Bayesian parameter estimation for heavy-ion collisions: inferring properties of the quark-gluon plasma

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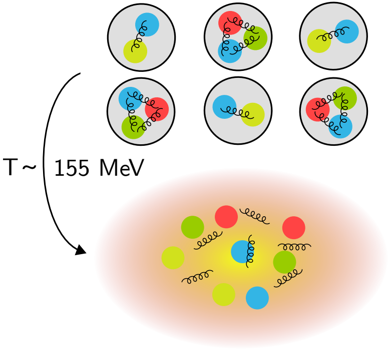
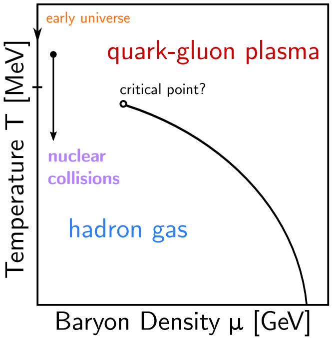
Lattice predicts existence of a quark-gluon plasma

Lattice QCD calculations find a pseudo-critical phase transition temperature $T \approx 155$ MeV, where hadrons melt to form a deconfined soup of quarks and gluons dubbed a quark-gluon plasma (QGP)

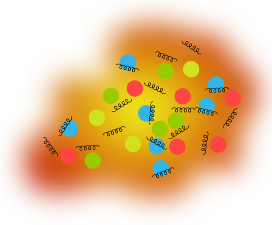




What are the quark-gluon plasma bulk properties?

Equation of state?
Relations between
thermal quantities,
e.g. $P = P(\epsilon)$

How and under what
conditions is it formed
in a nuclear collision?



Transport properties?
shear/bulk viscosity,
probe energy loss, etc

How does it recombine
to form colorless hadrons?

Formulating an inverse problem

Model-to-data comparison (in an ideal world)

Model A Model B Model C Model D Model E

Exp Data

Formulating an inverse problem

REALISTIC MODEL-TO-DATA COMPARISON

? ?

Model A Model B Model C Model D Model E

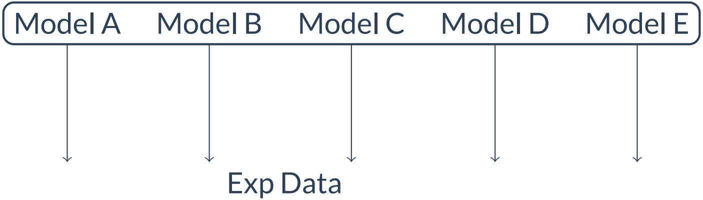
↓ ↓ ↓ ↓ ↓

Exp Data

I) Bayesian parameter estimation

Formulating an inverse problem

Parametrize Theory landscape



Formulating an inverse problem

Bayesian parameter estimation

$\boxed{\text{continuous model parameter: } x}$

$P(x_\star|\text{model, data})$

Exp Data

Bayes' Theorem:

$$\underbrace{P(x_\star|\text{model, data})}_{\text{posterior}} \propto \underbrace{P(\text{model, data}|x_\star)}_{\text{likelihood}} \underbrace{P(x_\star)}_{\text{prior}}$$

Formulating an inverse problem

BAYESIAN PARAMETER ESTIMATION

$$\boxed{\text{continuous model parameter: } x}$$

$$\uparrow \; P(x_\star|\text{model}, y_\text{exp})$$

Exp Data

BAYES' THEOREM:

$$\underbrace{P(x_\star|\text{model}, \text{data})}_{\text{posterior}} \propto \underbrace{P(\text{model}, \text{data}|x_\star)}_{\text{likelihood}} \underbrace{P(x_\star)}_{\text{prior}}$$

Formulating an inverse problem

BAYESIAN PARAMETER ESTIMATION

$\boxed{\text{continuous model parameter: } x}$

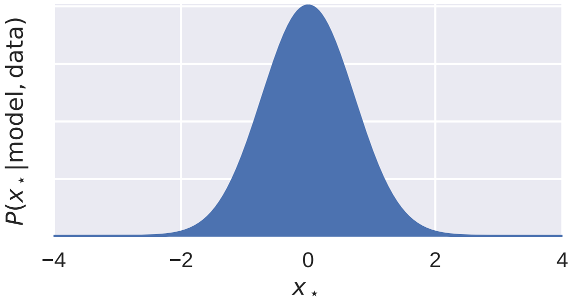
$$\text{Exp Data} \xrightarrow{P(x_\star|\text{model}, y_{\text{exp}})} \text{continuous model parameter: } x$$

BAYES' THEOREM:

$$\underbrace{P(x_\star|\text{model}, \text{data})}_{\text{posterior}} \propto \underbrace{P(\text{model}, \text{data}|x_\star)}_{\text{likelihood}} \underbrace{P(x_\star)}_{\text{prior}}$$

Formulating an inverse problem

YIELDS POSTERIOR DISTRIBUTION ON $x_\star$



Includes uncertainty in "best-fit value"

Multiple observables

posterior = likelihood $\times$ prior

More than one observable $f : x \mapsto (y_1, \ldots, y_n)$?
No problem, calculate likelihood using multivariate Gaussian

Log-likelihood

$$\ln(L) = -\frac{1}{2}(\ln(|\mathbf{\Sigma}|) + (\mathbf{y} - \mathbf{y}_{\text{exp}})^T \mathbf{\Sigma}^{-1}(\mathbf{y} - \mathbf{y}_{\text{exp}}) + k \ln(2\pi))$$

$$\mathbf{\Sigma} = \mathbf{\Sigma}_{\text{model}} + \mathbf{\Sigma}_{\text{exp}}^{\text{stat}} + \mathbf{\Sigma}_{\text{exp}}^{\text{sys}}$$

Multiple model parameters

posterior = likelihood $\times$ prior

Likelihood function $L(x) \to L(x_1, \ldots, x_n)$

Curse of dimensionality

Typically interested in marginalized probabilities

$L(x_1, \ldots, x_n)$ easy to calculate, hard to integrate.

Solution

Monte Carlo integration, e.g. importance sampling

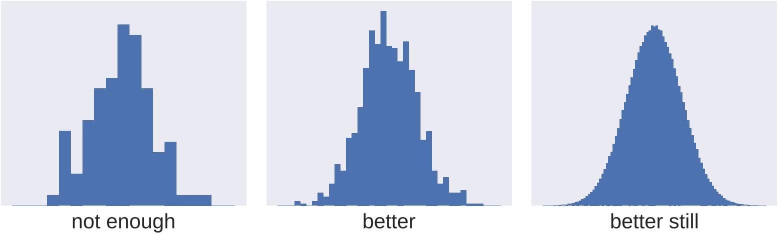
MCMC importance sampling:

1. large number of walkers in $\{x_1, \ldots, x_n\}$ space
2. update walker positions
3. accept new $\mathbf{x}$ with prob $P \sim L_{\text{new}}/L_{\text{old}}$

Marginalize by histogramming over flattened dimensions

MCMC and evaluating the likelihood

Number of likelihood samples needed for MCMC varies greatly



Several of the published results in this talk use $N_{\text{sample}} > 10^6$

If model is slow, e.g. 1 CPU hour per likelihood evaluation

...good luck

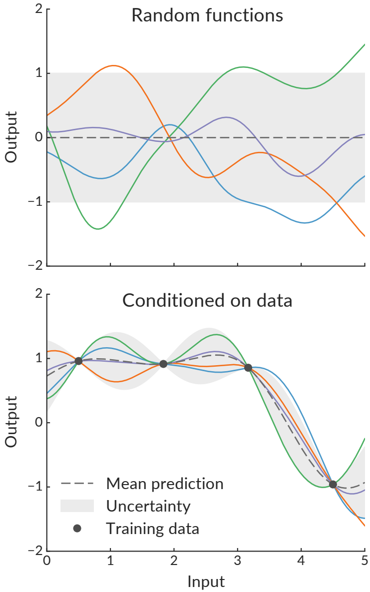
Training an emulator

Gaussian process:

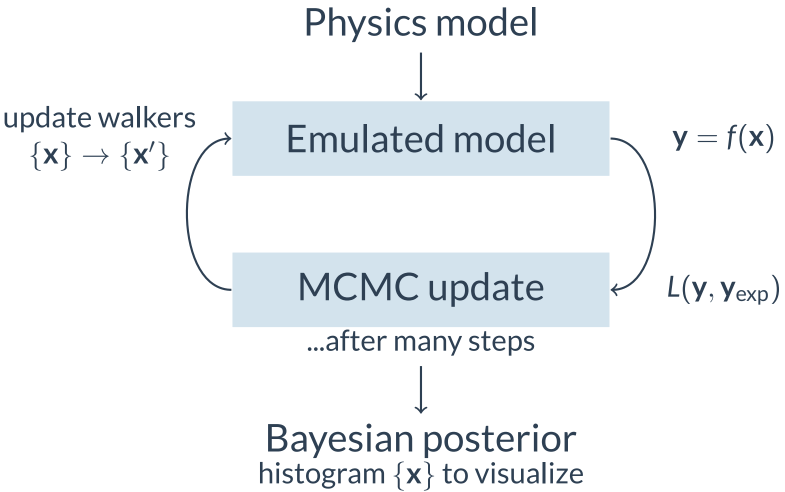
- Stochastic function: maps inputs to normally-distributed outputs
- Specified by mean and covariance functions

As a model emulator:

- Non-parametric interpolation
- Predicts *probability distributions*
  - Narrow near training points, wide in gaps
- Fast surrogate to actual model

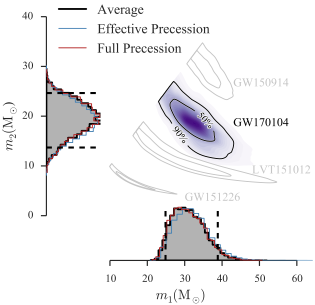


Workflow


Physics model
update walkers
$\{\mathbf{x}\} \to \{\mathbf{x}'\}$
Emulated model
$\mathbf{y} = f(\mathbf{x})$
MCMC update
$L(\mathbf{y}, \mathbf{y}_{\mathrm{exp}})$
...after many steps
Bayesian posterior
histogram $\{\mathbf{x}\}$ to visualize

Bayesian parameter estimation in physics

LIGO Experiment



est. black hole masses

PRL 118.221101

- **Planck Collaboration 2015: constraints on inflation**
  Astron. Astrophys. 594 (2016)
- **CKM parameters**
  Eur. Phys. J. C21 (2001)
- **Galaxy formation**
  Astron. Astrophys. 409 (2003)

...and many more examples not listed here

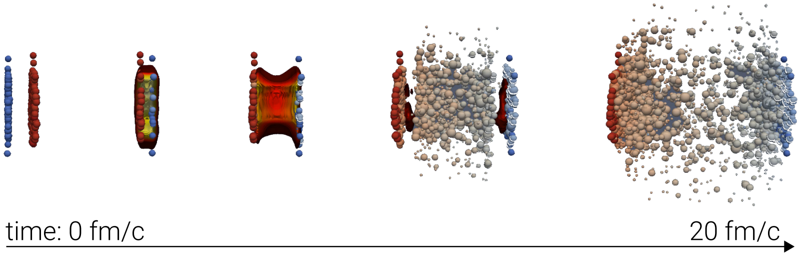
Adapt machinery to relativistic heavy-ion collisions?

II) Bayesian parameter estimation applied to heavy-ion physics

Bayesian methodology for heavy-ion collisions

Analogue

TRUSTED FRAMEWORK	EXPERIMENTAL DATA	FREE PARAMETER(S)
General relativity	gravitational waves	black hole masses
⚠ Relativistic hydro	particle yields & corr.	transport coefficients



Hydro framework imposes local energy and momentum conservation.
Clearly breaks in dilute limit. Should apply with care.

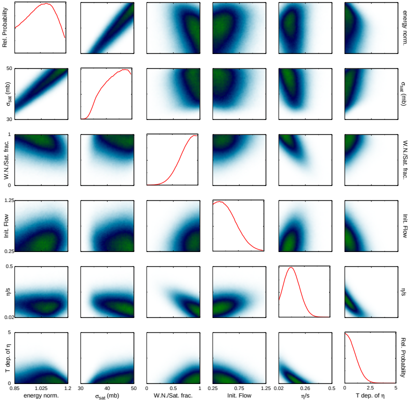
Bayesian methodology for heavy-ion collisions

Analogue

Trusted framework	Experimental data	Free parameter(s)
General relativity	gravitational waves	black hole masses
⚠ Relativistic hydro	particle yields & corr.	transport coefficients

- ⚠ Hydro for heavy-ion collisions not trusted on same level as e.g. GR for gravitational waves
- Posterior results always subject to framework credibility

Seminal Bayesian works in heavy-ion physics

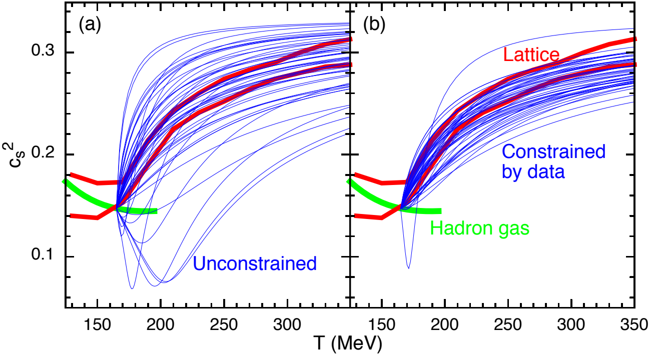


- Event-averaged hydro
- Parametric pre-flow
- Parametric initial state
- First Bayesian posterior on $(\eta/s)(T)$
- Omits bulk viscosity
- Two centrality bins

Determining Fundamental Properties of Matter Created in Ultrarelativistic Heavy-Ion Collisions, Novak, Novak, Pratt, Vredevoogd, Coleman-Smith, Wolpert PRC 89 (2014) 034917

Seminal Bayesian works in heavy-ion physics

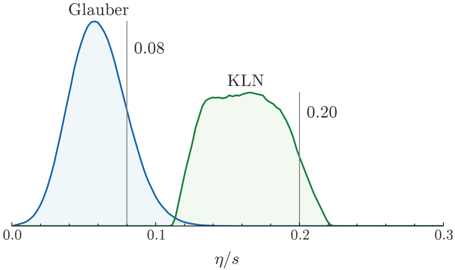
- Equation of state from lattice QCD is very close to parametric equation of state preferred by simulation



BNL

Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions, Pratt, Sangaline, Sorensen, Wang, PRL 114 (2015) 202301

Seminal Bayesian works in heavy-ion physics



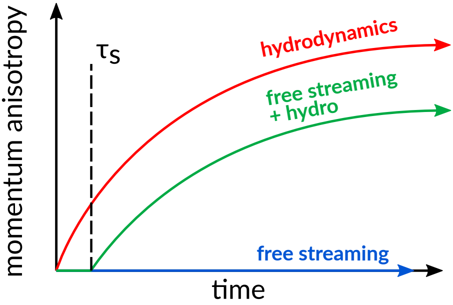
Theoretical biases affect preferred viscosity

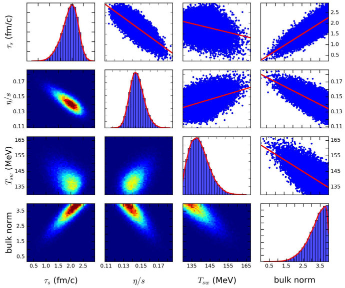
- Event-by-event hydro
- MC-Glauber & KLN initial conditions
- Centrality bins like experiment
- Constant η/s
- Omits bulk viscosity, pre-flow

Constraining the Eq. of State of Super-Hadronic Matter from Heavy-Ion Collisions, Bernhard, Marcy, Coleman-Smith, Huzurbazar, Wolpert, Bass, PRC 91 (2015) 054910

Initial stages and onset of hydrodynamic flow

Strong coupling limit $\rightarrow$ hydrodynamics
Weak coupling limit $\rightarrow$ freestreaming





Pre-equilibrium dynamics and heavy-ion observables,
Heinz, Liu, Nucl. Phys. A956 (2016) 549-552

Towards precision extraction of QGP properties

Applying Bayesian parameter estimation to relativistic heavy-ion collisions: simultaneous characterization of the initial state and QGP medium, Bernhard, Moreland, Bass, Liu, Heinz PRC 94 (2016) 024907

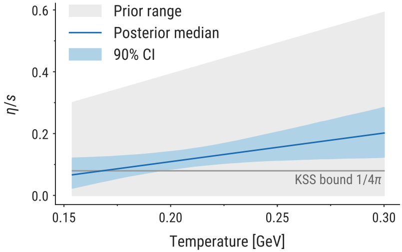
Generational improvements

- New **TRENTo** initial condition model: absorbs initial state uncertainties into several free parameters
- Full event-by-event hydro with hadronic afterburner
- Calculate observables exactly as experiment
- Bulk and shear viscous corrections
- *More* experimental observables

Physics insights

η/s min $= 0.07 \pm 0.05$

non-zero bulk viscosity



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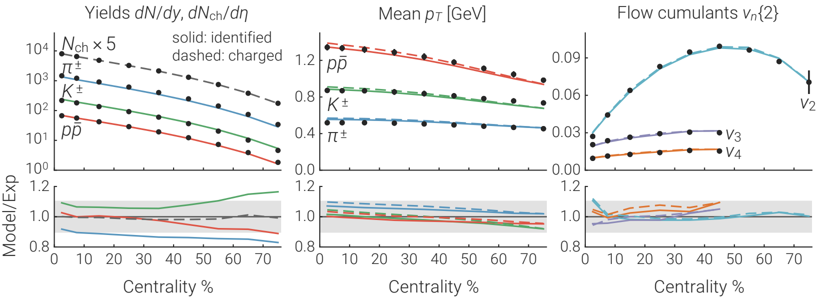
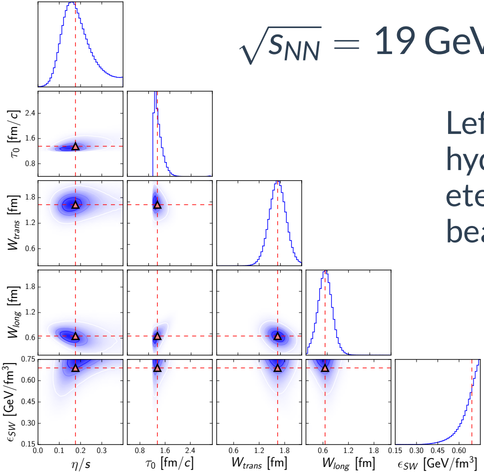
Model calculations with high-likelihood parameters from Bayesian posterior provide excellent description of bulk observables



Leveraging data from RHIC beam energy scan

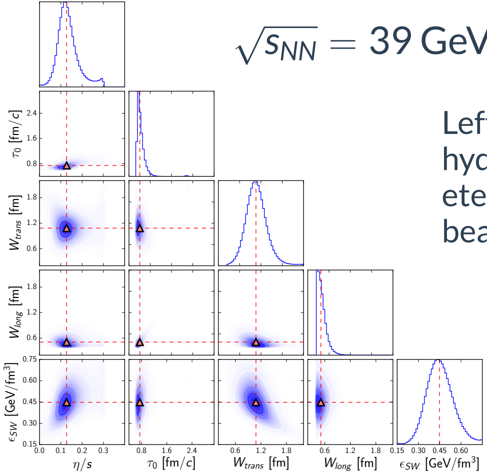


$$\sqrt{s_{NN}} = 19 \text{ GeV}$$

Left: Bayesian posterior for hydrodynamic model parameters calibrated at different beam energies $\sqrt{s_{NN}}$

Revealing the collision energy dependence of η/s in RHIC-BES Au+Au collisions using Bayesian statistics,
Auvinen, Karpenko, Bernhard, Bass, QM17 proceedings

Leveraging data from RHIC beam energy scan



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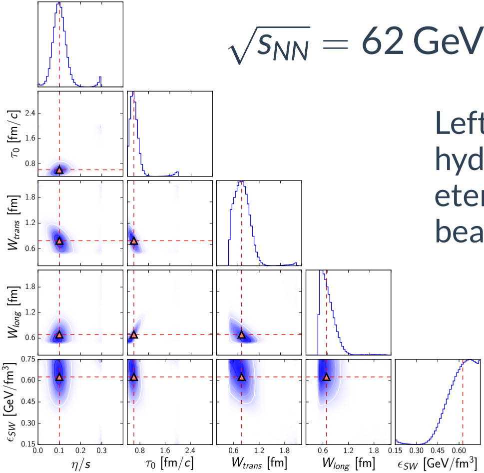

$$\sqrt{s_{NN}} = 62 \text{ GeV}$$

Left: Bayesian posterior for hydrodynamic model parameters calibrated at different beam energies $\sqrt{s_{NN}}$

Revealing the collision energy dependence of η/s in RHIC-BES Au+Au collisions using Bayesian statistics,

Auvinen, Karpenko, Bernhard, Bass, QM17 proceedings

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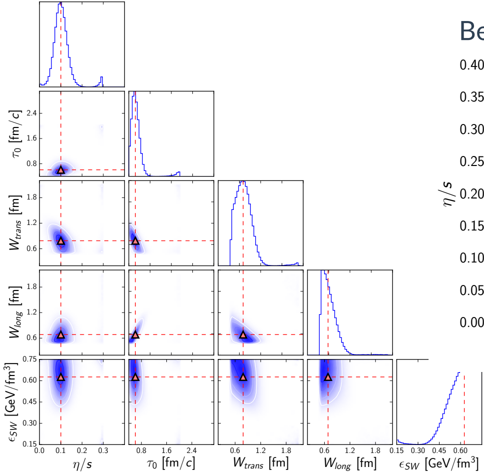
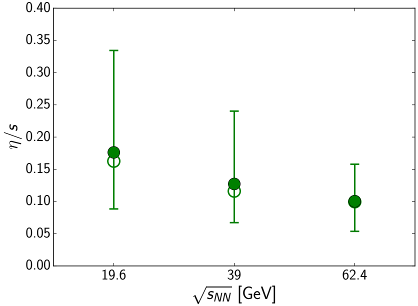




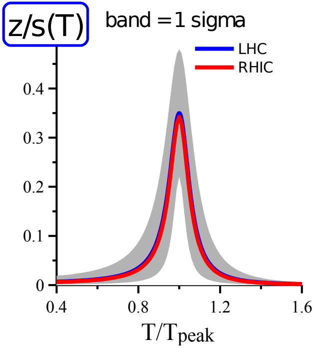
Revealing the collision energy dependence of η/s in RHIC-BES Au+Au collisions using Bayesian statistics,

Auvinen, Karpenko, Bernhard, Bass, QM17 proceedings

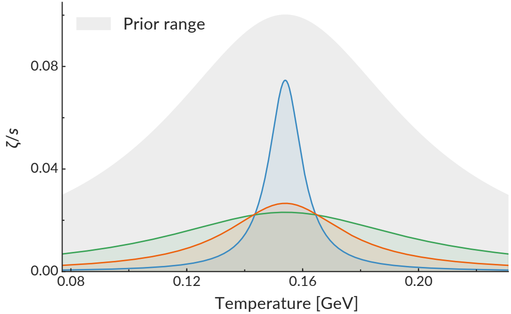
Bulk viscosity: a work in progress...

CHALLENGES

- Different methods for bulk viscous corrections at freezeout
- Less obvious parametric form for $(\zeta/s)(T)$
- Hydro cavitates if bulk is too large



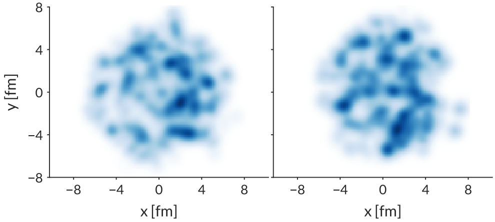
Denicol, Paquet, Gale, Jeon, Shen



Bernhard, Moreland, Bass

Studying the QGP fireball in 3D

Initial energy density (2D)



Initial energy density (3D)

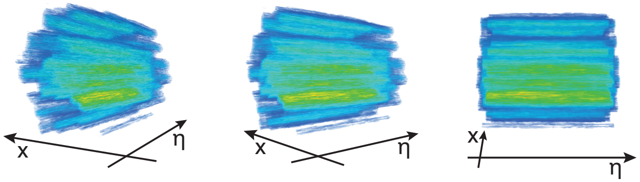


Figure credit: Schenke, Schlichting

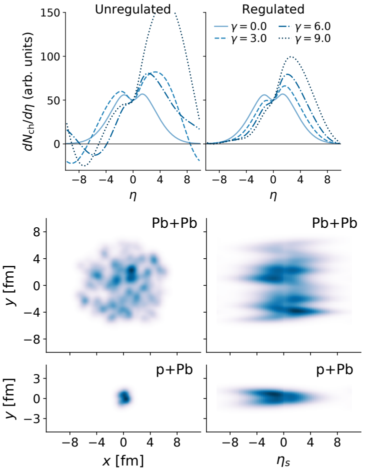
Studying the QGP fireball in 3D

Constraints on rapidity-dependent initial conditions from charged particle pseudorapidity densities and two-particle correlations,
Ke, Moreland, Bernhard, Bass (in prep)

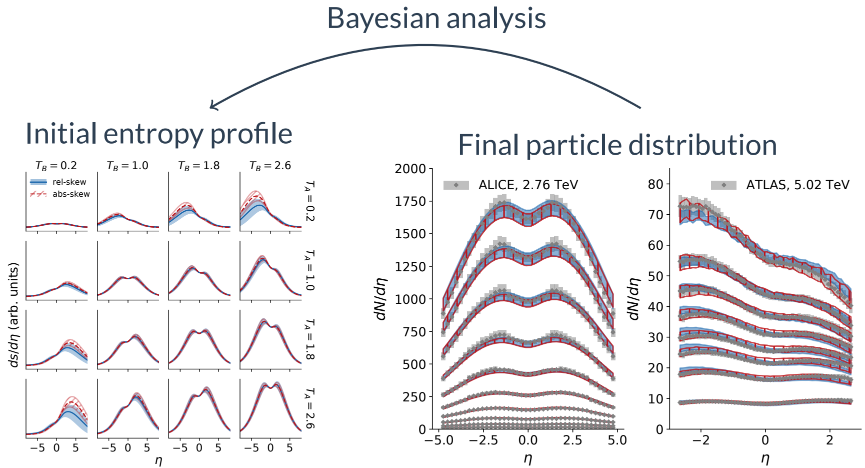
Optimization problem

Find initial energy density that evolves into final single particle distribution

- Parametrize initial longitudinal energy profile with moment-generating function
- Constrain form using charged particle rapidity distributions



Studying the QGP fireball in 3D



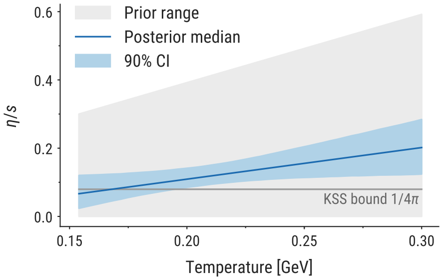
- Trust in hydro and Bayesian statistical machinery lets us deconvolve complex system evolution

QGP hard probes: open heavy-flavour

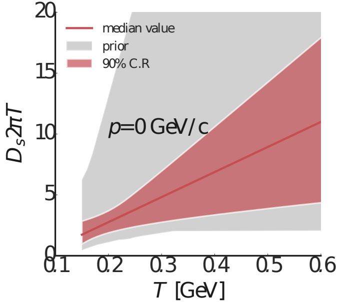
Analogue

Theory framework	Free parameter(s)
Hydrodynamics	QGP viscosity: $\eta/s, \zeta/s$
Langevin transport	charm diffusion coefficient: $D_{s,p}$

QGP shear viscosity



Charm diffusion coefficient



A data driven analysis for the temperature and momentum dependence of the heavy quark diffusion coefficient in relativistic heavy-ion collisions
Xu, Bernhard, Bass, Nahrgang, Cao (in preparation)

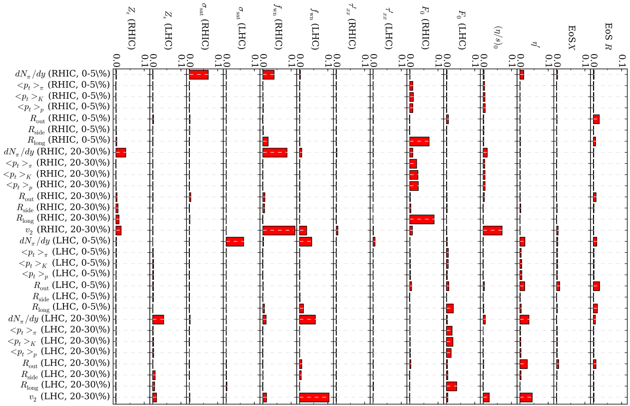
Summary

Virtues of Bayesian parameter estimation

- Works for models with multiple correlated parameters
- Rigorous accounting of errors and effect on quantities of interest
- Global analysis can promote and kill models

Backup slides

Seminal Bayesian works in heavy-ion physics



Sensitivity of experimental observables to model parameters

Towards a Deeper Understanding of How Experiments Constrain the Underlying Physics of Heavy-Ion Collisions, Sangaline, Pratt, PRC 93 (2016) 024908