Supported in part by the European Commission under Capacities 7th Framework Programme project EuCARD-2, grant agreement 312453.

L(λ)

Optimized Monochromatization for Direct Higgs Production at the FCCe+e-

o

Holiday Inn Pachuca 2016
Pachuca, Hidalgo, 101116

Motivation

Motivations

- Questions still unanswered about Higgs boson properties
- Post-LHC era requirements
- Upcoming beamstrahlung effects
- FCC proposal opportunity
- Monochromatization for high center-of-mass energy resolution

Objectives

Objectives

- Present a review of the field of accelerator physics and future proposals
- Characterize beamstrahlung at the FCCe+e-
- Obtain the self-consistent energy spread and self-consistent emittance
- Optimize the IP parameters to produce opposite dispersion
- Develop a monochromatization scheme for the FCCe+e-
- Describe the modification to the lattice design and the final focusing system

Accelerator Physics

Storage Ring A Qualitative Description

- Particles are injected into a vacuum chamber until the beam is stored.
- The guide field provides focusing capabilities driving particles toward the ideal designed.
- Stored particles loss energy by synchrotron radiation, compensated in average by the RFS.
- The periodic accelerating field collects the particles into circulating bunches.
- Energy loss by synchrotron radiation leads into damping of all oscillation amplitudes.
- Amplitude excitation occurs as an effect of the quantum nature of radiation.
- A balance between radiation damping and quantum excitation is reached.

Monochromatization principle A "simple explanation"

Standard collision Dispersion has the same sign in the IP

$$\begin{array}{ccccc} & \xrightarrow{E+\Delta E} & & \xleftarrow{E+\Delta E} & \\ e^- & \xrightarrow{E} & & \xleftarrow{E} & e^+ \\ & \xrightarrow{E-\Delta E} & & \xleftarrow{E-\Delta E} & \end{array}$$

$$w = 2(E_0 + \epsilon)$$

Monochromatization principle A “simple explanation”

Standard collision Dispersion has the same sign in the IP

$$
\begin{array}{ccc}
 & \xrightarrow{E+\Delta E} \quad \xleftarrow{E+\Delta E} & \\
e^- & \xrightarrow{E} \quad \xleftarrow{E} & e^+ \\
 & \xrightarrow{E-\Delta E} \quad \xleftarrow{E-\Delta E} &
\end{array}
$$

$$w = 2(E_0 + \epsilon)$$

Monochromatization Dispersion has opposite sign in the IP

$$
\begin{array}{ccc}
 & \xrightarrow{E+\Delta E} \quad \xleftarrow{E-\Delta E} & \\
e^- & \xrightarrow{E} \quad \xleftarrow{E} & e^+ \\
 & \xrightarrow{E-\Delta E} \quad \xleftarrow{E+\Delta E} &
\end{array}
$$

$$w = 2E_0 + 0(\epsilon)^2$$

After doing this we get an **Enhancement** of **energy resolution**, and sometimes increase of the relative frequency of the events at the centre of of the distribution.

Monochromatization principle In Optical Terms

- A special arrangement of elements or modification of the available ones is required
- In the context of this proposal, exploration of the current FCCee design is needed
- In literature, insertions to implement this idea is known as monochromator

Monochromatization Dispersion has opposite sign in the IP

$$
\begin{array}{ccccc}
 & \xrightarrow{E+\Delta E} & & \xleftarrow{E-\Delta E} & \\
e^- & \xrightarrow{E} & & \xleftarrow{E} & e^+ \\
 & \xrightarrow{E-\Delta E} & & \xleftarrow{E+\Delta E} &
\end{array}
$$

$$w = 2E_0 + 0(\epsilon)^2$$

Hill Differential Equation

$$x'' + K(s)x = 0 \qquad y'' - K(s)y = 0, \qquad K(s) \equiv \frac{1}{(B\rho)}\left(\frac{\partial B_y}{\partial x}\right)(s) \qquad K(s+C) = K(s)$$

Ansatz

$$x(s) = A\omega(s)\cos(\phi(x) + \phi_0)$$

Courant-Snyder Parameters

$$\beta(s) \equiv \frac{\omega^2(s)}{k} \qquad \alpha(s) \equiv -\frac{1}{2}\beta\prime(s) \qquad \gamma(s) \equiv \frac{1+\alpha^2(s)}{\beta(s)}$$

Courant-Snyder Invariant

$$\mathcal{W} = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s) \qquad \mathcal{W} \equiv A^2$$

Off-Momentum Particles

$$p = p_0(1+\delta)$$

Relative Momentum Deviation

$$\delta \equiv \frac{\Delta p}{p_0}$$

Momentum Contribution

$$x(s) = x_\beta(s) + x_\delta(s)$$

Dispersion Function

$$x_\delta(s) = D(s)\delta$$

Approximations

$$x \ll \rho \qquad \delta \ll 1$$

Dipole Contribution

$$\frac{\partial^2 x}{\partial \theta^2} + (1-n)x = \rho\delta$$

Dispersion Contribution

$$x(\theta) = A\cos\sqrt{1-n}\theta + B\sin\sqrt{1-n}\theta + \frac{\rho}{1-n}\delta$$

General Contribution

$$x'' + K(s)x = \frac{\delta}{\rho(s)}$$

Dispersion Contribution

$$x(s) = C(s)x_0 + S(s)x'_0 + D(s)\delta_0$$

Dispersion Function

$$D(s) = S(s)\int_0^s \frac{C(\tau)}{\rho(\tau)}d\tau - C(s)\int_0^s \frac{S(\tau)}{\rho(\tau)}d\tau$$

Area of the Ellipse

$$\epsilon = \pi\mathcal{W} = \pi[\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)]$$

Size of the Distribution

$$\sigma_{RMS} = \sqrt{\epsilon\beta(s)}$$

Normalized Emittance

$$\epsilon_N = \beta_r\gamma_r\epsilon$$

Synchrotron Radiation

Synchrotron radiation Energy Deviation Effects

Off-Energy Particles

$$E = E_0 + \epsilon,$$

Off-Energy Contribution

$$x = x_\beta + x_\epsilon$$

Radius of Curvature

$$G(s) = \rho^{-1}(s)$$

Equation of Motion for Off-Energy Particles

$$x'' = -K_x x + G\left(\frac{\epsilon}{E_0}\right)$$

Ultrarelativistic Limit

$$E \gg mc^2$$

$$p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} \approx \frac{E}{c}$$

$$\frac{\Delta p}{p_0} = \frac{\Delta E}{E_0}$$

$$\delta = \frac{\epsilon}{E_0}$$

Synchrotron radiation Off-Energy Equation of Motion

Off-Energy Contribution

$$x_\epsilon = D(s)\epsilon/E_0$$

Dispersion Equation

$$D''(s) = -K_x D(s) + G(s)$$

D $K_1 = 0$ $G = 0$ $D(s) = D'(s_0)s + D(s_0)$

FQ $K_1 \neq 0$ $G = 0$ $D(s) = A\cos(\sqrt{-K_x}x +$ if $K_1 < 0$

DQ $K_1 = 0$ $G \neq 0$ $D(s) = A\cosh(\sqrt{K_x}x + \theta)$ if $K_1 > 0$

BM $$D'' = -G^2\left(D - \frac{1}{G}\right)$$

Synchrotron radiation Dilation Factor

Length Increment

$$l_\epsilon = \oint dl = \oint (1 + G(s)x_\epsilon)ds = L + \delta l_\epsilon$$

$$\delta l_\epsilon = \oint G(s)x_\epsilon ds = \frac{\epsilon}{E_0} \oint G(s)D_x(s)ds$$

Dilation Factor

$$\frac{\delta l_\epsilon}{L} = \alpha \frac{\epsilon}{E_0} \qquad \alpha = \frac{1}{L} \oint G(s)D_x(s)ds$$

Ratio Orbit Length Revolution Time

$$\frac{L}{T_0} = \frac{L + \delta l}{T_0 + \delta T} \approx \frac{L}{T_0}\left(1 - \frac{\delta T}{T_0} + \frac{\delta l}{L}\right)$$

Relative Revolution Time Increment

$$\frac{\delta t}{T_0} = \frac{\delta l}{L} = \alpha \frac{\epsilon}{E_0}$$

Synchrotron radiation Energy Oscillations

Azimuthal Displacement

$$z(t) = s(s) - s_c(s)$$

Time Displacement

$$\tau = \frac{s(s) - s_c(s)}{c}$$

Time Displacement Evolution

$$\delta z = -\alpha \frac{\epsilon}{E_0} L \qquad \frac{d\tau}{dt} = -\alpha \frac{\epsilon}{E_0}$$

RF Energy Supply

$$eV(\tau) = U_{rf}(\bar{t}_s - \tau)$$

Energy Change

$$\delta U = eV(\tau_1) - U_{rad}(\epsilon)$$

Energy Evolution

$$\frac{d\epsilon}{dt} = \frac{1}{T_0}(e\dot{V}_0\tau - D\epsilon)$$

Synchrotron radiation Energy Oscillations

Time Displacement Evolution

$$\frac{d^2\tau}{dt^2} + 2\alpha_\epsilon \frac{d\tau}{dt} + \Omega^2\tau = 0 \qquad \alpha_\epsilon = \frac{D}{2T_0} \qquad \Omega^2 = \frac{\alpha e\dot{V}_0}{T_0 E_0}$$

Time displacement

$$\tau(t) = A\exp^{-\alpha_\epsilon t}\cos(\Omega t - \theta_0)$$

If $\alpha_\epsilon \ll \Omega$

In Complex Notation

$$\epsilon(t) = \widetilde{\epsilon} e^{-(\alpha_\epsilon - i\Omega)t} \qquad \tau(t) = \widetilde{\tau} e^{-(\alpha_\epsilon - i\Omega)t} \qquad \widetilde{\epsilon} = -i\frac{\Omega E_0}{\alpha}\widetilde{\tau}$$

Synchrotron radiation Quantum Radiation Effects

Instantaneous Power

$$P_\gamma = \int_0^\infty \frac{dP_\gamma}{d\omega}(\omega)d\omega = \int_0^\infty \int_{d\Omega} \frac{d^2P_\gamma}{d\Omega d\omega}(\Omega, \omega,)d\Omega d\omega$$

Spectrum Function

$$\frac{dP_\gamma}{d\omega}(\omega) = \frac{P_\gamma}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

Critical Frequency

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi \int_\xi^\infty K_{5/3}(\bar{\xi})d\bar{\xi}$$

Photon Spectrum Rate

$$\frac{dn_\gamma}{dt}(u) = \frac{P_\gamma}{u_c^2} F\left(\frac{u}{u_c}\right)$$

$$F(\xi) = S(\xi)/\xi$$

Synchrotron radiation Quantum Radiation Effects

Statistical Properties

$$\frac{dN_\gamma}{dt} = \int_0^\infty \frac{dn_\gamma}{dt}(u)du$$

$$\frac{dN_\gamma}{dt} = \frac{15\sqrt{3}}{8}\frac{P_\gamma}{u_c}$$

$$< u >= \left(\frac{dN_\gamma}{dt}\right)^{-1} \int_0^\infty u \frac{dn_\gamma}{dt} du.$$

$$< u >= \frac{8}{15\sqrt{3}} u_c$$

$$< u^2 >= \left(\frac{dN_\gamma}{dt}\right)^{-1} \int_0^\infty u^2 \frac{dn_\gamma}{dt} du$$

$$< u^2 >= \frac{11}{27} u_c^2$$

Excitation Term

$$< u^2 > \left(\frac{dN_\gamma}{dt}\right) = \frac{55}{24\sqrt{3}} r_e \hbar m c^4 \frac{\gamma^7}{\rho^3}$$

Synchrotron radiation Quantum Radiation Effects

Energy Deviation Oscillation

$$\epsilon = A_0 \exp^{i\Omega(t-t_0)}$$

Photon Emission

$$\epsilon = A_0 \exp^{i\Omega(t-t_0)} - u \exp^{i\Omega(t-t_i)} \qquad \epsilon = A_1 \exp^{i\Omega(t-t_1)}$$

New Amplitude

$$A_1^2 = A_0^2 + u^2 - 2A_0 \cos\Omega(t_i - t_0)$$

Probable Amplitude Change

$$< \delta A^2 > = < A_1^2 - A_0^2 > = u^2$$

Probable Amplitude Squared

$$\left\langle \frac{dA^2}{dt} \right\rangle = \frac{d\left\langle A^2 \right\rangle}{dt} = \frac{dN_\gamma}{dt} u^2 \qquad \frac{d\left\langle A^2 \right\rangle}{dt} = -2\frac{\left\langle A^2 \right\rangle}{\tau_\epsilon} \qquad \left\langle A^2 \right\rangle = \frac{1}{2}\tau_\epsilon \frac{dN_\gamma}{dt} u^2$$

For Sinusoidal Energy Oscillation

$$\sigma_\epsilon^2 = \left\langle \epsilon^2 \right\rangle = \frac{\left\langle A^2 \right\rangle}{2} = \frac{1}{4}\tau_\epsilon \frac{dN_\gamma}{dt} u^2$$

Beamstrahlung

Beamstrahlung Sokolov-Ternov Theory

Lorentz Invariant

$$\Upsilon \equiv \frac{e}{m_e^3}\sqrt{|(F_{\mu\nu}p^\nu)^2|} = \frac{B}{B_c} = \frac{2}{3}\frac{\hbar\omega_c}{E_e} \qquad \Upsilon_{\rm max} = 2\frac{r_e^2\gamma N_b}{\alpha\sigma_z(\sigma_x^* + \sigma_y^*)} \qquad \Upsilon_{\rm ave} \approx \frac{5}{12}\Upsilon_{\rm max} = \frac{5}{6}\frac{r_e^2\gamma N_b}{\alpha\sigma_z^*(\sigma_x^* + \sigma_y^*)}$$

Quantum Spectrum Formula

$$\frac{dW_\gamma}{d\omega\hbar} = \frac{\alpha}{\sqrt{3}\hbar\pi\gamma^2}\left(\int_\xi^\infty K_{5/3}(\xi')\,d\xi' + \frac{y^2}{1-y}K_{2/3}(\xi)\right) \qquad \xi = \frac{2\hbar\omega}{3\Upsilon(E-\hbar\omega)} \qquad y \equiv \omega/E_e$$

Classical Spectrum Formula

$$\Upsilon \to 0 \qquad \frac{dW_\gamma}{d\omega\hbar} = \frac{\alpha}{\sqrt{3}\pi\gamma^2}\int_\xi^\infty K_{5/3}(\xi')\,d\xi'$$

Beamstrahlung Sokolov-Ternov Theory

Photon Emission Rate

$$\frac{dN_\gamma}{dt} = \int_0^{E_e/\hbar} \frac{dW_\gamma}{d\omega}\, d\omega$$

Average Emitted Photons

$$n_\gamma \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x + \sigma_y} \approx \frac{12}{\pi^{3/2}} \frac{\alpha r_e N_b}{\sigma_x}$$

Relative Energy Loss

$$\delta_B \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 \gamma N_b^2}{\sigma_z(\sigma_x + \sigma_y)^2} \approx \frac{24}{3\sqrt{3}\pi^{3/2}} \frac{r_e^3 \gamma N_b^2}{\sigma_z \sigma_x^2}$$

Statistical Properties

$$\langle u \rangle = \frac{\delta_B}{n_\gamma} \approx \frac{2\sqrt{3}}{9} \frac{r_e^2 N_b \gamma}{\alpha \sigma_z \sigma_x}$$

Classical Formula

$$\langle u^2 \rangle \approx \frac{25 \times 11}{64} \langle u \rangle^2$$

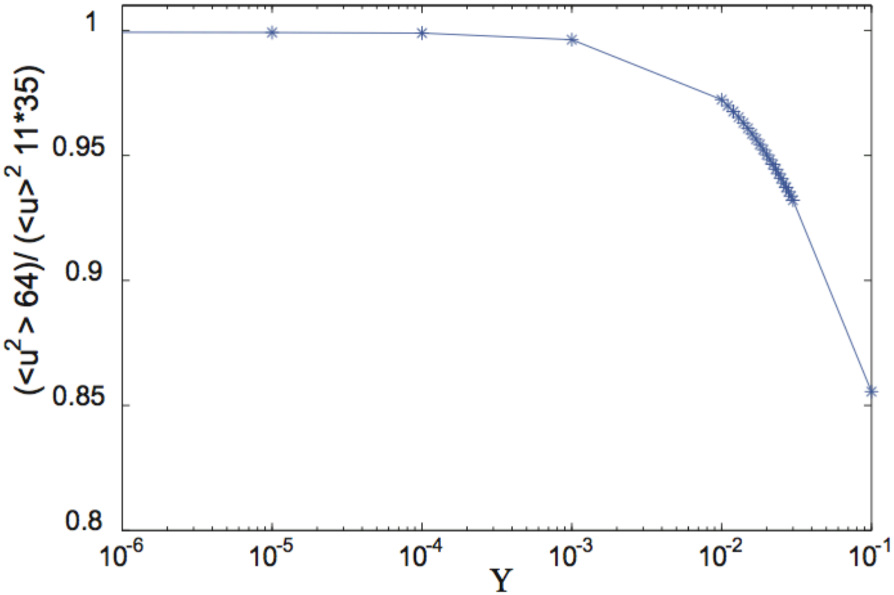
Beamstrahlung Sokolov-Ternov Theory

Photon Emission Rate

$$\frac{dN_{\gamma BS}}{dt} = \int_0^{E_e} (dW_\gamma/d\omega)d\omega$$

Statistical Properties

$$\langle u^2 \rangle = \left(\frac{dN_{\gamma BS}}{dt}\right)^{-1} \int_0^{E_e} \omega^2 (dW_\gamma/d\omega)d\omega \qquad \langle u \rangle = \left(\frac{dN_{\gamma BS}}{dt}\right)^{-1} \int_0^{E_e} \omega (dW_\gamma/d\omega)d\omega$$



Beamstrahlung Self-Consistant Energy Spread

Excitation Term

$$n_\gamma \langle u^2 \rangle \approx 1.4 \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 (\sigma_x + \sigma_y)^3} \approx 192 \frac{r_e^5 N_b^3 \gamma^2}{\sigma_z^2 \sigma_x^3}$$

Total Energy Spread

$$\sigma_{\mathrm{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \sigma_{\delta,\mathrm{BS}}^2$$

$$\sigma_{\delta,\mathrm{tot}}^2 - \sigma_{\delta,\mathrm{SR}}^2 = A \left(\frac{\sigma_{\delta,\mathrm{SR}}}{\sigma_{\delta,\mathrm{tot}}} \frac{1}{\sigma_{z,\mathrm{SR}}} \right)^2$$

$$\sigma_{\delta,\mathrm{tot}} = \left[\frac{1}{2} \sigma_{\delta,\mathrm{SR}}^2 + \left(\frac{1}{4} \sigma_{\delta,\mathrm{SR}}^4 + A \frac{\sigma_{\delta,\mathrm{SR}}^2}{\sigma_{z,\mathrm{SR}}^2} \right)^{1/2} \right]^{1/2}$$

Bunch Length

$$\sigma_{z,\mathrm{tot}} = \frac{\alpha_{\mathrm{C}} C}{2\pi Q_s} \sigma_{\delta,\mathrm{tot}}$$

Dispersion Invariant

$$\mathcal{H}_x^* \equiv \frac{\left(\beta_x^* D_x'^{\,*} + \alpha_x^* D_x^*\right)^2 + D_x^{*2}}{\beta_x^*}$$

Beamstrahlung Self-Consitant Equations

Self-Consistent Equations

$$\epsilon_{x,\mathrm{tot}} = \epsilon_{x,\mathrm{SR}} + \frac{\tau_x n_{\mathrm{IP}}}{4T_{\mathrm{rev}}}\{n_\gamma < u^2 >\}\mathcal{H}_x^*$$

$$\sigma_{\delta,\mathrm{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \frac{n_{\mathrm{IP}}\tau_{E,\mathrm{SR}}}{4T_{\mathrm{rev}}}\{n_\gamma < u^2 >\}$$

Monochromatization

$$D_x^*\sigma_{\delta,\mathrm{tot}} \gg \sqrt{\beta_x^*\epsilon_x}$$

$$\epsilon_{x,\mathrm{tot}} \approx \epsilon_{x,\mathrm{SR}} + \frac{2B\mathcal{H}_x^*}{D_x^{*3}\sigma_{\delta,\mathrm{tot}}^5}$$

$$\sigma_{\delta,\mathrm{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \frac{B}{D_x^{*3}\sigma_{\delta,\mathrm{tot}}^5}$$

$$B \equiv 48\,\frac{n_{\mathrm{IP}}\tau_{E,\mathrm{SR}}}{T_{\mathrm{rev}}}\,\frac{r_e^5 N_b^3\gamma^2}{(\alpha_{\mathrm{C}}C/(2\pi Q_s))^2}$$

Baseline

$$D_x^*\sigma_{\delta,\mathrm{tot}} \ll \sqrt{\beta_x^*\epsilon_x}$$

$$\epsilon_{x,\mathrm{tot}} \approx \epsilon_{x,\mathrm{SR}} + \frac{2B\mathcal{H}_x^*}{\sigma_{\delta,\mathrm{tot}}^2\beta_x^{*3/2}\epsilon_{x,\mathrm{tot}}^{3/2}}$$

$$\sigma_{\delta,\mathrm{tot}}^2 = \sigma_{\delta,\mathrm{SR}}^2 + \frac{B}{\sigma_{\delta,\mathrm{tot}}^2\beta_x^{*3/2}\epsilon_{x,\mathrm{tot}}^{3/2}}$$

Monochromatization

Beam Parameters Distributions

Transverse Displacement

$$x = x_\beta + D_x \epsilon_0, \qquad y = y_\beta + D_y \epsilon_0 \qquad \epsilon_0 = \frac{\epsilon}{E_0}$$

Phase Space Distribution

$$f^\pm(x, y, \epsilon) = \frac{f^\pm(p_x, p_y, z)}{\sqrt{8\pi^3 \beta_x^* \epsilon_{xc} \beta_y^* \epsilon_{yc} \sigma_\epsilon^2}} \exp\left\{ -\frac{(x_\beta + D_x\epsilon_0)^2}{2\beta_x^* \epsilon_{xc}} - \frac{(y_\beta + D_y\epsilon_0)^2}{2\beta_y^* \epsilon_{yc}} - \frac{\epsilon^2}{2\sigma_\epsilon^2} \right\}$$

Average at the IP

$$\langle \mathcal{A} \rangle^\pm = \int f^\pm(X^\pm) \mathcal{A}(X^+, X^-) dX^\pm \qquad \langle \mathcal{A} \rangle^* = \left\langle \langle \mathcal{A} \rangle^+ \right\rangle^- = \left\langle \langle \mathcal{A} \rangle^- \right\rangle^+$$

Average over distribution

$$\sigma_x^* = \sqrt{\langle x^2 \rangle^\pm} = \sqrt{\beta_x^* \epsilon_{xc} + D_x^{*2} \sigma_\epsilon^2} \qquad \langle x\epsilon_0 \rangle^\pm = D_x^{*\pm} \sigma_\epsilon^2 \qquad \sigma_\epsilon^* = \sqrt{\langle \epsilon_0^2 \rangle^\pm}$$

$$\sigma_y^* = \sqrt{\langle y^2 \rangle^\pm} = \sqrt{\beta_y^* \epsilon_{yc} + D_y^{*2} \sigma_\epsilon^2} \qquad \langle y\epsilon_0 \rangle^\pm = D_y^{*\pm} \sigma_\epsilon^2$$

Without Monochromatization Colliding Beams

Collision Energy Spread

$$\sigma_w = \sqrt{2} E_0 \sigma_\epsilon$$

Beam Energy Spread

$$\sigma_\epsilon{}^2 \propto \frac{55\hbar c E_0{}^2}{32\sqrt{3(mc^2)^3}} \frac{I_3}{I_2} \frac{1}{J_\epsilon}$$

Function of radius and J_ϵ

$$\sigma_w \propto (\rho J_\epsilon)^{-1/2}.$$

Typical Options

$$\left.\begin{array}{l} \rho >> \rho_0 \\ J_\epsilon > J_{0\epsilon} \end{array}\right\} \Rightarrow \sigma_w < \sigma_{0w}$$

$$J_\epsilon \in [0.5, 2.5]$$

Monochromatization

$$(\Sigma_w)_\lambda = \frac{\sigma_\delta}{\sqrt{2}} \frac{1}{\lambda}$$

$$\lambda \equiv \frac{\mathcal{L}_0}{\mathcal{L}}$$

$$\mathcal{L}_0 = \frac{k_b f_r N_+ N_-}{4\pi \sigma^*_{x\beta} \sigma^*_{y\beta}}$$

Without Monochromatization Standard Monochromatization

Baseline Scheme

$$D^*_{x+} = -D^*_{x-} = 0 \qquad \mathcal{L} = \mathcal{L}_0$$

Standard Monochromatization $D^*_{x+} = -D^*_{x-} = D^*_x \qquad D^*_{y+} = -D^*_{y-} = D^*_y$

$$\mathcal{L} = \frac{\mathcal{L}_0}{\sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}} + \frac{D^{*2}_y}{\sigma^{*2}_{y\beta}}\right)}} \qquad \Sigma_w = \frac{\sqrt{2}E_0\sigma_\epsilon}{\sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}} + \frac{D^{*2}_y}{\sigma^{*2}_{y\beta}}\right)}} \qquad \lambda = \sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}} + \frac{D^{*2}_y}{\sigma^{*2}_{y\beta}}\right)}$$

Standard Monochromatization $D^*_{x+} = -D^*_{x-} = D^*_x \qquad D^*_{y+} = D^*_{y-} = D^*_y$

$$\Sigma_w = \frac{\sqrt{2}E_0\sigma_\epsilon}{\sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}}\right)}} \qquad L = \frac{L_0}{\sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}}\right)}\sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_y}{\sigma^{*2}_{y\beta}}\right)}} \qquad \lambda = \sqrt{1+\sigma_\epsilon^2\left(\frac{D^{*2}_x}{\sigma^{*2}_{x\beta}}\right)}$$

Monochromatization factor $\lambda(D_x{}^*, \beta_x{}^* = 0.25\text{ m})$

Standard Monochromatization

$$D^*_{y+} = D^*_{y-} = 0 \qquad (\Sigma_w)_\lambda = \frac{\sigma_\delta}{\sqrt{2}}\frac{1}{\lambda} \qquad \mathcal{L} \propto \frac{1}{\lambda}$$

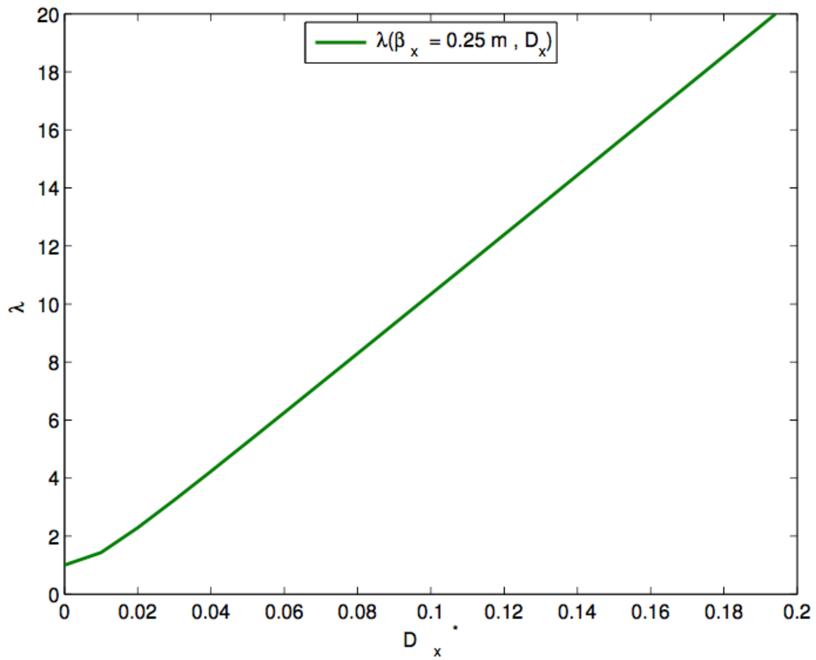


Figure 6.1: Monochromatization factor versus D_x^* at fixed $\beta_x^* = 0.25$ m, for constant emittance and energy spread.

Monochromatization factor $\lambda(D_{x*} = 0.11$ m, $\beta_x{}^*)$

Standard Monochromatization

$$D^*_{y+} = D^*_{y-} = 0 \qquad (\Sigma_w)_\lambda = \frac{\sigma_\delta}{\sqrt{2}}\frac{1}{\lambda} \qquad \mathcal{L} \propto \frac{1}{\lambda}$$

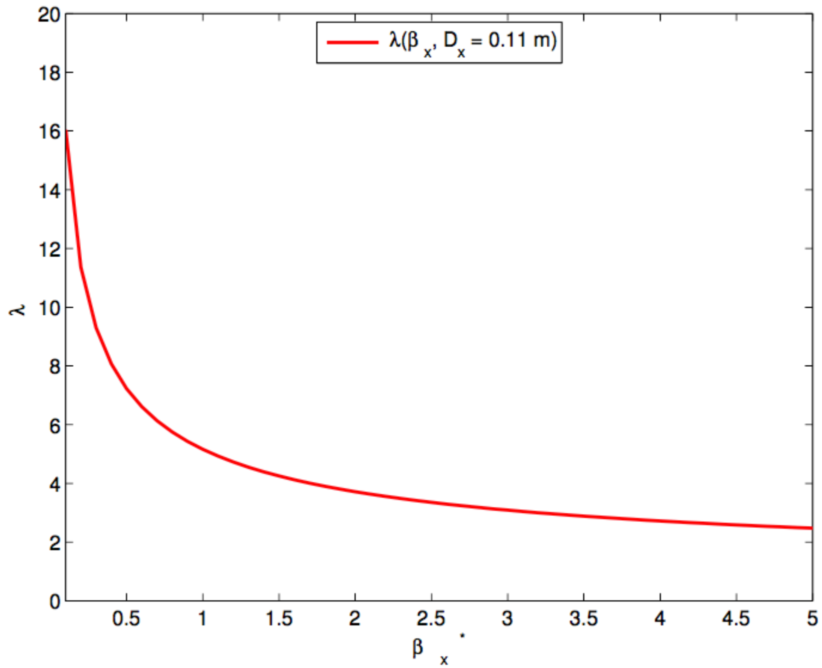


Figure 6.2: Monochromatization factor versus β^*_x at fixed $D^*_x = 0.11$ m, for constant emittance and energy spread.

Beamstrahlung Effects on Monochromatization

Baseline Monochromatization

$$D^*_{x+} = -D^*_{x-} = D^*_x \qquad D^*_{y+} = D^*_{y-} = 0$$

Self-Consistent Equations

$$\epsilon_{x,\mathrm{tot}} = \epsilon_{x,\mathrm{SR}} + \frac{\tau_x n_{\mathrm{IP}}}{4T_{\mathrm{rev}}}\{n_\gamma < u^2 >\}\mathcal{H}^*_x \qquad \sigma^2_{\delta,\mathrm{tot}} = \sigma^2_{\delta,\mathrm{SR}} + \frac{n_{\mathrm{IP}}\tau_{E,\mathrm{SR}}}{4T_{\mathrm{rev}}}\{n_\gamma < u^2 >\}$$

Statistical Properties

$$\sigma^*_x = \sqrt{\langle x^2\rangle^{\pm}} = \sqrt{\beta^*_x \epsilon_{xc} + D^{*2}_x \sigma^2_\epsilon} \qquad \langle x\epsilon_0\rangle^{\pm} = D^{*\pm}_x \sigma^2_\epsilon \qquad (\Sigma_w)_\lambda = \frac{\sigma_\delta}{\sqrt{2}}\frac{1}{\lambda}$$

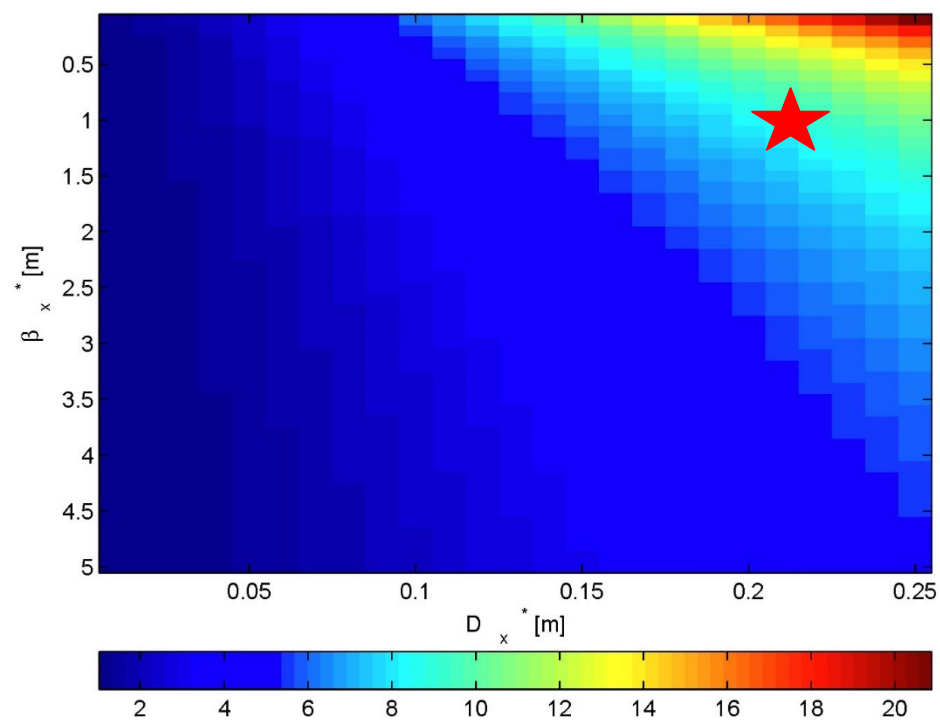
Baseline monochromatization

E_e [GeV]	62.5
scheme	m.c. basel.
I_b [mA]	408.3
N_b [10^{10}]	3.3
n_b [1]	25760
n_{IP} [1]	2
β_x^* [m]	1.0
β_y^* [mm]	2
D_x^* [m]	0.22
$\epsilon_{x,\mathrm{SR}}$ [nm]	0.17
$\epsilon_{x,\mathrm{tot}}$ [nm]	0.21
$\epsilon_{y,\mathrm{SR}}$ [pm]	1
$\sigma_{x,\mathrm{SR}}$ [μm]	132
$\sigma_{x,\mathrm{tot}}$ [μm]	144
σ_y [nm]	45
$\sigma_{z,\mathrm{SR}}$ [mm]	1.8
$\sigma_{z,\mathrm{tot}}$ [mm]	1.8
$\sigma_{\delta,\mathrm{SR}}$ [%]	0.06
$\sigma_{\delta,\mathrm{tot}}$ [%]	0.06
θ_c [mrad]	0
circ. C [km]	100
α_C [10^{-6}]	7
f_{rf} [MHz]	400
V_{rf} [GV]	0.4
$U_{0,\mathrm{SR}}$ [GeV]	0.12
$U_{0,\mathrm{BS}}$ [MeV]	0.01
τ_E/T_{rev}	509
Q_s	0.030
Υ_{max} [10^{-4}]	0.3
Υ_{ave} [10^{-4}]	0.1
θ_c [mrad]	0
ξ_x [10^{-2}]	1
ξ_y [10^{-2}]	4
λ [1]	9.2
L [10^{35} cm^{-2}s^{-1}]	1.0
σ_w [MeV]	5.8

Width of standard model Higgs 4-5 MeV requires $\lambda \geq 10$

$N_b = 3.3\times10^{10}$ $n_b = 25760$ $\beta_y^* = 2$ mm

Monochromatization Factor



★: $\lambda = 9.2$, $L = 1 \times 10^{35}$ cm^{-2}s^{-1}

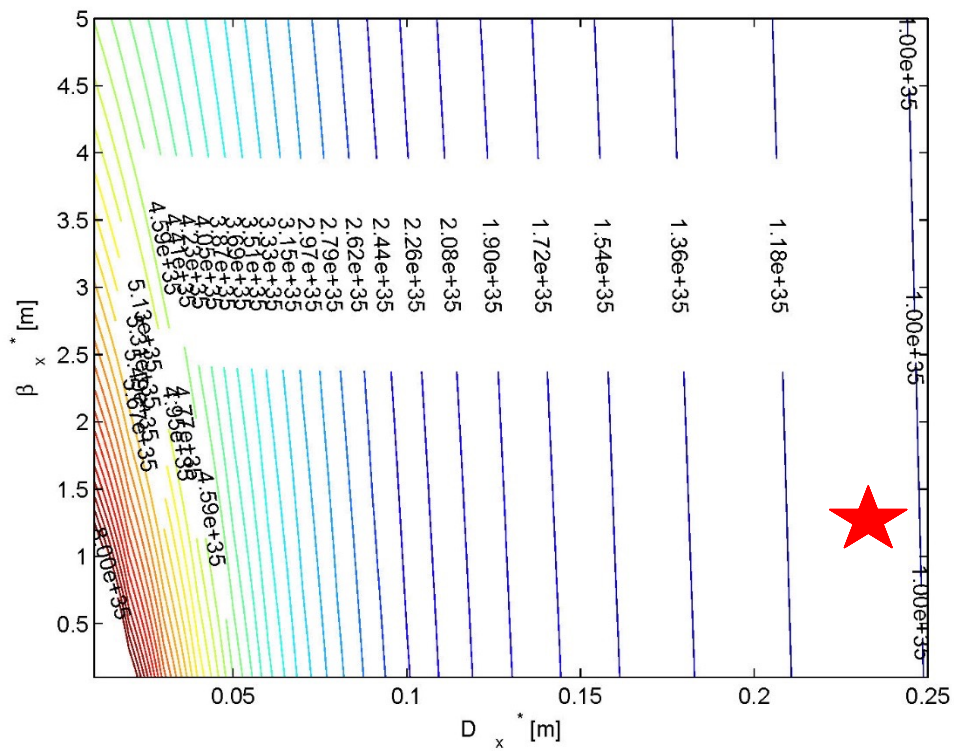
Baseline monochromatization

E_e [GeV]	62.5
scheme	m.c. basel.
I_b [mA]	408.3
N_b [10^{10}]	3.3
n_b [1]	25760
n_{IP} [1]	2
β_x^* [m]	1.0
β_y^* [mm]	2
D_x^* [m]	0.22
$\epsilon_{x,\mathrm{SR}}$ [nm]	0.17
$\epsilon_{x,\mathrm{tot}}$ [nm]	0.21
$\epsilon_{y,\mathrm{SR}}$ [pm]	1
$\sigma_{x,\mathrm{SR}}$ [μm]	132
$\sigma_{x,\mathrm{tot}}$ [μm]	144
σ_y [nm]	45
$\sigma_{z,\mathrm{SR}}$ [mm]	1.8
$\sigma_{z,\mathrm{tot}}$ [mm]	1.8
$\sigma_{\delta,\mathrm{SR}}$ [%]	0.06
$\sigma_{\delta,\mathrm{tot}}$ [%]	0.06
θ_c [mrad]	0
circ. C [km]	100
α_C [10^{-6}]	7
f_{rf} [MHz]	400
V_{rf} [GV]	0.4
$U_{0,\mathrm{SR}}$ [GeV]	0.12
$U_{0,\mathrm{BS}}$ [MeV]	0.01
τ_E/T_{rev}	509
Q_s	0.030
Υ_{max} [10^{-4}]	0.3
Υ_{ave} [10^{-4}]	0.1
θ_c [mrad]	0
ξ_x [10^{-2}]	1
ξ_y [10^{-2}]	4
λ [1]	9.2
L [10^{35} cm^{-2}s^{-1}]	1.0
σ_w [MeV]	5.8

Width of standard model Higgs 4-5 MeV requires $\lambda \geq 10$

$N_b = 3.3\mathrm{x}10^{10}$ $n_b = 25760$ $\beta_y^* = 2$ mm

Luminosity contours



★: $\lambda = 9.2$, $L = 1 \mathrm{~x~} 10^{35}$ cm^{-2}s^{-1}

Failed "pushed" monochromatization

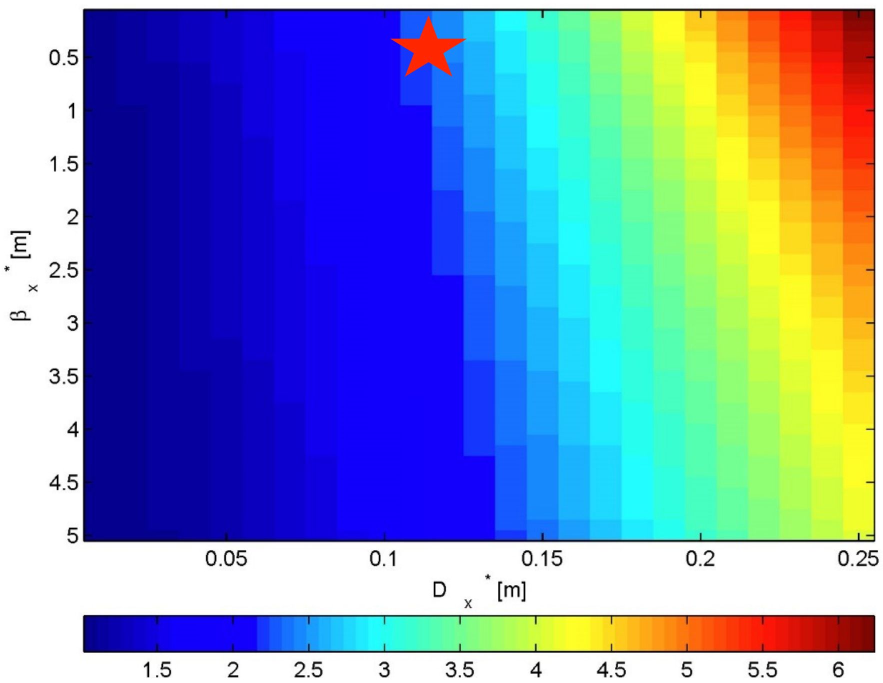
Pushed monochromatization:

$N_b = 8.5 \times 10^{10}$

$n_b = 10000$

$\beta_y^* = 1$ mm

Monochromatization Factor



★ : $\lambda = 2.3$, $L = 7.5 \times 10^{35}$ $cm^{-2}s^{-1}$

Failed “pushed” monochromatization

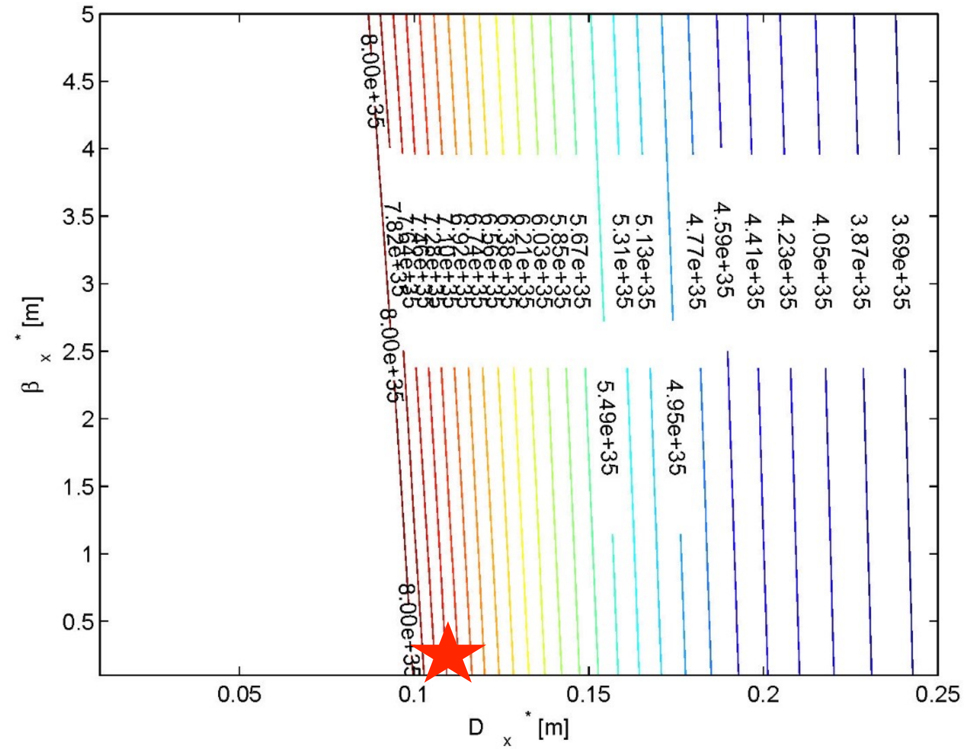
Luminosity contours

Pushed monochromatization:

$N_b = 8.5 \times 10^{10}$

$n_b = 10000$

$\beta_y^* = 1$ mm



★ : $\lambda = 2.3$, $L = 7.5 \times 10^{35}$ $cm^{-2}s^{-1}$

Optimized Monochromatization Luminosity_max and λ

$S = [\ 0.1\ ,\ 3\ ]$, $T = [\ 0.1\ ,\ 3\ ]$

$\beta_{0x} = 1.0$ m, $\beta_{0y} = 1.0$ mm

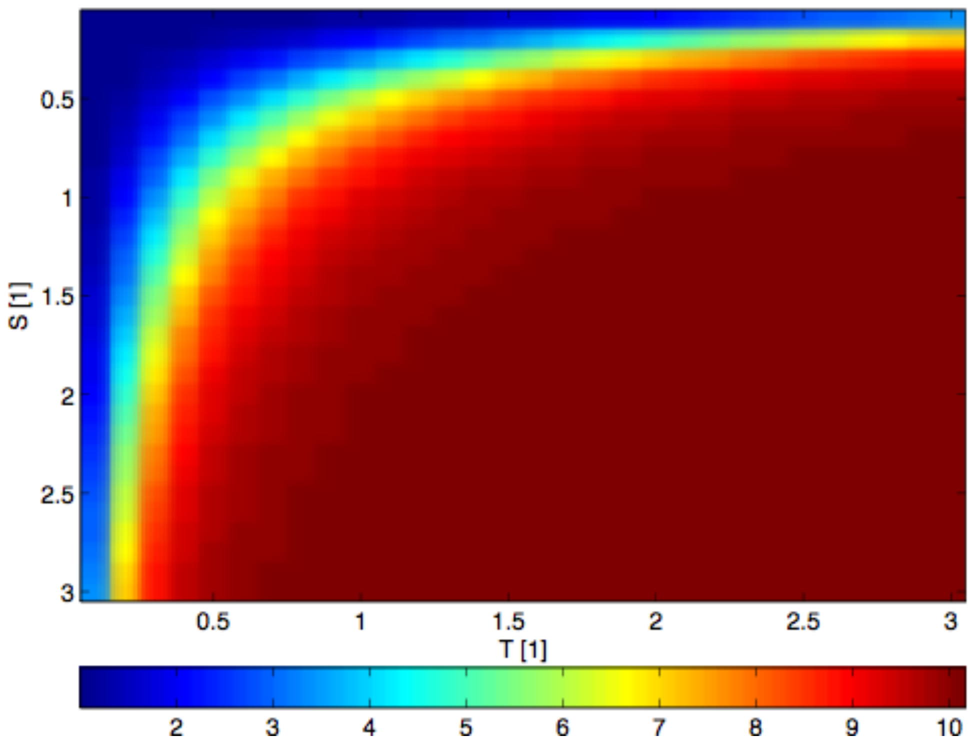
$D_{0x} = 0.22$ m

$N_{0b} = 3.3\text{x}10^{10}$, $n_{0b} = 25760$

$\beta_x = \beta_{0x} * S^2$, $D_x * = D_{0x} * S$

$N_b = N_{0b} / T$, $n_b = n_{0b} * T$

Monochromatization Factor



★ : $\lambda_0 = 10.17321$

Optimized Monochromatization Luminosity_max and λ

$S = [\ 0.1\ ,\ 3\ ]$, $T = [\ 0.1\ ,\ 3\ ]$

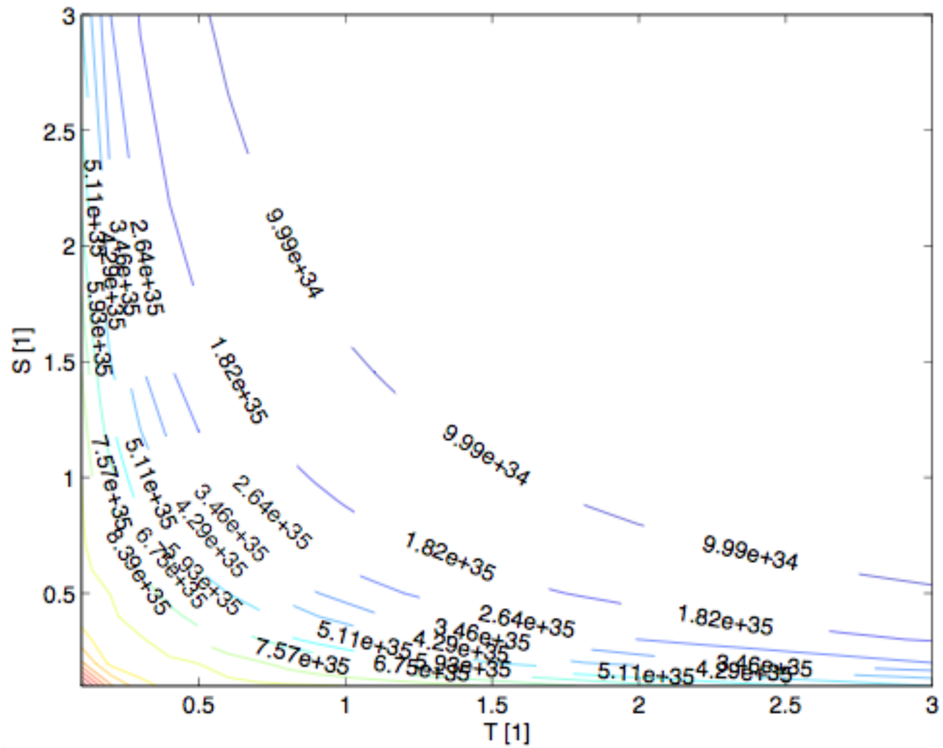
$\beta_{0x} = 1.0$ m, $\beta_{0y} = 1.0$ mm

$D_{0x} = 0.22$ m

$N_{0b} = 3.3 \times 10^{10}$, $n_{0b} = 25760$

$\beta_x = \beta_{0x} * S^2$, $D_x * = D_{0x} * S$

$N_b = N_{0b} / T$, $n_b = n_{0b} * T$



★ : $\lambda_0 = 10.17321$, $L \sim 1 \times 10^{36}\ \mathrm{cm^{-2}s^{-1}}$

Optimized Monochromatization Luminosity_max and λ

$S = [\ 0.1\ ,\ 3\ ]$, $T = [\ 0.1\ ,\ 3\ ]$

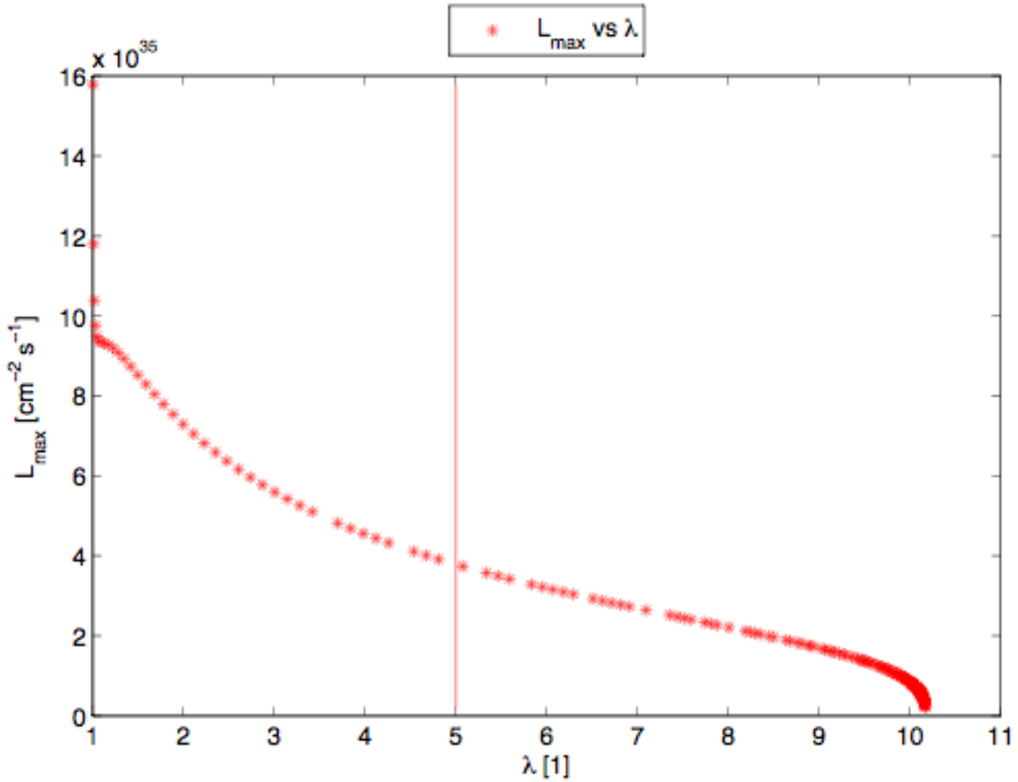
$\beta_{0x} = 1.0$ m, $\beta_{0y} = 1.0$ mm

$D_{0x} = 0.22$ m

$N_{0b} = 3.3\mathrm{x}10^{10}$, $n_{0b} = 25760$

$\beta_x = \beta_{0x} * S^2$, $D_x * = D_{0x} * S$

$N_b = N_{0b} / T$, $n_b = n_{0b} * T$



L_{max} vs λ

★ : $\lambda_0 = 10.17321$, $L \sim 1 \times 10^{36}$ cm^{-2}s^{-1}

Luminosity_max and λ

$S = [\ 0.1\ ,\ 3\ ]$, $T = [\ 0.1\ ,\ 3\ ]$

$\beta_{0x} = 1.0$ m, $\beta_{0y} = 1.0$ mm

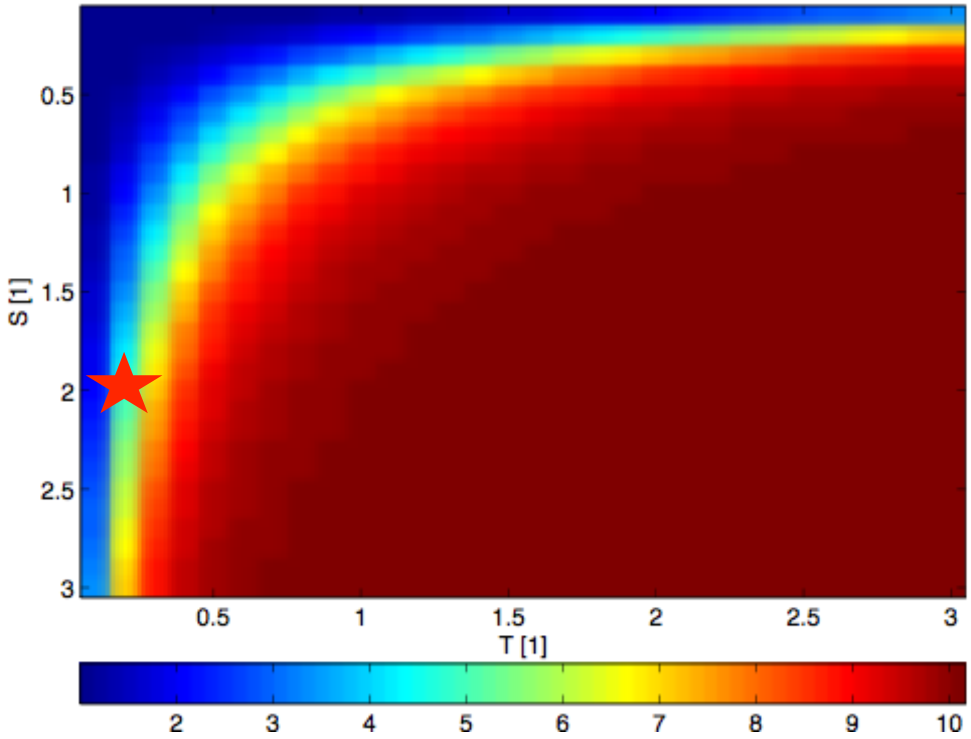
$D_{0x} = 0.22$ m

$N_{0b} = 3.3 \mathrm{x} 10^{10}$, $n_{0b} = 25760$

$\beta_x = \beta_{0x} * S^2$, $D_x * = D_{0x} * S$

$N_b = N_{0b} / T$, $n_b = n_{0b} * T$

Monochromatization Factor



★ : $\lambda = 5.07$, $\beta = 1.96$ m, $D_x = 0.308$ m, $L = 3.736 \mathrm{~x~} 10^{35}$ $cm^{-2}s^{-1}$

Conclusions

★ : $\lambda = 5.07$, $\beta = 1.96$ m, $D_x = 0.308$ m, $L = 3.736 \times 10^{35}$ $cm^{-2}s^{-1}$

E_e [GeV]	45.6	62.5	62.5	62.5	80
scheme	CW	h.-o.	m.c. basel.	m.c. opt'd	CW
I_b [mA]	1450.3	408.3	408.3	408.3	151.5
N_b [10^{10}]	3.3	1.05	3.3	11.1	6.0
n_b [1]	91500	80960	25760	7728	5260
n_{IP} [1]	2	2	2	2	2
β_x^* [m]	1	1.0	1.0	1.96	1
β_y^* [mm]	2	2	2	1	2
D_x^* [m]	0	0	0.22	0.308	0
$\epsilon_{x,SR}$ [nm]	0.09	0.17	0.17	0.17	0.26
$\epsilon_{x,tot}$ [nm]	0.09	0.17	0.21	0.70	0.26
$\epsilon_{y,SR}$ [pm]	1	1	1	1	1
$\sigma_{x,SR}$ [μm]	9.5	9.2	132	185.7	16
$\sigma_{x,tot}$ [μm]	9.5	9.2	144	188.5	16
σ_y [nm]	45	45	45	32	45
$\sigma_{z,SR}$ [mm]	1.6	1.8	1.8	1.8	2.0
$\sigma_{z,tot}$ [mm]	3.8	1.8	1.8	1.8	3.1
$\sigma_{\delta,SR}$ [%]	0.04	0.06	0.06	0.06	0.07
$\sigma_{\delta,tot}$ [%]	0.09	0.06	0.06	0.06	0.10
θ_c [mrad]	30	0	0	0	30
circ. C [km]	100	100	100	100	100
α_C [10^{-6}]	7	7	7	7	7
f_{rf} [MHz]	400	400	400	400	400
V_{rf} [GV]	0.2	0.4	0.4	0.4	0.8
$U_{0,SR}$ [GeV]	0.03	0.12	0.12	0.12	0.33
$U_{0,BS}$ [MeV]	0.5	0.05	0.01	0.01	0.21
τ_E/T_{rev}	1320	509	509	509	243
Q_s	0.025	0.030	0.030	0.030	0.037
Υ_{max} [10^{-4}]	1.7	0.8	0.3	0.85	4.0
Υ_{ave} [10^{-4}]	0.7	0.3	0.1	0.35	1.7
θ_c [mrad]	30	0	0	0	30
ξ_x [10^{-2}]	5	12	1	2.22	7
ξ_y [10^{-2}]	13	15	4	6.76	16
λ [1]	1	1	9.2	5.08	1
L [10^{35} cm^{-2}s^{-1}]	9.0	2.2	1.0	3.74	1.9
σ_w [MeV]	58	53	5.8	10.44	113

- **Monochromatization** scheme can be implemented.
- **Beamstrahlung** effects may be controlled.
- **Simulation** supports predictions.
- **Lattice** designed is still in progress and the required modification should be possible.
- **Theory** confirmation could be achieved at the FCCe+e-

Supported in part by the European Commission under Capacities 7th Framework Programme project EuCARD-2, grant agreement 312453.

L(λ)

Optimized Monochromatization for Direct Higgs Production at the FCCe+e-

Ing. M. Alan Valdivia G.
valdivia@ugto.mx

Holiday Inn Pachuca 2016
Pachuca, Hidalgo, 101116