Dynamics of inhomogeneous chiral condensates

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— Thiago Peixoto (IFT)

Motivation

QCD phase diagram at low temperature and finite baryon density might be more interesting than initially thought

Model calculations, large Nc arguments, predict that different kinds of inhomogeneous phases in QCD matter might exist at finite density

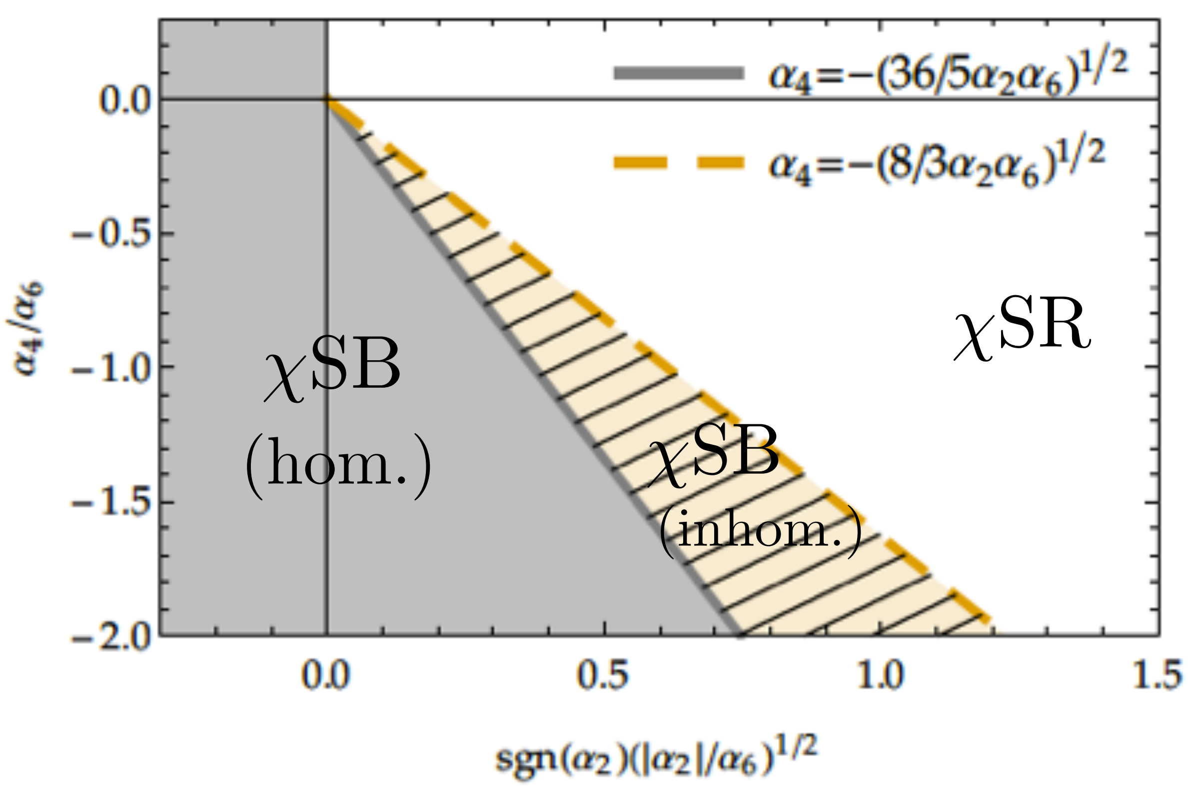
Chiral condensate might be inhomogeneous

NJL model:
Nakano & Tatsumi, Phys. Rev. D 71, 114006 (2005)
Basar, Dunne & Thies, PRD 79, 105012 (2009)
D. Nickel, Phys. Rev. Lett. 103, 072301 (2009)

NJL - inspired

Ginzburg-Landau framework

$$
\begin{aligned}
F[T,\mu;\phi(\mathbf{x})] &= \int d^3x\, \omega(T,\mu;\phi(\mathbf{x})) \\
\omega(T,\mu;\phi(\mathbf{x})) &= \frac{\alpha_2}{2}\phi(\mathbf{x})^2 + \frac{\alpha_4}{4}\left\{\phi(\mathbf{x})^4 + \left[\nabla\phi(\mathbf{x})\right]^2\right\} \\
&+ \frac{\alpha_6}{6}\left\{\phi(\mathbf{x})^6 + 5\left[\nabla\phi(\mathbf{x})\right]^2\phi(\mathbf{x})^2 + \frac{1}{2}\left[\nabla^2\phi(\mathbf{x})\right]^2\right\}
\end{aligned}
$$



Condensate profile (1-dim)

$$\phi(z) = \sqrt{\nu}\, q\, \mathrm{sn}(qz;\nu)$$

$$\Big\downarrow \; \alpha_4 = -\sqrt{36/5\,\alpha_2\alpha_6}, \quad \nu = 1$$

$$\phi(z) = q\tanh(qz)$$

Nickel, PRL 103, 072301 (2009)

Observable?

Compact stars:

— neutrino emissivity, larger than standard Urca cooling

— EOS supports stars with 2 $M_\odot$

Tatsumi & Muto, PRD 89, 103005 (2014),
Carignano, Ferrer, Incera & Paulucci, PRD 92, 105018 (2015)
Buballa & Carignano, EPJA 52, 57 (2016)

Observable?

Heavy-ion collisions:

— seem not yet fully explored

— inhomogeneous phase, different momentum distribution, eccentricities (geometric information)

— CBM, NICA, fragments of cold matter with inhomogeneous phase

— inhomogeneous phase in nuclear matter*

Important here is time evolution of condensate, formation of inhomogeneous condensate

*Heinz, Giacosa & Rischke, NPA 933, 34 (2014)

Phase Change
— time dependence

Typical situation:

— A system is forced to change from a thermodynamic equilibrium phase to another, out of equilibrium phase

— Evolution to new equilibrium through spatial fluctuations that take the system (initially homogeneous) through a sequence of highly (not in equilibrium) inhomogeneous states

Dynamics

— Coarse-graining

Rational:

1. It is hopeless to obtain a macroscopic description with microscopic d.o.f.
2. Focus on a small number of semi-macroscopic variables; the order parameters
3. Dynamics of the order parameters is slow in comparison to that of the (remaining) microscopic degrees of freedom

Coarse-graining

— cut off short wave lengths

A. Zee book

Dynamical equations

First-principles derivation:

— Schwinger-Keldysh effective action; real-time

Phenomenological:

— Ginzburg-Landau-Langevin equations

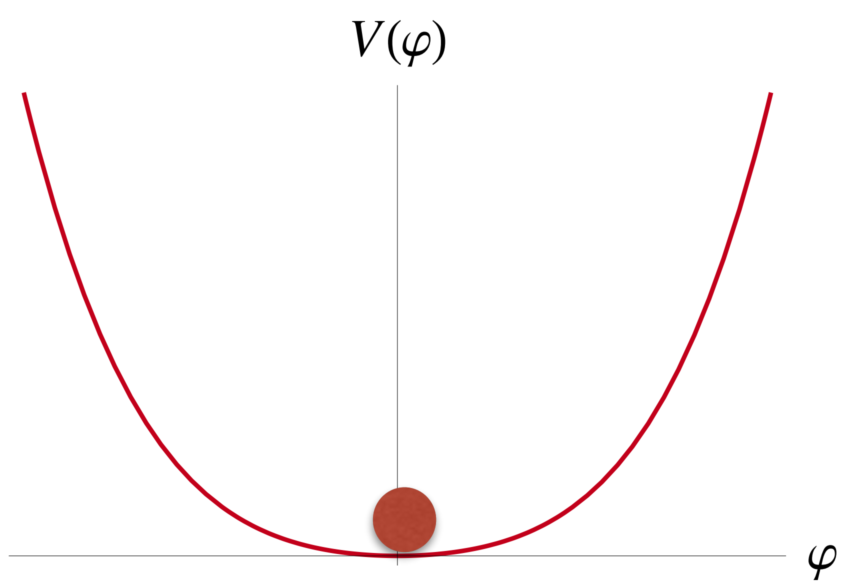
Smallish deviations from equilibrium

At equilibrium

At equilibrium, system described by a macroscopic free energy (Landau), functional of the order parameter:

$$F = F[\varphi(x)]$$

$\varphi(x)$: order parameter



Example:

$$F[\varphi] = \int d^3x \; \left[\kappa(\nabla\varphi)^2 + V(\varphi)\right]$$

$$V(\varphi) = \frac{1}{2}m^2\varphi^2(x) + \frac{1}{4}\lambda\varphi^4(x)$$

Equilibrium: $\dfrac{\delta F[\varphi]}{\delta\varphi(x)} = 0$

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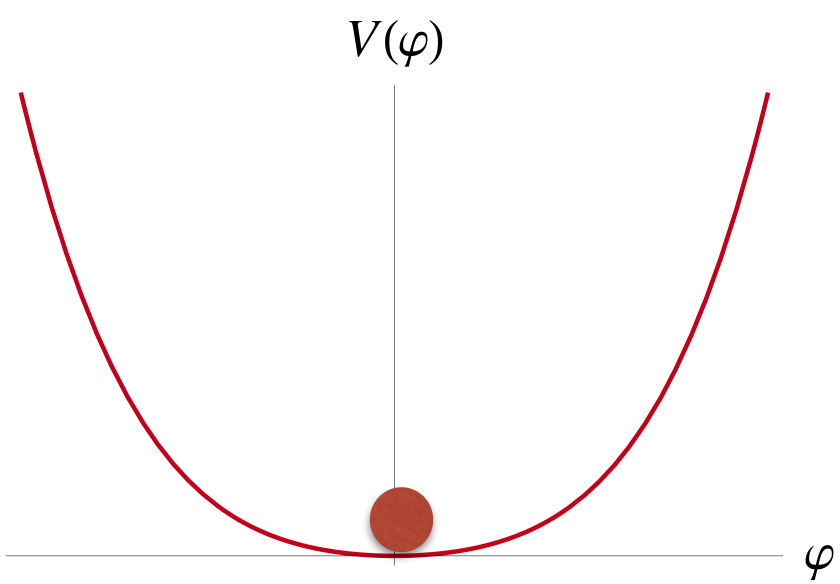

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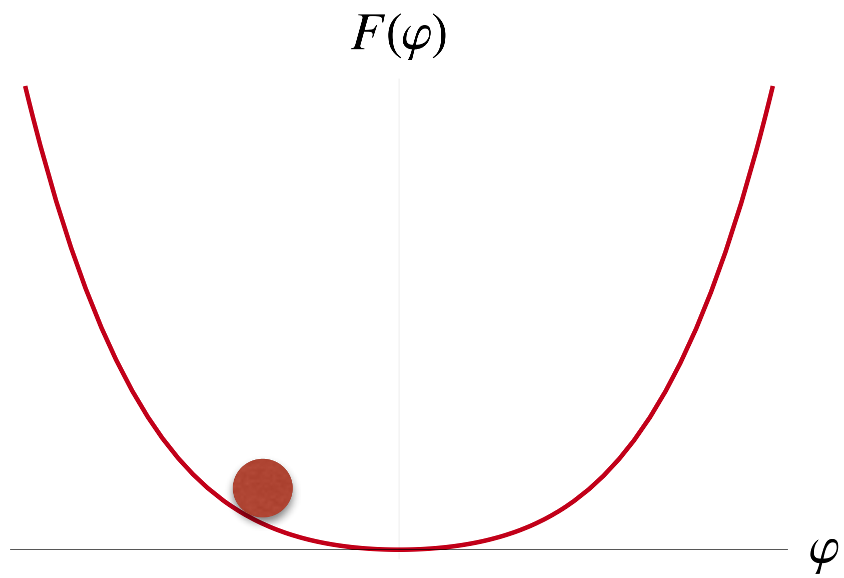
Equilibrium: $$\frac{\delta F[\varphi]}{\delta\varphi(x)} = 0$$

Mechanics: equilibrium, zero force, gradient of potential energy is zero

Thermodynamics: gradient of F is zero, $\frac{\delta F[\varphi]}{\delta\varphi(x)}$ is the thermodynamic force

Close to equilibrium



$$\varphi(x) \to \varphi(x,t)$$

Equation of motion $$\frac{\partial \varphi(x,t)}{\partial t} = -\Gamma \frac{\delta F[\varphi]}{\delta \varphi(x,t)}$$

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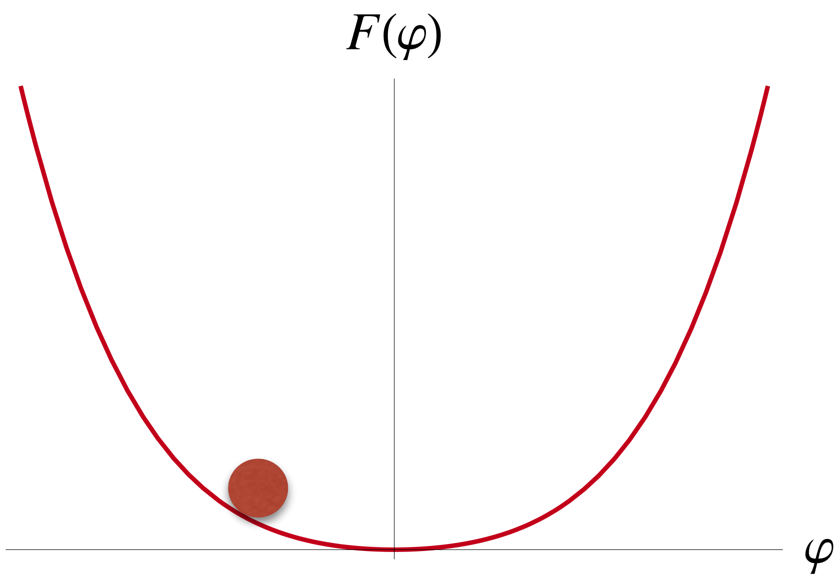
Close to equilibrium



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Equation of motion

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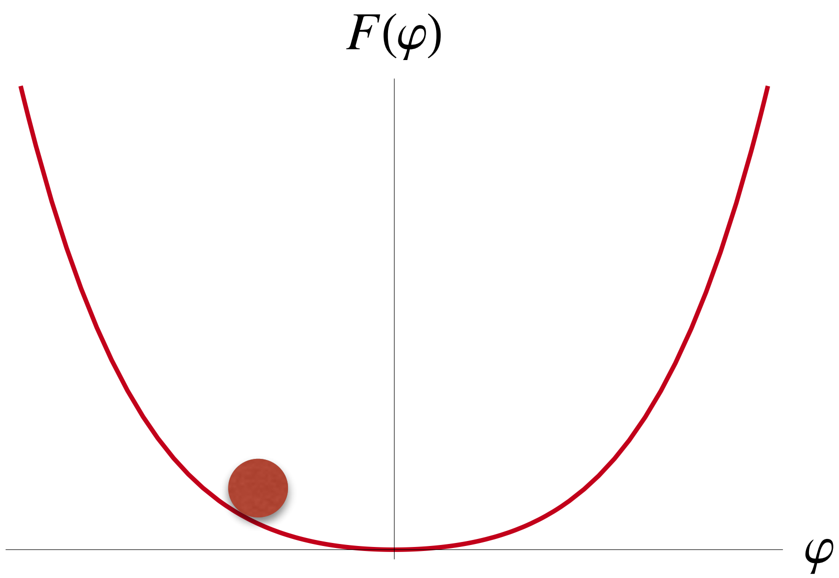
Near the minimum: $(\nabla \varphi)^2 \approx 0, \qquad V(\varphi) \approx \frac{1}{2} m^2 \varphi^2$

$$\frac{\partial \varphi(x,t)}{\partial t} = -\Gamma \frac{\delta F[\varphi]}{\delta \varphi(x,t)} \longrightarrow \varphi(x,t) \approx e^{-m^2 t/\Gamma}$$

Close to equilibrium



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Purely diffusive, FLUCTUATIONS are missing

Fluctuations

— noise fields

$$\frac{\partial \varphi(x,t)}{\partial t} = -\Gamma \frac{\delta F[\varphi]}{\delta \varphi(x,t)} + \xi(x,t)$$

Example: white noise

$$\langle \xi(x,t) \xi(x',t') \rangle = 2\,\Gamma\,T\,\delta(x-x')\delta(t-t')$$

Fluctuations

— noise fields

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Example: white noise

$$\langle \xi(x,t)\xi(x',t')\rangle = 2\,\Gamma\,T\,\delta(x-x')\delta(t-t')$$

What is this T?

Fluctuations

— noise fields

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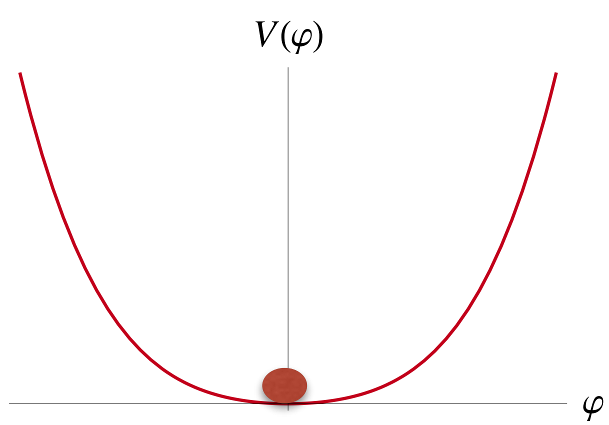
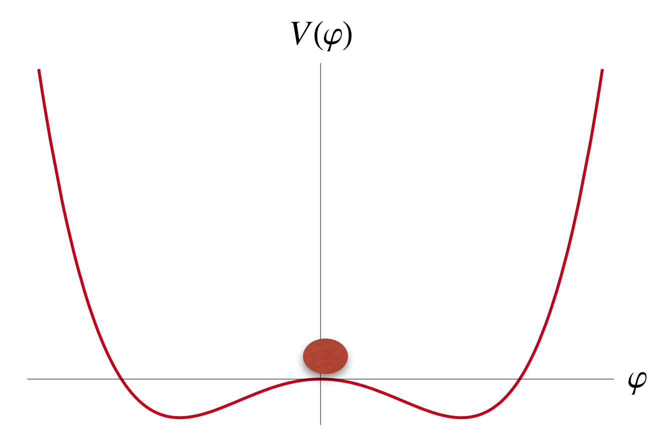


$T > T_c$

System is forced

to change phase



$T < T_c$

What is the input?

$$\Gamma \frac{\partial \varphi(x,t)}{\partial t} = -\frac{\delta F[\varphi]}{\delta \varphi(x,t)} + \xi(x,t)$$

$$\langle \xi(x,t)\xi(x',t')\rangle = 2\,\Gamma\, T\,\delta(x-x')\delta(t-t')$$

$$F[\varphi] = \int d^3x\, \left[\kappa(\nabla\varphi)^2 + V(\varphi)\right]$$

Need from elsewhere:

κ and Γ

Presently, rough estimates only

$V(\varphi)$: use equilibrium free energy

Example

— PNJL model

Two order parameters

— Chiral condensate: σ

— Polyakov loop: $\phi, \bar{\phi}$

$$
\begin{aligned}
\mathcal{J}_{CM}(T,\mu) &= T^4\left\{-\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}\left(\bar{\phi}^3+\phi^3\right) + \frac{b_4}{4}\left(\bar{\phi}\phi\right)^2\right\} + \frac{\sigma^2}{2G} - 6N_f\int_0^{\Lambda}\frac{d^3p}{(2\pi)^3}\omega_p - \\
&- 2N_fT\int\frac{d^3p}{(2\pi)^3}\left\{\ln[1+3\phi f_- + 3\bar{\phi}f_-^2 + f_-^3] + \ln[1+3\bar{\phi}f_+ + 3\phi f_+^2 + f_+^3]\right\}
\end{aligned}
$$

$$
f_{\pm} = \exp\left[-\frac{\omega_p \pm \mu}{T}\right] = \exp\left[-\frac{\sqrt{\vec{p}^2+M^2}\pm\mu}{T}\right] \qquad M = m_0 - \sigma
$$

Example

— PNJL model

Two order parameters

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$$\mathcal{J}_{CM}(T,\mu) = T^4\left\{-\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}\left(\bar{\phi}^3+\phi^3\right)+\frac{b_4}{4}\left(\bar{\phi}\phi\right)^2\right\}+\frac{\sigma^2}{2G}-6N_f\int_0^\Lambda\frac{d^3p}{(2\pi)^3}\omega_p -$$

$$- 2N_fT\int\frac{d^3p}{(2\pi)^3}\left\{\ln[1+3\phi f_- + 3\bar{\phi}f_-^2+f_-^3]+\ln[1+3\bar{\phi}f_+ + 3\phi f_+^2+f_+^3]\right\}$$

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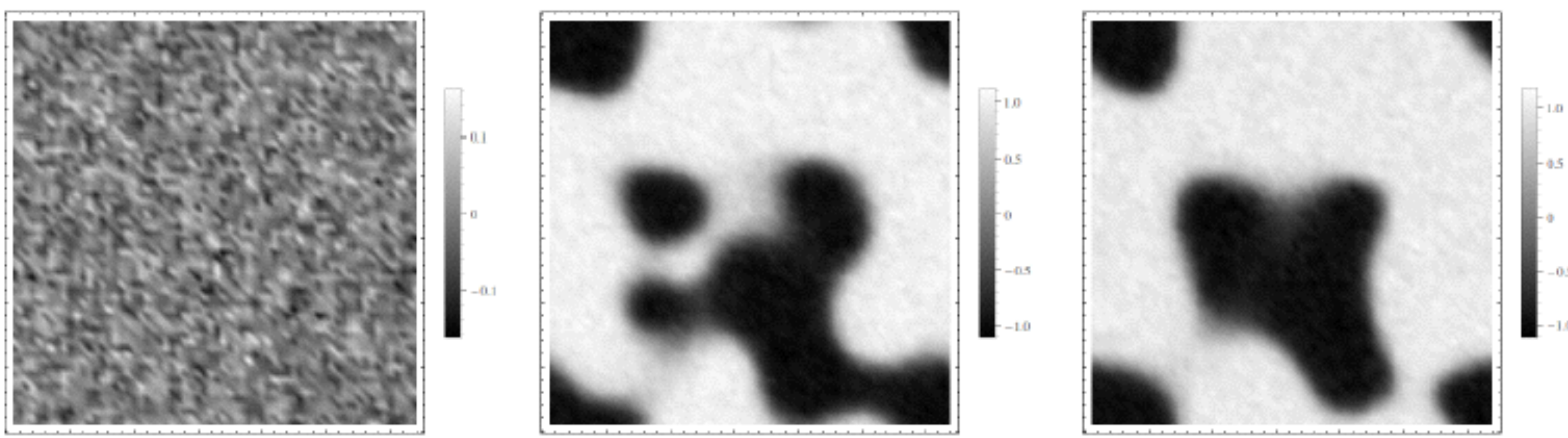
T. Peixoto

Domain formation

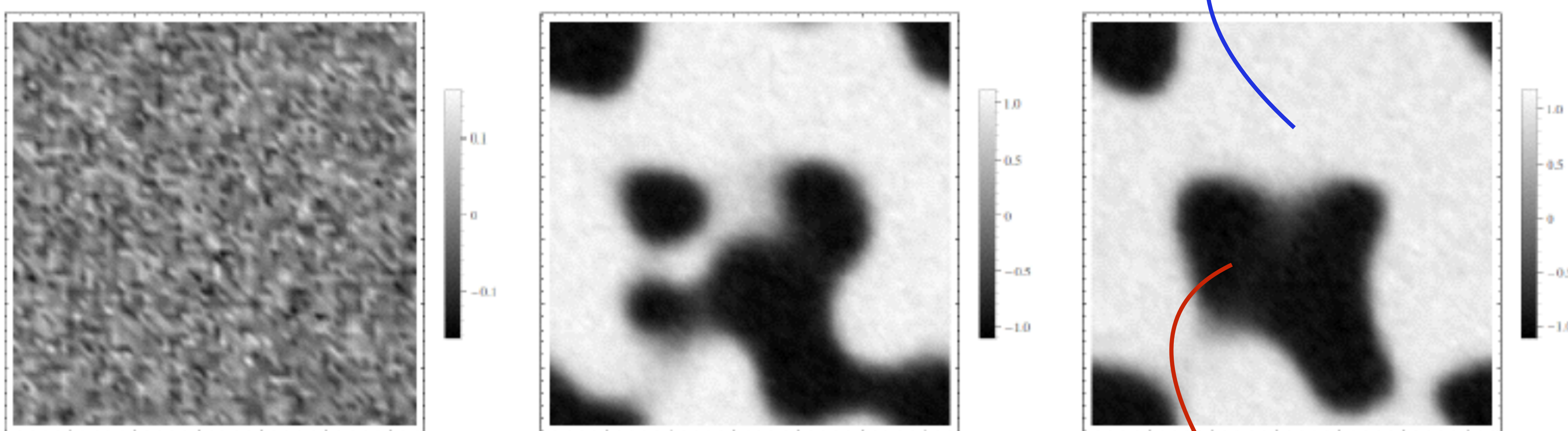
Time increases



Domain formation

Positive
order parameter

Time increases



Negative
order parameter

NJL - inspired

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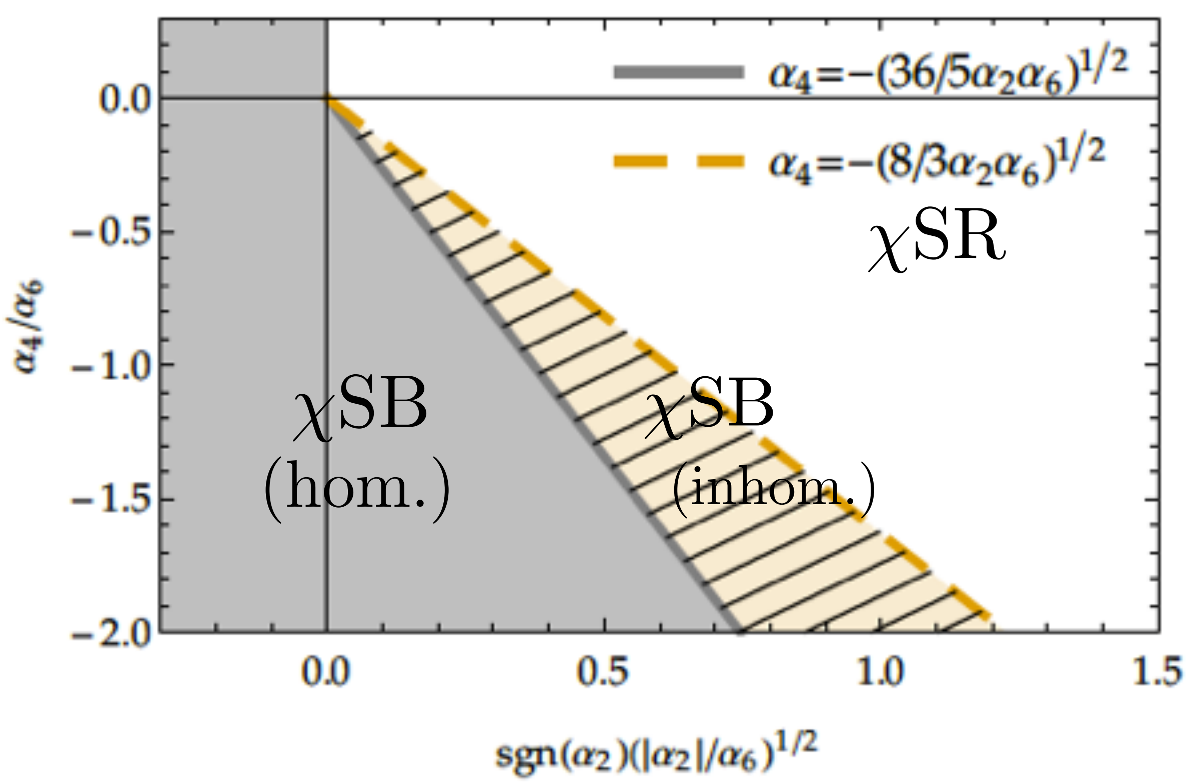

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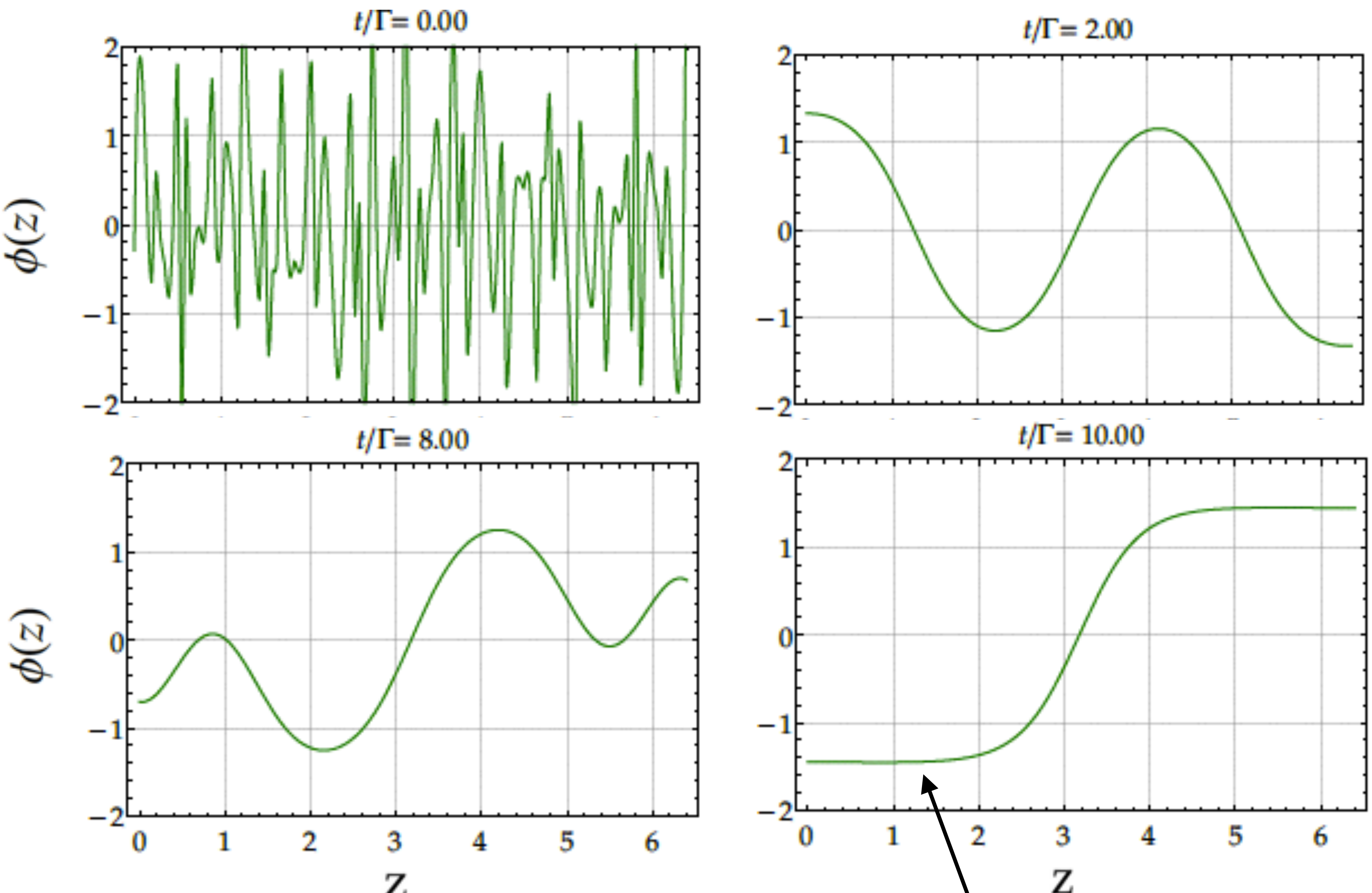
$$\phi(z) = q\tanh(qz)$$

Nickel, PRL 103, 072301 (2009)

Dynamics
— static medium

$$\boxed{\alpha_4 = -\sqrt{36/5\,\alpha_2\alpha_6}}$$



$$\phi(z) = q\tanh(qz)$$

Low T, noise has small effect

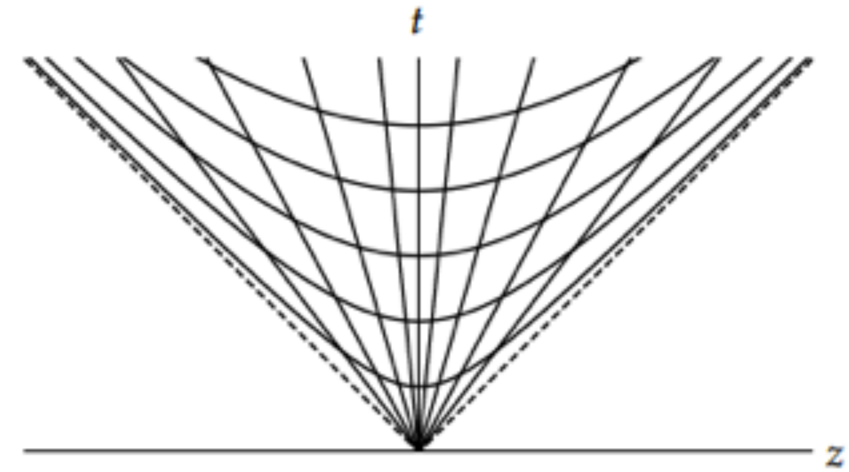
Typical value

(low temperatures)

$$\Gamma \simeq \frac{1}{3}\,\mathrm{fm}$$

Nahrgang, Leupold & Bleicher, PLB 711, 109 (2008)

Dynamics
— Bjorken expansion



$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \frac{1}{2}\log\frac{t+z}{t-z} \qquad z/t = \tanh\eta < 1$$

$$ds^2 = d\tau^2 - \tau^2 d\eta^2 = d\tau^2 - (\tau/\tau_0)^2 d(\tau_0\eta)^2$$

$\boxed{1/\tau_0 : \text{expansion rate}}$

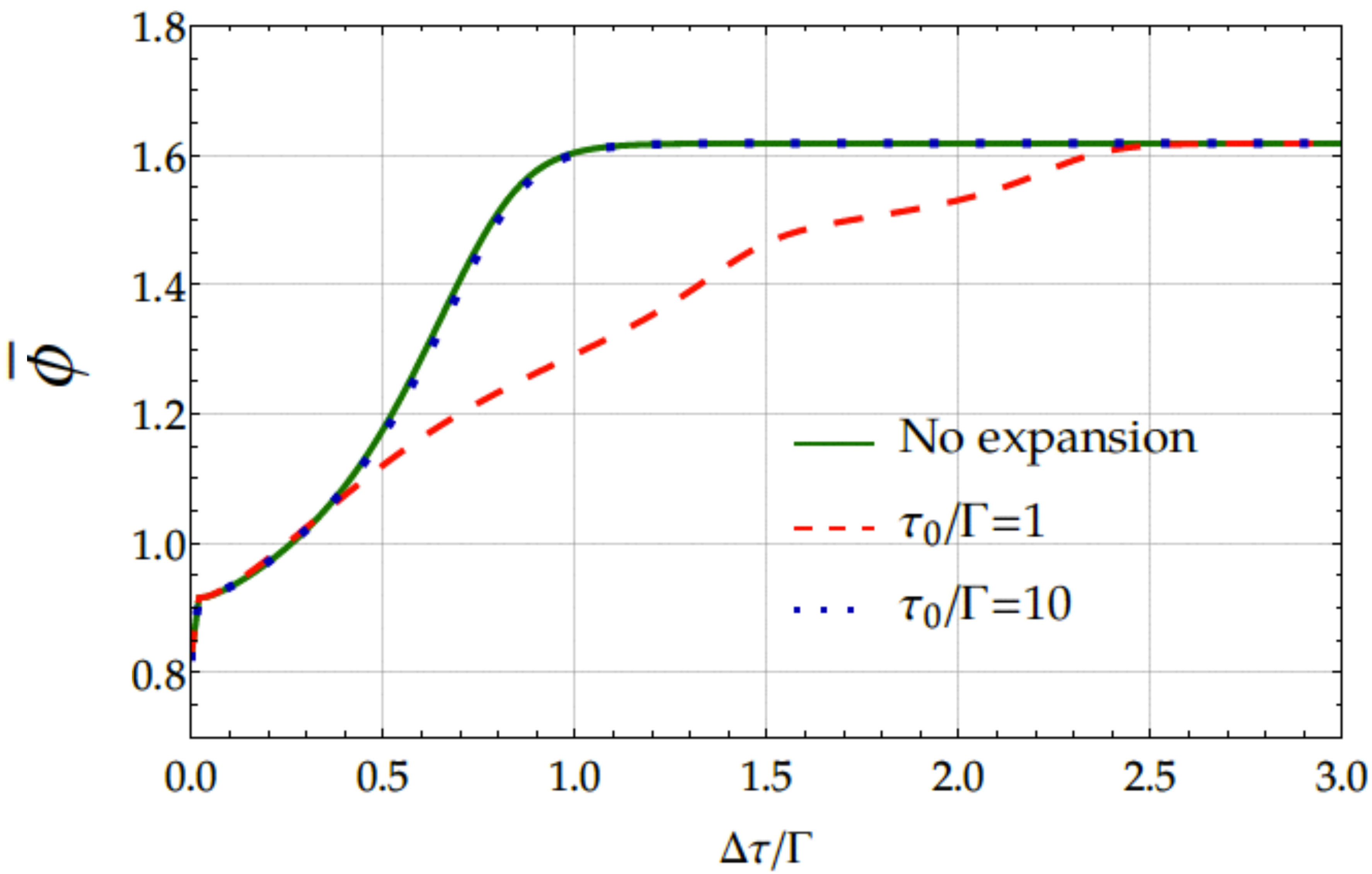
$$\nabla^2\phi \to -\partial^i\partial_i\phi = \left(\cancel{\nabla_\perp^2} + \frac{1}{\tau^2}\partial_\eta^2\right)\phi$$

$$(\nabla\phi)^2 \to -\partial^i\phi\partial_i\phi = (\cancel{\nabla_\perp\phi})^2 + \frac{1}{\tau^2}(\partial_\eta\phi)^2$$

$$\left\{ \frac{\partial\phi(\eta,\tau)}{\partial\tau} = -\Gamma\frac{\delta F[\phi]}{\delta\phi(\eta,\tau)} + \xi(\eta,\tau) \right.$$

Dynamics
— Bjorken expansion

Homogeneous phase



Volume average

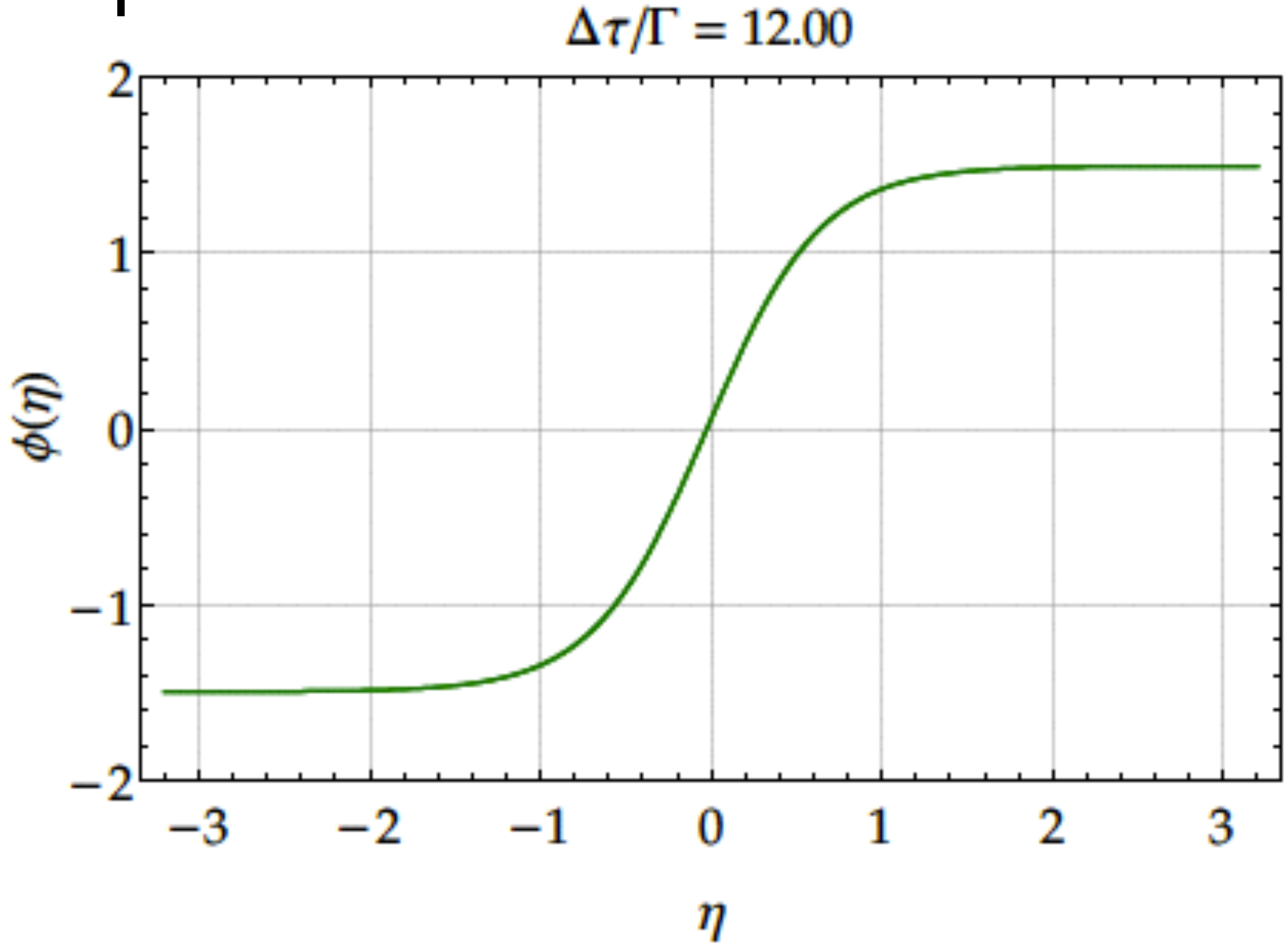
$$\overline{\phi}(\tau) = \frac{1}{L}\int_0^L dz\,\phi(\eta,\tau)$$

Dynamics
— Bjorken expansion

$$\boxed{\alpha_4 = -\sqrt{36/5\,\alpha_2\alpha_6}}$$

Inhomogeneous phase

Slow expansion



Fast expansion

$$\Delta\tau/\Gamma \sim 30$$

Qualitative same behavior
for different combinations
of parameters

Nonlocal NJL
— interaction has a range

$$S_E = \int d^4x \left[-i\bar{\psi}(x)\,\partial\!\!\!/\,\psi(x) - \frac{G}{2} j_a(x)\, j_a(x) \right]$$

$$j_a(x) = \int d^4z\, \mathcal{G}(z)\, \bar{\psi}(x + z/2)\, \Gamma_a\, \psi(-z/2)$$

$$\Gamma_a = (1, i\gamma_5\vec{\tau})$$

$\mathcal{G}(z)$: form factor

Diakonov & Petrov, JETP 62 (1985) 204; NPB 245, 259 (1989)
Bowler & Birse, NPA 582, 655 (1995)
Gomez Dumm & Scoccola, PRD D65, 074021 …
Carlomagno, Gomez Dumm & Scoccola, PLB 745, 1 (2015)

Free energy

$$\phi(\vec{x}) \;=\; (\sigma(\vec{x}), \vec{\pi}(\vec{x}))$$

$$\omega(T,\mu,\phi) = \frac{\alpha_2}{2}\,\phi^2 + \frac{\alpha_4}{4}(\phi^2)^2 + \frac{\alpha_{4b}}{4}(\nabla\phi)^2 + \frac{\alpha_6}{6}(\phi^2)^3 + \frac{\alpha_{6b}}{6}(\phi,\nabla\phi)^2$$

$$+\frac{\alpha_{6c}}{6}\left[\phi^2(\nabla\phi)^2 - (\phi,\nabla\phi)^2\right] + \frac{\alpha_{6d}}{6}\,(\triangle\phi)^2$$

Coefficients depend on temperature
and baryon chemical potential

Coefficients

— expressions

$$\alpha_2 = \frac{1}{G} - 8\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^2}{p_n^2}$$

$$\alpha_4 = 8\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^4}{p_n^4}$$

$$\alpha_{4b} = 8\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^2}{p_n^4} \left(1 - \frac{2}{3}\frac{g'}{g}\vec{p}^{\,2}\right)$$

$$\alpha_6 = -8\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^6}{p_n^6}$$

$$\alpha_{6b} = -40\, N_c \sum\!\!\!\!\!\!\int_{np} \left[\frac{g^4}{p_n^6}\left(1 - \frac{26}{15}\frac{g'}{g}\vec{p}^{\,2} + \frac{8}{5}\frac{g'^2}{g^2}\vec{p}^{\,2} p_n^2\right)\right]$$

$$\alpha_{6c} = -24\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^4}{p_n^6} \left(1 - \frac{2}{3}\frac{g'}{g}\vec{p}^{\,2}\right)$$

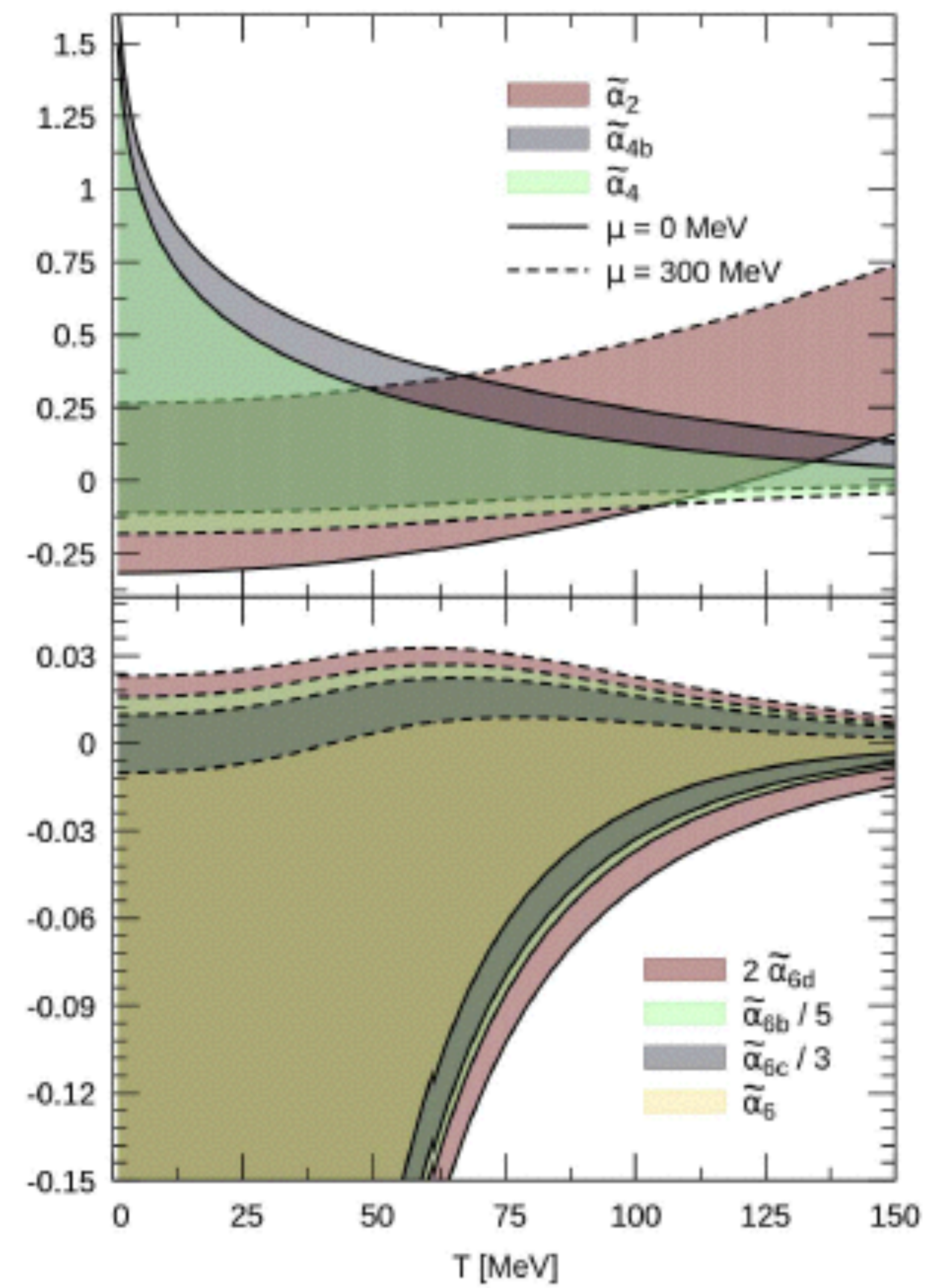
$$\alpha_{6d} = -4\, N_c \sum\!\!\!\!\!\!\int_{np} \frac{g^2}{p_n^6} \left[1 - \frac{2}{3}\frac{g'}{g}\vec{p}^{\,2} + \frac{1}{5}\left(\frac{g'^2}{g^2} + \frac{g''}{g}\right)\vec{p}^{\,4}\right]$$

$$\sum\!\!\!\!\!\!\int_{np} \equiv \frac{T}{2\pi^2} \sum_{n=-\infty}^{\infty} \int_0^{\infty} d|\vec{p}|\, \vec{p}^{\,2}$$

$$g(p) = \exp(-p^2/\Lambda^2)$$

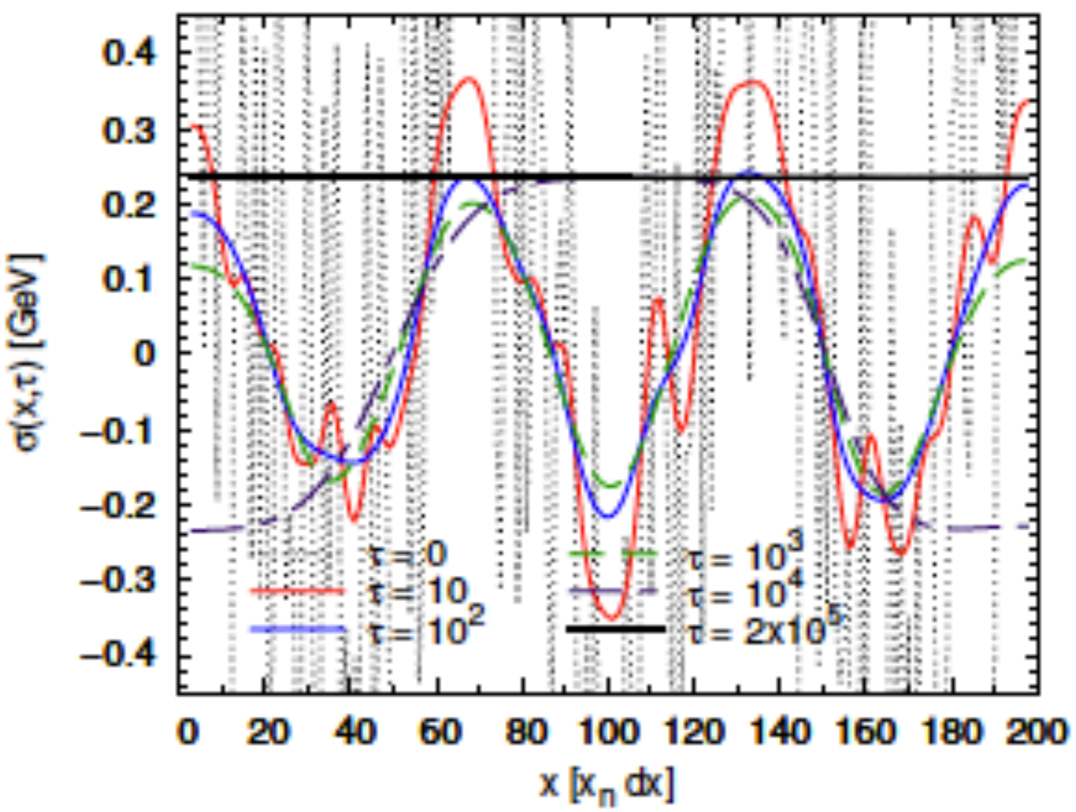
Coefficients

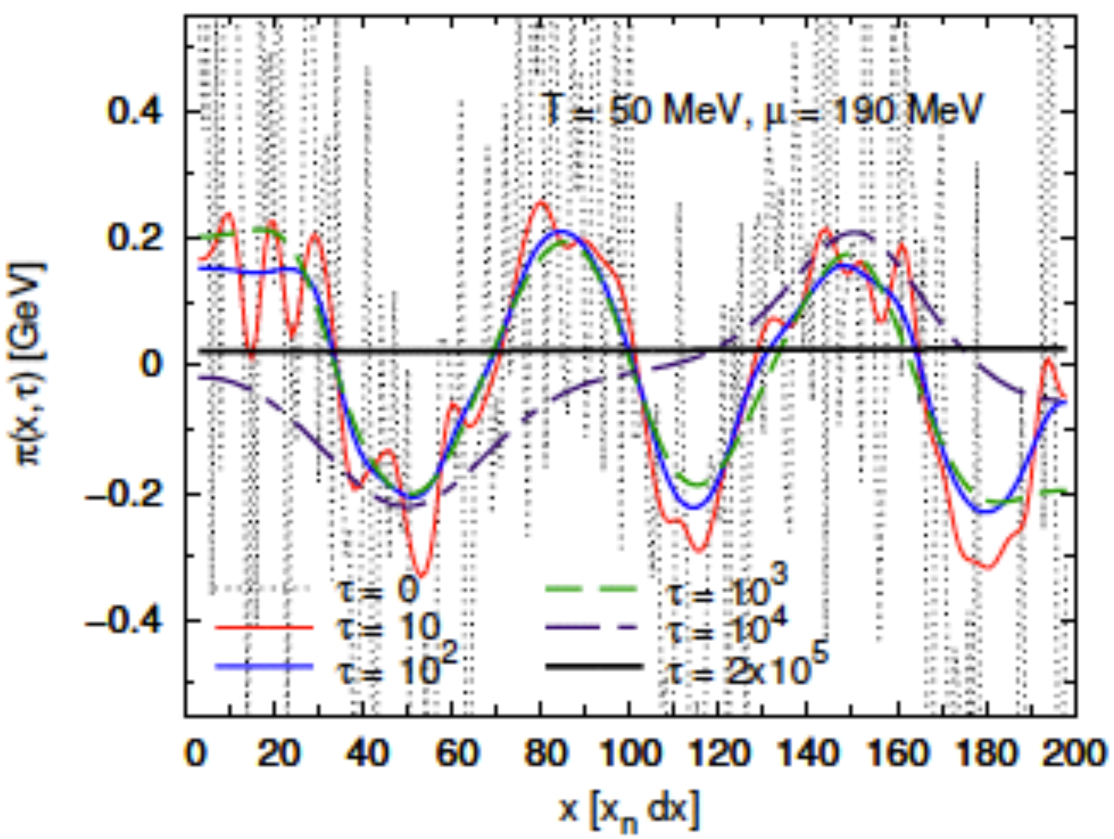
— T & μ dependences



Profiles

— homogeneous states reached after long time



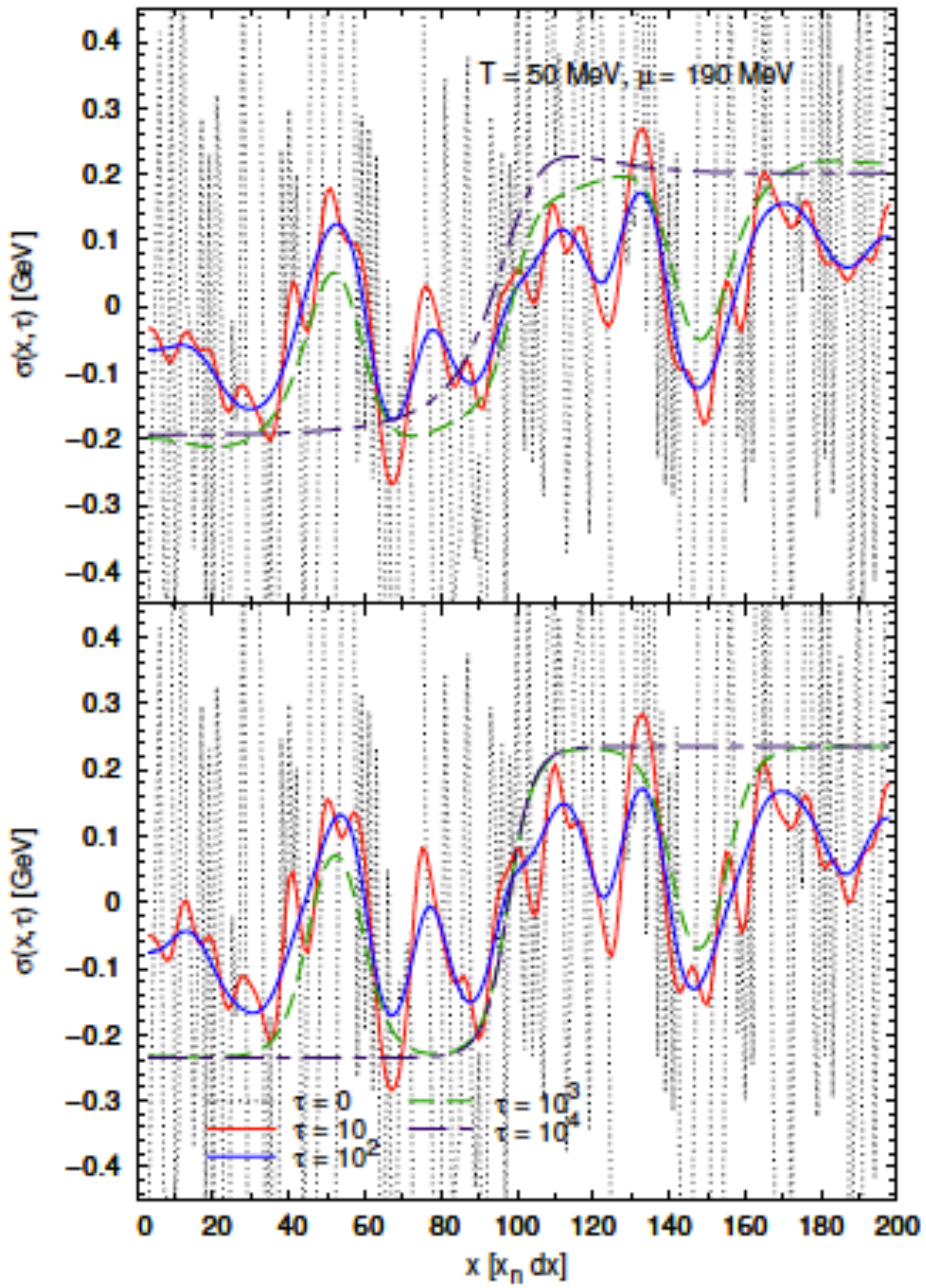


Reach homogeneous state

— passing through inhomogeneous states

Lifetime of inhomogeneous states

— can be increased, “right" initial conditions



σ only
$\pi = 0$

$\sigma + \pi$

Perspectives

— Results are qualitative, not quantitative

— Evolution probes inhomogeneous configurations

— Observable signatures in heavy-ion collision

— Need go to three dimensions

— More realistic expansion

— Derive GLL equations from microscopic model

Funding

FAPESP
SÃO PAULO RESEARCH FOUNDATION

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico