3 Parton production in DIS at small x

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IN COLLABORATION WITH

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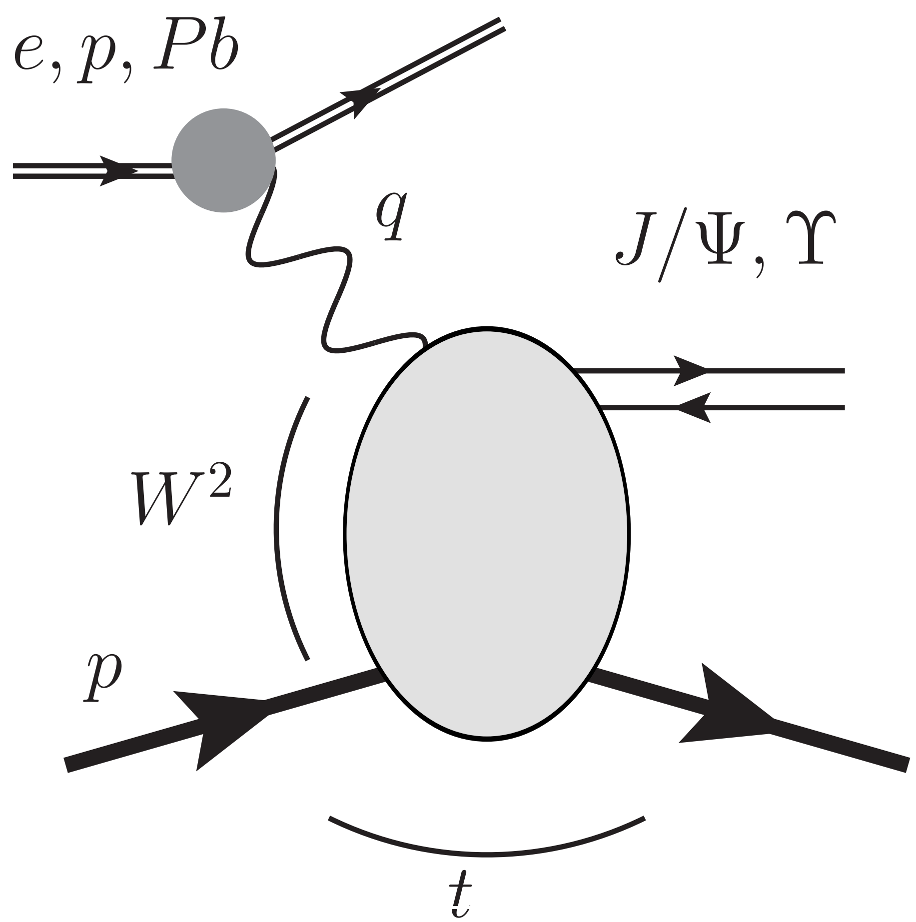
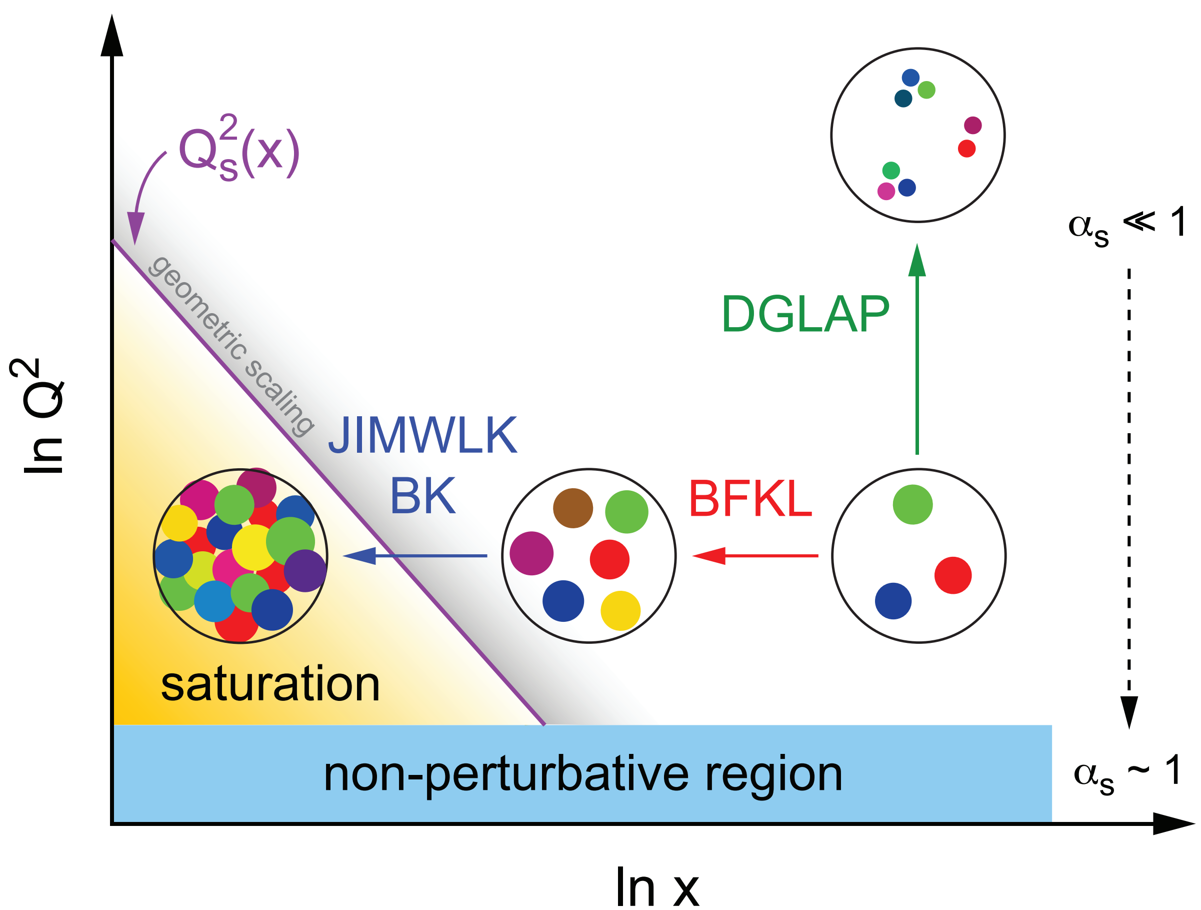
start with something else: exclusive VM production in UPC@LHC

[Bautista, Ferandez-Tellez, MH; 1607.05203]

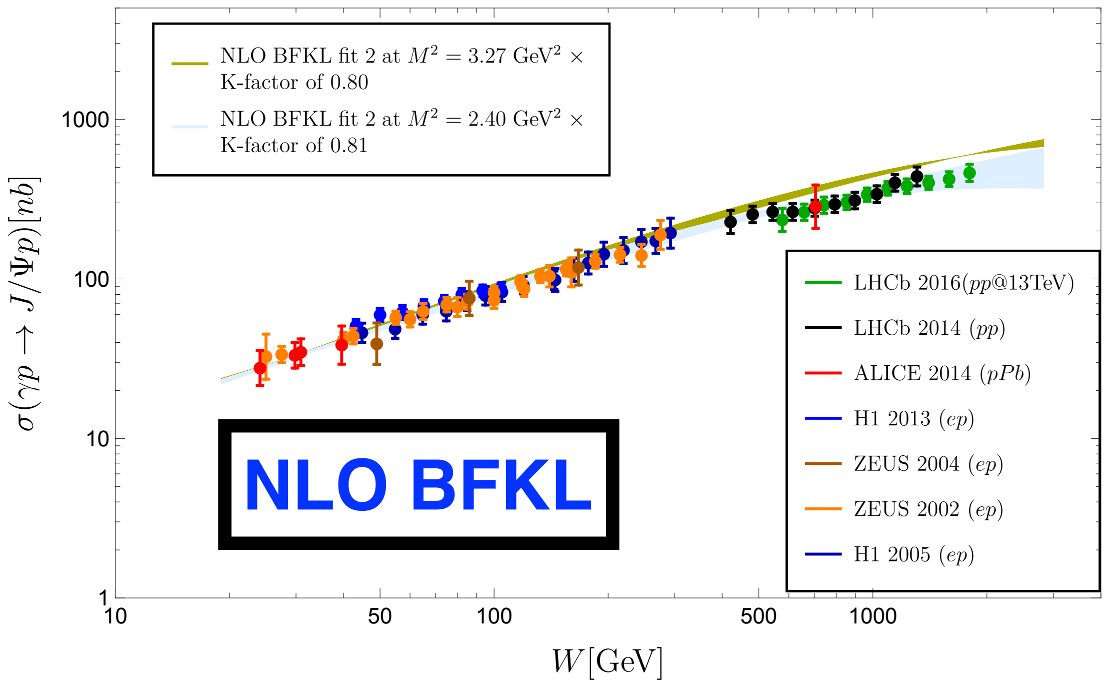


- measured at HERA (ep) and LHC (pp, ultra-peripheral pPb)
- charm and bottom mass provide hard scale ➡ pQCD
- exclusive process, but allows to relate to inclusive gluon

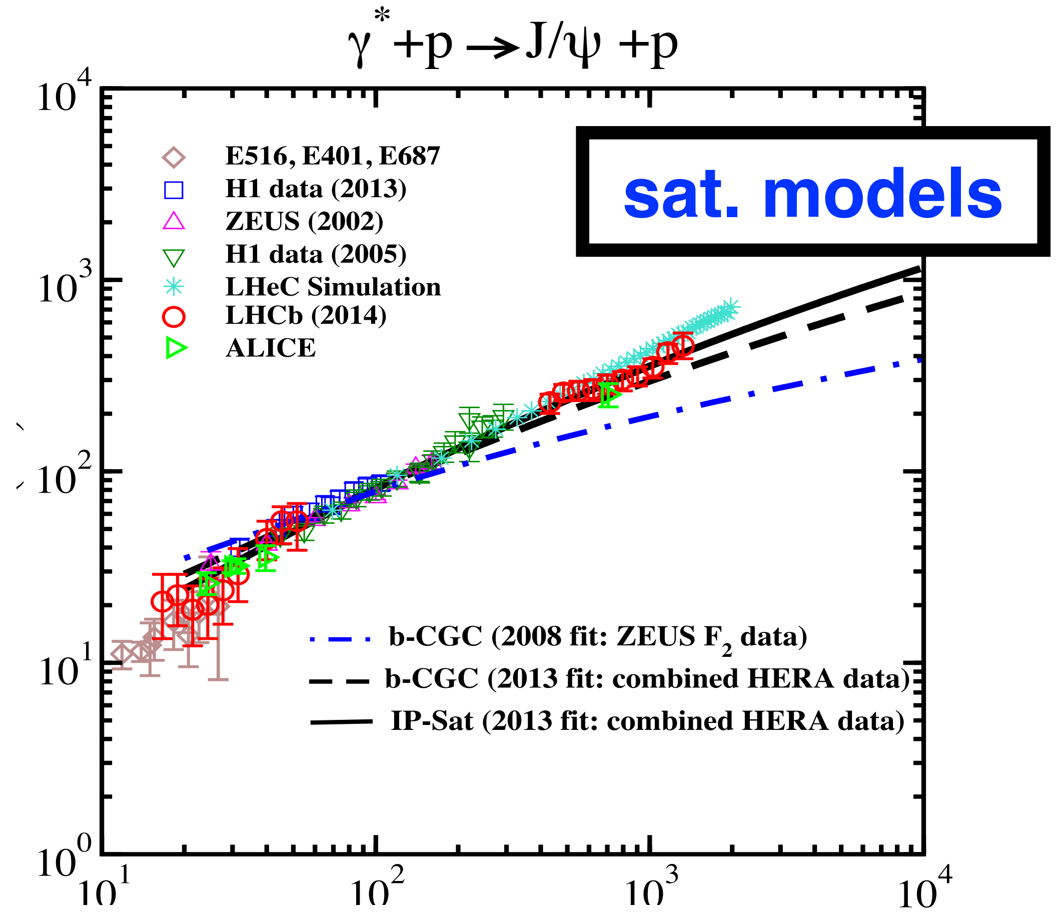
reach values down to $x = 4 \times 10^{-6} \rightarrow$ (unique ?) opportunity to explore the low x gluon in particular: test low x evolution and look for possible onset of saturation



- gluon grows like a power at low x
- at some x_0: saturation/high density will set in → slow down the growth
- when will it happen? do we reach this region already in UPCs@LHC? is it already there



[Bautista, Fernandez-Tellez, MH; 1607.05203]



[Armesto, Rezaeian; 1402.4831],
[Goncalves, Moreira, Navarra; 1405.6977]

observation: both non-linear saturation models & linear NLO BFKL describe data; 2 potential explanations:

a) saturation still far away
b) BFKL can mimic effects in "transition region"→both connected!

$$2\int d^2\boldsymbol{b}\,\mathcal{N}(x,r,b) = \frac{4\pi}{N_c}\int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)\alpha_s G(x,\boldsymbol{k}^2)\,.$$

dipole amplitude/includes saturation — BFKL unintegrated gluon

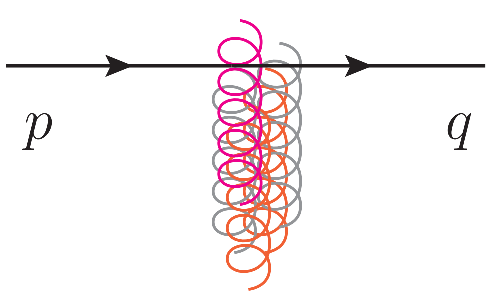
evolution differs (presence or absence of nonlinear terms),

.... but essentially same object

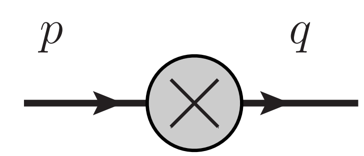
technical reason:

- interaction of a single quark line with infinitely many gluons is somehow equivalent to the interaction with a single high energy gluon (“reggeization”) **[Bartels, Wüsthoff, Z.Phys. C66 (1995) 157-180], others …**

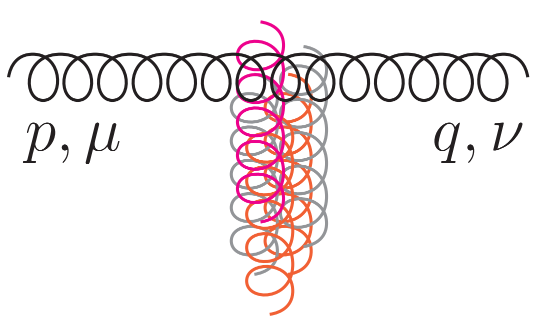
- Color Glass Condensate formalism: interaction collected into a single Wilson line→one effective vertex



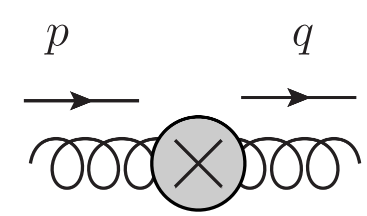
$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$



$\tilde{S}_F^{(0)}(q)$



$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$



$\tilde{G}_{\alpha\nu}^{(0)}(q)$

- to manifest non-linear effects, need to evolve over (relatively large) regions of phase space
  - BFKL:
    $\partial_{\ln 1/x} G(x, k) = K \otimes G$
  - BK:
    $\partial_{\ln 1/x} G(x, k) = K \otimes G - G \otimes G$

not clear how fast the non-linear term becomes relevant

- an alternative: observables which reveal non-linear effects without evolution

Observable ~ $G + \#G^2 + \#G^4 + ...$

a possibility: observables which depend on the quadrupole

$$\mathcal{N}^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) = \frac{1}{N_c}\mathrm{Tr}\left(1 - V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\right)$$

$$\sim \quad G + \#G^2 + \#G^4 + ...$$

(= 4 gluon exchange doesn't reduce to effective 2 gluon exchange on Xsec. level)

$$\mathcal{N}(\boldsymbol{r}, \boldsymbol{b}) = \frac{1}{N_c}\mathrm{Tr}\left(1 - V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right)$$

contains also 4 gluon exchange, but gathered in 2 Wilson lines

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z})t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z})T^c$$

well known example where this happens:

production of 2 partons in DIS

[Dominguez, Marquet, Xiao,Yuan; 1101.0715]

believe: worthwhile to go a step beyond (→ extra constrains on so far little studied quadrupole)

this project: calculate

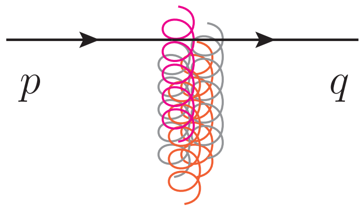
a) inclusive 3-parton production at LO (real part of NLO corrections to di-partons)
b) question: how to organise calculation in effective way; develop techniques for complex calculation?
c) related calculation for diffraction (includes already virtual)
[Boussarie, Grabovsky, Szymanowski, Wallon; 1405.7676, 1606.00419]

NEW: re-vive idea of momentum space calculations within the CGC

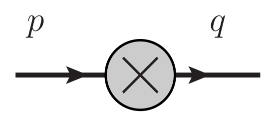
Theory: Propagators in background field

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

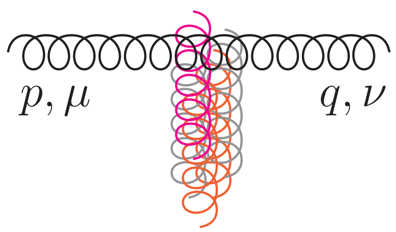


$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

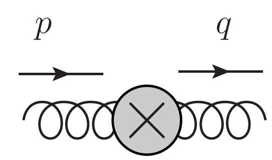


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



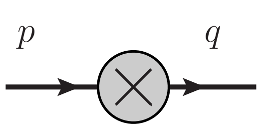
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

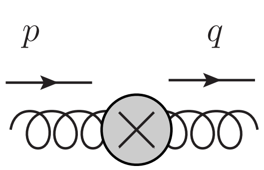


$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\,\not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right]\right\}$$

strong background field resummed into path ordered exponentials (Wilson lines)

$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$



$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\,(-2p^+)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right]\right\}$$

momentum vs. configuration space

	conventional pQCD (use known techniques)	inclusion of finite masses (charm mass!)	intuition: interaction at t=0 with Lorentz contracted target
momentum space	well explored	complication, but doable	lose intuitive picture at first -> large # of cancelations
configuration space	poorly explored	very difficult	many diagrams automatically zero

our approach:
work in momentum space + exploit configuration space to set a large fraction of all diagrams to zero

How to do that?

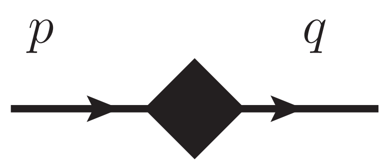
Essentially: re-install configuration space rules at the level of a single diagram

essential results: can use configuration space simplification also for momentum space calculations

Result: New effective rules for momentum space

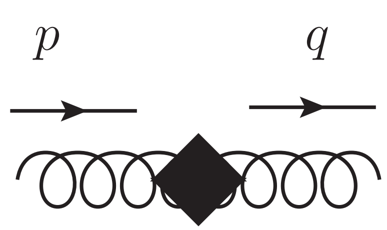
A. Determine zero light-cone time cuts of a given diagram

B. Place new vertices at these cuts



$$= \bar{\tau}_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\cdot \not{n}$$

$$\cdot \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)V_{ij}(\boldsymbol{z}) - \theta(-p^+)V^{\dagger}_{ij}(\boldsymbol{z})\right\}$$



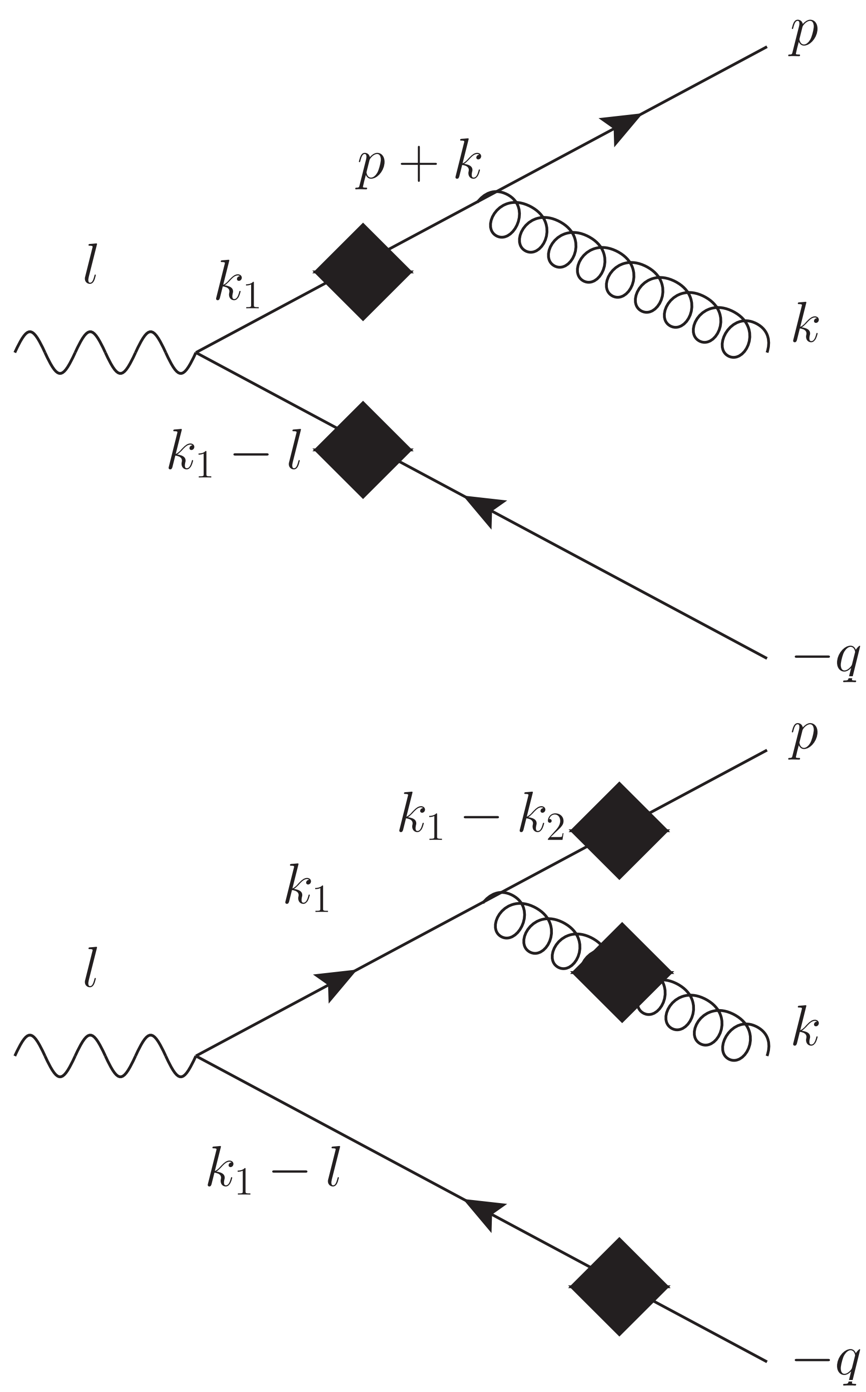
$$= \bar{\tau}^{ab}_{G}(p,q) = 2\pi\delta(p^+ - q^+)\cdot(-2p^+)$$

$$\cdot \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)U^{ab}(\boldsymbol{z}) - \theta(-p^+)\left(U^{ab}\right)^{\dagger}(\boldsymbol{z})\right\}$$

verified by explicit calculation for tree level diagrams; in general also extendable to loop diagrams …

First result: minimal set of amplitudes

(nothing new if you're used to work in coordinate space, momentum space: reduction by factor of 4)



What do we win with new momentum space rules?

can use techniques explored in (conventional) Feynman diagram calculations

- loop integrals (d-dimensional, covariant) → won’t talk about this today …. in general: complication due to Fourier factors remain

- **spinor helicity techniques** (calculate amplitudes not Xsec. + exploit helicity conservation in mass-less QCD) → compact expressions (→ for a different application to h.e.f. see **[van Hameren, Kotko, Kutak, 1211.0961]**)

Spinor-helicity formalism

see *e.g.* [Mangano, Parke; Phys. Rept. 200, 301 (1991)] ,[Dixon; hep-ph/9601359]

central idea: express both external spinors & polarisation vectors in terms of spinors of **massless** momenta of definite helicity

$$u_\pm(k) = \frac{1\pm\gamma_5}{2}u(p) \qquad v_\mp(k) = \frac{1\pm\gamma_5}{2}v(p)$$
$$\bar{u}_\pm(k) = \bar{u}(k)\frac{1\mp\gamma_5}{2} \qquad \bar{v}_\pm(k) = \bar{v}(k)\frac{1\pm\gamma_5}{2}$$

$$|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv u_\pm(k_i) = v_\mp(k_i)$$

$$\langle i^\pm| \equiv \langle k_i^\pm| \equiv \bar{u}_\pm(k_i) = \bar{v}_\mp(k_i)$$

$$\epsilon_\mu^{(\lambda=+)}(k,n) \equiv +\frac{\langle k^+|\gamma_\mu|n^+\rangle}{\sqrt{2}\langle n^-|k^+\rangle} = \left(\epsilon_\mu^{(\lambda=-)}(k,n)\right)^*$$

$$\epsilon_\mu^{(\lambda=-)}(k,n) \equiv -\frac{\langle k^-|\gamma_\mu|n^-\rangle}{\sqrt{2}\langle n^+|k^-\rangle} = \left(\epsilon_\mu^{(\lambda=+)}(k,n)\right)^*$$

… and make heavy use of various IDs

→many cancelations already at amplitude level

A reminder from before we realised that …

Dirac traces from Computer Algebra Codes

- possible to express elements of Dirac trace in terms of scalar, vector and rank 2 tensor integrals
- Evaluation requires use of computer algebra codes;
  use 2 implementations:
  FORM [Vermaseren, math-ph/0010025] &
  Mathematica packages FeynCalc and FormLink
- result (3 partons) as coefficients of "basis"-functions $f_{(a)}$ and $h_{(a,b)}$;
  result lengthy (~100kB), but manageable
- currently working on further simplification through integration by parts relation between basis function (work in progress)

the large N_C result

$$
\frac{d\sigma^{T,L}}{d^2\boldsymbol{p}\, d^2\boldsymbol{k}\, d^2\boldsymbol{q}\, dz_1 dz_2} = \frac{\alpha_s \alpha_{em} e_f^2 N_c^2}{z_1 z_2 z_3 (2\pi)^2} \prod_{i=1}^{3} \prod_{j=1}^{3} \int \frac{d^2\boldsymbol{x}_i}{(2\pi)^2} \int \frac{d^2\boldsymbol{x}'_j}{(2\pi)^2} e^{i\boldsymbol{p}(\boldsymbol{x}_1-\boldsymbol{x}'_1)+i\boldsymbol{q}(\boldsymbol{x}_2-\boldsymbol{x}'_2)+i\boldsymbol{k}(\boldsymbol{x}_3-\boldsymbol{x}'_3)}
$$

$$
\Big\langle (2\pi)^4 \Big[\Big(\delta^{(2)}(\boldsymbol{x}_{13})\delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_2)
$$

$$
+ \Big(\delta^{(2)}(\boldsymbol{x}_{23})\delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{2;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}'_1 | \boldsymbol{x}'_2, \boldsymbol{x}_2) \Big]
$$

$$
+ (2\pi)^2 \Big[\delta^{(2)}(\boldsymbol{x}_{13}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(24)}(\boldsymbol{x}_{3'}, \boldsymbol{x}_{1'} | \boldsymbol{x}_{2'}, \text{-}_2, \boldsymbol{x}_1, \boldsymbol{x}_{3'}) + \{1\} \leftrightarrow \{2\}
$$

$$
+ \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_3 | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_{1'}) + \{1'\} \leftrightarrow \{2'\} \Big]
$$

$$
+ \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{3'}, \boldsymbol{x}_3 | \boldsymbol{x}_3, \boldsymbol{x}_{3'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) \Big\rangle_{A\,\text{-}},
$$

in terms of correlators of Wilson lines
& wave functions

the details: correlators of Wilson lines

- written in terms of dipoles and quadrupoles

$$S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\right]$$

$$S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\right]$$

- quadrupole $S^{(4)}$ linear & quadratic
  → extra handle to explore it wrt. 2 partons

$$N^{(4)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4) \equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)}\,,$$
$$N^{(22)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4) \equiv$$
$$\equiv \left[S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - 1\right]\left[S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)} - 1\right]$$
$$N^{(24)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4,\boldsymbol{x}_5,\boldsymbol{x}_6) \equiv$$
$$1 + S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(4)}_{(\boldsymbol{x}_3\boldsymbol{x}_4\boldsymbol{x}_5\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_4\boldsymbol{x}_5)}\,,$$
$$N^{(44)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4|\boldsymbol{x}_5,\boldsymbol{x}_6,\boldsymbol{x}_7,\boldsymbol{x}_8) \equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)}S^{(4)}_{(\boldsymbol{x}_5\boldsymbol{x}_6\boldsymbol{x}_7\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_4)}S^{(2)}_{(\boldsymbol{x}_5\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_2\boldsymbol{x}_3)}S^{(2)}_{(\boldsymbol{x}_6\boldsymbol{x}_7)}$$

(quadrupole only linear)

the details: wave functions & amplitudes

$$\psi^{L}_{j,hg} = -2\sqrt{2}QK_0\left(QX_j\right)\cdot a^{(L)}_{j,hg}, \qquad j=1,2$$

$$\psi^{T}_{j,hg} = 2ie^{\mp i\phi_{\boldsymbol{x}_{12}}}\sqrt{(1-z_3-z_j)(z_j+z_3)}QK_1\left(QX_j\right)\cdot a^{\pm}_{j,hg} \qquad j=1,2$$

$$\psi^{L}_{3,hg} = 4\pi iQ\sqrt{2z_1z_2}K_0\left(QX_3\right)(a^{(L)}_{3,hg}+a^{(L)}_{4,hg}),$$

$$\psi^{T}_{3,hg} = -4\pi Q\sqrt{z_1z_2}\frac{K_1\left(QX_3\right)}{X_3}(a^{\pm}_{3,hg}+a^{\pm}_{4,hg}).$$

symmetry relation between amplitudes

$$a^{T,L}_{k+1,hg} = -a^{T,L}_{k,-hg}(\{p,\boldsymbol{x}_1\}\leftrightarrow\{q,\boldsymbol{x}_2\}), \qquad k = \text{ 1,3}$$

$$a^{T,L}_{j,hg} = a^{(-T,L)*}_{j,-h-g}, \quad j=1,\ldots,4.$$

longitudinal photon

$$a^{(L)}_{1,++} = -\frac{(z_1 z_2)^{3/2}\,(z_1 + z_3)}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(L)}_{3,++} = \frac{z_1 z_2}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(L)}_{1,-+} = -\frac{\sqrt{z_1}\, z_2^{3/2}\,(z_1 + z_3)^{\,2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(L)}_{3,-+} = \frac{z_2(1 - z_2)}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

transverse photon

for precise def. see paper

take away message:

very compact expressions

$$a^{(+)}_{1,++} = -\frac{(z_1 z_2)^{3/2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,+-} = \frac{\sqrt{z_1}(z_2)^{\frac{3}{2}}(z_1+z_3)}{z_1 e^{i\theta_k}|\boldsymbol{k}| - z_3 e^{i\theta_p}|\boldsymbol{p}|},$$

$$a^{(+)}_{1,-+} = \frac{\sqrt{z_1 z_2}(z_1+z_3)^2}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,--} = \frac{z_1^{3/2}\sqrt{z_2}(z_1+z_3)}{z_3 e^{i\theta_p}|\boldsymbol{p}| - z_1 e^{i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{3,++} = \frac{z_1 z_2(z_2 z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} + z_3|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}} - z_1 z_2|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{(z_1+z_3)|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,+-} = \frac{z_2^2(z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}|e^{i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,-+} = -\frac{z_2(z_1+z_3)(z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,--} = \frac{z_1 z_2(z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}} - z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}})}{|\boldsymbol{x}_{13}|e^{i\phi_{\boldsymbol{x}_{13}}}}.$$

First attempts in phenomenology

- differential Xsec: given in terms of dipole and quadrupole operators
- need to be evaluated for a given background field configuration = represents dynamics of target

$$\langle \ldots \rangle_{A^-} = \int D[\rho] \ldots e^{-W[\rho]}$$

$\boldsymbol{\rho}$: color carge, relates to back-ground field through Yang-Mills equation

$$-\boldsymbol{\partial}^2 A^{c,-}(z^+, \boldsymbol{x}) = g_s \rho_c(z^+, \boldsymbol{x})$$

- in general: weight function W[$\boldsymbol{\rho}$] not known … what can be extracted from inclusive DIS data is the dipole amplitude

$$\langle S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2)\rangle_{A^-} = \frac{1}{N_c}\langle \mathrm{tr}\left(V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\right)\rangle_{A^-}$$

→ higher correlators not known; way out: “Gaussian approximation” (McLerran-Venugopalan model) for weight function with width $\boldsymbol{\mu}$

$$W[\rho] = \int d^2\boldsymbol{x} \int d^2\boldsymbol{y} \int dz^+ \, \frac{\rho_c(z^+, \boldsymbol{x})\rho_c(z^+, \boldsymbol{y})}{2\mu^2(z^+)}$$

can argue: good approximation in dilute limit

- allows to calculate dipole in terms of μ^2 and 2 point correlator of fields → fix this combination from DIS inclusive fits of $S^{(2)}$

- calculate quadrupole correlator in terms of dipole correlator
[Dominguez, Marquet, Xiao,Yuan; 1101.0715]

$$S^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) = S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}) - \frac{\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_{2'}; \boldsymbol{x}_2, \boldsymbol{x}_{1'})}{\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{x}_{2'}, \boldsymbol{x}_{1'})} S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}) S^{(2)}(\boldsymbol{x}_2, \boldsymbol{x}_{2'})$$

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_{2'}; \boldsymbol{x}_2, \boldsymbol{x}_{1'}) = \ln \frac{S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_{2'}) S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_2)}{S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_{1'}) S^{(2)}(\boldsymbol{x}_2, \boldsymbol{x}_{2'})}$$

- numerical study: a good approximation to full expression
[Dumitru, Jalilian-Marian, Lappi, Schenke, Venugoplana; 1108.4764]

- in general: known for finite N_C; here: large N_C limit→ argue that expectation values of combinations of $S^{(2)}$ and $S^{(4)}$ factorise

- our treatment: use $S^{(2)}=1 - N^{(2)}$ and expand for small $N^{(2)}$ to linear and quadratic order → large quadratic corrections: sensitive to non-linear effects

- For $S^{(2)}$ use model with parameters fitted to rcBK DIS fit
[Quiroga-Arias,Albacete, Armesto, Milhano, Salgado, 1107.0625]

$$S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \int d^2\boldsymbol{l}\, e^{-i\boldsymbol{l}\cdot\boldsymbol{x}_{12}}\ \Phi(\boldsymbol{l}^2)$$

$$= 2\left(\frac{Q_0|\boldsymbol{x}_{12}|}{2}\right)^{\alpha-1} \frac{K_{\alpha-1}(Q_0|\boldsymbol{x}_{12}|)}{\Gamma(\alpha-1)},$$

$$\Phi(\boldsymbol{l}^2) = \frac{\Gamma(\alpha)}{Q_0^2\pi\Gamma(\alpha-1)}\left(\frac{Q_0^2}{Q_0^2+\boldsymbol{l}^2}\right)^{\alpha},$$

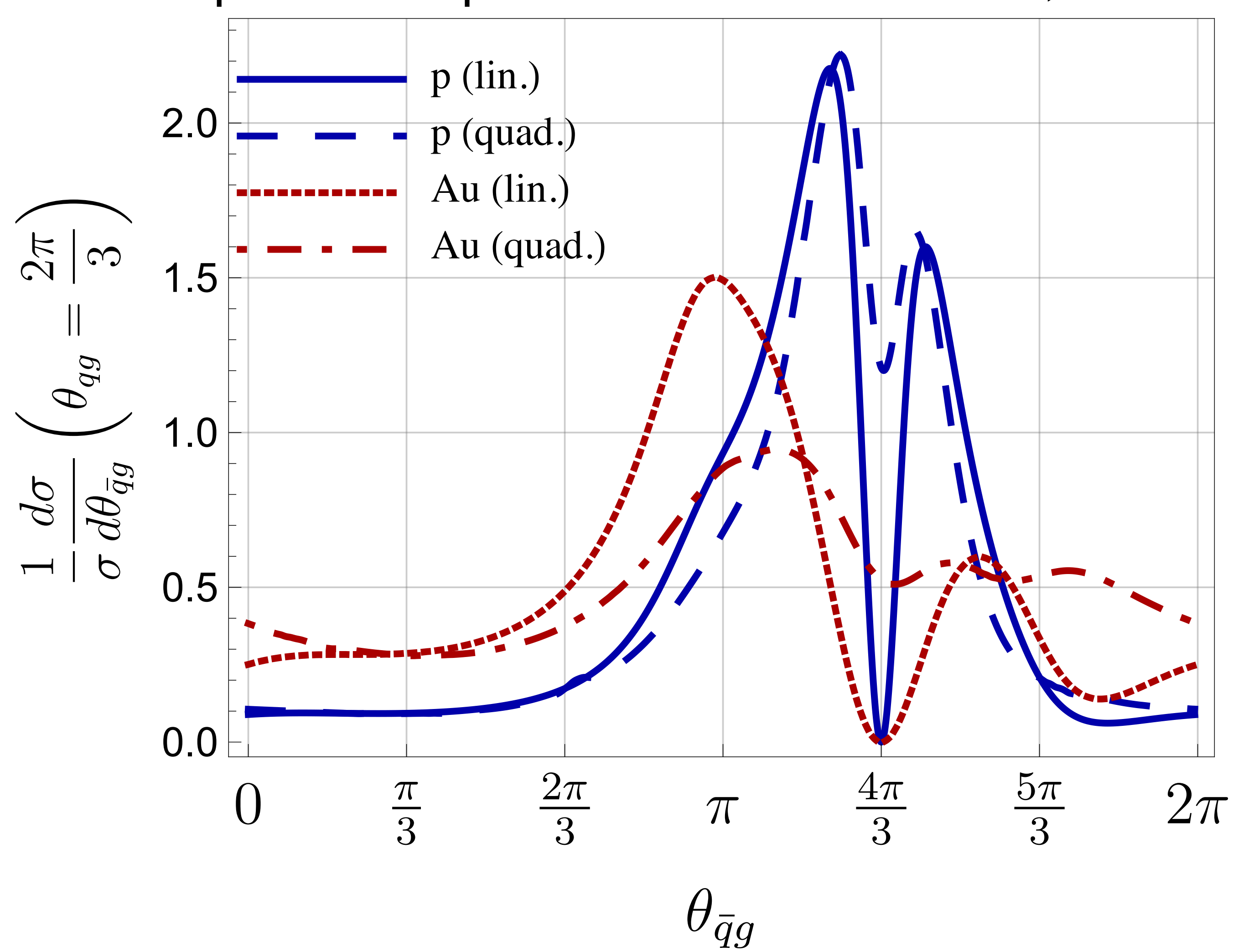
- parameters: $\boldsymbol{\alpha} = 2.3$

  proton: $Q_0^{prot.} = 0.69$ GeV   correspond to $x = 0.2 \cdot 10^{-3}$

  gold: $Q_0^{gold} = A^{1/6}\, Q_0^{prot.} = 1.67$ GeV

First study at partonic level

- explore deviations from Mercedes star configuration→back-to-back for three particles
- parton p_T fixed to 2 GeV, Q=3 GeV



- fix one angle (quark-gluon), vary antiquark-gluon
- sizeable quadratic corrections for gold

Summary:

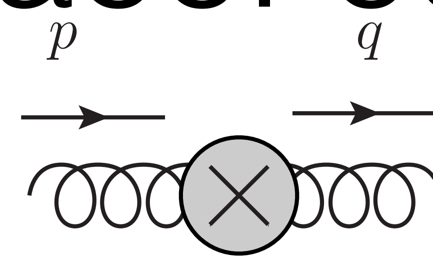
a more detailed phenomenological study is needed … ,
so far:

- possible to use momentum space calculations for CGC calculations→ access to momentum space techniques
- helicity spinor formalism can greatly simplify calculations within high energy factorisation
- to detect high gluon density effects, observables directly sensitive to such effects should help ("evolution only" might require too much phase space)
  → we studied such an observables and showed that this could actually work (at partonic level so far)

Gracias!

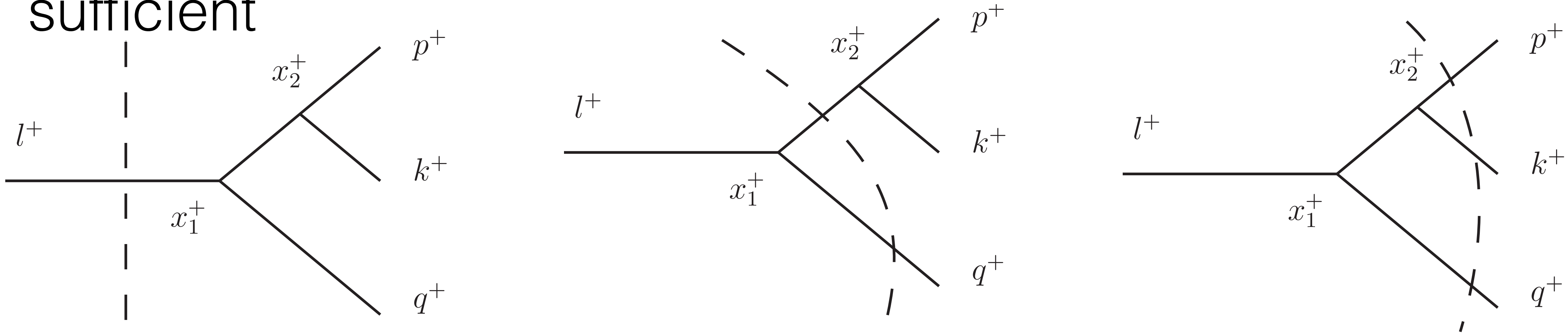
Configuration space: cuts at x^+=0

- start without special vertices



- divide x_i^+ integral $\int_{-\infty}^{\infty} dx^+ \to \int_{-\infty}^{0} dx^+ + \int_{0}^{\infty} dx^+$ + theta functions in plus momenta & coordinates → each of our diagrams cut by a line separating positive & negative light-cone time (left: negative; right: positive)

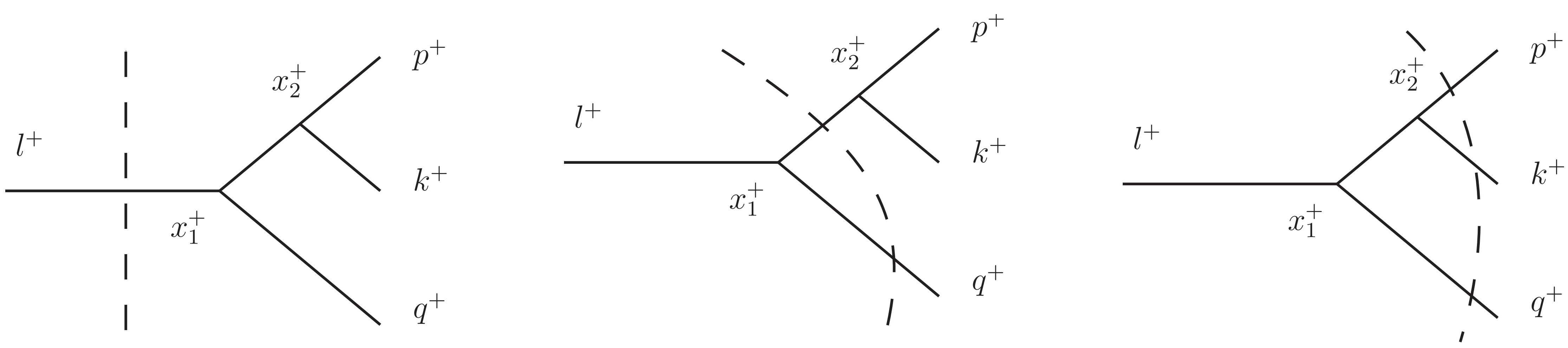
- only plus coordinates & momenta → skeleton diagrams sufficient



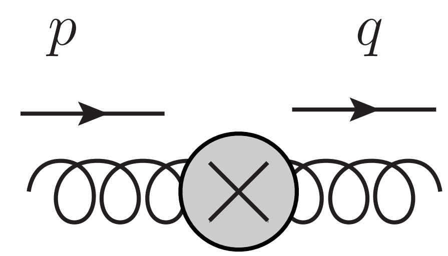
- a "cut" propagator crosses light-cone time x^+=0

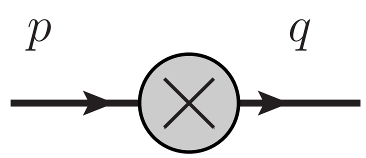
Which cuts are possible?

- in general: any line through the diagram
- fix kinematics to s-channel kinematics [$l^+ = p^+ + q^+ + k^+$, all plus momenta positive always]
  → only s-channel type cuts possible (~vertical cuts)



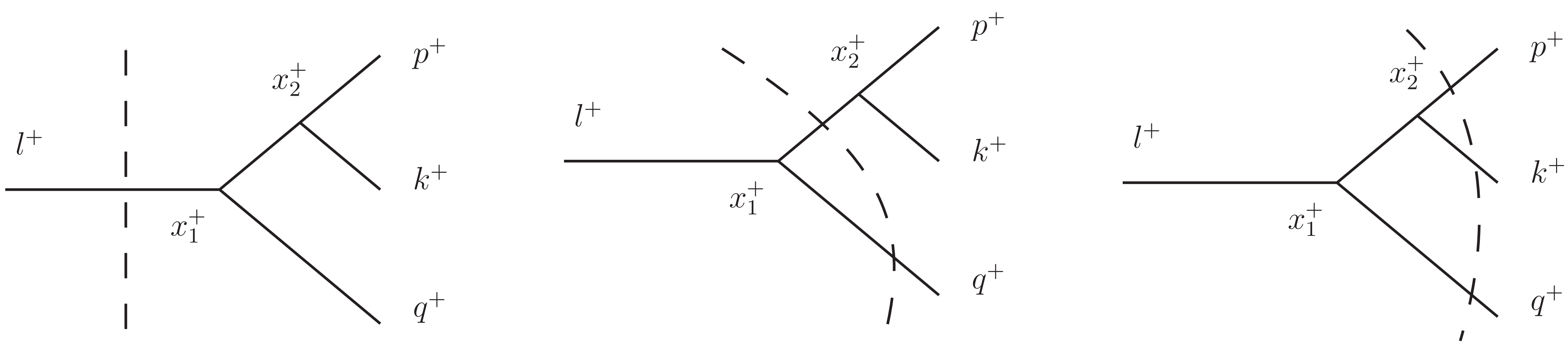
- for this topology, these are the only possible cuts

- NEXT: add special vertices 

- recall: 

$\sim \delta(p^+ - q^+)$ plus momentum flow not altered + placed at $z^+=0$ ⇒ by default on the cut

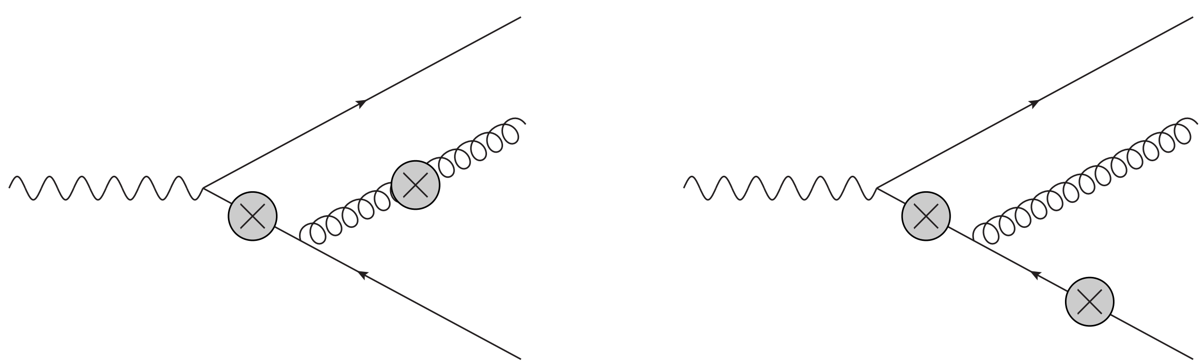
- go back to momentum space: special vertices still must be aligned along the cut



- at a cut: “propagator ⊗ special vertex ⊗ propagator” or “propagator” only; no special vertex anywhere else

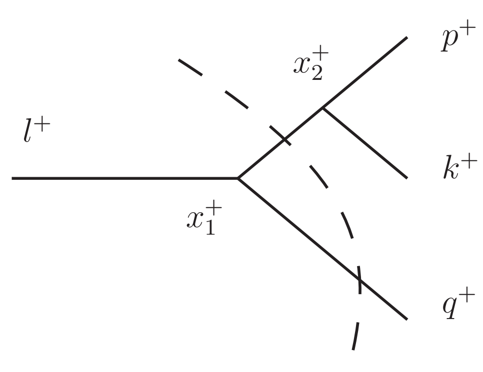
How does it help?

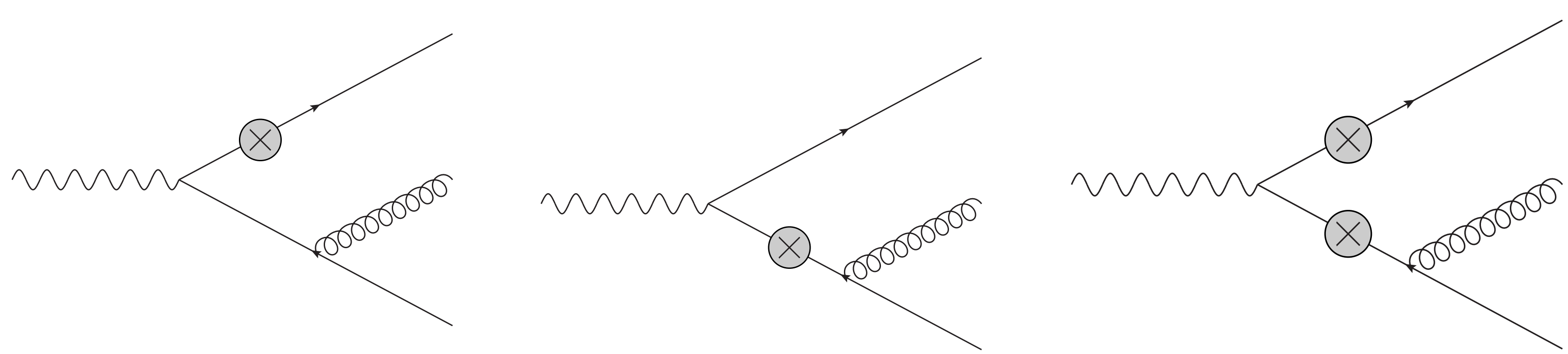
- evaluates 50% of possible momentum diagrams to zero



not possible for s-channel kinematics

- ….. but each cut contains still several diagrams



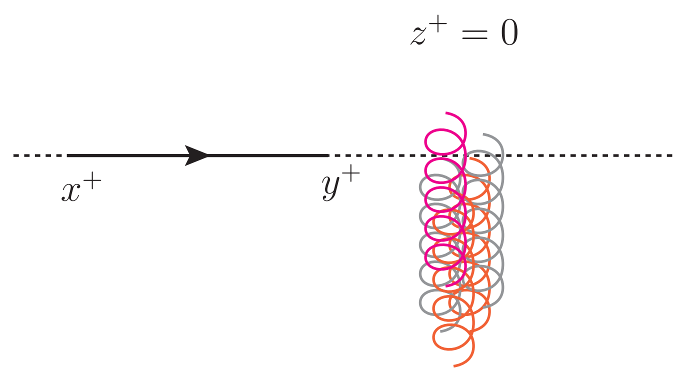


Configuration space knows more …
(partial) Fourier transform for complete propagator

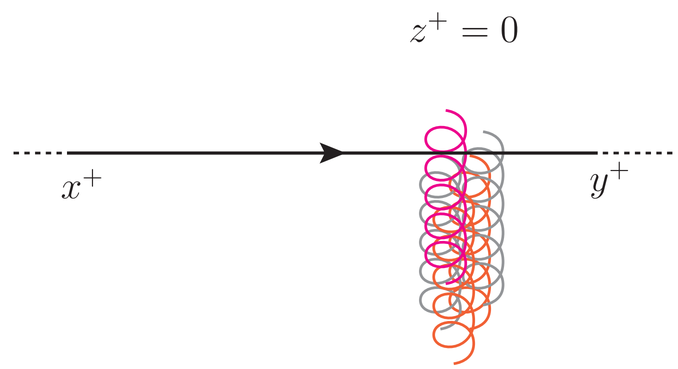
$$\int \frac{dp^-}{2\pi} \int \frac{dq^-}{2\pi} e^{-ip^- x^+} e^{iq^- y^+} \left[S^{(0)}_{F,il}(p)(2\pi)^4 \delta^{(4)}(p-q) + S^{(0)}_{F,ij}(p) \cdot \tau_{F,jk}(p,q) \cdot S^{(0)}_{kl}(q) \right]$$

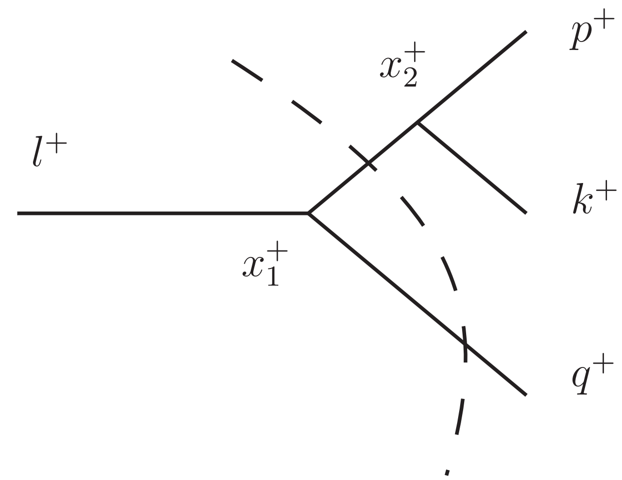
obtain free propagation for

- $x^+, y^+ < 0$ ("before interaction")
- $x^+, y^+ > 0$ ("after interaction")

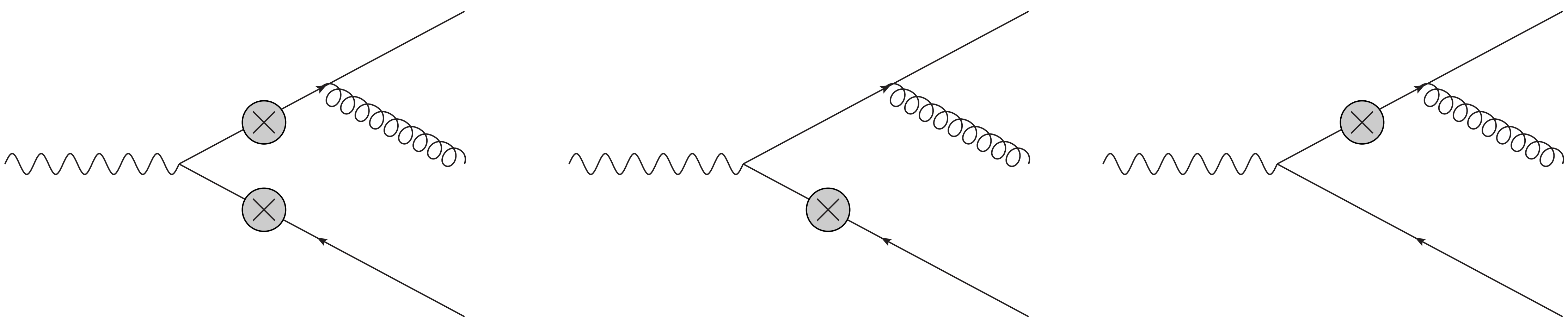


propagator proportional to complete Wilson line V (fermion) or U (gluon) if we cross light-cone time $z^+=0$
→ must pass through the cuts

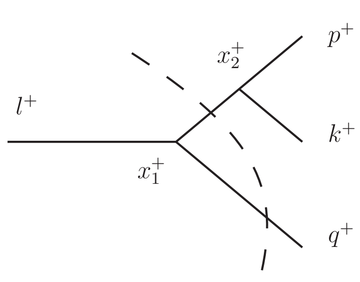


- for a single cut: 

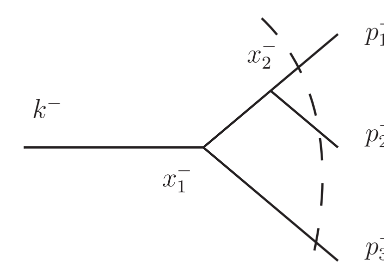
effectively adds up



- reality: more complicated due to mixing of different cuts



vs.



- crucial: positive plus momenta in all lines for tree diagrams
- allows to formulate a new set of effective "Feynman rules"