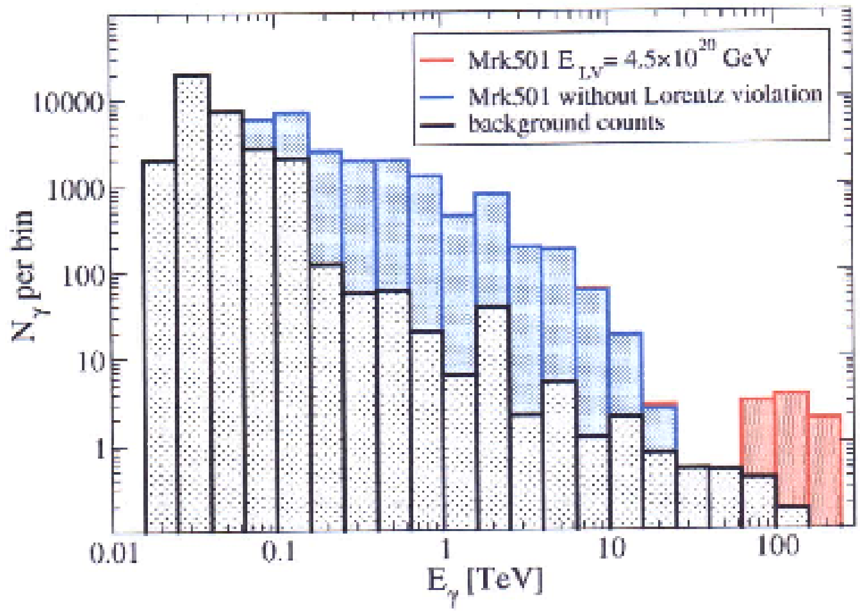

Figure 3. *The expected number of signal events (blue and red columns) compared with the expected number of background events alone (black columns), calculated for 50 hours of observation of the AGN Markarian 501, assuming the power-law spectrum (3.3). The red columns represent the expected flux assuming a Lorentz violating energy scale $M_{LV1} = 4.5 \times 10^{20}$ GeV, whereas the blue columns denote the flux expected in the absence of Lorentz violation, and are identical to the red columns below 15 TeV.*

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La esructura básica es

$$x_{\gamma\gamma}(E_\gamma)^{-1} = \frac{1}{8E_\gamma^2\beta_\gamma}\int_{\epsilon_{min}}^{\infty} d\epsilon \frac{n(\epsilon)}{\epsilon^2}\int_{s_{min}}^{s_{max}} ds(s - m_\gamma^2(E_\gamma))\sigma(s)$$

THE INTEGER QUANTUM HALL EFFECT (IQHE)

- **Huge interest in topological aspects of quantum matter, enhanced by the recent Nobel prize to M. Kosterlitz, D. Haldane and D. J. Thouless.**

- **....., Quantum integer Hall effect, Quantum fractionally Hall effect, Topological insulators, etc. etc.**

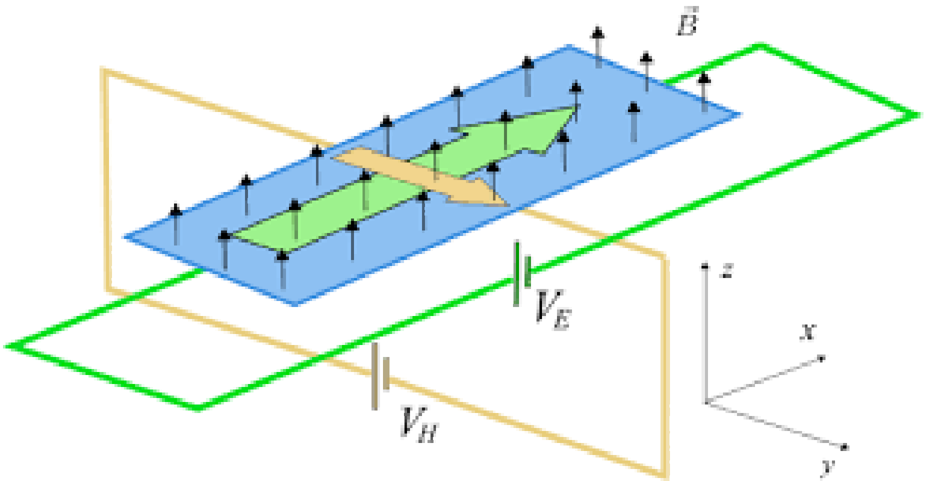
- **Microscopic approach (people from condensed matter) and Effective field theory approach (people from high energy).**

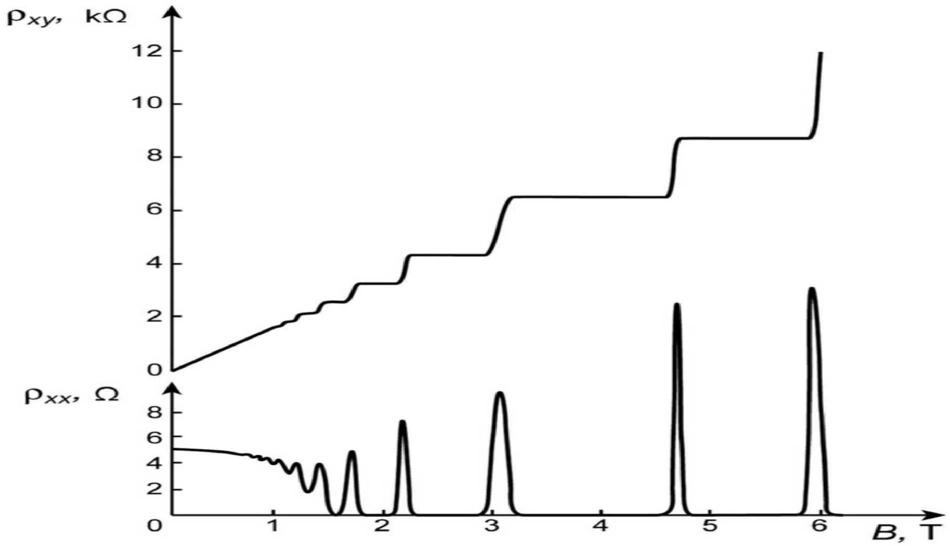
- **Recent work by A. Martin-Ruiz, L. Urrutia (ICN) and M. Cambiaso (UNAB-Santiago) on the electromagnetic response of topological insulators: [PRD92(2015)125015, EPL113(2016)6005, PRD93(2016)045022, PRD94(2016)085019.**

- **Try to give a feeling on how does the basic topology enters in the simplest case of the IQHE.**

- **Require: (a) Basic properties of the IQHE, (b) The Kubo formula for the conductivities, (c) The idea of a Berry connection and (d) putting all together.**

PROPERTIES OF THE IQHE





- $\rho_{xx} = 0$ at the plateaux, $B = \frac{2\pi\hbar n}{Ne}$ in the center of the plateaux, $\rho_{xy} = \frac{2\pi\hbar}{e^2}\frac{1}{N}$, N integer. Also $\sigma_{xy} = \frac{e^2}{2\pi\hbar}N$

THE KUBO FORMULA

- This is a particular case of the linear response theory of a system to a perturbation.

- In this case the perturbation is the electric field E in the direction x and the response of the system is the current J_x, J_y which determines the conductivity matrix σ such that $\mathbf{J} = \sigma \mathbf{E}$

- The unperturbed Hamiltonian H_0 is arbitrary. The perturbation is $\Delta H = \mathbf{J} \cdot \mathbf{A}$. We work in a gauge with $A_0 = 0$, so that $\mathbf{E} = -\partial \mathbf{A}/\partial t$. The dimensions of $\mathbf{J}$ are Amp/m^2.

- We take $\mathbf{E}(t) = \mathbf{E} e^{-i\omega t}$, such that $\mathbf{A}(t) = \mathbf{E} e^{-i\omega t}/i\omega$ with $\omega \to 0$ at the end.

- We work in the interaction picture. The system is in the the ground state $|0\rangle$ at $t \to -\infty$ and evolves with the operator

$$U(t) = T \exp\left(-\frac{i}{\hbar}\int_{-\infty}^{t} dt' \, \Delta H(t')\right).$$

- We want to calculate

$$\langle \mathbf{J}(t) \rangle = \langle 0(t) | \mathbf{J}(t) | 0(t) \rangle.$$

- To first order in perturbation theory we get

$$\langle J_i(t)\rangle = \frac{1}{\hbar\omega}\left(\int_0^\infty dt'' e^{i\omega t''}\langle 0|\left[J_j(0),\ J_i(t'')\right]|0\rangle\right) E_j e^{-i\omega t}.$$

- From here we read

$$\sigma_{xy} = \frac{1}{\hbar\omega}\int_0^\infty dt e^{i\omega t}\langle 0|\left[J_y(0),\ J_x(t)\right]|0\rangle.$$

- Recalling that operators evolve according to $J_i(t) = e^{iH_0 t/\hbar} J_i(0) e^{-iH_0 t/\hbar}$ it is possible to rewrite σ_{xy} in terms of eigenstates $|n\rangle$ of H_0 as

$$\sigma_{xy} = i\hbar \sum_{n\neq 0}\left[\frac{\langle 0|J_y(0)|n\rangle\langle n|J_x(0)|0\rangle - \langle 0|J_x(0)|n\rangle\langle n|J_y(0)|0\rangle}{(E_n - E_0)^2}\right]$$

in the limit $\omega \to 0$.

- The challenge now is to show how the above result is related to topology , with the further consequence of its quantization.

- For our purposes it is convenient to associate V_H to the flux $\Phi_V = \Phi_y$ and the electric field to the flux $\Phi_J = \Phi_x$.
- In this way, the Hall arrangement becomes a torus.

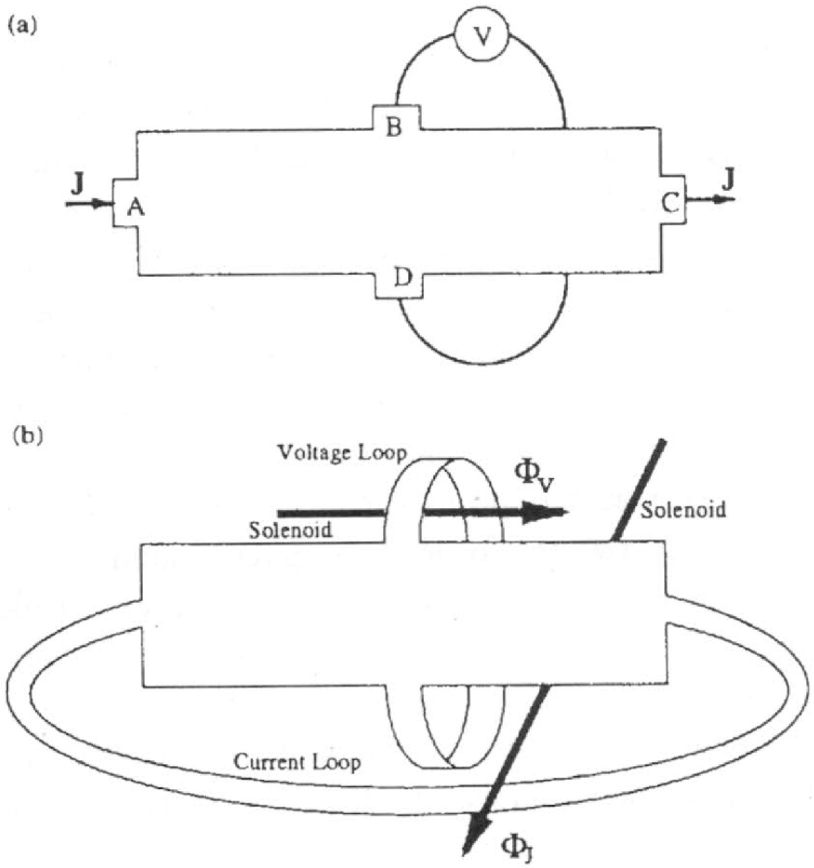
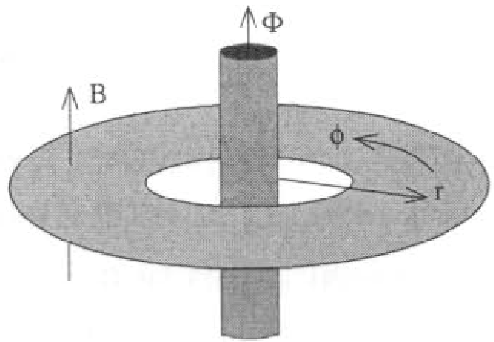


Fig. 14. The Hall bar, with current and voltage leads, shown in (a) can be replaced by the arrangement shown in (b), where the voltage is supplied by changing flux Φ_V through one loop, and the current is monitored by observing changes of the flux Φ_J through the other loop.

- In order to appreciate the effect of contant solenoidal fluxes Φ, let us consider the following situation



- The Hamiltonian is

$$H = \frac{1}{2m}\left[-\hbar^2 \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right) + \left(-i\frac{\hbar}{\rho}\frac{\partial}{\partial \phi} + \frac{eBr}{2} + \frac{e\Phi}{2\pi\rho}\right)^2\right].$$

- The flux produces a local gauge transformation. We can factor it as

$$\Psi(\rho, \phi) \rightarrow \exp\left(\frac{ie\Phi\phi}{2\pi\hbar}\right)\Psi(\rho, \phi).$$

- But the wave function must satisfy periodicity in ϕ, which requires $\frac{e\Phi}{2\pi\hbar} = N$. In other words Φ must be an integer multiple of the quantum of a flux $\Phi_0 = \frac{2\pi\hbar}{e}$.
- Only under this condition the spectrum remains invariant.

- Next we apply the addition of fluxes to our Hall torus, which we characterize as a rectangle with sides L_x and L_y, with extremes identified. Here we work in the Landau gauge $A_x = \frac{\Phi_x}{L_x}, \quad A_y = \frac{\Phi_y}{L_y} + Bx$. These define our unperturbed Hamiltonian H_0.

- Following the idea of the linear response aproximation, we add the following perturbation in order to identify the resulting currents

$$\Delta H = -j_x \frac{\Phi_x}{L_x} - j_y \frac{\Phi_y}{L_y},$$

Let us notice that in two dimensions $j = -env$ with n the electron density (#/cm2). The dimension of j is then Amp/m

- The effect of the perturbation upon the ground state of the system $\Psi_0(\Phi_i)$ is

$$\begin{aligned}
|\Psi_0(\Phi_i + \delta\Phi_i)\rangle &= |\Psi_0\rangle + \sum_{n\neq 0} \frac{\langle n|\Delta H|\Psi_0\rangle}{E_n - E_0}|n\rangle \\
&= |\Psi_0\rangle - \sum_i \frac{1}{L_i} \sum_{n\neq 0} \frac{\langle n|j_i|\Psi_0\rangle}{E_n - E_0}|n\rangle\delta\Phi_i \\
|\frac{\partial\Psi_0}{\partial\Phi_i}\rangle &= -\frac{1}{L_i}\sum_{n\neq 0} \frac{\langle n|j_i|\Psi_0\rangle}{E_n - E_0}|n\rangle, \quad where \quad J_i = \frac{j_i}{L_i}
\end{aligned}$$

- Substituting in the general expression for σ_{xy} we find

$$\sigma_{xy} = i\hbar \left(\frac{\partial}{\partial \Phi_y} \langle \Psi_0 | \frac{\partial \Psi_0}{\partial \Phi_x} \rangle - \frac{\partial}{\partial \Phi_x} \langle \Psi_0 | \frac{\partial \Psi_0}{\partial \Phi_y} \rangle \right) .$$

- Recalling that to maintain global gauge invariant we require that the Φ_i to be periodic in terms of the quantum of flux. In this way, the parameter space is also a torus which we denote by T^2_Φ, which we can parametrize with adimensional angular variables

$$\theta_i = \frac{2\pi \Phi_i}{\Phi_0}, \qquad \theta_i \in [0, 2\pi), \qquad \Phi_0 = \frac{2\pi\hbar}{e}$$

- After this change of variables, we identify the Berry connection $\mathcal{A}_i$ and curvature $\mathcal{F}_{ij}$, respectively

$$\mathcal{A}_i = \langle \Psi_0 | \frac{\partial \Psi_0}{\partial \theta_i} \rangle, \qquad \mathcal{F}_{ij} = \frac{\partial \mathcal{A}_i}{\partial \theta_j} - \frac{\partial \mathcal{A}_j}{\partial \theta_i}.$$

- In this way

$$\sigma_{xy} = -\frac{e^2}{\hbar} \mathcal{F}_{xy}$$

- The Berry phase is a two-form and thus can be integrated over a two-surface. The fundamental property is

$$\oint_S F_{ij}\, dS^{ij} = 2\pi C, \qquad C \qquad integer,$$

for any closed two-surface S. The number C is called the Chern number.

- Taking the average over the previous expression for σ_{xy} one obtains

$$\frac{1}{(2\pi)^2}\int d\theta_x d\theta_y \sigma_{xy} = \sigma_{xy} = \frac{1}{(2\pi)^2}\int d\theta_x d\theta_y \left(\frac{e^2}{\hbar}\right)\mathcal{F}_{xy} = -\frac{e^2}{2\pi\hbar}C.$$

which shows the topological origin of the IQHE quantization.

- This is not the whole story, there are many relevant features which I did not consider, for example: (1) what is the relevance of the edge states, (2) why is the result independent of the impurities, (3) why the plateaux are extended in some range of the magnetic field, (4) which is the effective theory that describes the IQHE, etc.