Kicks of magnetized strange quarks stars induced by anisotropic emision of neutrinos

Daryel Manreza Paret,
ICN-UNAM
dmanreza@gmail.com

Collaborators
Alejandro Ayala, ICN-UNAM México,
A. Perez Martinez, ICIMAF La Habana Cuba,
G. Piccinelli, FES Aragón-UNAM, México,
A. Sanchez, Facultad de Ciencias UNAM México,
J. Salvador Ruiz Montaño, Universidad Autónoma de Sinaloa, México.

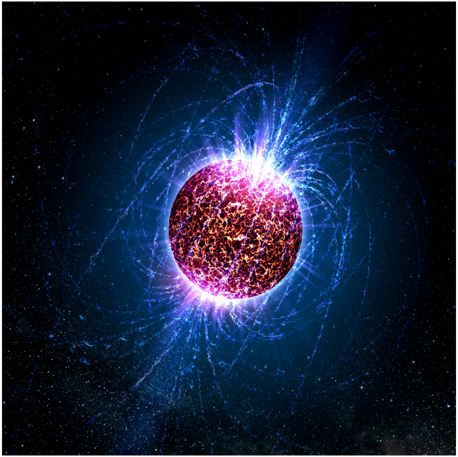
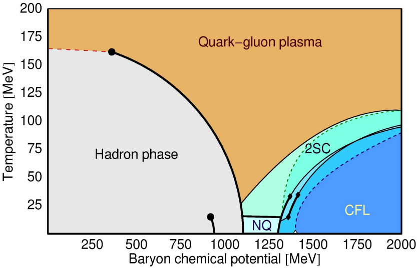


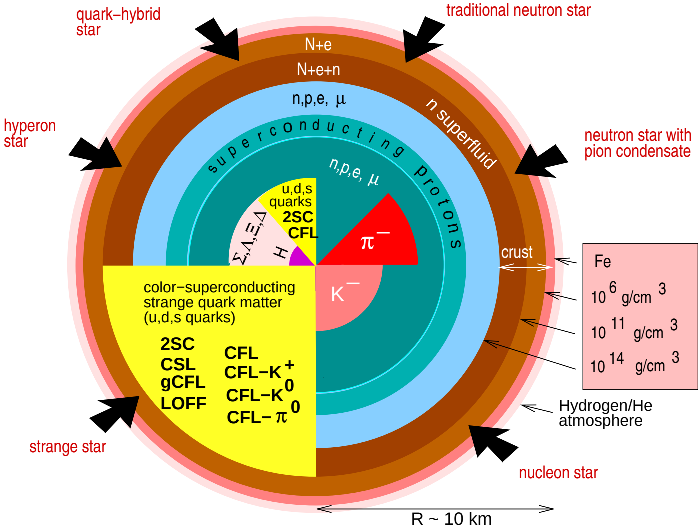
Figure: Artist's illustration of an isolated neutron star. Author: Casey Reed - Penn State University

Neutron Stars

- $M \sim 1.4 M_{\odot}$
- $R \sim 12\,\mathrm{km}$
- $\rho \sim 10^{14}\,\mathrm{g/cm^3}$
- $B \sim 10^{12} - 10^{15}\,\mathrm{G}$



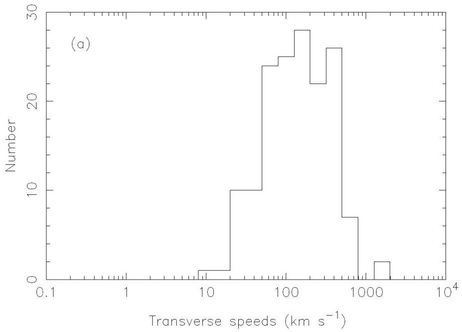
Introduction
Compact Objects



† F. Weber. doi:10.1016/j.ppnp.2004.07.001.

Introduction
Observational evidences for NSs kicks

- Pulsar kicks refers to peculiar translational velocities observed on pulsars with respect to surrounding stars and with respect to their progenitors.
- Kicks can be natal or post-natal: a natal kick is imparted to the NS at birth while post-natal kicks is due to some inner process of the pulsar.
- Hobbs et al.† have studied the data from the proper motion of 233 pulsars, obtaining velocities as high as 1000 km s^{-1} and that the mean velocity of young pulsar is 400 km s^{-1}.



†Hobbs, G., Lorimer, D., Lyne, A., & Kramer, M. 2005, Mon. Not. R. Astron. Soc., 360, 974.

Introduction
Models to describe NSs kicks

Some mechanism to explain the kicks are:

1. **Hydrodynamically Driven Kicks:** This mechanism explain a natal kick during the core collapse and supernova explosion due to hydrodynamical perturbations that could lead to asymmetric matter ejection.

2. **Electromagnetic rocket effect:** Electromagnetic radiation from an off-centered rotating magnetic dipole imparts a kick to the pulsar along its spin axis.

3. **Neutrino–Magnetic Field Driven Kicks:** Asymmetric neutrino emission induced by strong magnetic fields.

Introduction
Models to describe NSs kicks

Some mechanism to explain the kicks are:

1. **Hydrodynamically Driven Kicks:** This mechanism explain a natal kick during the core collapse and supernova explosion due to hydrodynamical perturbations that could lead to asymmetric matter ejection.

2. **Electromagnetic rocket effect:** Electromagnetic radiation from an off-centered rotating magnetic dipole imparts a kick to the pulsar along its spin axis.

3. **Neutrino–Magnetic Field Driven Kicks:** Asymmetric neutrino emission induced by strong magnetic fields.
   - Neutrino emissivity from the process $d \to u + e + \bar{\nu}_e, \;\; u + e \to d + \nu_e,$

Introduction
Models to describe NSs kicks

Some mechanism to explain the kicks are:

1. **Hydrodynamically Driven Kicks:** This mechanism explain a natal kick during the core collapse and supernova explosion due to hydrodynamical perturbations that could lead to asymmetric matter ejection.

2. **Electromagnetic rocket effect:** Electromagnetic radiation from an off-centered rotating magnetic dipole imparts a kick to the pulsar along its spin axis.

3. **Neutrino–Magnetic Field Driven Kicks:** Asymmetric neutrino emission induced by strong magnetic fields.
   - Neutrino emissivity from the process $d \to u + e + \bar{\nu}_e, \;\; u + e \to d + \nu_e,$
   - The polarisation of the electron spin will fix the neutrino emission in one direction along the magnetic field

Introduction
Models to describe NSs kicks

Some mechanism to explain the kicks are:

1. **Hydrodynamically Driven Kicks:** This mechanism explain a natal kick during the core collapse and supernova explosion due to hydrodynamical perturbations that could lead to asymmetric matter ejection.

2. **Electromagnetic rocket effect:** Electromagnetic radiation from an off-centered rotating magnetic dipole imparts a kick to the pulsar along its spin axis.

3. **Neutrino–Magnetic Field Driven Kicks:** Asymmetric neutrino emission induced by strong magnetic fields.
   - Neutrino emissivity from the process $d \to u + e + \bar{\nu}_e, \;\; u + e \to d + \nu_e,$
   - The polarisation of the electron spin will fix the neutrino emission in one direction along the magnetic field
   - The neutrinos work as a propulsion mechanism for the neutron star

Pulsar kick velocity

The idea that neutrinos can be the cause for the kick is easily understood with the following estimation:

- Energy released in the emission of neutrinos $\sim 10^{53}$ erg.
- Kinetic energy of a $1.4M_\odot$ NS moving at 1000 km s^{-1} $\sim 10^{49}$ erg.
- The momentum of neutrinos (p_ν) and the NS (p_{NS}) are

$$\begin{aligned} p_\nu &= \frac{E_\nu}{c} \sim 10^{43} \frac{\mathrm{erg \cdot s}}{\mathrm{cm}} \\ p_{NS} &= M_{NS} \cdot v_{kick} \sim 2.8 \times 10^{41} \frac{\mathrm{erg \cdot s}}{\mathrm{cm}} \sim 0.03 p_\nu \end{aligned}$$

Pulsar kick velocity

To compute the kick velocity of the NS we have to take into account that the acceleration of the NS is due to the luminosity of the fraction of asymmetric emitted neutrinos:

$$\frac{dv}{dt}M_{NS} = \chi\frac{L}{c}, \quad L = \frac{4}{3}\pi R^3\epsilon$$

where χ is the electron polarization, L is the neutrino luminosity and ϵ the neutrino emissivity.
The cooling equation give a relation between the emissivity and the specific heat

$$C_v dT = -\epsilon dt.$$

In this way we have for the kick velocity

$$dv = -\frac{\chi_e}{M_{NS}}\frac{4}{3}\pi R^3 C_v dT.$$

We have to compute the electron polarization and the specific heat in a magnetic field!

Pulsar kick velocity
Electron polarization in a magnetic field

The energy spectrum of electrons in a magnetic field is quantized by the so called Landau Levels

$$E_l^2 = m_e^2 + p_3^2 + 2leB,$$

and the number density reads

$$n_e = \frac{d_e m_e^3}{2\pi^2}\frac{B}{B_c^e}\sum_{l=0}^{\infty}(2-\delta_{l0})\int_0^{\infty} dp_3 \frac{1}{e^{(E_l-\mu_e)/T}+1},$$

where $l = \nu + \frac{1}{2} + s$, are the Landau level quantum numbers, $\nu = 0, 1, 2, \ldots, \quad s = \pm 1/2$ and $B_c^e = m_e^2/e = 4.41 \times 10^{13}$ G.

Pulsar kick velocity

Electron polarization in a magnetic field

The electron spin polarization χ is given by

$$\chi = \frac{n_- - n_+}{n_- + n_+},$$

where $n_\pm$ are the number densities of electrons with spin parallel ($s = +1$) or anti–parallel ($s = -1$) to the magnetic field direction respectively, given by

$$n_- = \frac{d_e m_e^3}{2\pi^2}\frac{B}{B_c^e}\sum_{\nu=0}^{\infty}\int_0^\infty dx_3 \frac{1}{e^{\left(\frac{m_e}{T}\sqrt{x_3^2+1+2\nu B/B_c^e}-x_e\right)}+1},$$

$$n_+ = \frac{d_e m_e^3}{2\pi^2}\frac{B}{B_c^e}\sum_{\nu=1}^{\infty}\int_0^\infty dx_3 \frac{1}{e^{\left(\frac{m_e}{T}\sqrt{x_3^2+1+2\nu B/B_c^e}-x_e\right)}+1},$$

where $x_e = \mu_e/m_e$ and we have used the relation between l, ν and s, changing the summation over l by the summation over ν (its important to noticed that the change is $\sum_{l=0}^{\infty}(2-\delta_{l0}) \to \sum_{s=\pm 1}\sum_{\nu=0}^{\infty}$).

Pulsar kick velocity
Electron polarization in a magnetic field

We can numerically compute the dependence of χ with the parameters B, T, and μ from the following expression

$$\chi_e = \left\{ 1 + \frac{2\sum\limits_{\nu=1}^{\infty}\int_0^\infty dx_3 \frac{1}{e^{(\frac{m_e}{T}\sqrt{x_3^2+1+2\nu B/B_c^e}-x_e)}+1}}{\int_0^\infty dx_3 \frac{1}{e^{(\frac{m_e}{T}\sqrt{x_3^2+1}-x_e)}+1}} \right\}^{-1} .$$

Pulsar kick velocity
Electron polarization in a magnetic field

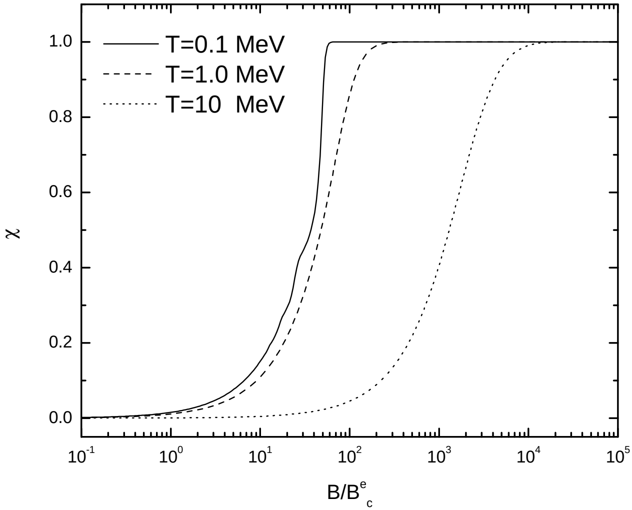


Figure: Polarization of electrons χ as function of the magnetic field and temperature, for a fixed chemical potential $x_e = 10$.

Pulsar kick velocity
Electron polarization in a magnetic field

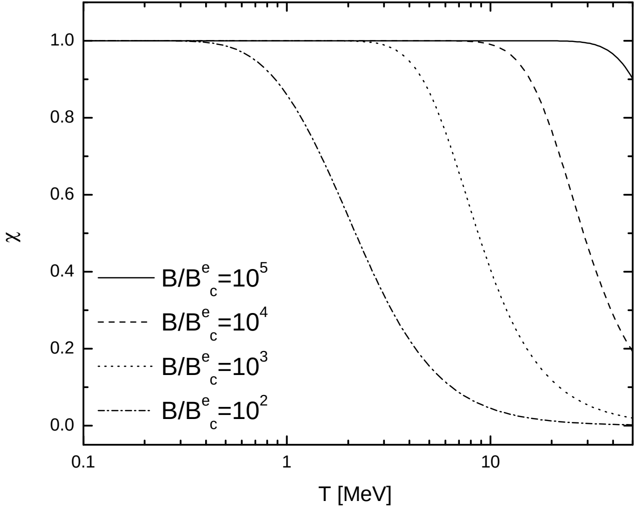


Figure: Polarization of electrons χ as function of the temperature for several values of the magnetic field and fixed chemical potential $x_e = 10$.

Pulsar kick velocity

Electron polarization in a magnetic field

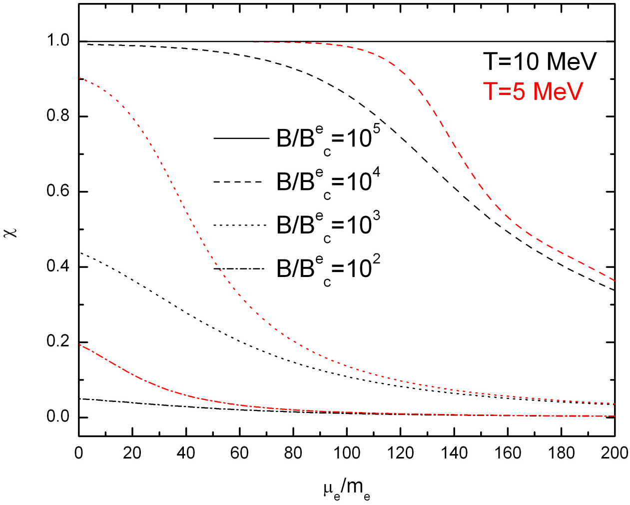


Figure: Polarization of electrons χ as function of the chemical potential for several values of the magnetic field and fixed temperature.

The thermodynamical potential of a magnetized Fermi gas is given by

$$\Omega_f(B,\mu,T) = -\frac{e_f d_f B}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty}(2-\delta_{l0})\left[\frac{1}{\beta}\ln\left(1+e^{-\beta(E_{lf}-\mu_f)}\right)\right],$$

being f the fermion species.
From the thermodynamical potential we can compute the specific heat as

$$C_{vf} = T\frac{\partial S_f}{\partial T}, \quad S_f = -\frac{\partial \Omega_f}{\partial T},$$

obtaining

$$C_{vf} = \frac{e_f d_f B}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty}(2-\delta_{l0})\frac{(E_{lf}-\mu_f)^2}{2T^2[1+\cosh\frac{E_{lf}-\mu_f}{T}]}.$$

Pulsar kick velocity
Stellar equilibrium equations

As we have seen, $\chi_e = \chi_e(B, T, \mu_i)$ and $C_v = C_v(B, T, \mu_i) = \sum_i C_{vi}$, $i = e, u, d, s$.
Strange quark matter consist of u, d, s quarks and electrons in weak equilibrium

$$\begin{aligned} d &\rightarrow u + e + \bar{\nu}_e, \quad u + e \rightarrow d + \nu_e, \\ s &\rightarrow u + e + \bar{\nu}_e, \quad u + d \rightarrow u + s. \end{aligned}$$

If we impose stellar equilibrium conditions (beta equilibrium, charge neutrality and barionic number conservation), then we will need to solve the system of equations

$$\begin{aligned} \mu_u + \mu_e - \mu_d = 0 \ , \ \mu_d - \mu_s &= 0, \\ 2n_u - n_d - n_s - 3n_e &= 0, \\ n_u + n_d + n_s - 3n_B &= 0. \end{aligned}$$

Te solution of this system of equations will fix the chemical potentials as a function of temperature and baryon density (n_B).

Pulsar kick velocity
Stellar equilibrium equations

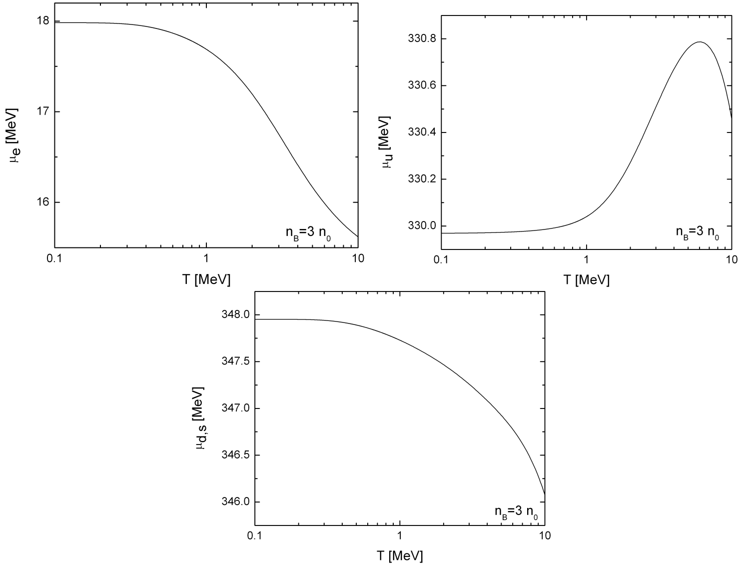


Figure: Chemical potentials of SQM as function of the temperature ($B = 10^5 B_c^e$).

Pulsar kick velocity

The pulsar kick velocity can be rewritten in the following form

$$v = -803.925\,\frac{\text{km}}{\text{s}}\left(\frac{1.4\,M_\odot}{M_{NS}}\right)\left(\frac{R}{10\,\text{km}}\right)^3\left(\frac{I}{\text{MeV fm}^{-3}}\right),$$

where

$$I = \int_{T_i}^{T_f} \chi_e(B,T,\mu_i)\,C_v(B,T,\mu_i)dT, \quad i = e,u,d,s.$$

We can compare our result with the one obtained by I. Sagert and J. Schaffner-Bielich†

$$v \sim 40\,\frac{\text{km}}{\text{s}}\left(\frac{1.4\,M_\odot}{M_{NS}}\right)\left(\frac{R}{10\,\text{km}}\right)^3\left(\frac{\mu_q}{400\,\text{MeV}}\right)^2\left(\frac{T_0}{\text{MeV}}\right)^2,$$

where $\mu_q = 400$ MeV, $M_{NS} = 1.4\,M_\odot$, $\chi_e = 1$ and $T_0 = 10$ MeV.

† I. Sagert and J. Schaffner-Bielich, Astron. Astrophys. Astron. Astrophys. **489**, 281 (2008) doi:10.1051/0004-6361:20078530 [arXiv:0708.2352 [astro-ph]]

Pulsar kick velocity

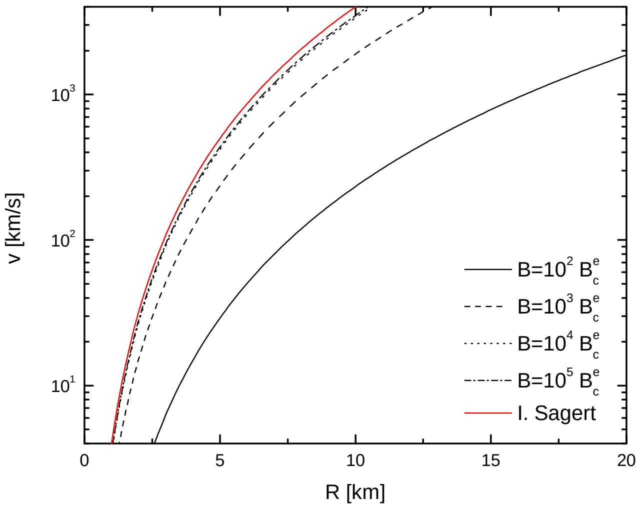


Figure: Pulsar velocity as a function of the star radius for different values of the magnetic field and a baryon density of $n_B = 5\, n_0$. We have taken $T_i = 10$ MeV and $T_f = 0.1$ MeV.

Pulsar kick velocity

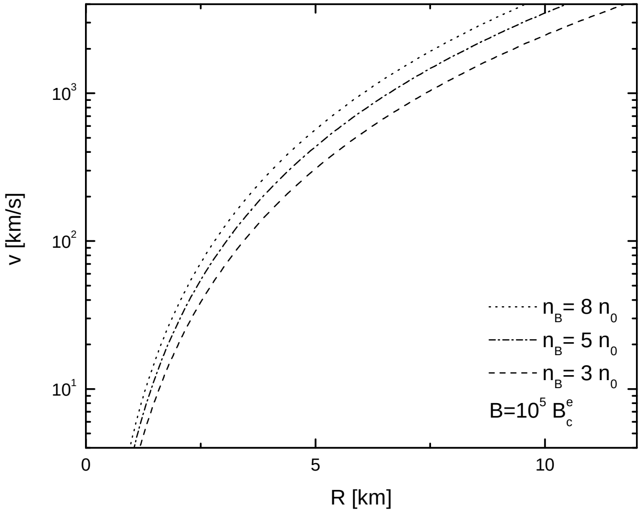


Figure: Pulsar velocity as a function of the star radius for different values of the baryon density and a magnetic field of $B = 10^5 B_c^e$.

Conclusions

1. We have studied pulsar kicks by the emission of neutrinos taking into account the effects of a strong magnetic field.
2. The polarizations of electron was obtained exactly (numerically) as a function of all the parameters (μ_e, T, B).
3. The specific heat of an electron, u, d, s quark gas was obtained exactly (numerically) as a function of all the parameters (μ_i, T, B).
4. The velocity of NS was computed for SQM in presence of a magnetic field in stellar equilibrium.
   - We have obtained kick velocities $v_{\text{kick}} \sim 1000$ km s^{-1} for different values of magnetic fields and star radius.
   - We have studied the dependence of the kick velocity with the central densities of the star obtaining that when the central density increases the stars can reach higher velocities for the same value of magnetic fields.

Future work

1. Analyze the absorption and scattering of neutrinos in quark matter under strong magnetic fields.
2. Incorporate color superconducting phases.

MUCHAS GRACIAS