

Ultra-peripheral collisions (UPCs)

or

The growth with energy of exclusive J/ψ and Y photo-production cross-sections and BFKL evolution

or

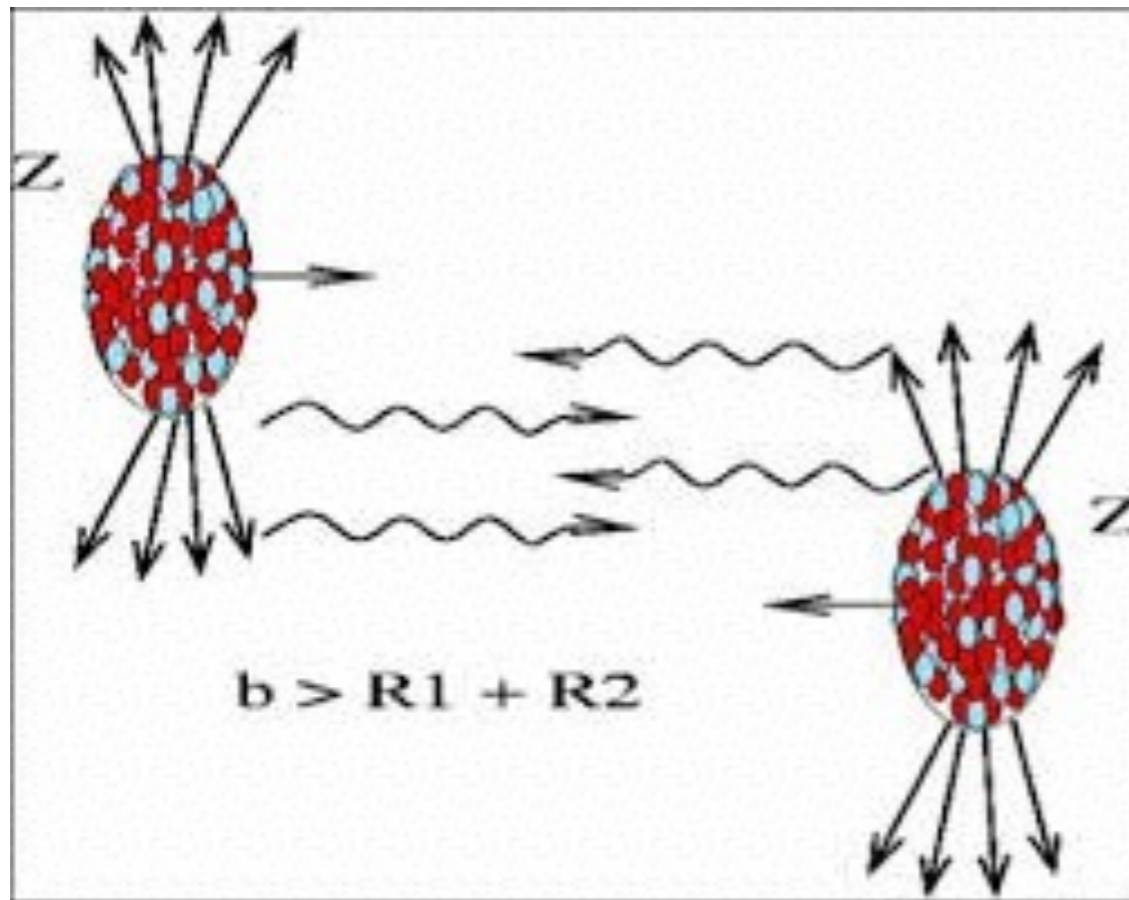
some QCD theory

Martin Hentschinski
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based on:

I. Bautista, A. Fernandez Tellez, MH. [arXiv:1607.05203] (PRD 94 054002)

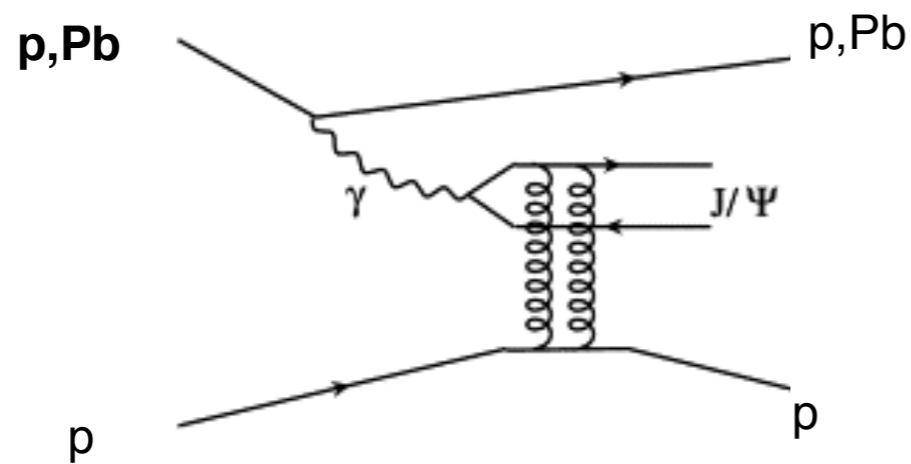
LHC: the most energetic photon source ever built



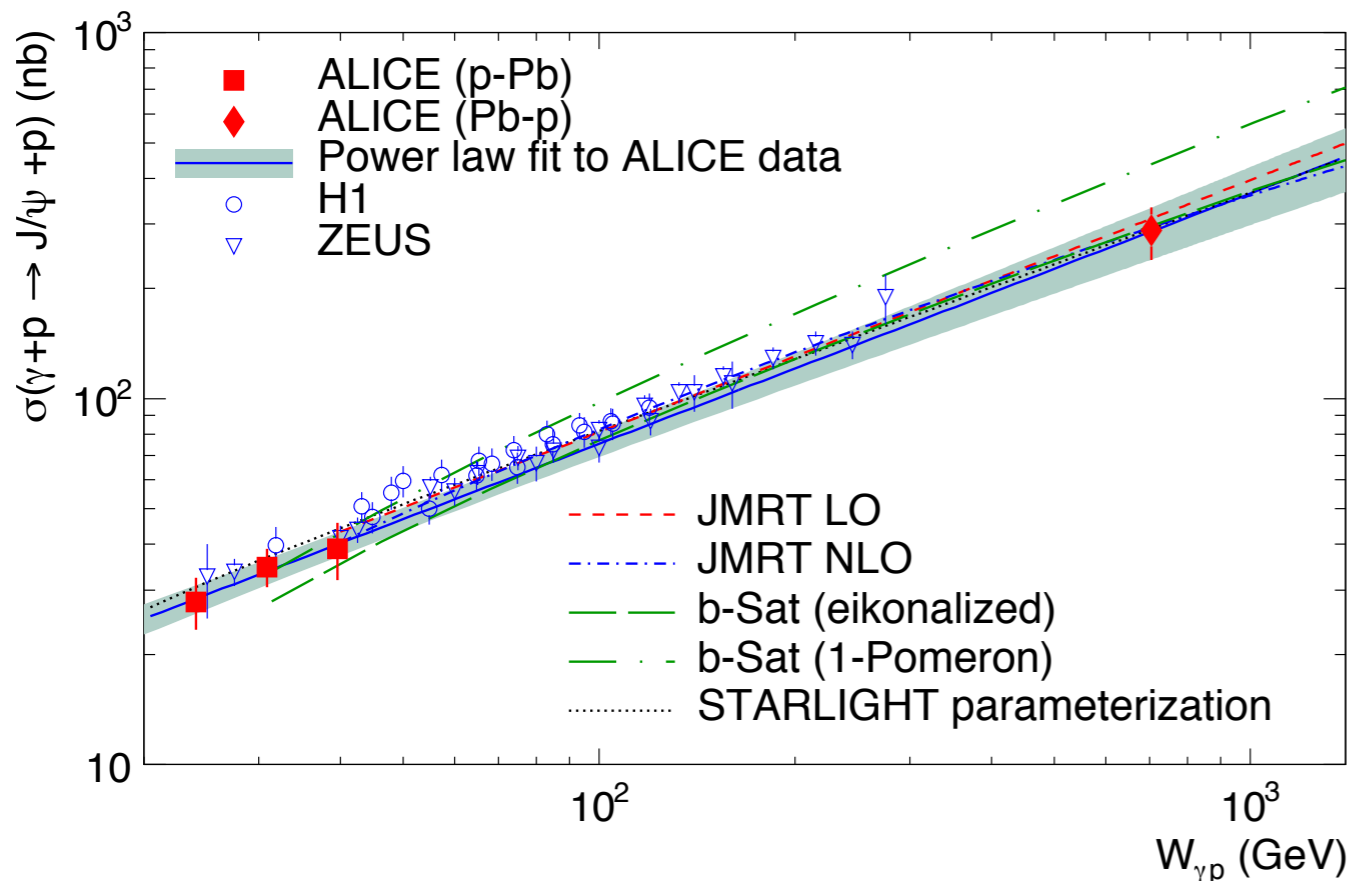
photon induced collisions
at the TeV scale

- two ions (protons) pass each other with impact parameters $b > 2R \rightarrow$ hadronic interactions strongly suppressed
- high photon flux $\sim Z^2$ well described by Weizsäcker-Williams approximation (electromagnetic field = a beam of quasi real photons)

probe the proton at small x

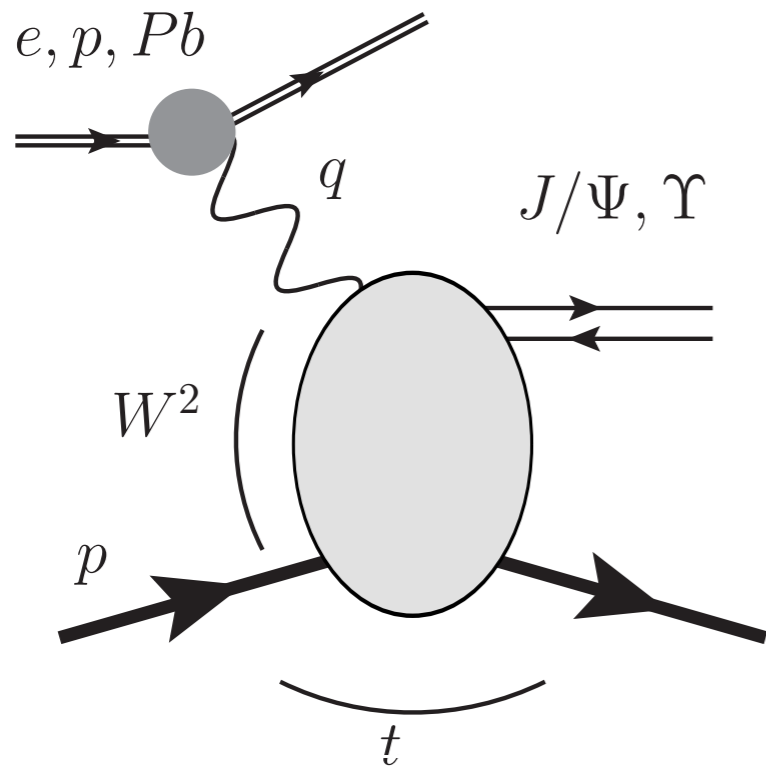


- exclusive photo production
Xsec. of $J/\psi \sim \text{gluon}^2$
- ultra small $x \sim$ region
 10^{-2} - 10^{-5} accessible at the LHC
- UPC vector meson production (VM) = a tool to access the potentially saturated proton

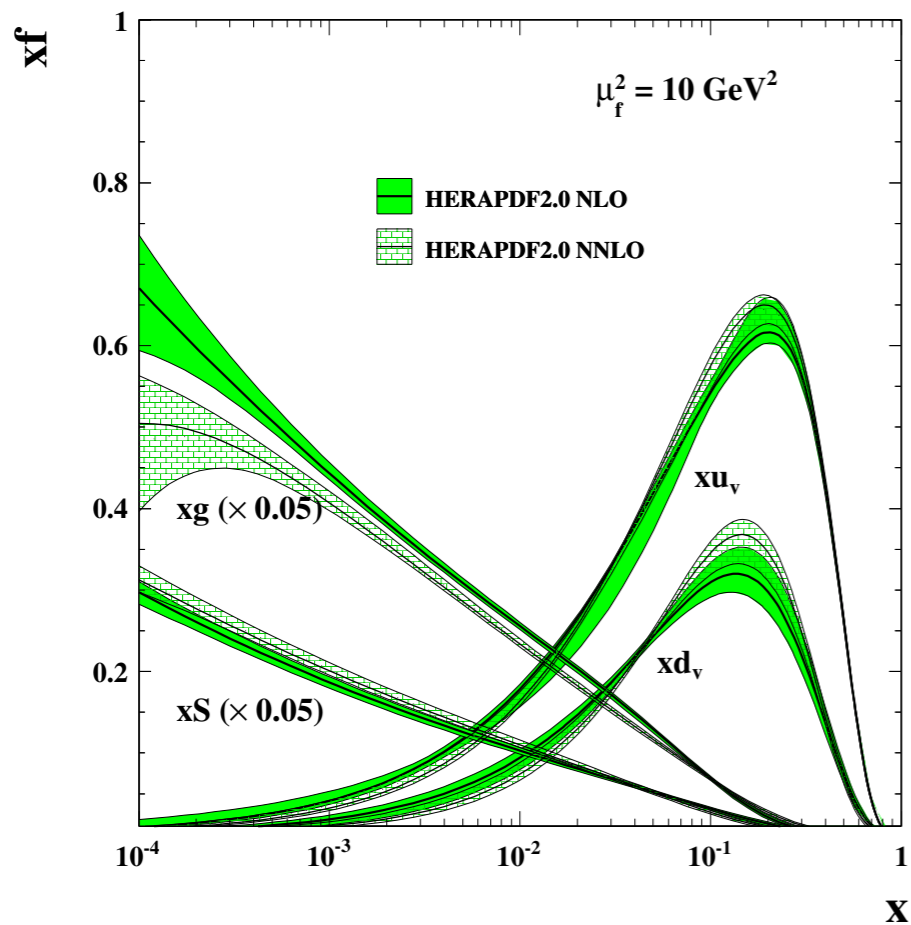


shown: ALICE data (arXiv: 1406.7819) and HERA data (ep scattering)

kinematics



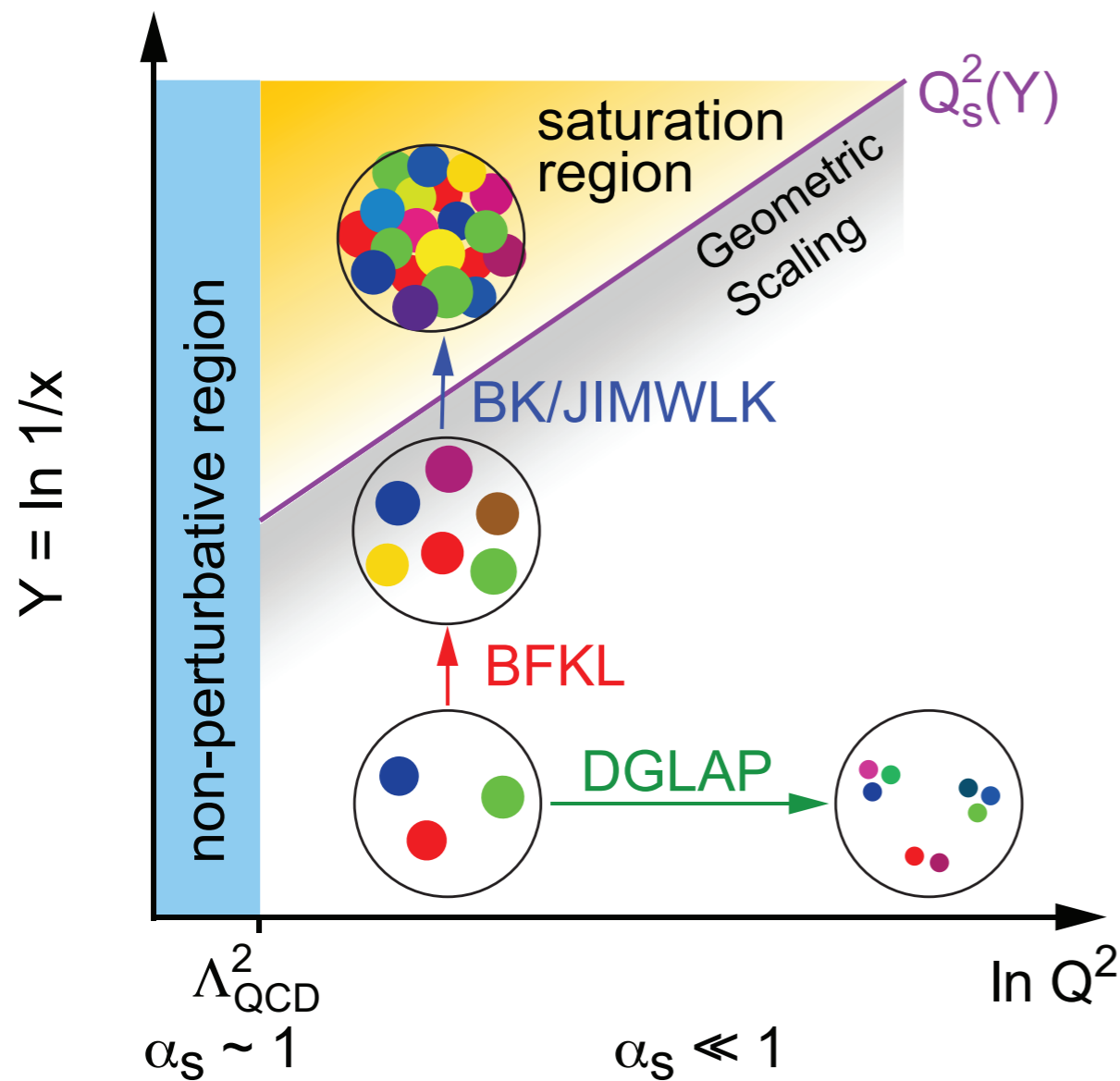
H1 and ZEUS



- energy squared $W^2 = (p + q)^2$
 t : momentum transfer
- probe proton and therefore gluon at $x = M_{VM}^2 / (W^2 - m_p^2)$
- HERA constrains gluon down to $x = 10^{-4}$,
 UPCs: gain order of magnitude
 \rightarrow learn about the low x gluon

and explore (maybe?) a new regime:

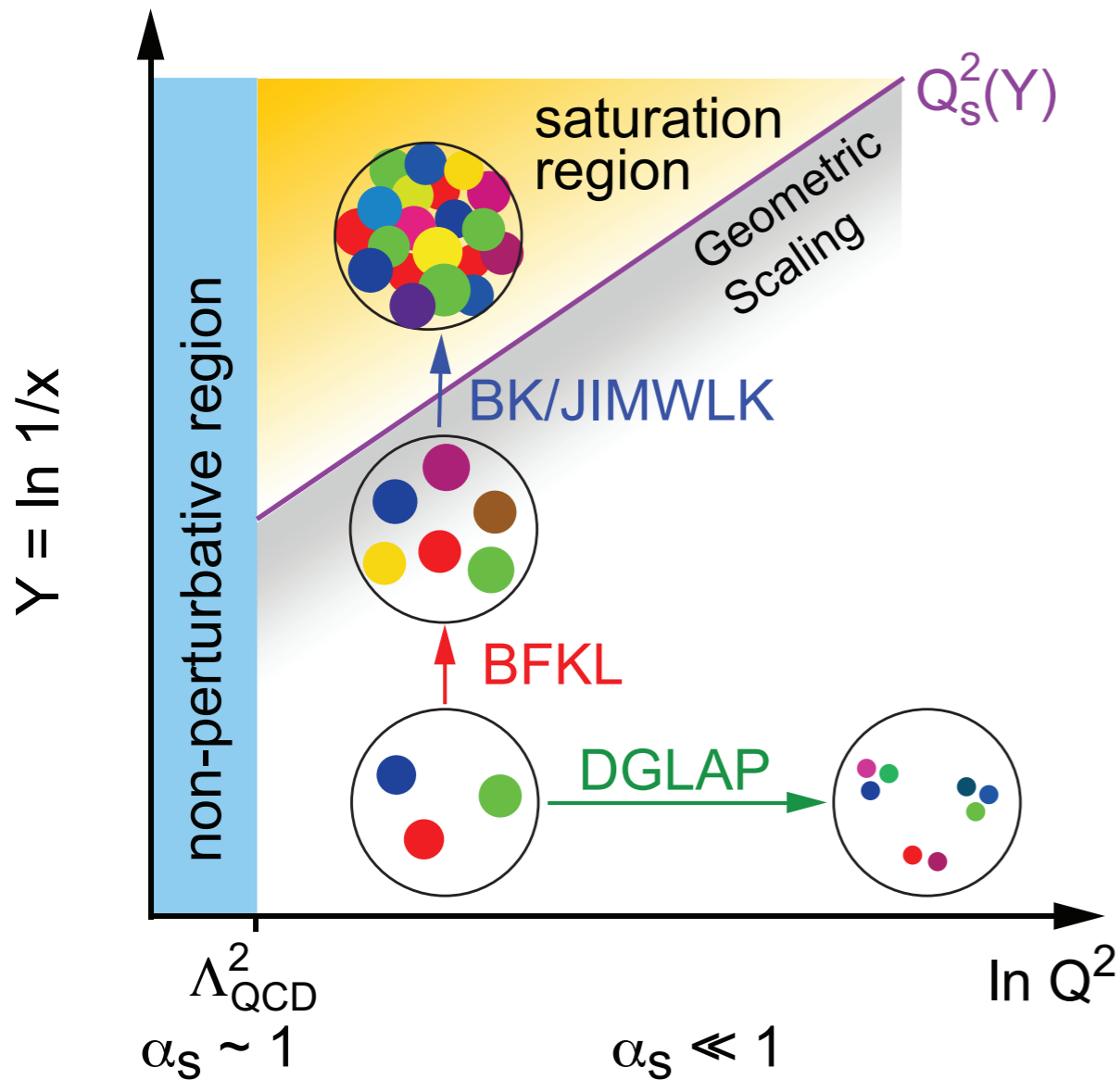
saturation?



- gluon distribution grows like a power at low x
- at some x : low density approximation invalid, patrons “overlap”, recombination effects
- consequence: growth with $1/x$ slows down

already reached in UPCs at LHC? (saturation models describe data ...)

low x evolution (very schematic)



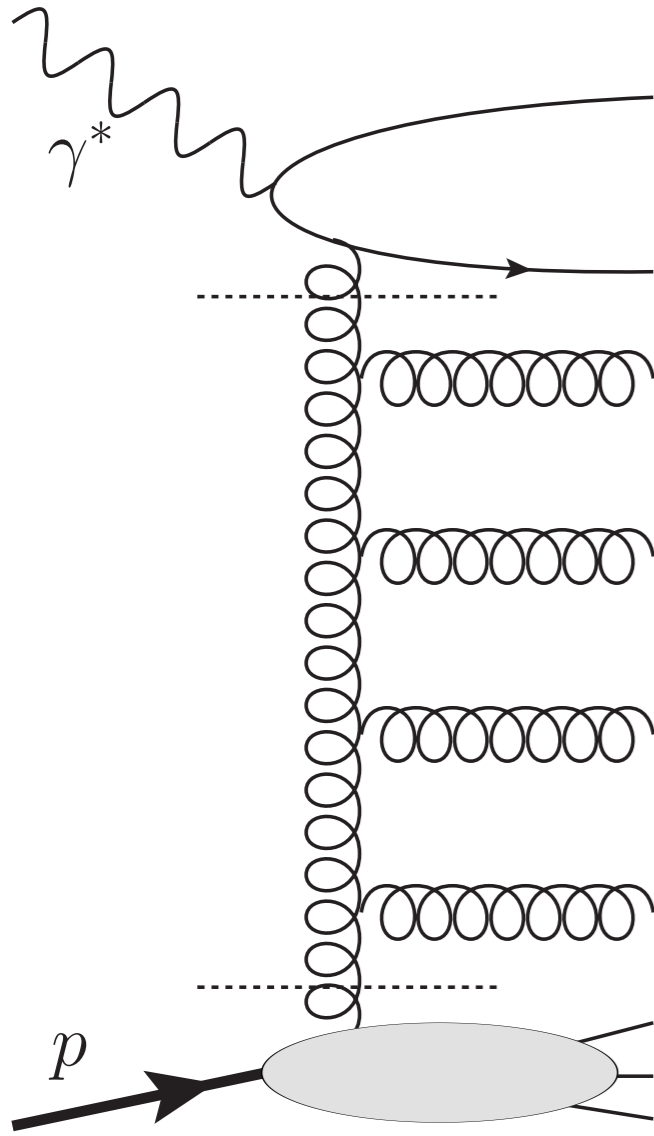
- BFKL:

$$\partial_{\ln 1/x} G(x, k) = K \otimes G$$
- BK:

$$\partial_{\ln 1/x} G(x, k) = K \otimes G - G \otimes G$$
- K: the BFKL kernel
 LL: [Fadin, Kuraev, Lipatov; PLB 60 (1975) 50],
 [Balitsky, Lipatov, SJNP (1978 822)]
 NLL: [Fadin, Lipatov; PLB 429 (1998) 127];
 [Ciafaloni, Camici; PLB 430 (1998) 349]
- BFKL = low density limit of BK evolution

Question: Can BFKL evolve 'HERA gluon' to LHC values and describe UPC data on VM production or do already require non-linear effects?

The underlying NLO BFKL fit



$$F_2(x, Q^2) = \int_0^\infty dk^2 \int_0^\infty \frac{dq^2}{q^2} \Phi_2 \left(\frac{k^2}{Q^2} \right) \mathcal{F}_{\text{BFKL}}^{\text{DIS}}(x, k^2, q^2) \Phi_p \left(\frac{q^2}{Q_0^2} \right)$$

virtual photon: quarks mass-less, $n_f = 4$ fixed

$$\text{proton impact factor: } \Phi_p \left(\frac{q^2}{Q_0^2}, \delta \right) = \frac{\mathcal{C}}{\pi \Gamma(\delta)} \left(\frac{q^2}{Q_0^2} \right)^\delta e^{-\frac{q^2}{Q_0^2}}$$

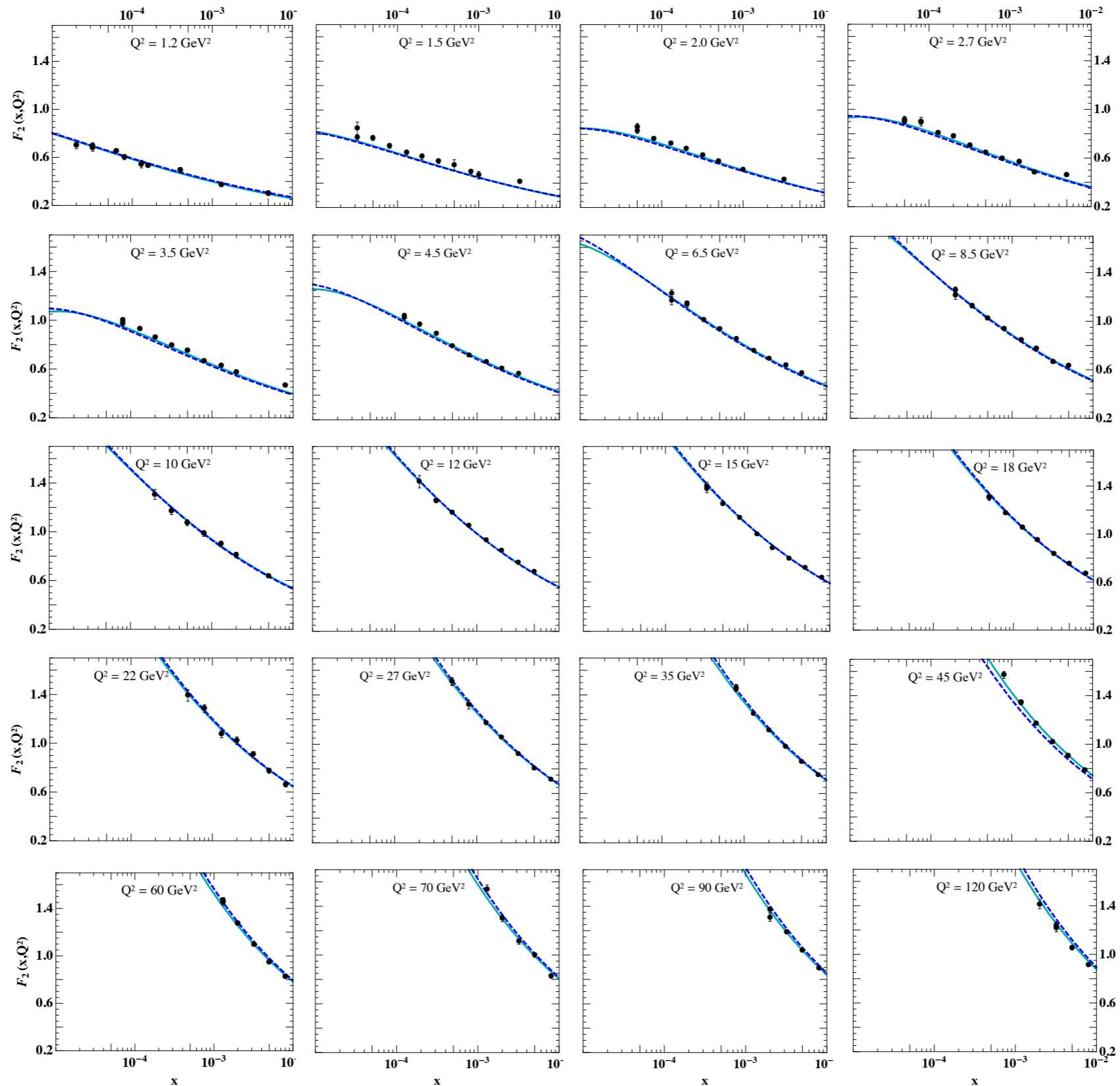
free parameters of proton impact factor from fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

→ allows for definition of unintegrated gluon density [Chachamis, Deak, MH, Rodrigo, Sabio Vera; 1507.05778]

$$G(x, k^2, Q_0^2) = \int \frac{dq^2}{q^2} \mathcal{F}_{\text{BFKL}}^{\text{DIS}}(x, k^2, q^2) \Phi_p \left(\frac{q^2}{Q_0^2} \right)$$

	virt. photon impact factor	Q_0/GeV	δ	\mathcal{C}	$\Lambda_{\text{QCD}}/\text{GeV}$
fit 1	leading order (LO)	0.28	8.4	1.50	0.21
fit 2	LO with kinematic improvements	0.28	6.5	2.35	0.21

Good description of cominbed HERA [H1 & ZEUS collab. 0911.0884]



idea:

use BFKL HERA fit &

apply it to UPC data

task: relate **exclusive** photoproduction to **inclusive** gluon distribution (*use DGLAP procedure*)

The framework of this BFKL study

procedure:

a) calculate diff. Xsec. at $t = 0$

→ *exclusive* scattering amplitude can be expressed through *inclusive* gluon distribution

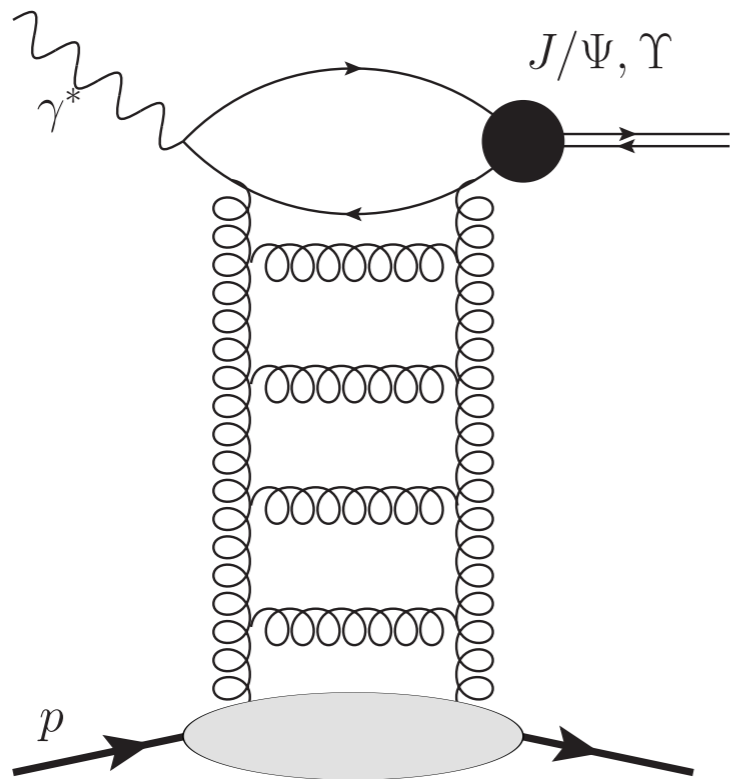
b) parametrize t dependence $\frac{d\sigma(t)}{dt} = \frac{d\sigma(t=0)}{dt} \cdot e^{-|t|B_D(W)}$,

slope $B_D(W) = b_0 + 4\alpha' \ln \frac{W}{W_0}$ + fix parameters by (HERA) data
(here: values proposed by [\[Jones, Martin, Ryskin, Teubner; 1307.7099, 1312.6795\]](#))

→ cross-section:
$$\sigma^{\gamma p \rightarrow V p}(W) = \underbrace{\frac{1}{B_D(W)}}_{\text{phenomenological}} \underbrace{\left. \frac{d\sigma^{\gamma p \rightarrow V p}}{dt} \right|_{t=0}}_{\text{BFKL / theory}}$$

relate 2 pictures of the BFKL Pomeron

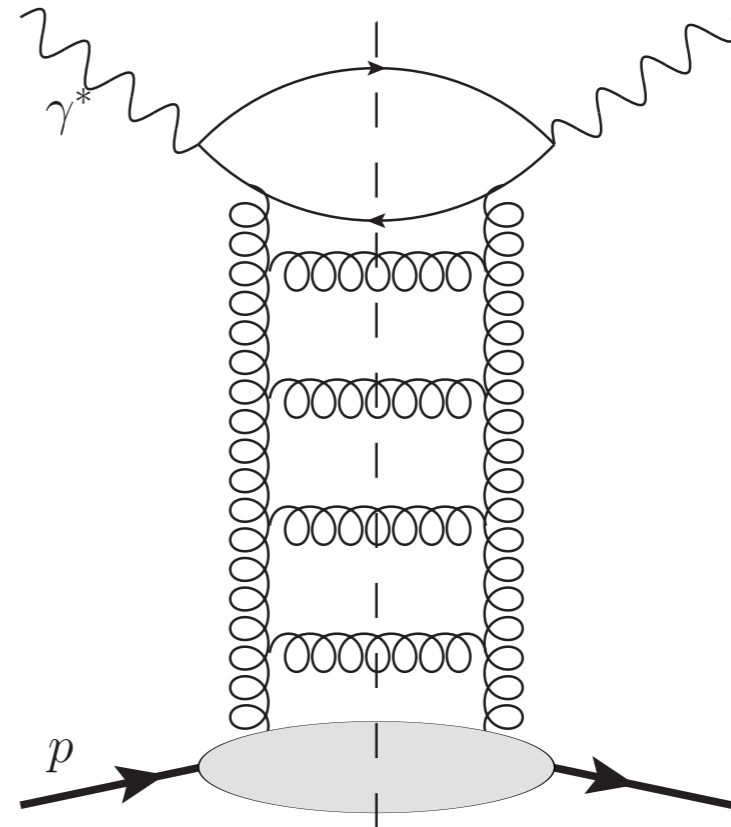
a) exclusive photo-production of vector mesons:



'uncut' Pomeron: diffractive/elastic scattering (amplitude level)

$$\mathcal{A}(s, t)$$

b) proton structure functions:



'cut' Pomeron: high multiplicity events (total X-sec.)

$$\sigma_{\text{tot}} = \frac{1}{s} \Im m \mathcal{A}(s, t = 0)$$

The setup: diff. Xsec. at $t = 0$

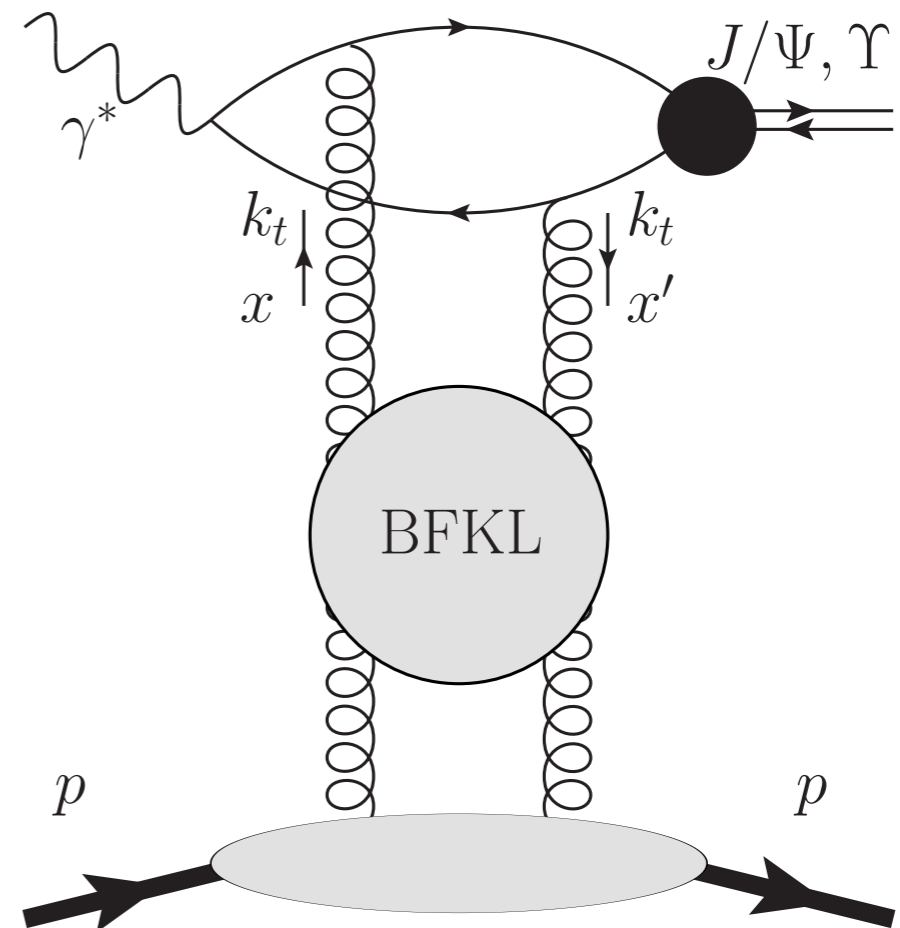
a) imaginary part of scattering amplitude:

- ▶ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ▶ impact factor $\gamma \rightarrow J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies

[Kowalski, Motyka, Watt; hep-ph/0606272]

b) real part:

- ▶ $\Im \mathcal{A}(W^2, t)$ dominant, real part can be numerically large
 → recover real part using dispersion relation



Vector mesons & dipole models ...

factorization into light-front wave function & dipole amplitude

e.g. [Kowalski, Motyka, Watt; hep-ph/0606272]

$$\Im m \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(W, t=0) = 2 \int d^2 \mathbf{r} \int d^2 \mathbf{b} \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_{T,L} \mathcal{N}(x, r, b),$$

light-front wave function overlap

$$(\Psi_V^* \Psi)_T = \frac{\hat{e}_f e N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(m_f r) \phi_T(r, z) - [z^2 + (1-z)^2] m_f K_1(m_f r) \partial_r \phi_T(r, z) \right\}$$

scalar parts of VM wave function: boosted Gaussian wave-functions with Brodsky-Huang-Lepage prescription

$$\phi_T^{1s}(r, z) = \mathcal{N}_T z(1-z) \exp \left(-\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2} \right).$$

free parameters fixed through normalization condition & leptonic decay width $\Gamma_{e^- e^+}$:

Meson	m_f/GeV	\mathcal{N}_T	$\mathcal{R}^2/\text{GeV}^{-2}$	M_V/GeV	$8\mathcal{R}^{-2}/\text{GeV}^2$	$\frac{1}{4}M_V^2/\text{GeV}^2$
J/ψ	$m_c = 1.27$	0.596	2.45	3.097	3.27	2.40
Υ	$m_b = 4.2$	0.481	0.57	9.460	15.38	22.42

use parameters obtained by [Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1408.1344]

From wave functions to impact factors

BFKL study requires impact factor in γ space $\Phi_{V,T}(\gamma)$:

$$\begin{aligned} \Im \mathcal{A}_T^{\gamma* p \rightarrow V p}(W, t=0) &= 2 \int d^2 \mathbf{b} \int d^2 \mathbf{r} \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_T(r) \cdot \sigma_0 N(x, r) \\ &= \alpha_s (\overline{M} \cdot Q_0) \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \int_0^1 \frac{dz}{4\pi} \hat{g} \left(x, \frac{M^2}{Q_0^2}, \frac{\overline{M}^2}{M^2}, Q_0, \gamma \right) \cdot \Phi_{V,T}(\gamma, z, M) \cdot \left(\frac{M^2}{Q_0^2} \right)^\gamma \end{aligned}$$

can be derived using relation dipole \leftrightarrow unintegrated gluon e.g. [\[Kutak, Stasto; hep-ph/040811\]](#)

$$2 \int d^2 \mathbf{b} \mathcal{N}(x, r, b) = \frac{4\pi}{N_c} \int \frac{d^2 \mathbf{k}}{k^2} \left(1 - e^{i\mathbf{k} \cdot \mathbf{r}} \right) \alpha_s G(x, \mathbf{k}^2).$$

yields

$$\begin{aligned} \Phi_{V,T}(\gamma, z, M) &= e \hat{e}_f 8\pi^2 \mathcal{N}_T \frac{\Gamma(\gamma)\Gamma(1-\gamma)}{m_f^2} \left(\frac{m_f^2 \mathcal{R}^2}{8z(1-z)} \right)^2 e^{-\frac{m_f^2 \mathcal{R}^2}{8z(1-z)}} e^{\frac{m_f \mathcal{R}^2}{2}} \left(\frac{8z(1-z)}{M^2 \mathcal{R}^2} \right)^\gamma \\ &\left[U \left(2-\gamma, 1, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)} \right) + [z^2 + (1-z)^2] \frac{(2-\gamma)}{2} U \left(3-\gamma, 2, \frac{m_f^2 \mathcal{R}^2}{8z(1-z)} \right) \right], \end{aligned}$$

[$U(a, b, z)$ hypergeometric function of the second kind or Kummer's function]

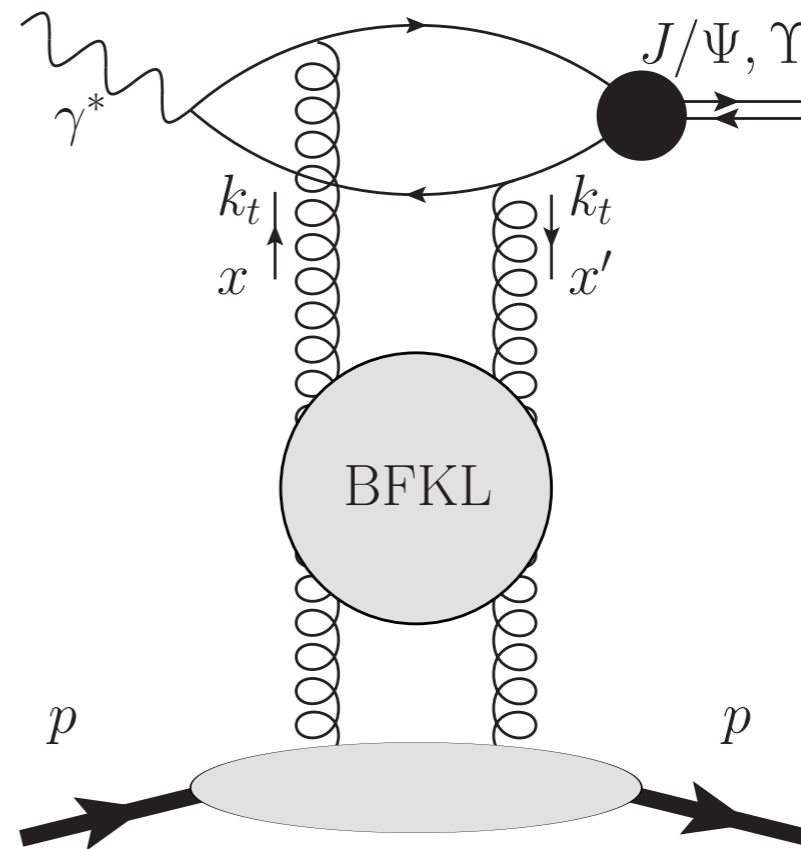
The setup: diff. Xsec. at $t = 0$

a) imaginary part of scattering amplitude:

- ✓ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ✓ impact factor $\gamma \rightarrow J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies

b) real part:

- ✗ $\Im \mathcal{A}(W^2, t)$ dominant, real part can be numerically large
➔ recover **real part** using dispersion relation



- ▶ for $t = 0$ the (pseudo-)inclusive process fixes imaginary part of $\mathcal{A}(W^2, t = 0)$ of the exclusive process;

- ▶ Common approach: $\frac{\Re\mathcal{A}(W^2, t)}{\Im\mathcal{A}(W^2, t)} = \tan \frac{\lambda\pi}{2}$, with $\lambda = \frac{d \ln \mathcal{A}(W^2, t)}{d \ln W^2}$

follows from analytic representation of scattering amplitudes of (scalar) particles in the Regge limit for positive signature

- ▶ Often: $\lambda = \text{const.}$ \rightarrow constant ratio of real & imaginary part

Real part from imaginary part

here: reconstruct real part through using representation in ω -Mellin space, conjugate to W^2 :

$$\mathcal{A}(W^2, t) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^\omega \left(i + \tan \frac{\omega\pi}{2}\right) a(\omega, t), \quad x = \frac{M_V^2}{W^2 - m_p^2}$$

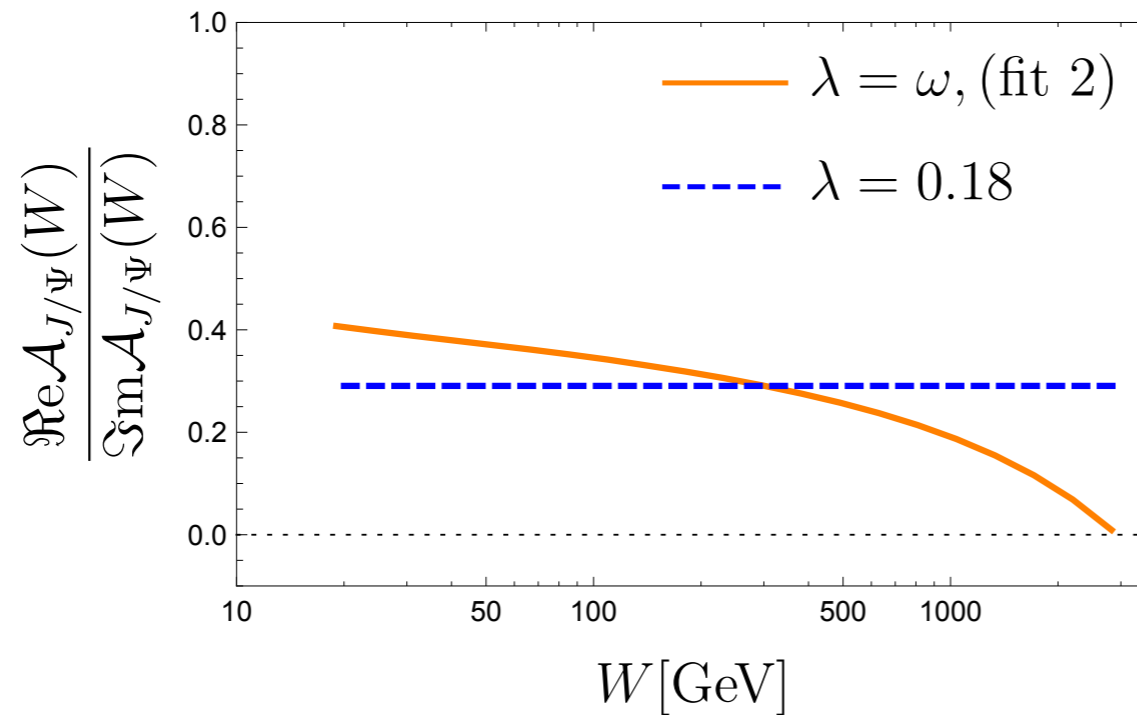
partial wave $a(\omega, t)$ can be fixed from imaginart part

$$a(\omega, 0) = \alpha_s \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{M^2}{Q_0^2}\right)^\gamma \int_0^1 \frac{dz}{4\pi} \Phi_{V,T}(\gamma, z) \frac{\mathcal{C} \cdot \Gamma(\delta - \gamma)}{\pi\Gamma(\delta)} \cdot \left\{ \frac{1}{\omega - \chi\left(\gamma, \frac{\bar{M}^2}{M^2}\right)} + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)/(8N_c)}{\left[\omega - \chi\left(\gamma, \frac{\bar{M}^2}{M^2}\right)\right]^2} \left[-\psi(\delta - \gamma) - \frac{d \ln [\Phi_{V,T}(\gamma, z)]}{d\gamma} \right] \right\}$$

yields *energy dependent* ratio of real & imaginary part

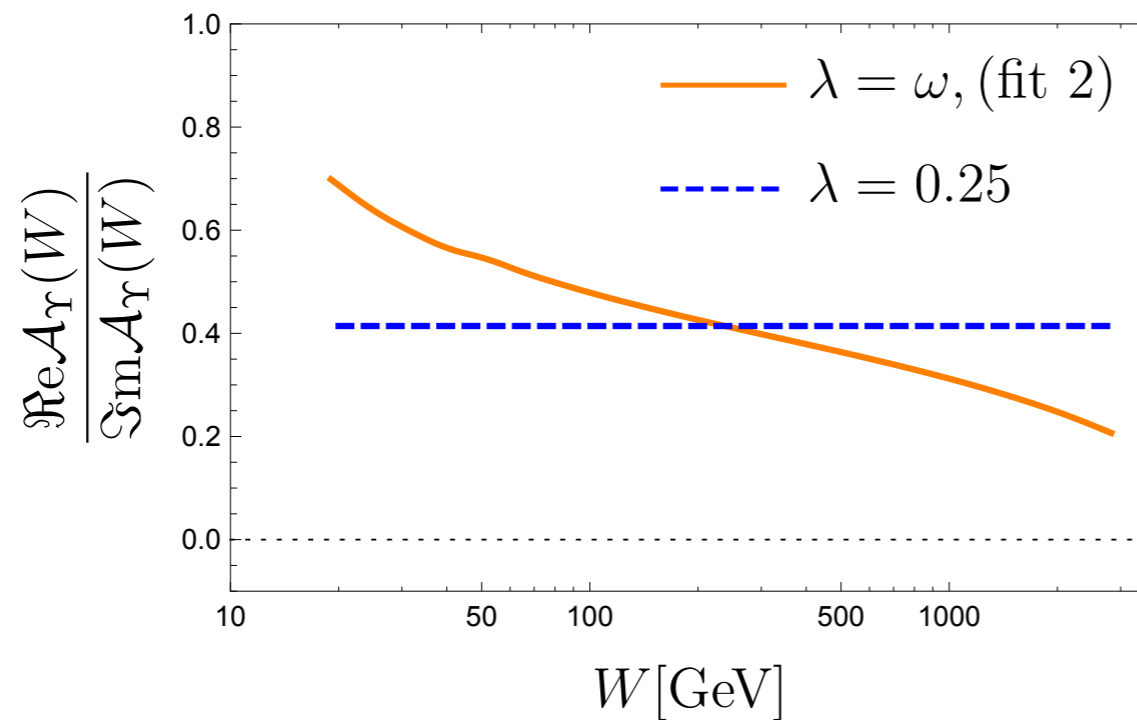
energy dependence of $r(W) = \Re A / \Im A$

J/Ψ :



ratio decreases with energy

Υ :



sizeable effect on overall energy dependence

→ slows down the growth

Comparison to data

- ▶ provide results for both HERA fits (standard (fit 1) & kinematic improved (fit 2) LO impact factor)
- ▶ hard scale M^2 :
 - photoproduction scale $M_{pp} = M_V/2$

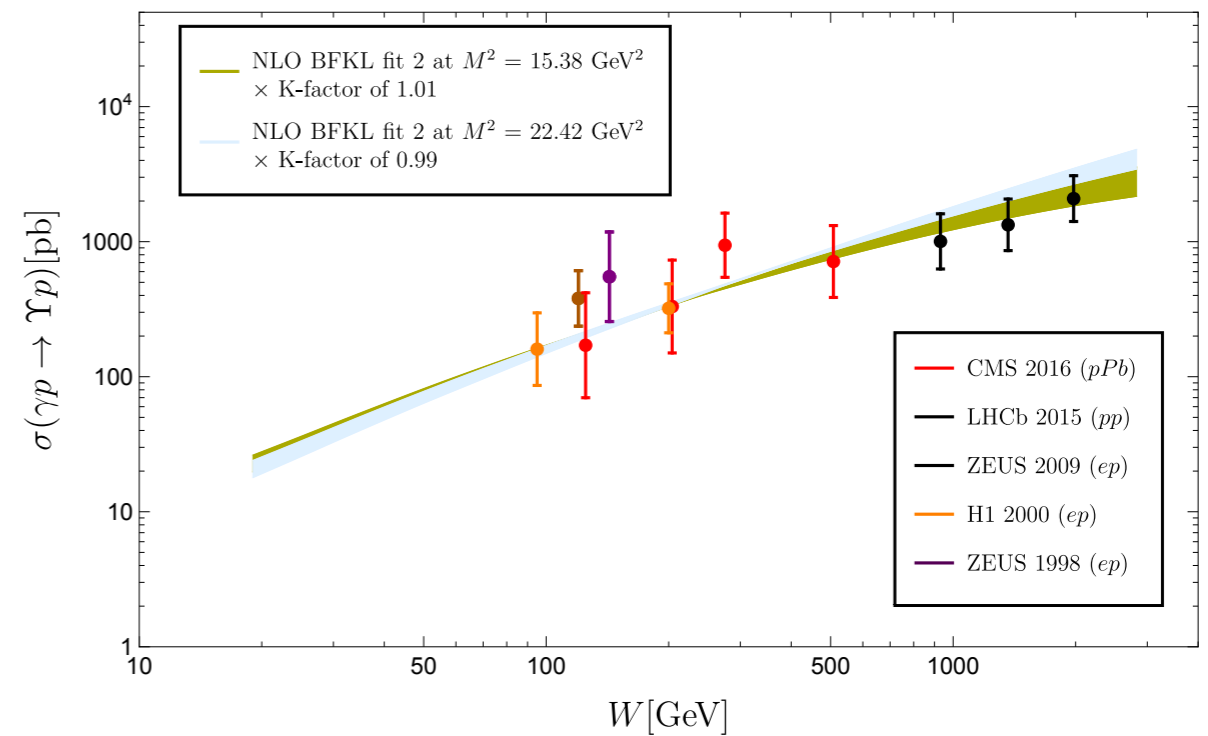
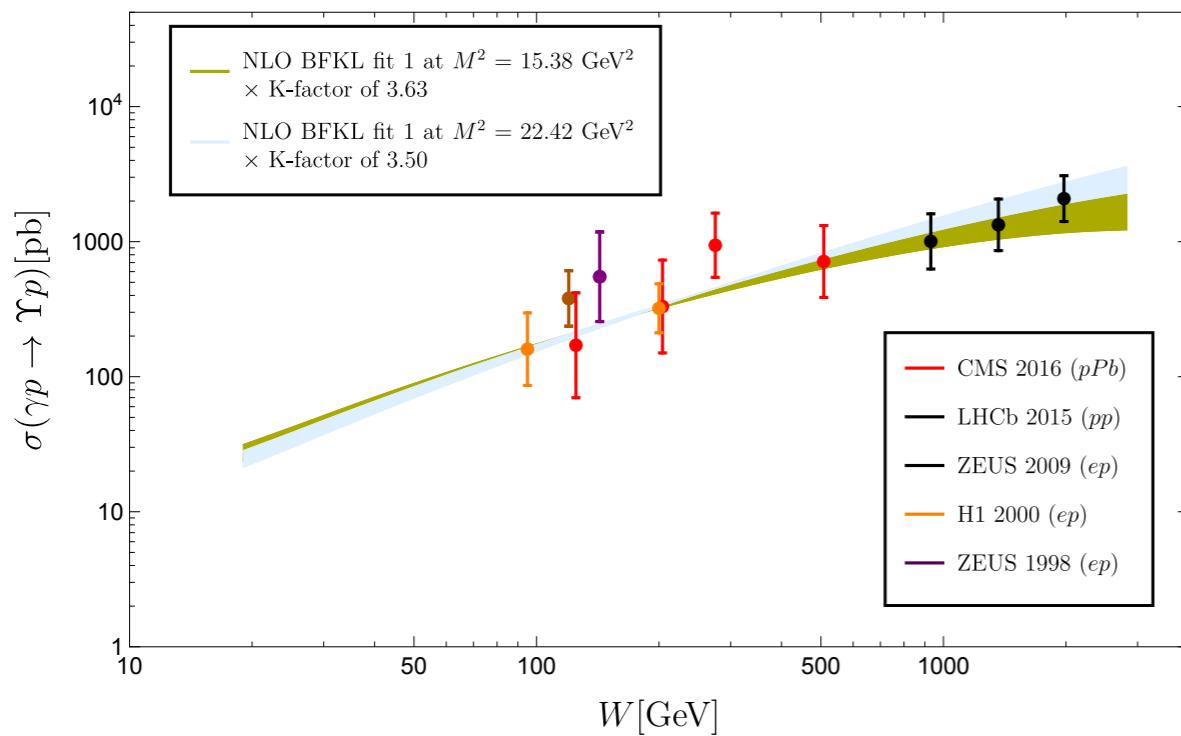
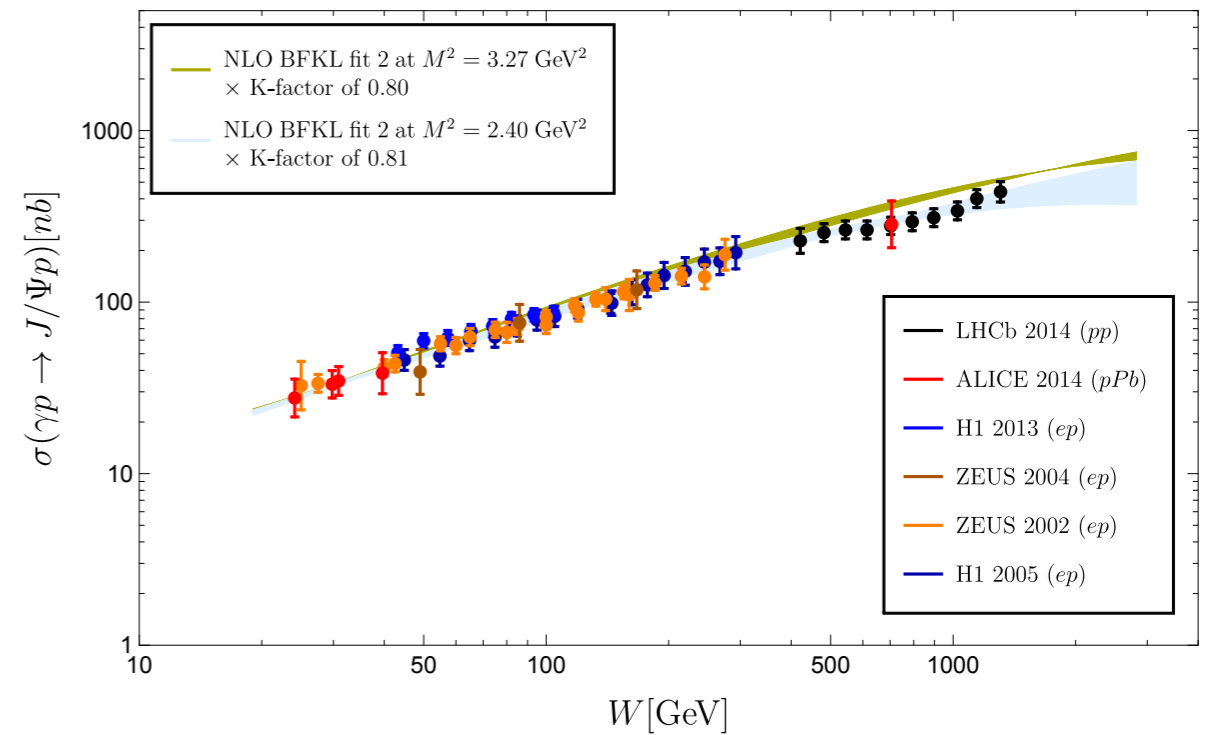
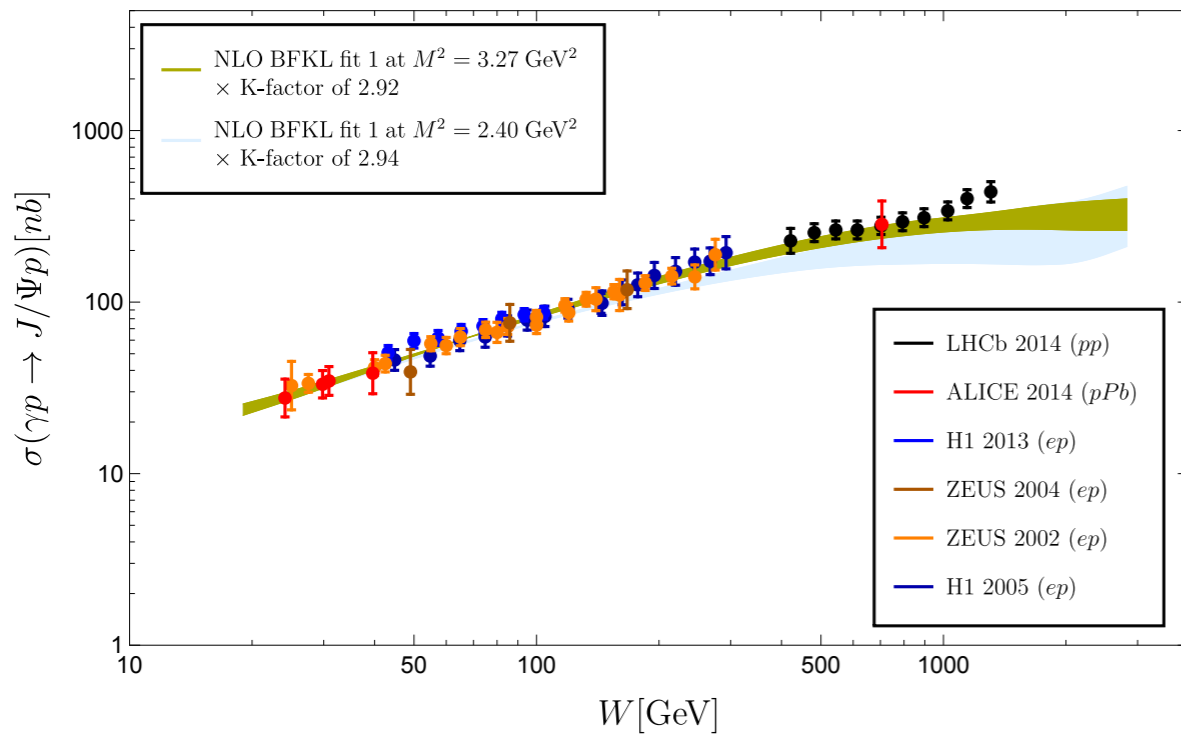
$$(M_{pp}^2)_{J/\Psi} = 2.40 \text{ GeV}^2$$

$$(M_{pp}^2)_{\Upsilon} = 22.42 \text{ GeV}^2$$
 - impact factor motivated: $M_{if}^2 = 8\mathcal{R}_V^{-2}$ – eliminates $(\dots)^\gamma$ factor & minimizes NLO running coupling correction related to impact factor

$$(M_{if}^2)_{J/\Psi} = 3.27 \text{ GeV}^2$$

$$(M_{if}^2)_{J/\Psi} = 15.38 \text{ GeV}^2$$
- ▶ (hard) running coupling scale $\overline{M} = M$, but vary in range $[M^2/2, M^2 \cdot 2]$ to check stability of result
- ▶ fix normalization by low energy ALICE (J/Ψ) and H1 (Υ) data point → K-factor

Results & Conclusions



Caveats

- ▶ both BFKL HERA fit & VM photoproduction use LO impact factor
➔ large corrections at NLO possible
- ▶ BFKL HERA fit for $n_f = 4$ mass-less quarks

both effects should affect the normalization, not so much W -dependence

- ▶ unintegrated gluon density can develop instability at ultra-small x :

$$G(x, \mathbf{k}^2, M) = \frac{1}{\mathbf{k}^2} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \hat{g}(x, \gamma) \left(\frac{\mathbf{k}^2}{Q_0^2} \right)^\gamma$$

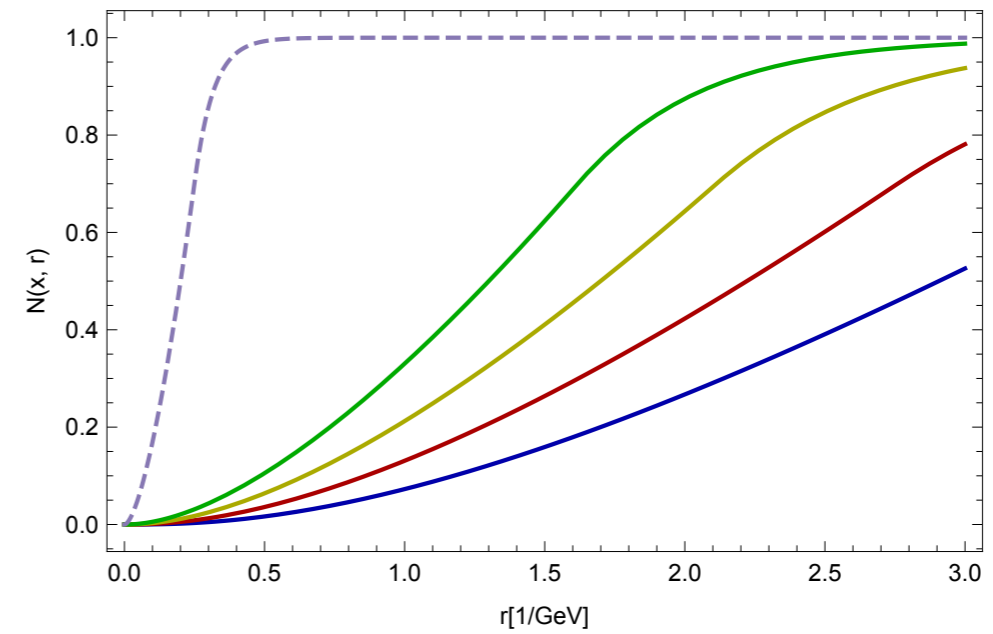
$$\hat{g}(x, \gamma) \sim \left(\frac{1}{x} \right)^{\chi\left(\gamma, \frac{M^2}{M^2}\right)} \cdot \left\{ 1 + \frac{\bar{\alpha}_s^2 \beta_0 \chi_0(\gamma)}{8N_c} \log\left(\frac{1}{x}\right) \left[-\psi(\delta - \gamma) + \log\frac{M^2}{Q_0^2} - \partial_\gamma \right] \right\},$$

- ▶ will enter at some point region $\alpha_s^2 \ln(1/x) \sim 1$ ➔ control of such terms will become necessary

instead of conclusions: final considerations

- the given description is the most straight-forward: take HERA fit, calculate necessary impact factor/coefficient, evolve to lower x
→ works very good without tuning etc.
- Question: do we hide on-set of saturation in the linear NLO term?
Answer: We don't know
- Way out 1: re-fit HERA data with BFKL Green's function which provides eternal growth (if possible) and see what happens
- Way out 2: Look at different observables ...

inclusive DIS, exclusive photo-
production: \sim to $N(x,r,b)$
(dipole amplitude = prob. to
scatter with proton gluon field)



$$\text{Im} \mathcal{A}_{T,L}^{\gamma^* p \rightarrow V p}(W, t=0) = 2 \int d^2 \mathbf{r} \int d^2 \mathbf{b} \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_{T,L} \mathcal{N}(x, r, b)$$

recent theory study:

[Ayala, MH, Jalilian-Marian, Tejada-Yeomans, arXiv:1604.08526]

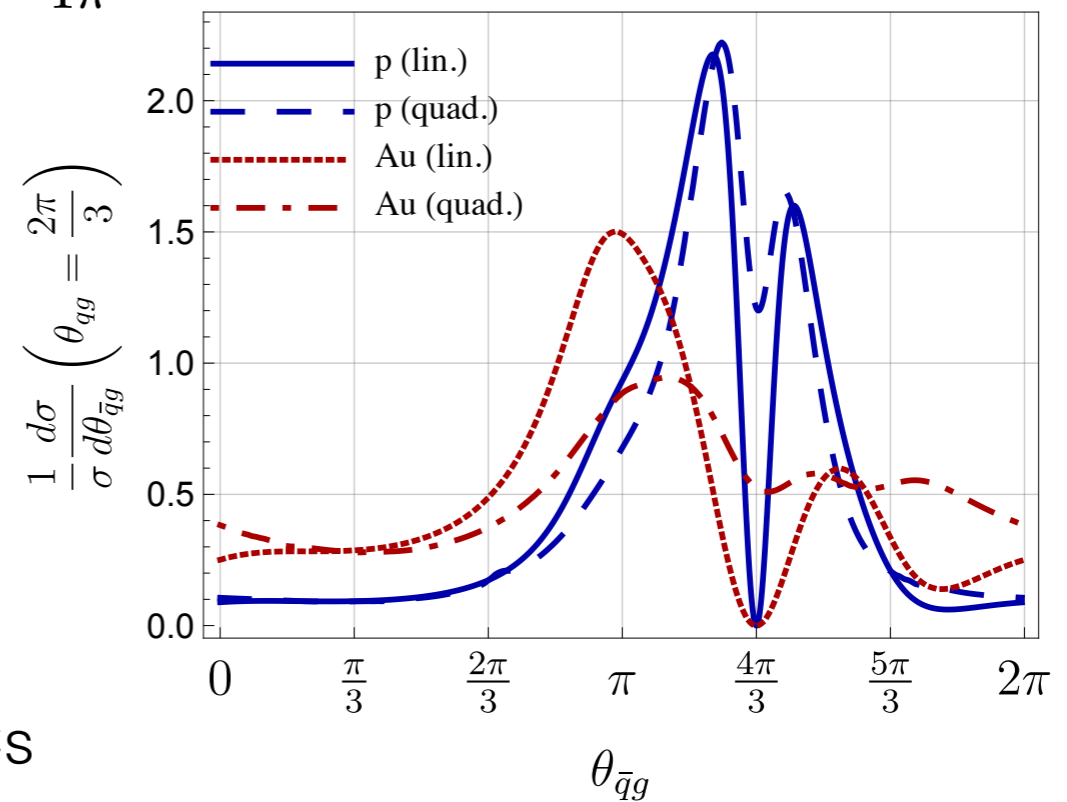
3 parton production in DIS

involves higher correlators

→ in Gaussian approximation as function of
 $N(x,r)$ + expand $O(N^2)$

there are potential similar processes in UPCs

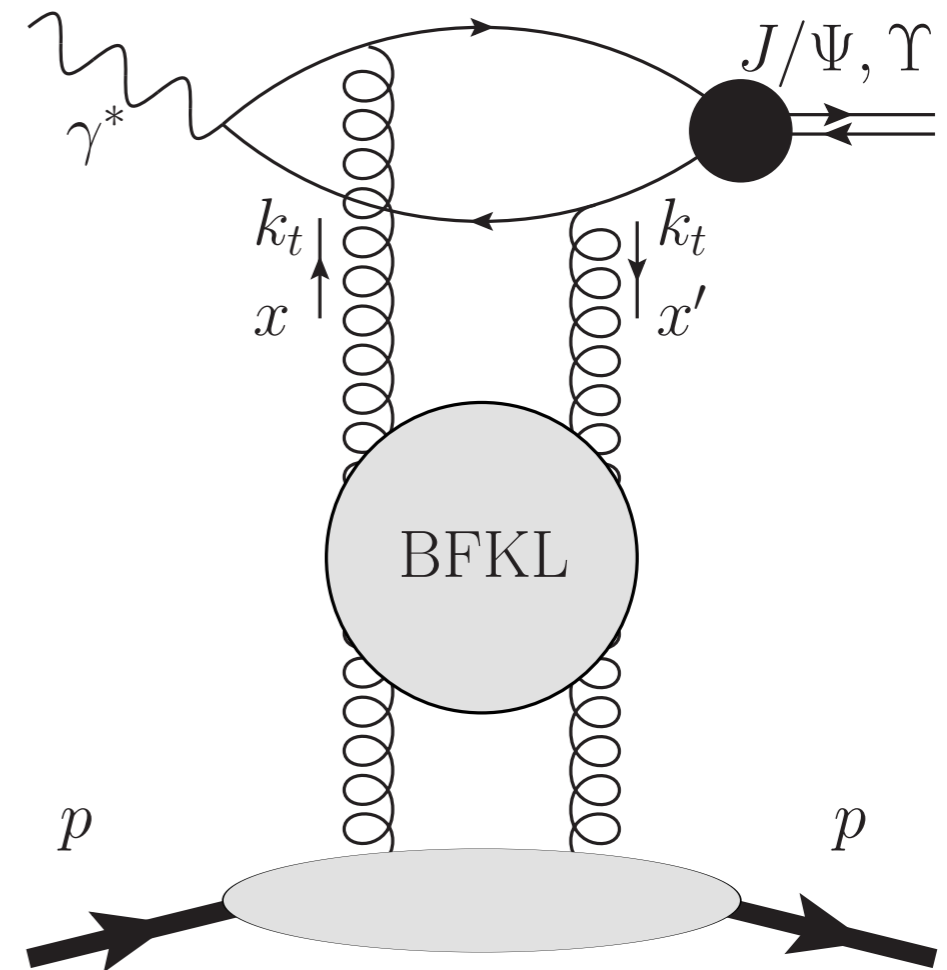
→ need to explore, can be measured? ...



extra slides

Observations:

- ▶ K -factor: small for fit 2, sizeable for fit 1 – likely related to the impact factors in used in the HERA fit (massless, $n_f = 4$, $(\mathcal{C}_1/\mathcal{C}_2)^2 = 2.45$)
- ▶ common correction not included: GPD motivated factor to take into account $x' \neq x$; currently calculated for collinear pdf [Shuvaev, Golec-Biernat, Martin, Ryskin, hep-ph/9902410] \rightarrow to be calculated for k_T factorized BFKL impact factor



very good description of W -dependence

$W_{J/\psi} > 471 \text{ GeV}$ & $W_{\Upsilon} > 669 \text{ GeV} \equiv$ beyond region of incl. HERA fit
(from $x = 4.3 \cdot 10^{-5}$ to $x = 3.5 \cdot 10^{-6}$) \rightarrow direct test of BFKL evolution

Existing description of data

... work pretty well

- ▶ J/Ψ : power-law fit to HERA data $\sigma \sim W^{0.67}$ [LHCb Collaboration; 1401.3288]
- ▶ collinear factorization: NLO fits [Jones, Martin, Ryskin, Teubner; 1307.7099]
- ▶ saturation models: IPsat, bCGC, rcBK
[Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1405.6977, 1408.1344]
- ▶ see also [Fiore, Jenkovszky, Libov, Machado; 1408.0530], [Cisek, Schäfer, Szczurek; 1405.2253]

BFKL special:

don't fit W -dependence, but calculate from perturbative low x evolution
don't evoke saturation (= effects beyond BFKL)

LHC: reach ultra-small x values $\simeq 4 \cdot 10^{-6}$ not constrained by HERA

The setup: diff. Xsec. at $t = 0$

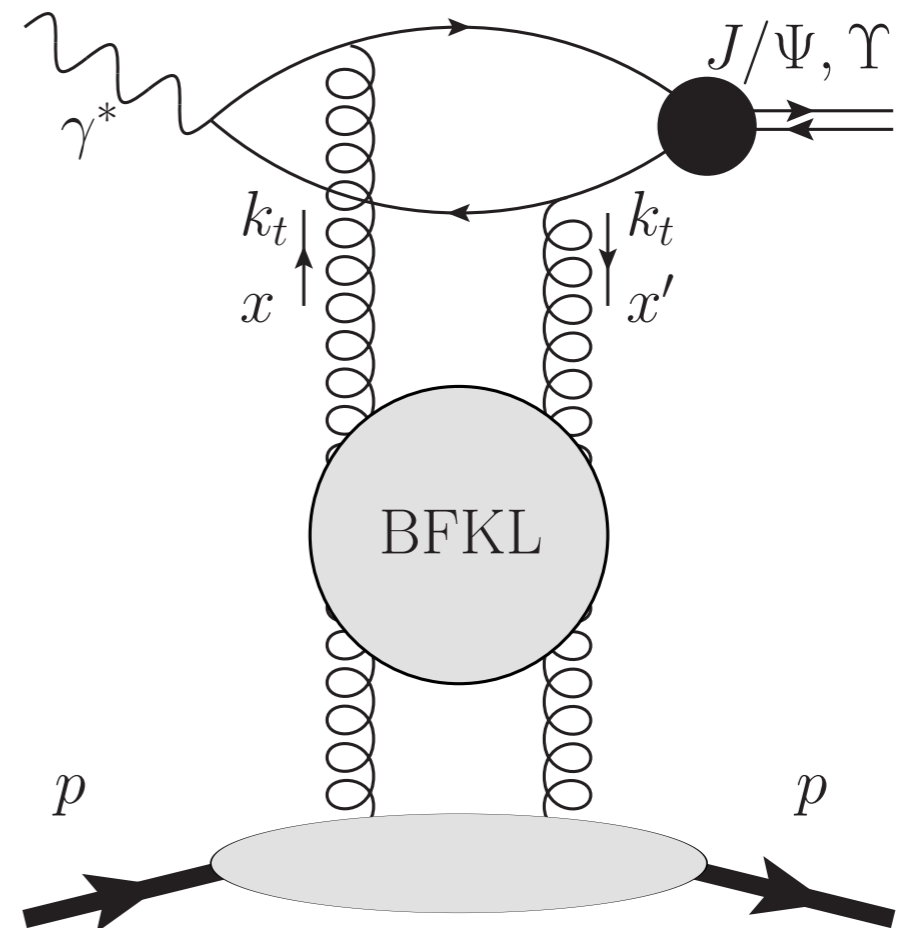
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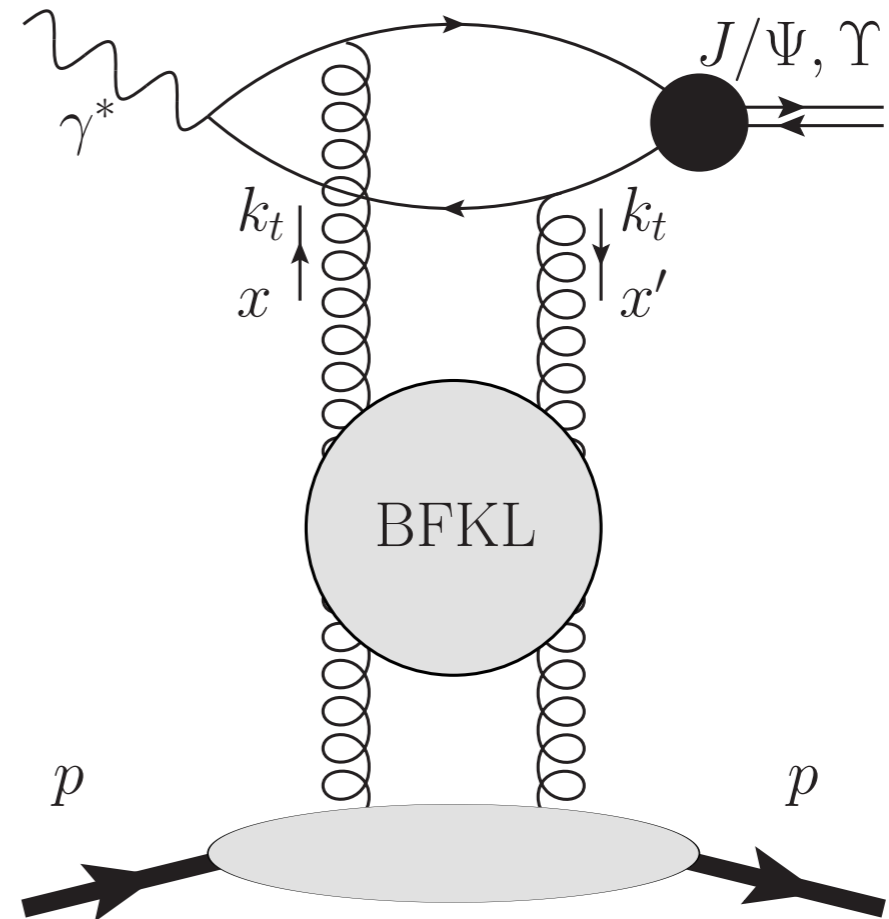


The setup: diff. Xsec. at $t = 0$

a) imaginary part of scattering amplitude:

calculate diff. Xsec. at $t = 0$

- ✓ unintegrated gluon density from NLO BFKL fit to combined HERA data [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]
- ✗ **impact factor** $\gamma \rightarrow J/\Psi, \Upsilon$ from light-front wave function used in dipole model studies
- ✗ ...



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likely related to the impact factors used in the HERA fit
(massless quarks, $n_f=4$, $(C_1/C_2)^2=2.45$)
- common “GPD” correction not included; known for collinear
pdf, need to be re-calculated within high energy
factorization
- very good description of W -dependence, going beyond
HERA region