Transverse Momentum Dependent Parton Distribution Functions from Parton Shower in Pythia.

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Abstract

We present an approach to obtain Transverse Momentum Dependent Parton Distribution Functions (TMDPDF) from Parton Shower (PS) from Pythia Monte Carlo (MC) generator. We implement a special process in Pythia where we fix Bjorken x of one of the incoming partons to make the calculations of k_T of the second incoming parton possible. We perform detailed studies on the effects of using different definitions of internal evolution variable (the longitudinal momentum fraction of the proton carried by the second incoming parton). We conclud that depending on the definition of internal evolution variable one gets different TMDPDFs. We also check the relation between TMDPDFs and PDFs, i.e. we study the integrated TMDPDFs and compar them with $cteq61l$ PDFs.

Contents

1 Introduction

1.1 Parton Distribution Functions

PDFs represent the probability densities to find inside a proton at scale Q^2 a parton i carrying a longitudinal fraction x of the proton momentum

$$f_i\left(x, Q^2\right). \tag{1}$$

PDFs can not be obtained from QCD calculations, we have to measure them but if we know PDF at a given x and Q^2, we can get them at other x or Q^2 using evolution equations. This is shown in the following figure 1.

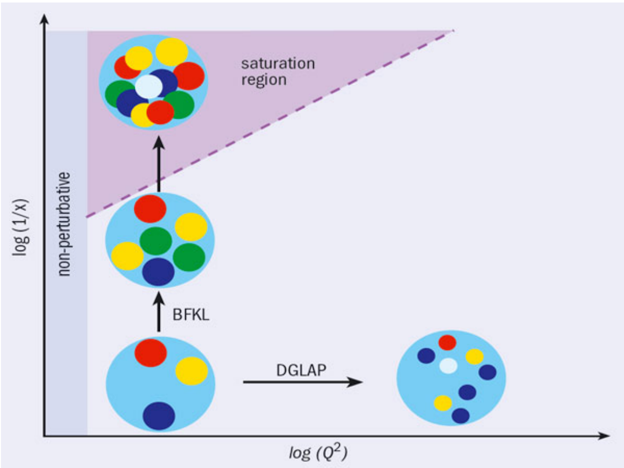


Figure 1: QCD evolution equations[1].

From figure 1, we can learn that if we know a PDF at a given Q_0^2, we can get them at another Q^2 using DGLAP evolution equation. On the other hand, if we know a PDF at a given x_0, we can get them at another x using BFKL evolution equation.

PDFs sets are obtained from fits to cross section data from many experiments.

An example of the parton distribution functions from HERAPDF1.5 at NNLO is shown in figure 2, where we can see that:

- at large values of x valence quarks predominate inside a proton.

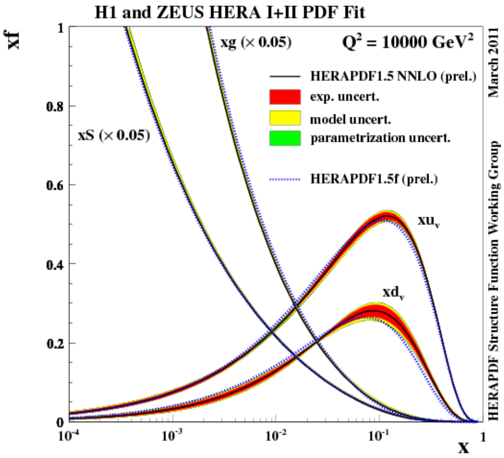


Figure 2: The parton distribution functions from HERAPDF1.5 NNLO at $Q^2 = 10000 GeV^2$[2].

- at small values of x sea quarks and gluons predominate.

A precise knowledge of PDFs of the proton is essential in order to make predictions for the Standard Model and beyond the Standard Model processes at hadron colliders[2].

1.2 QCD Parton Model.

1.2.1 Introduction to the formalism.

In our project we use the following variables: P_1 and P_2, are the four momenta of colliding protons, p_1 and p_2 are the four momenta of colliding partons. In collinear case: $p_1 = x_1 P_1$, $p_2 = x_2 P_2$, where x_1 and x_2 are the x Bjorken.

We use the four vector convention:

$$\begin{aligned}(E, \mathbf{p}) &= \left(p^0, p^1, p^2, p^3\right) \\ &= p^\mu = p.\end{aligned}$$

The invariant mass of a particle is defined as

$$m^2 = E^2 - \mathbf{p}^2$$

Any four-vector can be represented in Light Cone Variables (LCV):

$$
\begin{aligned}
p^+ &= \frac{1}{\sqrt{2}}(p^0 + p^3) \\
p^- &= \frac{1}{\sqrt{2}}(p^0 - p^3) \\
p_\perp &= (p^1, p^2)
\end{aligned}
$$

with $p_\perp$ being a two-component vector.

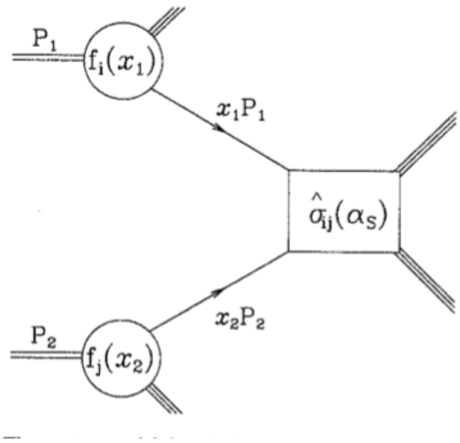


Figure 3: The parton model description of a hard scattering process[3].

1.2.2 Parton Model

In Parton Model partons are point-like constituents of hadrons. This model describes the high energy interactions of hadrons as an interaction between the quarks and gluons (partons)[3].

The basis of any QCD calculations lies in the collinear factorization theorem which says that the cross section can be calculated as a convolution of parton density functions and a hard scattering cross section[3],[4]:

$$\sigma\left(P_1, P_2\right) = \Sigma_{ij} \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \widehat{\sigma}_{ij} \left(p_1, p_2, \alpha_s(Q^2), \frac{Q_0^2}{Q^2}\right)$$

where Q_0 is the characteristic scale and $\widehat{\sigma}_{ij}$ is the hard matrix element, $\widehat{\sigma}_{ij}$ represents the perturvative part and PDFs represents the non perturvative part. For the scheme of the process see figure 3.

1.3 Transverse Momentum Dependent PDF

Since in collinear approach the information about transverse momentum (k_T) of the partons is missing, a transverse momentum dependency is added to generalize the concept of PDFs by new types of parton distributions: the TMD parton distributions (TMDPDF)[5],[6].

A classic example, that illustrates why we need to add a k_T dependency, is given by Drell-Yan hadro-production of electroweak gauge bosons. Figure 4 shows the differential cross section for Z-boson production in pp collision at the LHC as a function of the Z-boson transverse momentum q_T[6].

We can distinguish two regions for q_T:

- High q_T: expected to be well represented by an evaluation of the partonic Z-boson cross section to finite order in QCD perturbation theory, combined with factorization in terms of ordinary (collinear) parton distribution functions (pdfs).

- Decreasing and Turn Over q_T: the physical behavior of the Z-boson spectrum near the peak region and below is controlled by multi-parton QCD radiation, which is not well approximated by truncating the QCD perturbation series.

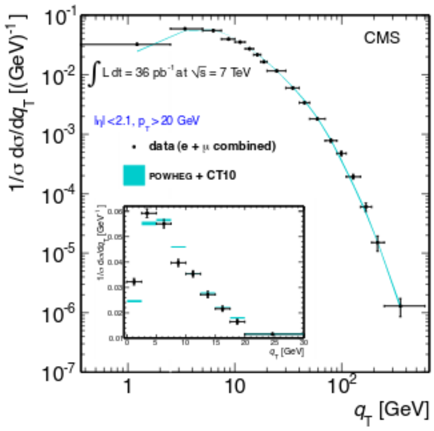


Figure 4: The Z-boson transverse momentum q_T spectrum in pp collisions at the LHC[6].

The cross section predicted from any finite order perturbation theory, convoluted with ordinary parton distributions, will diverge as q_T decreases. One way to manage this situation is to add explicitly a dependency on transverse momentum to parton distribution functions (TMDPDF) and to use TMD factorization theorem. After this generalization the physical behaviour of the $Z - boson$ spectrum can be predicted[6].

2 Our Project

We want to obtain TMDPDF from MC generator from Parton Shower (PS)[7].

Since, we are interested in parton evolution (we are not interested in final state), we use Pythia[8] to generate special non physical hard process $gg \longrightarrow Z$. In order to make the calculation of k_T possible, because its information is not stored in Pythia, x_1 is fixed (to .98). Thanks to that we forbid the evolution of the first parton and the whole $\overrightarrow{k}_T$ comes from the evolution of the second parton. From momentum conservation, we know that $p_{T\,final} = k_T$ of the second parton ($p_{T\,final}$ information is stored in Pythia).

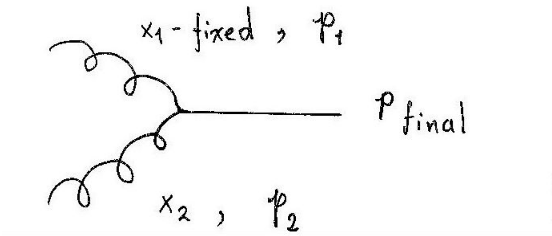


Figure 5: Non-physical process generated by Pythia to obtain TMD from PS.

2.1 First step: $\widehat{\sigma}$

We ask Pythia to generate the hard process. Then, it generates collinear $\widehat{\sigma}$, it means $\overrightarrow{k}_{T_0} = 0$.

We have to define x_2. One way to define x_2 is using the following relation:

$$\begin{aligned} \widehat{s} = (p_1 + p_2)^2 &= p_{final}^2 \\ &= x_1 x_2 s \end{aligned}$$

$$\longrightarrow x_2 = \frac{\widehat{s}}{x_1 s} \tag{2}$$

where in the CM: $|\overrightarrow{P_1}| = |\overrightarrow{P_2}| = \overrightarrow{P}$, and $M_1 = M_2 = M$, thus $s = (P_1 + P_2)^2 = 4E_{beam}^2 = E_{CM}^2$.

Pythia uses this definition, we check it our project. Another way to define x_2 is using LCV.

$$x_2 = \frac{p_2^+}{P_2^+}$$

$$= \frac{\frac{1}{\sqrt{2}} \cdot (p_2^0 + p_2^3)}{\frac{1}{\sqrt{2}} \cdot (P_2^0 + P_2^3)} \tag{3}$$

In collinear case these two definitions of x_2 are equivalent.

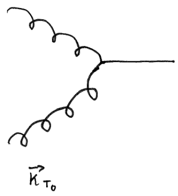


Figure 6: We turn on intrinsic k_T.

2.2 Second step in Pythia: evolution.

2.2.1 k_T On

Now, we turn on intrinsic $\overrightarrow{k}_T$, i.e. (*id est.*), the initial $\overrightarrow{k}_{T_0} \neq 0$, but small.
The energy of the partons' interaction does not change when we turn on k_T, *i.e.*, $\widehat{s}$ does not change. But components of p_2 has to be changed. Therefore $x_2 = \frac{\widehat{s}}{x_1 s}$ (eq.(2))- is the same as in collinear case. And $x_2 = \frac{p_2^+}{P_2^+}$ (eq.(3)) is different than in collinear case.

When we plot TMDPDFs as a function of k_T, for different definitions of x_2, we obtain that all curves are the same, due to the fact that $\overrightarrow{k}_T$ is small. See figure 7.

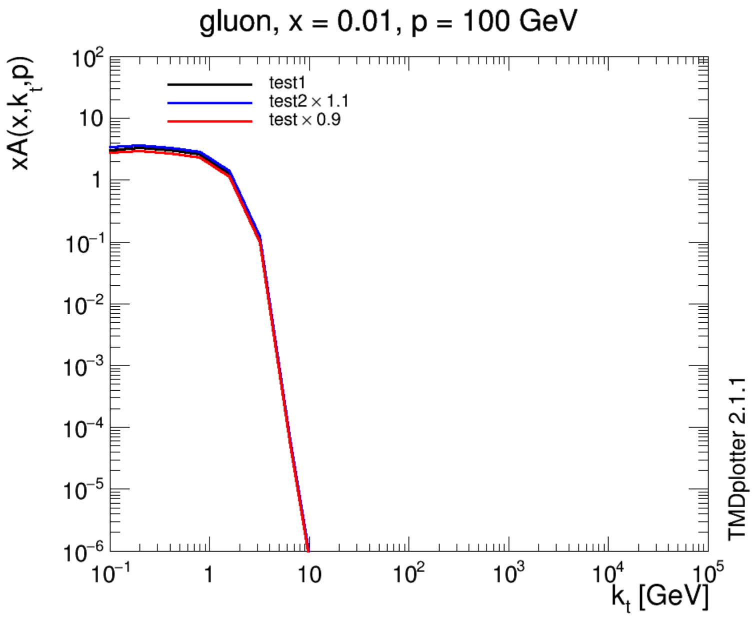


Figure 7: Results for TMDPDF with intrinsic k_T on, at $x = 0.01$ and $p = Q = 100 GeV$.

2.2.2 k_T and PS On

We turn on Parton Shower (PS), and accumulate $\overrightarrow{k}_T$ in the evolution process.

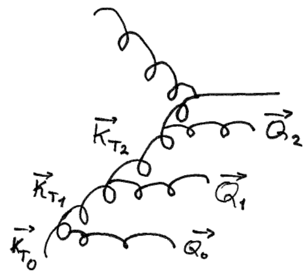


Figure 8: Evolution process with k_T and PS on.

It means that if we want to calculate $\overrightarrow{k_{T_1}}$, we have to sum the previous $\overrightarrow{k_{T_0}}$ and the transverse momentum of the emitted parton $\overrightarrow{Q_0}$:

$$\begin{aligned}\overrightarrow{k_{T_1}} &= \overrightarrow{k_{T_0}} + \overrightarrow{Q_0},\\ \overrightarrow{k_{T_2}} &= \overrightarrow{k_{T_1}} + \overrightarrow{Q_1},\end{aligned}$$

This is shown in the figure 8.

Once again, the energy of the partons' interaction does not change when we turn on k_T and PS, so

- $x_2 = \frac{\widehat{s}}{x_1 s}$ is the same as in collinear case.

But components of p_2 has to be changed,

- $x_2 = \frac{p_2^+}{P_2^+}$ is different than in collinear case.

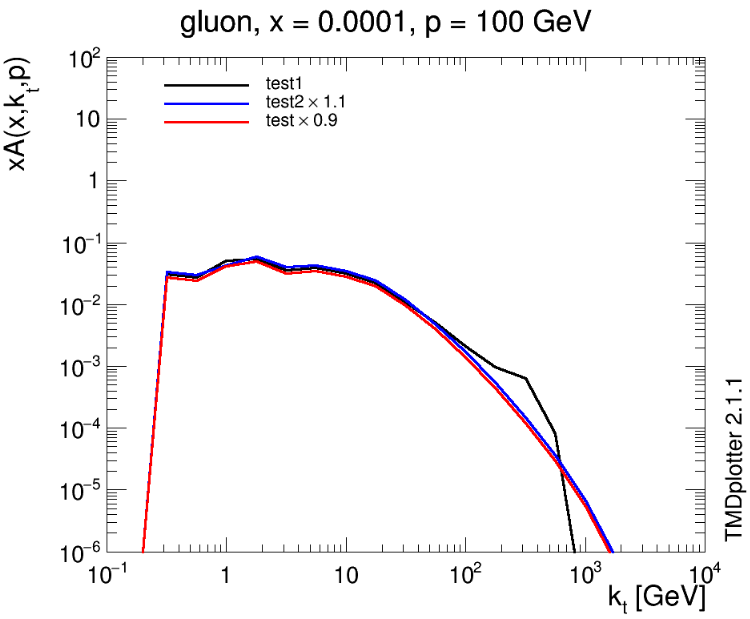


Figure 9: Results for three definitions of x_2, with k_T on and PS on at $x = 0.0001$ and $p = Q = 100 GeV$

Hence, with different definitions of x_2 we get diffent $TMDPDFs$. See figure 9

In table 1 and 2, we present our results for x_2 and k_T distributions for different definitions of x_2 and different Pythia options. (In the first column, we present collinear case, with PS and intrinsic k_T off. In the second column we turn on intrinsic k_T and PS off. In the third column we turn on PS and k_T, finally in the last column we put all cases in the same plot.)

For LCV definition of x_2 (eq.3), we see that when we turn on PS the distribution of x_2 changes. Whereas for Pythia definition of x_2 (eq.2)it remains the same.

k_T distributions do not depend on x_2 definitions.

Table 1: Distributions of x_2

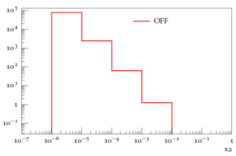
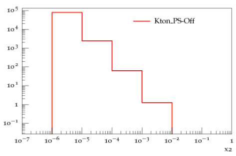
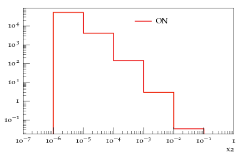
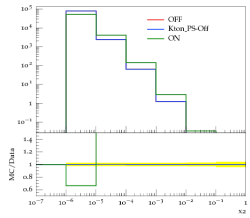
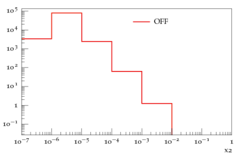
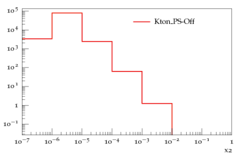
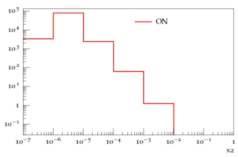
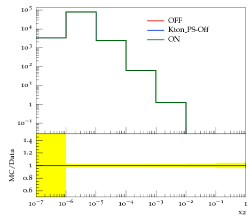
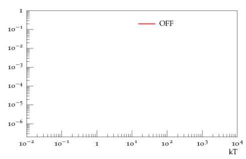
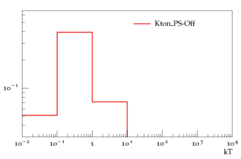
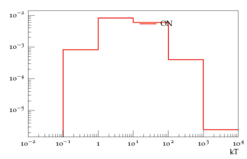
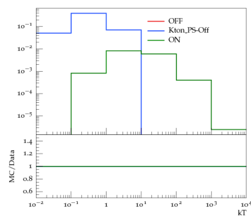
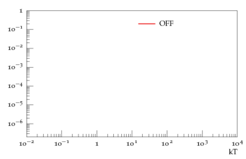
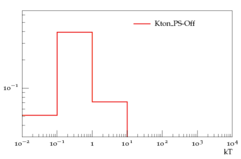
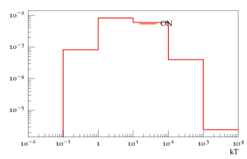
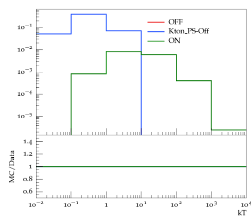
x_2	PS and k_T off	PS off and k_T on	PS and k_T on	All together
$x_2 = \frac{p_2^+}{P_2^+}$				
$x_2 = \frac{\widehat{s}}{x_1 s}$				

Table 2: Distributions of k_T

x_2	PS and k_T off	PS off and k_T on	PS and k_T on	All together
$x_2 = \frac{p_2^+}{P_2^+}$				
$x_2 = \frac{\widehat{s}}{x_1 s}$				

2.3 Integrated TMDs

It is necessary to remind you that PDFs can not be obtained from QCD calculations, but if we know a PDF at a given x and Q_0^2, we can get them at another x and Q^2 using DGLAP evolution equation. In our project we use PDF set *cteq*61*l*, as a starting distribution at scale Q_0^2. We let Pythia to evolve, *i.e.*, turn on PS and accumulate k_T.

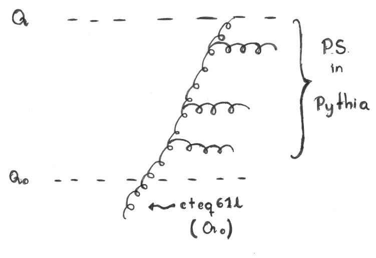


Figure 10: We take PDF set *cteq*61*l*, as a starting distribution at scale Q_0^2. Then, we let Pythia to evolve and accumulate k_T. We stop the evolution at Q^2.

We stop the evolution at scale Q^2 and we check the relation:

$$''PDF(x, Q^2) = \int dk_T TMDPDF(x, Q^2, k_T)'' \tag{4}$$

I.e. We want to check, if integrating over k_T $TMDPDF$ from Pythia at scale Q^2, gives the same as PDF from *cteq*61*l* at the same Q^2 scale. The procedure is explained in figure 10.

At graph 11, we present our results.

On the graph we have four different curves:

- The blue one $x_2 = \frac{\widehat{s}}{x_1 s}$.
- The black one $x_2 = \frac{p_2^+}{P_2^+}$.

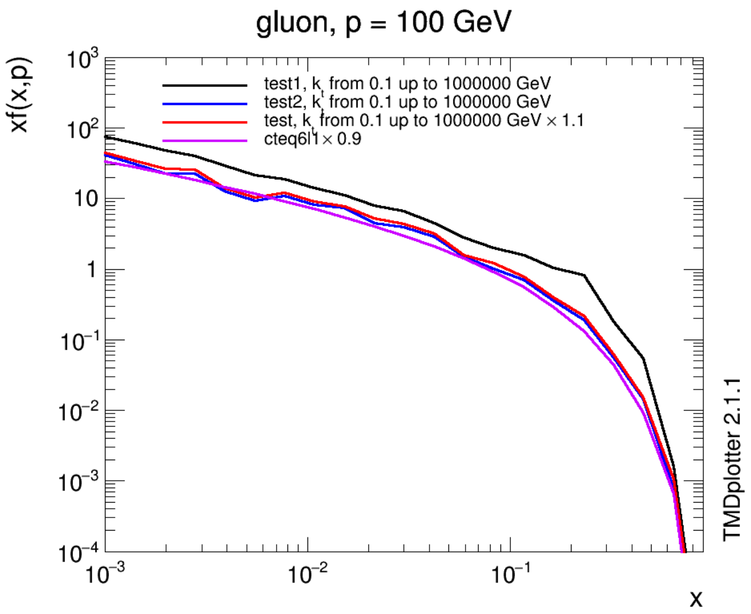


Figure 11: Result with k_T on and PS on at $x = 0.0001$ and $p = Q^2 = 100$ for three definitions of x_2

- The red one is the internal Pythia definition, the same as eq.2.

- The magenta one is the $cteq61l$.

The relation $''PDF(x, Q^2) = \int dk_T TMDPDF(x, Q^2, k_T)''$ works with Pythia definition of x_2 (eq.2) and LCV definition of x_2 (eq.3) does no agree with $cteq61l$.

Depending on how we deffine x_2, we get different PDFs.

3 Conclusions

With the special nonphysical process implemented in Pythia we are able to obtain TMDs from PS.
After integrating TMD over $\overrightarrow{k}_T$ we obtain collinear PDF which agrees with $cteq61l$ if we take the internal Pythia definition of x_2 (eq.2). If we deffine x_2 (eq.3)with light cone variables, we obtain different PDFs.

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