Hydrodynamical Description of the QGP Using Energy-momentum In-medium Deposition By An Extended Source

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Introduction

How does a parton travel through the QGP and deposits energy-momentum?

I address this questions about the QGP medium's response using linearized viscous hydrodynamics.

What can we learn from the energy-momentum deposition into the medium?

Deposition could be on the sides of the path traveled, or along the direction of the traveling parton.

This is work in progress and I will briefly explain the beginning of my work.

Equilibrium Medium

The medium 4-velocity,

$$u^\mu = (1, \mathbf{u})\,, \quad \mathbf{u} = \frac{\mathbf{g}}{\epsilon_0(1 + c_s^2)} \tag{1}$$

with $\mathbf{g}$ the momentum density related with the perturbation. ϵ_0 and c_s static background's energy density and sound velocity.

By equilibrium conditions, local conservation laws are presented,

$$\partial_\mu \Theta_0^{\mu\nu} = 0, \qquad \Theta_0^{\mu\nu} = -Pg^{\mu\nu} + (\epsilon + P)u^\mu u^\nu \tag{2}$$

Introducing a disturbance

Assuming that the disturbance introduced by the parton is small, the medium's energy-momentum tensor can be written as [1]

$$\Theta^{\mu\nu} = \Theta_0^{\mu\nu} + \delta\Theta^{\mu\nu} \tag{3}$$

where $\Theta_0^{\mu\nu}$ is the equilibrium energy-momentum tensor and $\delta\Theta^{\mu\nu}$ is the perturbation made by the parton.

Considering at first order in shear (η) and bulk (ξ) viscosity,

$$\delta\Theta^{00} = \delta\epsilon, \quad \delta\Theta^{0i} = \mathbf{g} \tag{4}$$

$$\delta\Theta^{ij} = \delta_{ij} c_s^2 \delta\epsilon - \frac{3}{4}\Gamma_s(\partial^i \mathbf{g}^j + \partial^j \mathbf{g}^i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{g}) - \xi\delta_{ij}\nabla\cdot\mathbf{g} \tag{5}$$

with $\Gamma_s \equiv 4\eta/3\epsilon_0(1+c_s^2)$, the sound attenuation length.

Introducing a disturbance

The hydrodynamics equations are,

$$\partial_0 \delta\epsilon + \nabla \cdot \mathbf{g} = J^0 \tag{6}$$

$$\partial_0 g^i + \partial_j \delta\Theta^{ij} = J^i \tag{7}$$

with J^μ the source of the disturbance.

Substituting the tensor components, and using the Fourier transform for these equations, the energy and momentum density are written in terms of the source components,

$$\delta\epsilon = \frac{i\mathbf{k}\cdot\mathbf{J}(\mathbf{k},\omega) + J^0(\mathbf{k},\omega)(i\omega - \Gamma_s k^2)}{\omega^2 - c_s^2 + i\Gamma_s \omega k^2} \tag{8}$$

$$\mathbf{g}_L = \frac{i\left[\frac{\omega}{k^2}\mathbf{k}\cdot\mathbf{J}(\mathbf{k},\omega) + c_s^2 J^0(\mathbf{k},\omega)\right]}{\omega^2 - c_s^2 k^2 + i\Gamma_s\omega k^2}, \qquad \mathbf{g}_T = \frac{i\mathbf{J}_T}{\omega + i\frac{3}{4}\Gamma_s k^2} \tag{9}$$

Localized disturbance source

The parton can be represented by a localized source, as it's chosen in [1]

$$J^{\nu}(\mathbf{x}, t) = \left(\frac{dE}{dx}\right) v^{\nu} \delta(\mathbf{x} - \mathbf{v}t) \tag{10}$$

dE/dx is the parton's energy loss per unit length and it's taken as a constant along the parton's path. $v^{\nu} = (1, \mathbf{v})$ is the parton's velocity.

Its Fourier's transform,

$$J^{\nu}(\mathbf{x}, t) = \frac{2\pi}{(2\pi)^4} \left(\frac{dE}{dx}\right) v^{\nu} \delta(\mathbf{k} \cdot \mathbf{v} - \omega)$$

The last integration is rewritten using dimensionless quantities,

$$\xi \equiv \left(\frac{3\Gamma_s}{2v}\right) k_T, \quad \alpha \equiv z \left(\frac{3\Gamma_s}{2v}\right), \quad \beta \equiv \left(\frac{3\Gamma_s}{2v}\right)^{-1} \tag{11}$$

Taking as parameters α and β, the plots were performed from an alpha minimum value of 0.5, to 6, and from -6 to 6 for β.

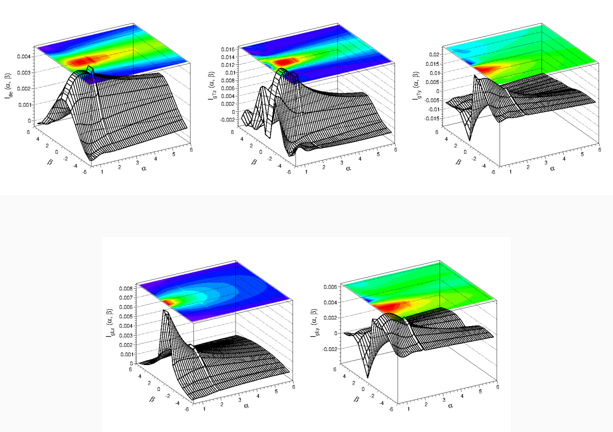

Figure 1: $(\vec{g}_T)_z$, $(\vec{g}_T)_y$ $(\vec{g}_L)_z$, $(\vec{g}_T)_y$ and $\delta\epsilon$ quantities for a localized source with $\alpha_{\text{min}} = 0.5$

An extended source

An extended source was proposed, inspired by [2]. The main idea is the comparison with the previously localized source,

$$J^{\nu}(\vec{x},t) = \left(\frac{dE}{dx}\right) v^{\nu} \frac{1}{\left(\sqrt{2\pi}\sigma\right)^{3}} e^{-\frac{(\vec{x}-\vec{v}t)^2}{2\sigma^2}} \tag{12}$$

Parton's velocity $\vec{v}$ makes a constant angle with the position vector $\vec{x}$, represented by $\vec{x}\cdot\vec{v} = |\vec{x}||\vec{v}|\cos\gamma$. Again, the source is transform to the Fourier space,

$$J^{\nu}(\vec{k},\omega) = \frac{1}{(2\pi)^4}\left(\frac{dE}{dx}\right)\frac{\sqrt{2\pi}\sigma}{v\sin^3\gamma} v^{\nu} e^{-\frac{\sigma^2}{2v^2}\left[\left(1+\frac{4}{\sin^2\gamma}\right)\omega^2 - \frac{8\vec{v}\cdot\vec{k}}{\sin^2\gamma}\omega + \frac{4v^2}{\sin^2\gamma}k^2\right]} \tag{13}$$

It is feasible to calculate $(\vec{g}_T)_z$ performing the integration over ω and k_z using a contour integral that contains at least one of the function poles. The final expression was rewritten using the variables α, β and ξ,

$$(\vec{g}_T)_z = \frac{1}{(2\pi)^{\frac{11}{2}}} \left(\frac{dE}{dx}\right) \frac{\sigma}{2\sin^2\gamma} \left(\frac{2v}{3\Gamma_s}\right)^3 \int_0^\infty d\xi \xi^2 J_0(\beta\xi)\, e^{-\alpha\xi} \tag{14}$$

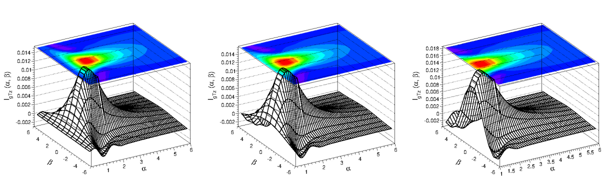

Figure 2: From left to right, $(\vec{g}_T)_z$ for an extended source with $\alpha_{\text{min}} = 0.1$, $\alpha_{\text{min}} = 0.5$, $\alpha_{\text{min}} = 1.0$

Final Remarks

It's possible to obtain similar plots for the other energy-momentum components.

The final integration could be performed using a Monte-Carlo integration method, and then, to compare with the integration already done for a localized source.

If we want to consider a σ value that is far away from the one we are expecting in the delta approximation, it would be convenient to consider non-linear terms in the hydrodynamics equations.

This is work in progress and eventually we want to generate initial conditions with energy and momentum maps that can be used as input on numerical simulations in different hydrodynamical set-ups.

References i

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