The Weizsäcker-Williams distribution of linearly polarized gluons (and its fluctuations) at small x

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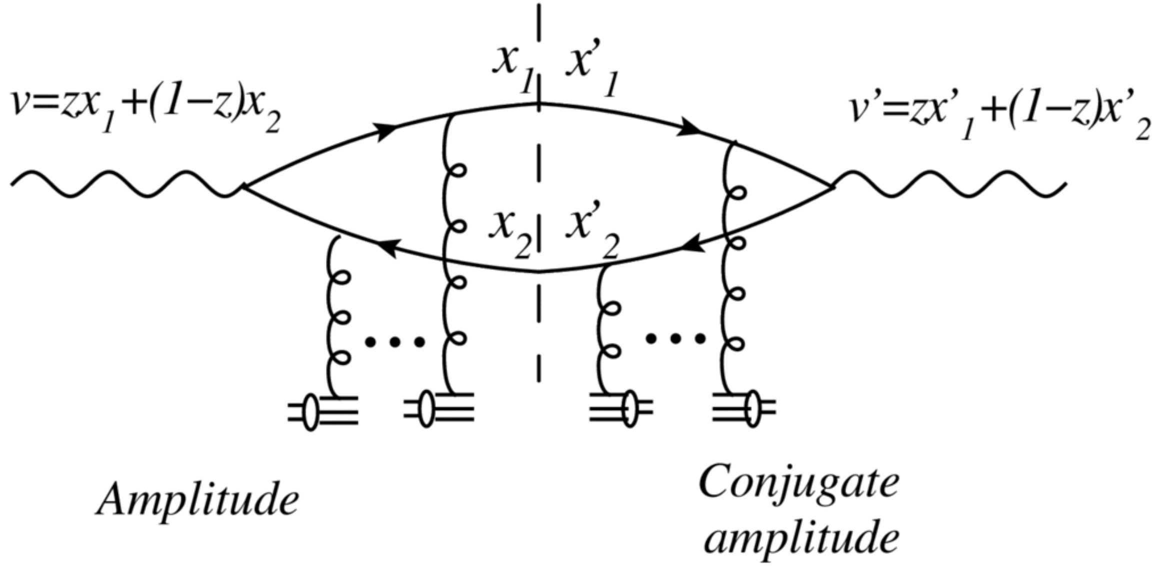
based on: A.D., T. Lappi, V. Skokov, 1508.04438 / PRL 115 (2015)
A.D., V. Skokov, 1704.05917 / PRD (2017)

Dijets in γ*A :

(Dominguez, Marquet, Xiao, Yuan, PRD 2011)



CM tr. momentum:

$$\vec{P} = \frac{1}{2}\left(\vec{k}_1 - \vec{k}_2\right) \quad \text{or} \quad \widetilde{P} = (1-z)\vec{k}_1 - z\vec{k}_2$$

and momentum imbalance: $\vec{q} = \vec{k}_1 + \vec{k}_2$

- Both dijets in the hemisphere of γ* (y≥0)

“correlation limit” $P \gg q$ involves only 2-point functions / TMDs (leading power)

WW gluon distribution, unpolarized target

(Mulders, Rodrigues, PRD 2001
Metz, Zhou, PRD 2011,
Dominguez, Qiu, Xiao, Yuan, PRD 2012)

$$\int d^2\xi \, d\xi^- e^{ixP^+\xi^- - i\vec{q}_\perp\cdot\vec{\xi}} \left\langle \mathrm{tr}\ F^{i+}(\xi) U_\xi^{[+]\dagger} F^{j+}(0) U_0^{[+]} \right\rangle$$

$$\sim \quad \delta^{ij}\ xG^{(1)}(x, q_\perp) + \left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij} \right)\ xh^{(1)}(x, q_\perp)$$

$$\delta^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (e_x^i e_x^j + e_y^i e_y^j) = \left[\varepsilon_+^{*i} \varepsilon_+^j + \varepsilon_-^{*i} \varepsilon_-^j \right]$$

$$\left(\frac{2q_\perp^i q_\perp^j}{q_\perp^2} - \delta^{ij} \right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (e_x^i e_y^j + e_y^i e_x^j) = -i \left[\varepsilon_+^{*i} \varepsilon_-^j - \varepsilon_-^{*i} \varepsilon_+^j \right]$$

(in frame where $q_x = q_y$)

compare to gluon helicity distribution

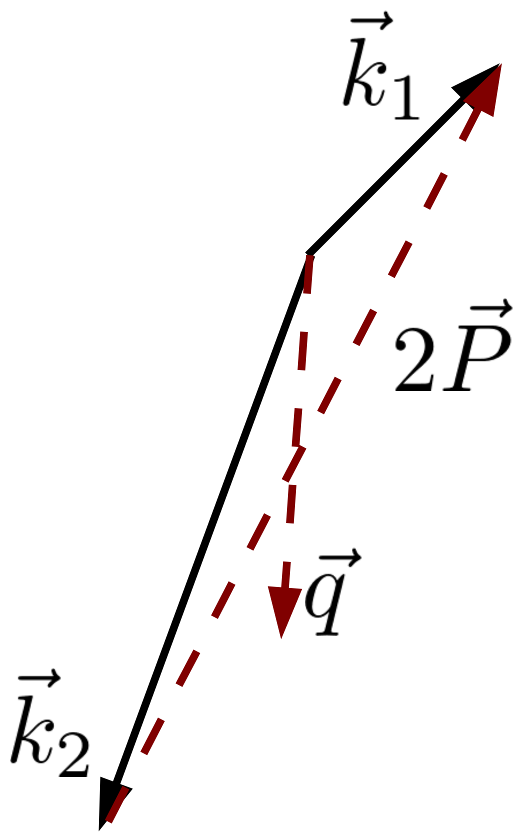
$$i\epsilon^{ij} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = ie_x^i e_y^j - e_y^i e_x^j = \varepsilon_+^{*i} \varepsilon_+^j - \varepsilon_-^{*i} \varepsilon_-^j$$

Azimuthal anisotropy

Dominguez, Qiu, Xiao, Yuan, PRD 2012
Boer, Mulders, Pisano, PRD 80 (2009) – 2016
Boer, Brodsky, Mulders, Pisano, PRL 106 (2011)

$$d\sigma^{\gamma_L^* A \to q\bar{q}X} = e_q^2 \alpha \alpha_s z^2 (1-z)^2 \frac{8\epsilon_f^2 \widetilde{P}^2}{(\widetilde{P}^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x,q) + \cos(2\phi)\, xh^{(1)}(x,q) \right)$$

ϕ = angle between $\vec{P}$ and $\vec{q}$



→ rotate net transverse momentum vector q around and measure amplitude of cos(2ϕ) modulation

$$v_2(q,x) = \langle \cos 2\phi \rangle = \frac{1}{2} \frac{h_\perp^{(1)}(x,q)}{G^{(1)}(x,q)}$$

The distribution of linearly polarized gluons

(in terms of L.C. gauge field correlator)

Metz, Zhou: PRD 2011;
Dominguez, Qiu, Xiao, Yuan, PRD 2012

$$xG^{(1)}(x,k) = -\frac{2}{\alpha_s L^2}\delta^{ij}\left\langle \mathrm{Tr}\left[A_i(\vec{k})A_j(-\vec{k})\right]\right\rangle$$

$$xh^{(1)}(x,k) = \frac{2}{\alpha_s L^2}\left(\delta^{ij} - 2\frac{k^i k^j}{k^2}\right)\left\langle \mathrm{Tr}\left[A_i(\vec{k})A_j(-\vec{k})\right]\right\rangle$$

$$A_i(\vec{k}) = \int \frac{d^2 y}{(2\pi)^2} e^{-i\vec{k}\cdot\vec{y}}\, U^\dagger(\vec{y})\partial_i U(\vec{y})$$

$$U(\vec{y}) = \mathcal{P}\, e^{-ig\int dz^+ A^-(z^+,\vec{y})}$$

$$\partial_i U(\vec{y}) = ig\int_{-\infty}^{\infty} dz^+ U(-\infty, z^+; \vec{y})\, \partial_i A^-(z^+,\vec{y})\, U(z^+,\infty;\vec{y})$$

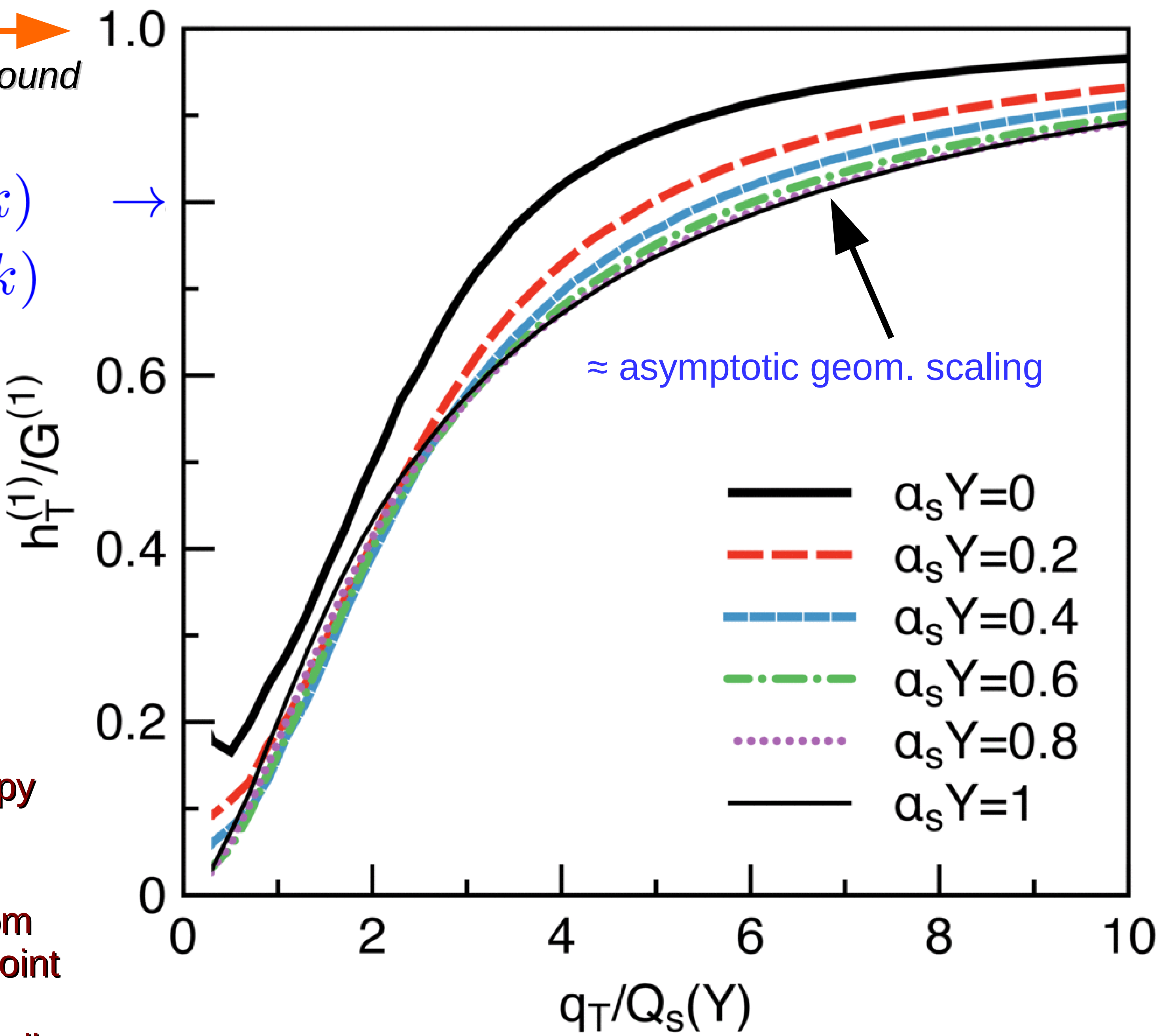
F^{i-}

We have computed these functions at small x
by solving JIMWLK from MV model initial conditions

(A.D., T. Lappi, V. Skokov: 1508.04438)

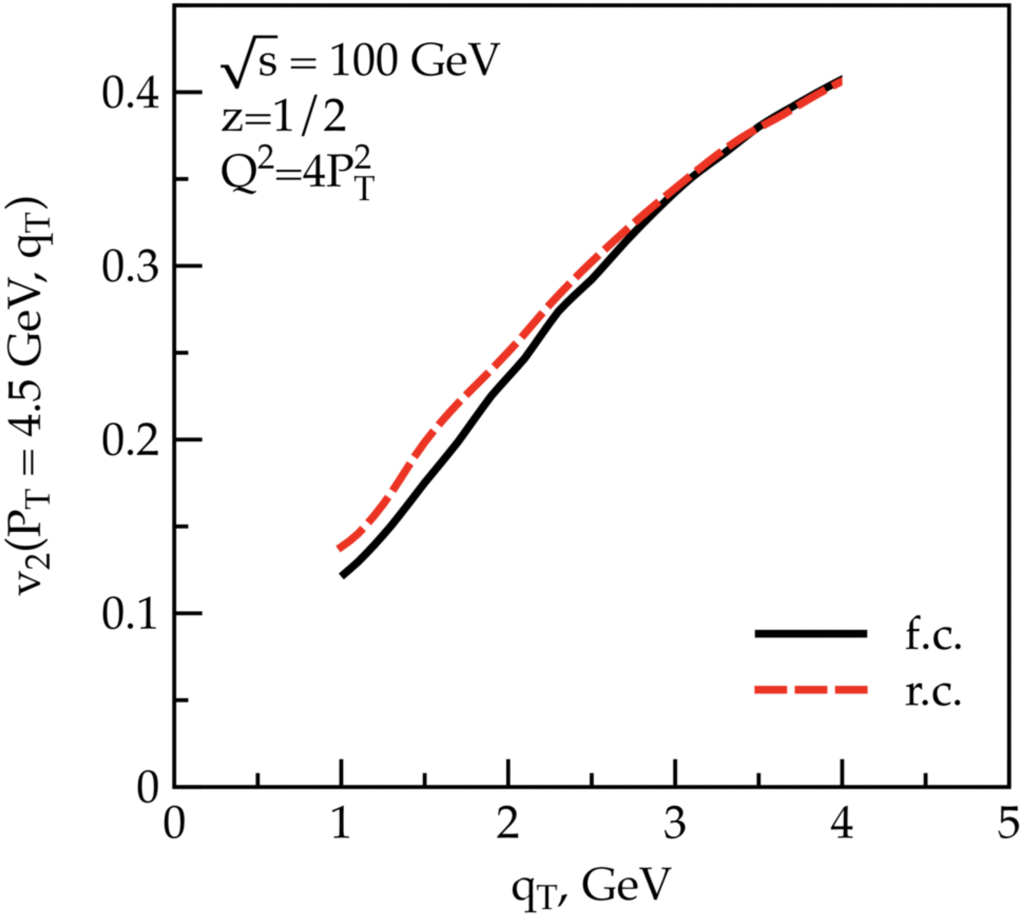
saturation of positivity bound

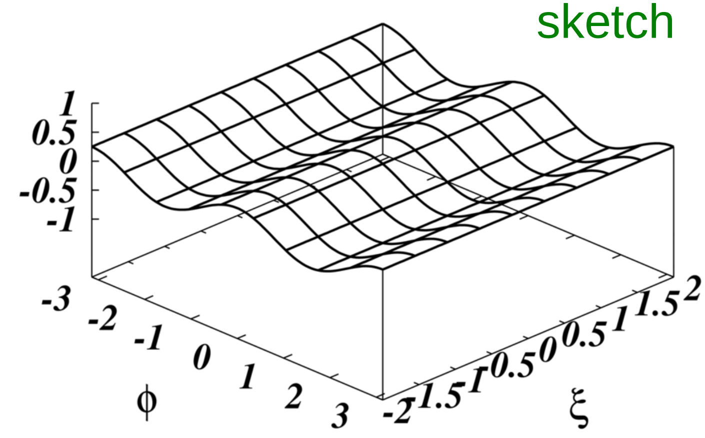
$$A^i(k) \simeq \frac{ik^i}{k^2}\,\rho(k) \quad \rightarrow$$
$$xG^{(1)}(k) \simeq xh^{(1)}(k)$$



- Large ~cos(2φ) anisotropy at $q_T > Q_s$
- rapid initial flow away from MV model → RG fixed point
- asymptotically: geom. scaling

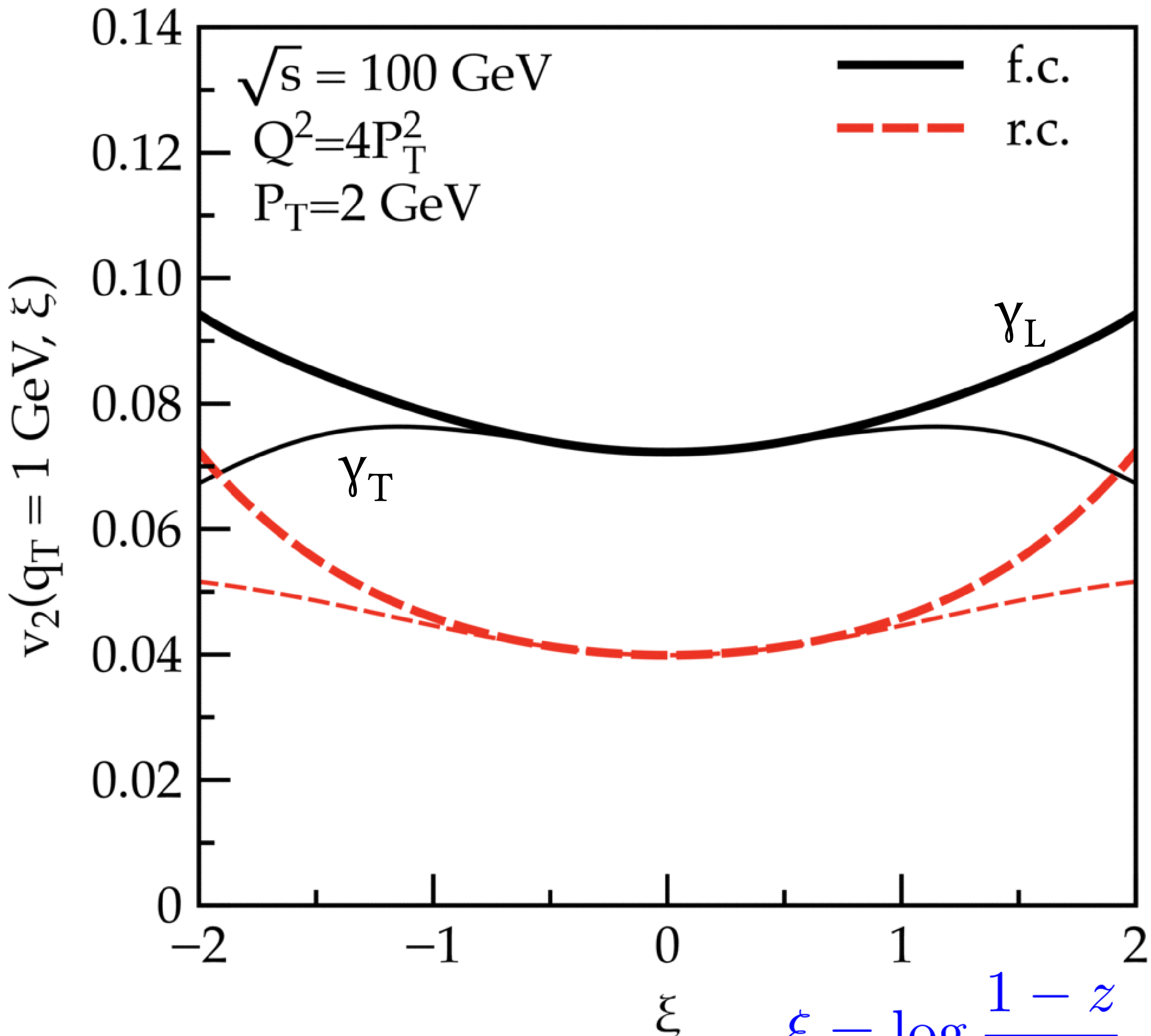
Large cos(2ϕ) amplitudes...





$$\xi = \log \frac{1-z}{z}$$

Amplitude of cos(2Φ) is long range in rapidity

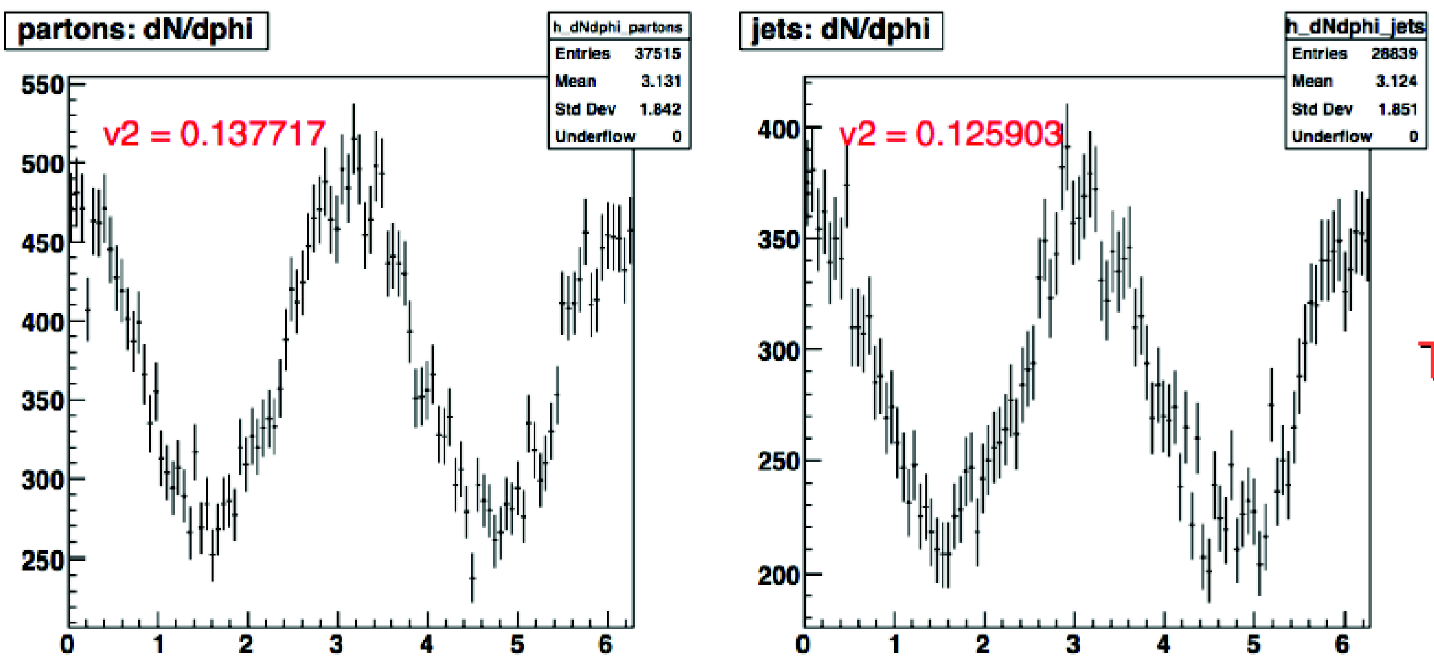


$$\xi = \log \frac{1-z}{z}$$

Monte Carlo Event generator

- DIS event with random Q^2, W^2, photon polarization, as well as $P_\perp$ and $q_\perp$
- Input: $\sqrt{s}$ and A
- Q_s and target area are adjusted according to A
- Output: Parton 4-momentum etc
- Pythia afterburner $\rightarrow$ particles
- Jet reconstruction

- 1 < qt < 1.5
- 2 < Pt < 2.5
- pol=1 (L)



T. Ullrich & V. Skokov

A. Dumitru, V. S. and T. Ullrich work in progress

Constraint effective action:

integrate out fluctuations which do not affect observable O[A^+]
→ obtain effective action / potential for that observable

$$e^{-V_{\rm eff}[X(q)]} = \int \mathcal{D}A^+(q)\, W[A^+(q)]\, \delta(X(q) - O[A^+(q)])$$

$$\frac{\delta V_{\rm eff}[X(k)]}{\delta X(q)} = 0 \to X_s(q) \equiv \langle X(q) \rangle$$ (at large Nc)

$$\text{for } O[A^+] = g^2\, {\rm tr}\, |A^+(q)|^2 \text{ in Gaussian model :}$$

$$V_{\rm eff}[X(q)] = \int \frac{d^2q}{(2\pi)^2} \left[\frac{q^4}{g^4\mu^2} X(q) - \frac{1}{2} A_\perp N_c^2 \log X(q) \right]$$

$$X_s(q) = \frac{1}{2} N_c^2 A_\perp \frac{g^4\mu^2(q^2)}{q^4}$$ ✓

(cov. gauge gluon distribution at q>Qs)

field redefinition: $e^{\Phi(q)} \equiv X(q) / X_s(q)$ → Liouville action

$$V_{\rm eff}[\phi(q)] = \frac{1}{2} A_\perp N_c^2 \int \frac{d^2q}{(2\pi)^2} \left[e^{\phi(q)} - \phi(q) \right]$$

to exhibit fluctuations of X(q) in other correlators use:

$$\left\langle \rho^a(x^-, q) \rho^b(y^-, k \right\rangle = \delta^{ab} \delta(x^- - y^-) \, (2\pi)^2 \delta(q+k) \, \mu^2(q)$$

→

$$\begin{aligned} \left\langle \rho^a(x^-, q) \rho^b(y^-, k) \right\rangle &= \delta^{ab} \delta(x^- - y^-) \, (2\pi)^2 \delta(q+k) \, \mu^2(q) \\ &\times \int \mathcal{D}X(\ell) \, e^{-V_{\rm eff}[X(\ell)]} \, X(q)/X_s(q) \end{aligned}$$

access rare configurations :

pick any gluon distribution $X(q)$ you like*,

its weight relative to the average $X_s(q)$ is

$$w[X(q)] = \exp - (V[X(q)] - V[X_s(q)])$$

* i) $X(q) \sim A^{1/3}$ or else it corresponds to a higher order correction at fixed $g^4 A^{1/3} = O(1)$

Yu. Kovchegov, PRD (2000)

ii) $X(q) \sim N_c^2$ or else need $O(N_c^0)$ contribution to $V_{eff}[X]$

WW (light cone gauge) gluon distribution

$$g^2 \mathrm{tr}\, A^i(q) A^j(-q)$$

at order $(gA^+)^4$

$$\begin{aligned}
\delta^{ij}\, g^2 \mathrm{tr}\, A^i(q) A^j(-q) &= \frac{1}{2} q^2 g^2 A^{+a}(q) A^{+a}(-q) \\
&+ \frac{g^4}{8} f^{abe} f^{cde} \left(\frac{q^l q^m}{q^2} - \delta^{lm} \right) \\
&\int \frac{d^2 k}{(2\pi)^2} \frac{d^2 p}{(2\pi)^2} k^l p^m A^{+a}(q-k) A^{+b}(k) A^{+c}(-q-p) A^{+d}(p)
\end{aligned}$$

$$\begin{aligned}
\left(2 \frac{q^i q^j}{q^2} - \delta^{ij} \right) g^2 \mathrm{tr}\, A^i(q) A^j(-q) &= \frac{1}{2} q^2 g^2 A^{+a}(q) A^{+a}(-q) \\
&- \text{same as above}
\end{aligned}$$

* on average over **all** configurations, contribution at order $\langle (g^2 A^+ A^+)^n \rangle \sim (Q_s^2/q^2)^n$ at $q^2 > Q_s^2$
* power suppression guarantees that $xh^{(1)}(x,q^2) > 0$ at $q^2 > Q_s^2$
* [all twist resummation makes **average** $xh^{(1)}(x,q^2) > 0$ at all q^2]

* even though average over **all** configurations is a positive definite function, this needs not be true for particular subclasses of configurations

$$\left\langle g^2 \mathrm{tr}\, A^i(q) A^i(-q) \right\rangle \;=\; \int \mathcal{D}X(p)\, q^2 X(q)\, e^{-V_{\mathrm{eff}}[X(p)]}$$

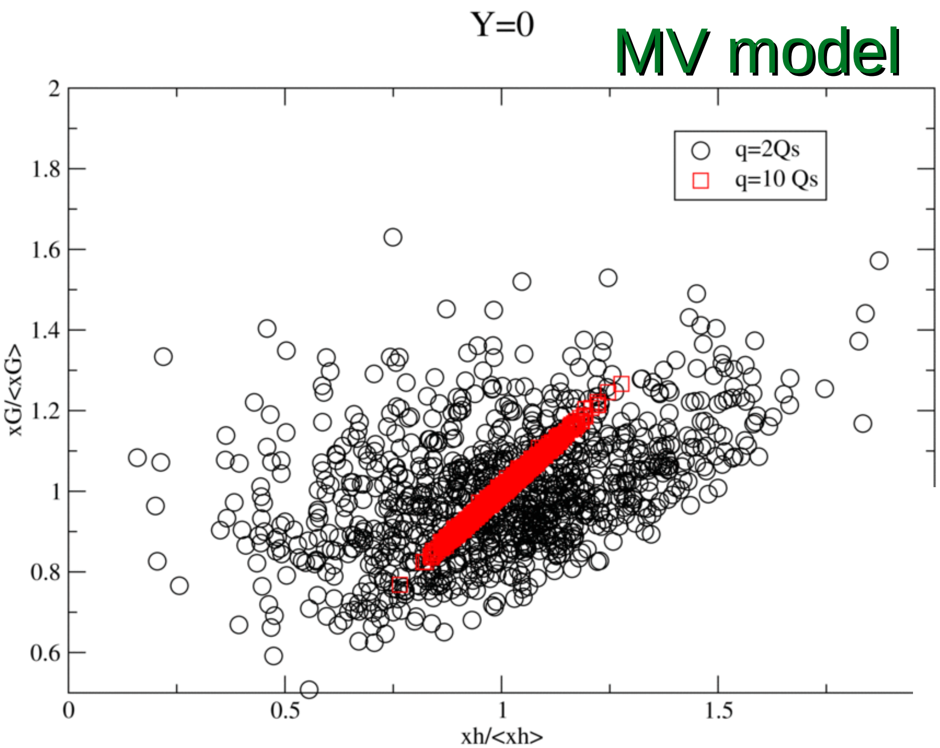
$$\pm \quad \frac{1}{N_c A_\perp} \int \mathcal{D}X(p)\, e^{-V_{\mathrm{eff}}[X(p)]} \int \frac{d^2k}{(2\pi)^2} \left[k^2 - (\hat{q}\cdot k)^2\right] X(q-k) X(k)$$

* for some X(q) this can go negative

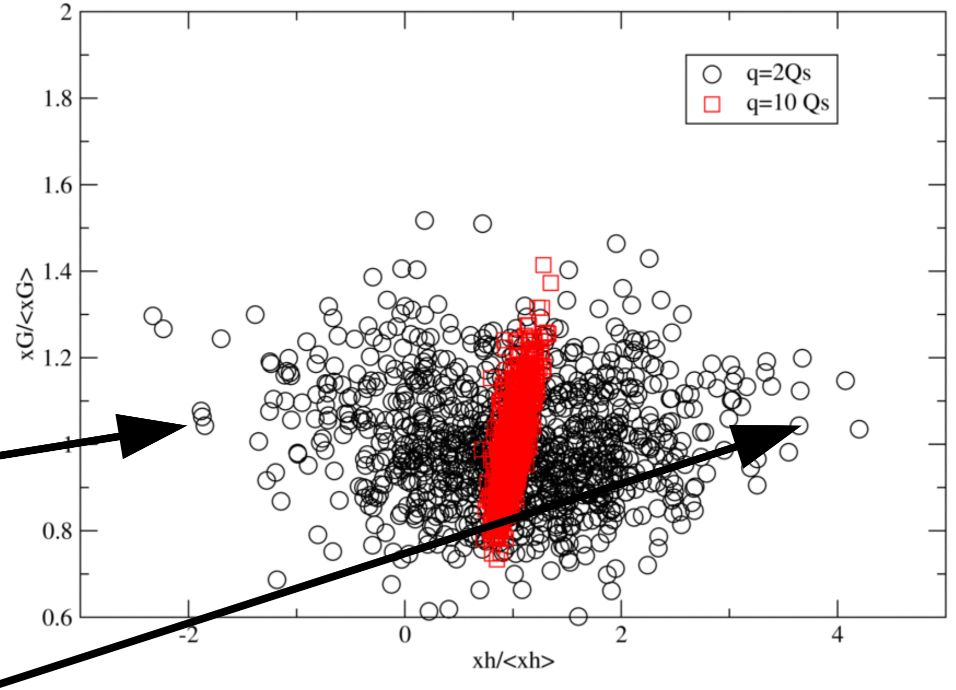
Fluctuations of WW gluon distributions (MV vs. f.c. JIMWLK)

(all orders in A^+)

MV model



JIMWLK αY=1



note negative xh here

and large positive xh here

Observables ?

$\mathrm{tr}\, F\tilde{F}(x)$ divergence of Chern-Simons current

$$\left\langle \mathrm{tr}\, F\tilde{F}(x)\ \mathrm{tr}\, F\tilde{F}(y) \right\rangle \sim [xG_P^{(1)}(r)]^2\,[xG_T^{(1)}(r)]^2 - [xh_P^{(1)}(r)]^2\,[xh_T^{(1)}(r)]^2$$

Lappi, Schlichting, 1708.08625

projectile / target WW distributions

* for r ~ 1 / Qs this average is over a wide distribution of $xh^{(1)}(r)$ at small x

Summary:

- Dijet production in eA probes WW gluon TMD (in P_T » q_T limit)

- WW distribution can be decomposed into **two** UGDs / TMDs
  i) conventional gluon probability $xG^{(1)}(x,q_T)$
  ii) linearly polarized distribution $xh^{(1)}(x,q_T)$
  (corresponds to orthogonal polarizations in amplitude vs. conj. amplitude

- Classical field gives large ~cos(2Φ) anisotropies at $q_T > Qs$

- JIMWLK small-x evolution: $xG^{(1)}(x,q_T)$ and $xh^{(1)}(x,q_T)$ evolve similarly, (~ geometric scaling)

- Fluctuations of (impact parameter integrated) cov-gauge gluon distribution $\Phi(q) \sim \log g^2 \, \mathrm{tr} \, |A^+(q)|^2$ is described by Liouville action, in small-x Gaussian theory for $A^+(q)$ or $\rho(q)$

- distribution of linearly polarized gluons $xh^{(1)}(x,q)$ exhibits large fluctuations at small x!
  (in particular for q ~ Qs)

Backup slides

Resummation of boost-invariant quantum fluctuations (JIMWLK):

classical ensemble at Y = log x_0/x = 0:

$$P[\rho] \sim e^{-S_{\rm cl}[\rho]} \ , \ S_{\rm MV} \quad = \quad \int d^2x_\perp \, dx^+ \, \frac{1}{2\mu^2} \rho^a \rho^a \ ,$$

$$V(x_\perp) = \mathcal{P} \exp \ ig^2 \int dx^+ \frac{1}{\nabla_\perp^2} \rho(x_\perp)$$

JIMWLK quantum evolution: functional RG equation

$$\frac{\partial}{\partial Y} W[V] = -H \left[V, \frac{\delta}{\delta A^-} \right] W[V]$$

distribution in space of Wilson lines

quantum evolution to Y>0: Langevin / random walk in space of Wilson lines

$$\partial_Y V(x_\perp) = V(x_\perp)\; it^a \left\{ \int d^2y_\perp\; \varepsilon_k^{ab}(x_\perp, y_\perp)\; \xi_k^b(y_\perp) + \sigma^a(x_\perp) \right\} \; .$$

$$\varepsilon_k^{ab} = \left(\frac{\alpha_s}{\pi}\right)^{1/2} \; \frac{(x_\perp - y_\perp)_k}{(x_\perp - y_\perp)^2} \; \left[1 - U^\dagger(x_\perp) U(y_\perp)\right]^{ab}$$

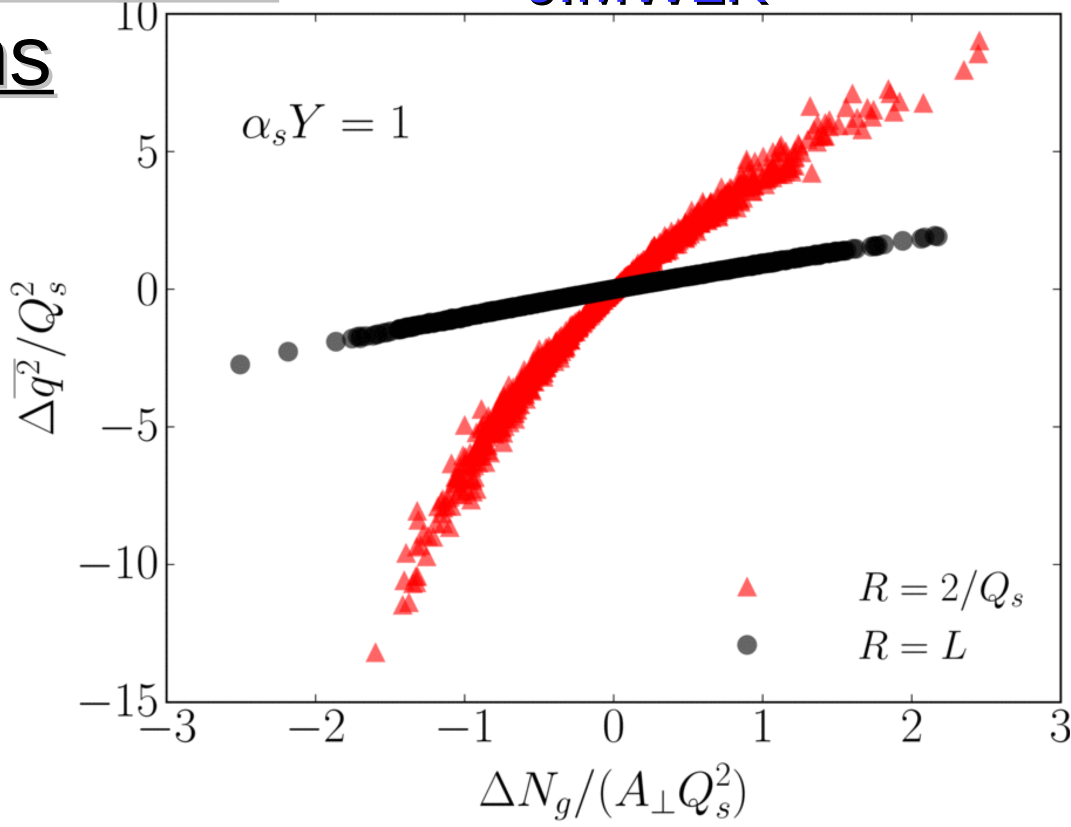
$$\langle \xi_i^a(x_\perp)\; \xi_j^b(y_\perp) \rangle = \delta^{ab} \delta_{ij} \delta^{(2)}(x_\perp - y_\perp)$$

$$\sigma^a(x_\perp) = -i \frac{\alpha_\mathrm{s}}{2\pi^2} \int d^2z_\perp \, \frac{1}{(x_\perp - z_\perp)^2} \mathrm{tr} \; \left(T^a U^\dagger(x_\perp)\, U(z_\perp)\right)$$

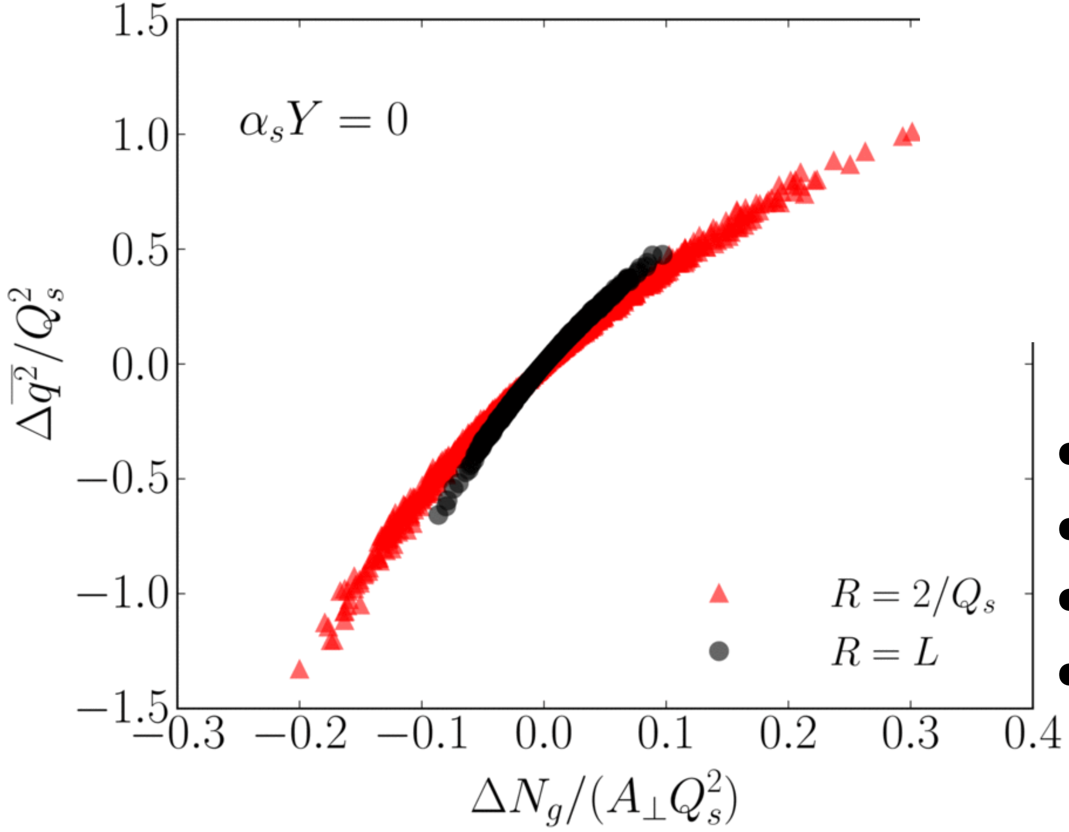
Correlation of transv. momentum and multiplicity fluctuations

A.D., V. Skokov: 1704.05917

JIMWLK



MV model



- ΔN_g ~ 10 – 100 for A = 0.1 fm^2, Q_s^2 = (1-2.5 GeV)^2
- tight correlation (~ single curve, few outliers)
- JIMWLK shows correlations in b-space
- strong increase of Δq^2 with ΔN_g in small area "patches"

General expression for $\gamma^* A \rightarrow q\bar{q}X$ to all orders in q/P

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^3k_1 d^3k_2} =$$

$$N_c \alpha_{em} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2x_1}{(2\pi)^2}\frac{d^2x_2}{(2\pi)^2}\frac{d^2x_1'}{(2\pi)^2}\frac{d^2x_2'}{(2\pi)^2} e^{-i\vec{k}_1(\vec{x}_1-\vec{x}_1')-i\vec{k}_2(\vec{x}_2-\vec{x}_2')}$$

$$\sum_{\gamma\alpha\beta} \psi^{\mathrm{T,L}}_{\alpha\beta}(\vec{x}_1-\vec{x}_2)\psi^{\mathrm{T,L}*}_{\alpha\beta}(\vec{x}_1'-\vec{x}_2')$$

$$\Big[1 + \frac{1}{N_c}\Big(\langle \mathrm{Tr}\, U(\vec{x}_1)U^\dagger(\vec{x}_2)U(\vec{x}_2')U^\dagger(\vec{x}_1')\rangle$$

Quadrupole

$$-\langle \mathrm{Tr}\, U(\vec{x}_1)U^\dagger(\vec{x}_2)\rangle - \langle \mathrm{Tr}\, U^\dagger(\vec{x}_1')U(\vec{x}_2')\rangle\Big)\Big]$$

write $e^{-i\vec{k}_1(\vec{x}_1-\vec{y}_1)-i\vec{k}_2(\vec{x}_2-\vec{y}_2)} = e^{-i\vec{P}(\vec{u}-\vec{u}')-i\vec{q}(\vec{v}-\vec{v}')}$

$$\vec{u} = \vec{x}_1 - \vec{x}_2 \quad , \quad \vec{v} = (\vec{x}_1 + \vec{x}_2)/2$$

and expand in powers of u, u'

$$\mathcal{Q}(\vec{x}_1,\vec{x}_2;\vec{x}_2',\vec{x}_1') = 1+\frac{\langle \mathrm{Tr}\, U(\vec{x}_1)U^\dagger(\vec{x}_1')U(\vec{x}_2')U^\dagger(\vec{x}_2)\rangle - \langle \mathrm{Tr}\, U(\vec{x}_1)U^\dagger(\vec{x}_2)\rangle - \langle \mathrm{Tr}\, U(\vec{x}_1')U^\dagger(\vec{x}_2')\rangle}{N_c}$$

$$= u_i u_j' \mathcal{G}^{i,j}(v,v') + u_i u_j' u_k' u_l' \mathcal{G}^{i,jkl}(v,v') + u_i u_j u_k u_l' \mathcal{G}^{ijk,l}(v,v') + u_i u_j u_k' u_l' \mathcal{G}^{ij,kl}(v,v') + \cdots$$

$$\mathcal{G}^{i,j}(v,v') = -\frac{1}{N_c}\langle \mathrm{Tr}\, V^\dagger(v)\partial_i V(v)\; V^\dagger(v')\partial_j V(v')\rangle$$

$$\mathcal{G}^{ij,mn}(v,v') = \frac{1}{16N_c}\langle \mathrm{Tr}\,\left[V^\dagger(v)\partial_i\partial_j V(v) + (\partial_i\partial_j V^\dagger(v))V(v)\right]\left[(\partial_m\partial_n V^\dagger(v'))V(v') + V^\dagger(v')\partial_m\partial_n V(v')\right]\rangle\ ,$$

$$\mathcal{G}^{ijm,n}(v,v') = -\frac{1}{24N_c}\langle \mathrm{Tr}\,\left[V^\dagger(v)\partial_i\partial_j\partial_m V(v) + 3(\partial_i\partial_j V^\dagger(v))\partial_m V(v)\right] V^\dagger(v')\partial_n V(v')\rangle\ ,$$

$$\mathcal{G}^{n,ijm}(v,v') = -\frac{1}{24N_c}\langle \mathrm{Tr}\,\left[V^\dagger(v)\partial_n V(v)\right]\left[V^\dagger(v')\partial_i\partial_j\partial_m V(v') + 3(\partial_i\partial_j V^\dagger(v'))\partial_m V(v')\right]\rangle$$

The dijet X-section involves the following combination:

$$\mathcal{G}^{ijmn}(x,q^2) = \mathcal{G}^{i,jmn}(x,q^2) + \mathcal{G}^{ijm,n}(x,q^2) - \frac{2}{3}\mathcal{G}^{ij,mn}(x,q^2)$$

introduce projectors $\mathfrak{P}_1^{ijmn},\ \mathfrak{P}_2^{ijmn},\ \mathfrak{P}_3^{ijmn}$
which project out cos(0φ), cos(2φ), cos(4φ)

$$\Phi_2(x,q^2) = -\frac{2N_c}{\alpha_s}\mathfrak{P}_3^{ijmn}\mathcal{G}^{ijmn}(x,q^2)$$

explicit evaluation in Gaussian large-Nc model

$$\mathcal{Q}^G = 1 + e^{-\frac{C_F}{2}\left[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')\right]} - e^{-\frac{C_F}{2}\left[\Gamma(x_1-x_2)\right]} - e^{-\frac{C_F}{2}\left[\Gamma(x_2'-x_1')\right]}$$
$$- \frac{\Gamma(x_1-x_1') - \Gamma(x_1-x_2') + \Gamma(x_2-x_2') - \Gamma(x_2-x_1')}{\Gamma(x_1-x_1') - \Gamma(x_1-x_2) + \Gamma(x_2-x_2') - \Gamma(x_2'-x_1')}$$
$$\times \left(e^{-\frac{C_F}{2}\left[\Gamma(x_1-x_2)+\Gamma(x_2'-x_1')\right]} - e^{-\frac{C_F}{2}\left[\Gamma(x_1-x_1')+\Gamma(x_2'-x_2)\right]}\right)$$

Jalilian-Marian, Kovchegov: hep-ph/0405266
Blaizot, Gelis, Venugopalan: hep-ph/0402257
Dominguez, Marquet, Xiao, Yuan: 1101.0715

perform same expansion in powers of u, u' :

$$xG^{(1)}(x,q^2) = \frac{4N_c}{\alpha_s}\frac{S_\perp}{(2\pi)^3}\int dr\, r J_0(qr)\left(1-[S^{(2)}(r^2)]^2\right)\left(\frac{\Gamma^{(1)}(r^2)}{\Gamma(r^2)} + r^2\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)}\right)$$

$$xh^{(1)}(x,q^2) = \frac{4N_c}{\alpha_s}\frac{S_\perp}{(2\pi)^3}\int dr\, r^3 J_2(qr)\left(1-[S^{(2)}(r^2)]^2\right)\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)}$$

$$\Phi_2(x,q^2) = -\frac{N_c}{\sqrt{2}\,3\pi\alpha_s}\frac{S_\perp}{(2\pi)^2}\int dr\, J_4(rq)\, r^5$$
$$\times\left[\frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)}\left(1-\left[S^{(2)}(r^2)\right]^2\right) - 5\left(\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)}\right)^2\left[1-\left[S^{(2)}(r^2)\right]^2(1+C_F\Gamma(r^2))\right]\right]$$

BK small-x fixed point: $\Gamma(r^2) \sim \left(r^2 Q_s^2(x)\right)^{\gamma_c}$

anomalous dimension γ_c~0.63 near saturation boundary

$\chi(\gamma_c)/\gamma_c = \chi'(\gamma_c)$

Mueller & Triantafyllopoulos (2002)
S. Munier & R. Peschanski (2005)

γ=1-O(α_s) in DGLAP regime

more generally: when $S^{(2)}(r,x) = S^{(2)}(r\, Q_s(x))$

$xG^{(1)}(x,q)$, $xh^{(1)}(x,q)$, and $\Phi_2'(x,q) \equiv \Phi_2(x,q)/q^2$ exhibit "geometric scaling",

i.e. functions only of $q/Q_s(x)$

DiJet cross section

DiJet cross section to this order

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{d^2k_1dz_1d^2k_2dz_2}$$

$$= \alpha_s\alpha_{em}e_q^2\left(z_1^2+z_2^2\right)\left[\frac{P^4+\epsilon_f^4}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)-\frac{2\epsilon_f^2P^2}{P^4+\epsilon_f^4}xh^{(1)}(x,q^2)\cos 2\phi+\mathcal{O}\left(\frac{1}{P^2}\right)\right)\right.$$

$$\left.-\frac{48\epsilon_f^2P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right]$$

$$\frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d^2k_1dz_1d^2k_2dz_2}$$

$$= 8\alpha_s\alpha_{em}e_q^2z_1z_2\epsilon_f^2\left[\frac{P^2}{(P^2+\epsilon_f^2)^4}\left(xG^{(1)}(x,q^2)+xh^{(1)}(x,q^2)\cos 2\phi+\mathcal{O}\left(\frac{1}{P^2}\right)\right)\right.$$

$$\left.+\frac{48P^4}{\sqrt{2}\,(P^2+\epsilon_f^2)^6}\Phi_2(x,q^2)\cos 4\phi\right].$$

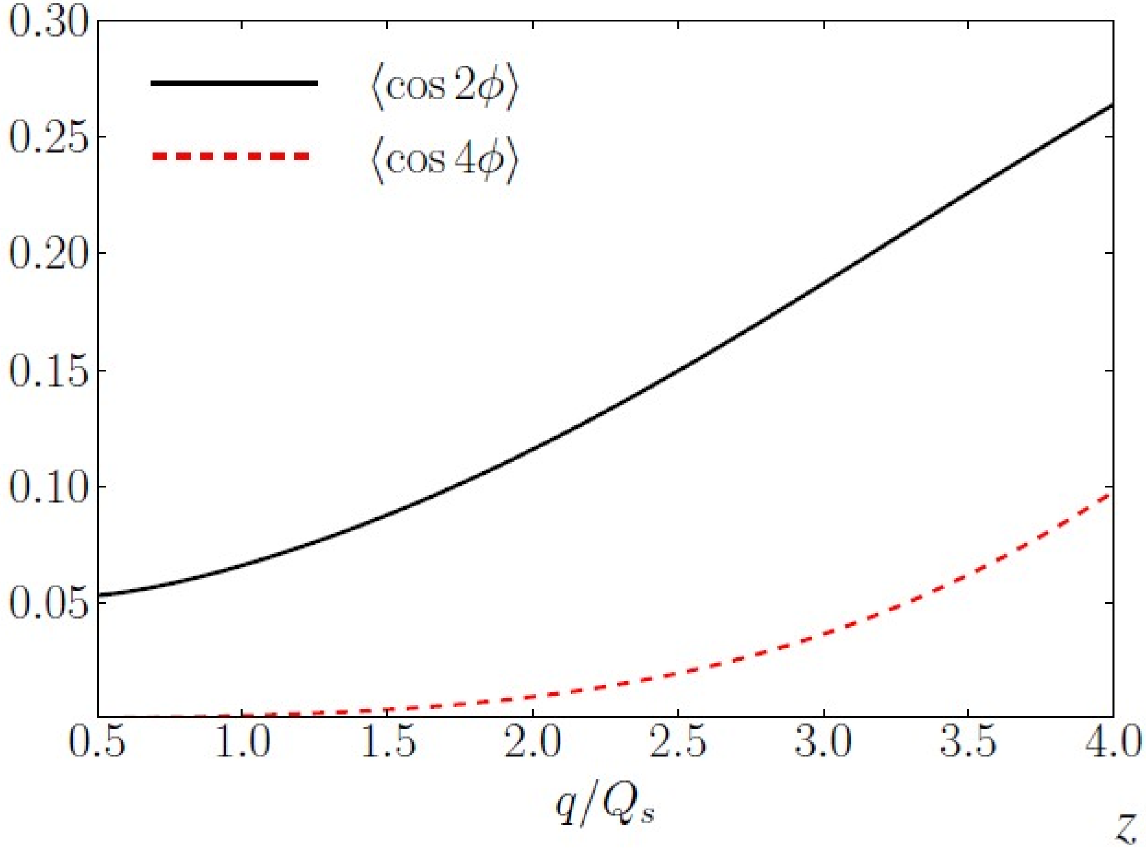
$$\text{DIS}: \epsilon_f^2 = z_1z_2Q^2$$

$$Q^2 \sim P^2$$

A. Dumitru and V. S., arXiv:1605.02739

MV RESULTS

$\langle \cos 2\phi \rangle$ and $\langle \cos 4\phi \rangle$ in $\gamma_L^* + A \to q + \bar{q}$ dijet production from MV model:



$z = 1/2,\ P = 4.5 Q_s$

A. Dumitru and V. S., arXiv:1605.02739