Strong magnetic fields in a non-local Polyakov chiral quark model

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PLAN OF THE TALK

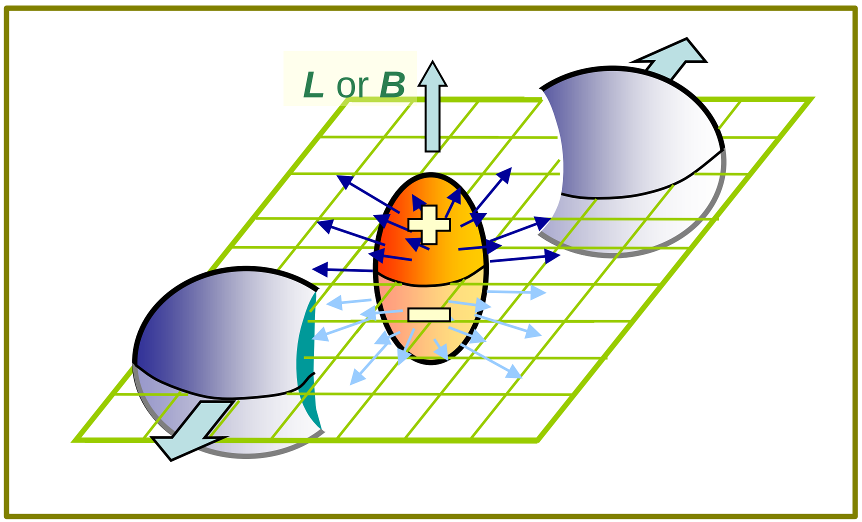
- Introduction
- Magnetic field in non-local Polyakov chiral quark models
- Results
- Outlook & Conclusions

Refs: Pagura, Gomez Dumm, Noguera & NNS, Phys.Rev. D95 (2017) 034013
Gomez Dumm, Izzo Villafañe, Noguera, Pagura & NNS, in preparation

Introduction

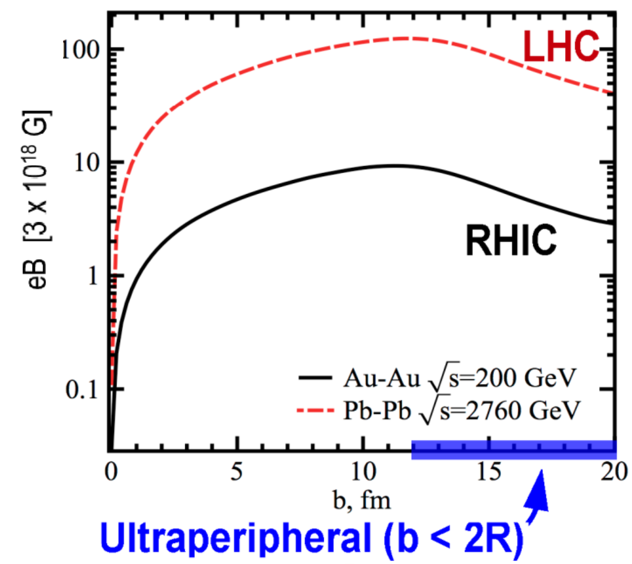
Recently, there has been quite a lot of interest in investigating how the QCD phase diagram is affected by the presence of strong magnetic fields. Motivation: their possible existence in physically relevant situations:

High magnetic fields in non-central relativistic heavy ion collisions



Voloshin, QM2009

Magnetic field at t=0



(A. Bzdak, V. Skokov (12))

Compact Stellar Objects: magnetars are estimated to have B ~10^{14}-10^{15} G at the surface. It could be much higher in the interior (Duncan and Thompson (92/93))

Features of strongly interacting matter under intense magnetic fields has been investigated in a variety of approaches.

For example [certainly incomplete list !]

- NJL and relatives (Klevansky, Lemmer (89); Klimenko et al. (92,..); Gusynin, Miransky, Shokovy (94/95); Ferrer, Incera et al (03..), Hiller, Osipov (07/08); Menezes et al (09);Fukushima, Ruggieri, Gatto (10) [PNJL]; …)

- χPT (Shushpanov, Smilga (97); Agasian, Shushpanov (00); Cohen, McGady, Werbos (07);….)

- Linear Sigma Model and MIT bag model: (Fraga, Mizher (08), Fraga, Palhares (12)…)

- Lattice QCD (D'Elia (10/11), Bali et al (11/12),…)

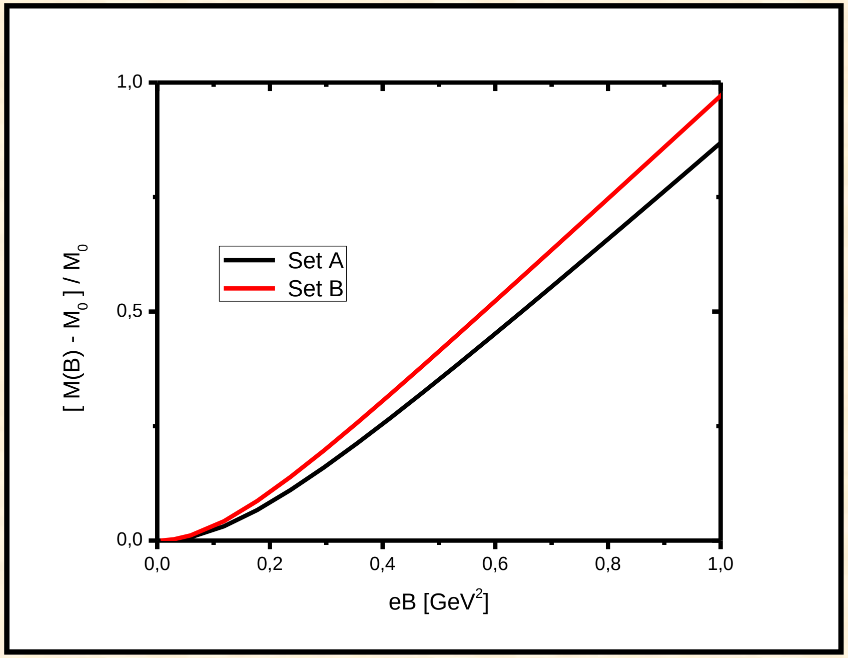
Recent reviews:

Kharzeev, Landsteiner, Schmitt,Yee, Lect. Notes Phys. 871, 1 (2013).

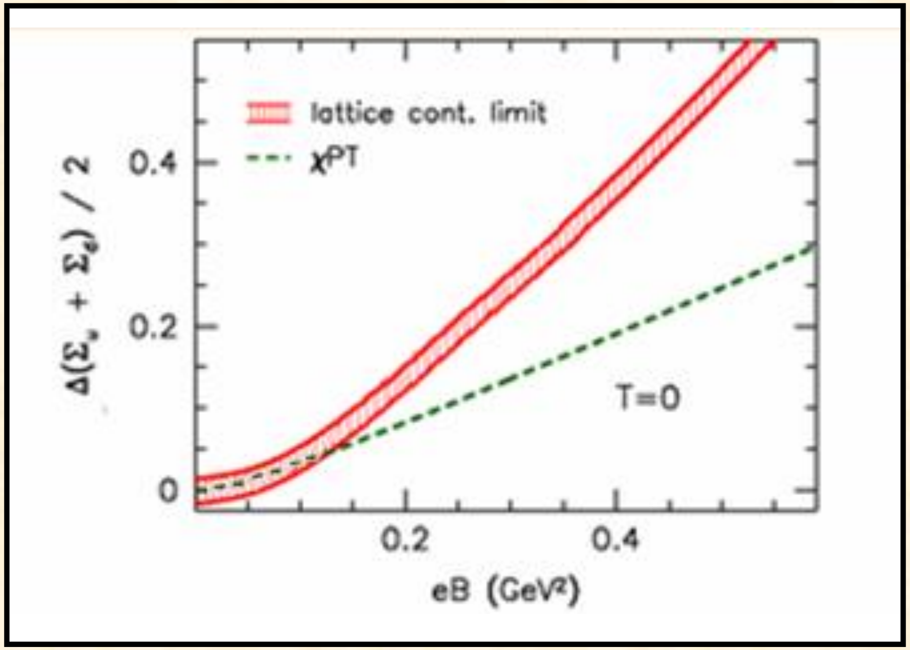
Miransky, Shovkovy, Phys. Rept. 576, 1 (2015).

Andersen, W. R. Naylor, A. Tranberg, Rev. Mod. Phys. 88, 025001 (2016).

Magnetic catalysis (μ=T=0)



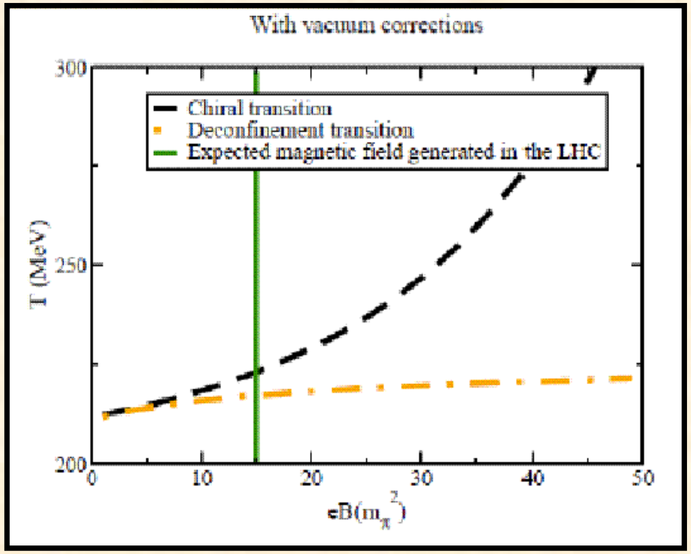
Typical NJL model result



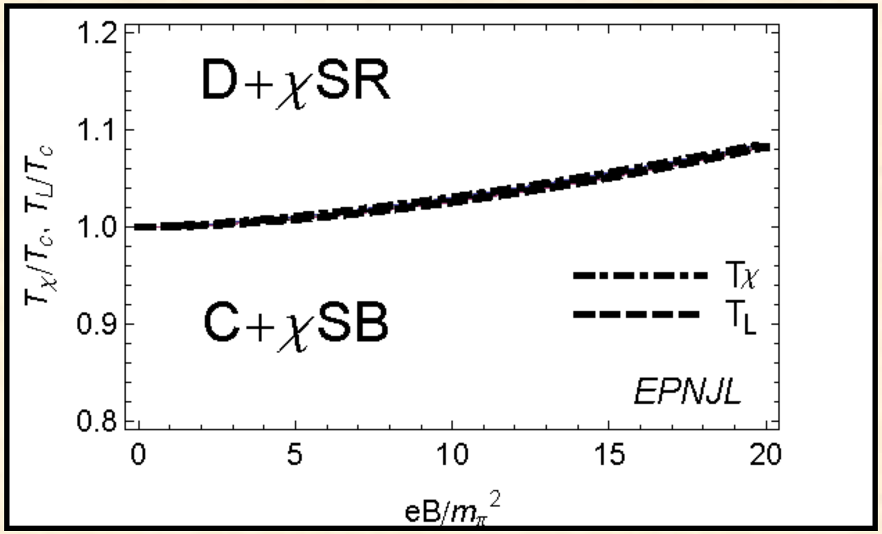
Lattice Bali et al (12)
χPT Cohen et al(07)

At T=0 there is an enhancement of the condensate with B: Magnetic catalysis
(Gusynin, Miransky, Shokovy (94/95))

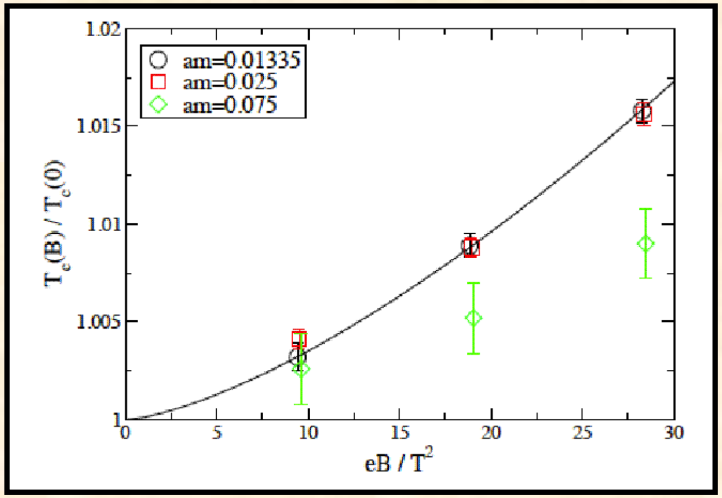
Critical temperatures for deconfinement and chiral transitions



LSM Mizhner, Fraga (10)



E-PNJL Gatto-Ruggieri (12)

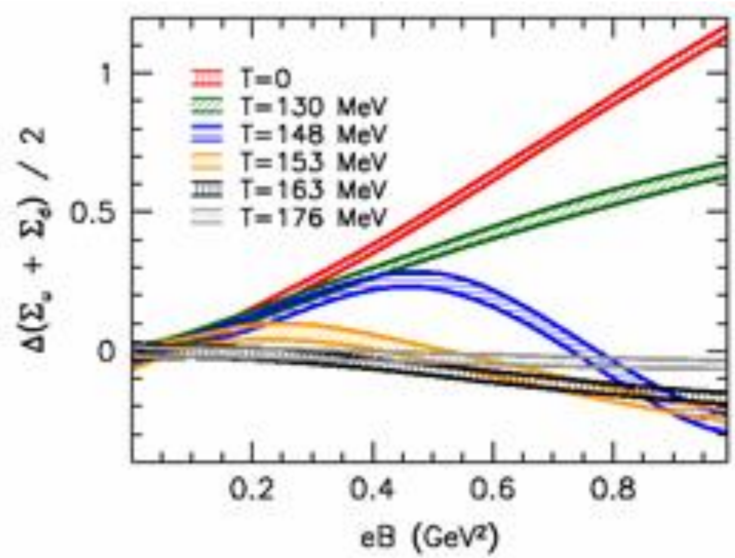


LQCD D'Elia et al (10)

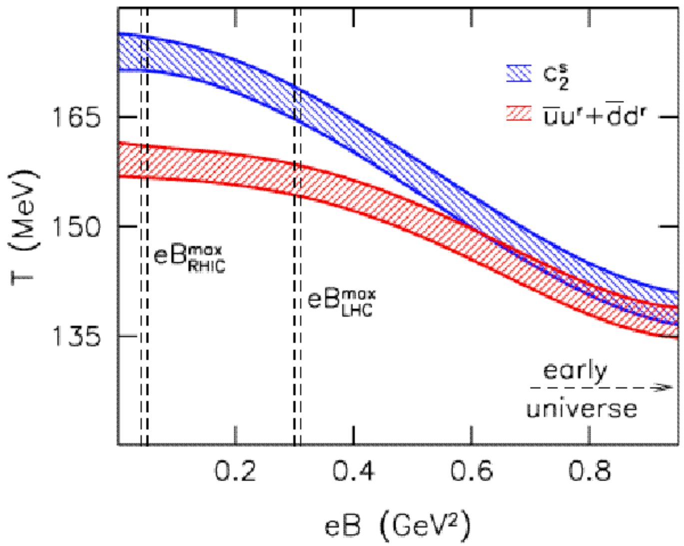
Most models and early LQCD results foresee an enhancement of critical temperatures for chiral transition with B

Lattice results with smaller lattice spacings (Bali et al (12))

Condensates as functions of B for various *T*



Critical temperatures as functions of B



At that time most models failed to predict these lattice results for the behavior of the condensates as functions of B for T close and above T_c

Many scenarios have been considered in the last few years to account for the Inverse Magnetic Catalysis (IMC). E.g. [certainly incomplete list !]

• T and B dependence on the NJL coupling constant (Ayala et al (14); Farias et al (14), Ferrer et al (15))

• B dependence of PL parameters in EPNJL models (M. Ferreira et al (14))

• Holography: (Rougemont, R. Critelli and J. Noronha (16))

• Effects beyond MFA (K. Fukushima and Y. Hidaka (13), S. Mao (16)…)

•Schwinger-Dyson methods (N. Mueller and J. M. Pawlowski (15), Braun, W. A. Mian and S. Rechenberger (16))

Yet, the physics behind IMC at finite T is not fully understood.

Non-local quark models

Compared to NJL, non-local quark models represent a step towards a more realistic modeling of the QCD interactions:

Nonlocal quark couplings present in the many approaches to low-energy q dynamics: i.e. instanton liquid model, Schwinger-Dyson resummation techniques, etc. Also in LQCD. Some advantages over the local NJL model:

- No need to introduce sharp momentum cut-offs
- Small next-to-leading order corrections
- Successful description of meson properties at T = μ= B= 0

Euclidean action for two flavors

$$\boxed{S_E = \int d^4x \left\{ \bar{\psi}(x)\left(-i\,\partial\!\!\!/ + m_c\right)\psi(x) - \frac{G}{2}\, j_a(x)\, j_a(x) \right\}}$$

Where

$$\boxed{j_a(x) = \int d^4z\, \mathcal{G}(z)\; \bar{\psi}(x+\frac{z}{2})\, \Gamma_a\, \psi(x-\frac{z}{2})}$$

$$\boxed{\Gamma_a = (1, i\gamma_5 \vec{\tau})}$$

$\mathcal{G}(z)$ nonlocal, well behaved covariant form factors

Since we are interested in studying the influence of a magnetic field, we introduce in the effective action a coupling to an external electromagnetic gauge field $\mathcal{A}_\mu$

For a local theory this can be done by performing the replacement

$$\boxed{\partial_\mu \to \partial_\mu - i\,\hat{Q}\,\mathcal{A}_\mu(x)}$$

where $\hat{Q}$ = diag(q_u, q_d), with q_u / 2 = - q_d = e / 3.

In the case of the nonlocal model the situation is more complicated since the inclusion of gauge interactions implies a change not only in the kinetic terms of the Lagrangian but also in the nonlocal currents. One has

$$\boxed{\psi(x - z/2) \to W\left(x, x - z/2\right)\psi(x - z/2)}$$

where

$$\boxed{W(s,t) = \mathrm{P}\exp\left[-i\hat{Q}\int_s^t dr_\mu\,\mathcal{A}_\mu(r)\right]}$$

r runs over an arbitrary path connecting s with t. We take a straight line path.

We bosonize the fermionic theory introducing scalar and pseudoscalar field and integrating out the fermion fields. The gauged bosonized action is

$$\boxed{S_{\mathrm{bos}} = -\ln\det\mathcal{D} + \frac{1}{2G}\int d^4x\left[\sigma(x)\sigma(x) + \vec{\pi}(x)\cdot\vec{\pi}(x)\right]}$$

where

$$\mathcal{D}\left(x+\frac{z}{2}, x-\frac{z}{2}\right) = \gamma_0\, W\left(x+\frac{z}{2}, x\right)\gamma_0 \left\{\delta^{(4)}(z)\left(-i\partial\!\!\!/ + m_c\right) + \mathcal{G}(z)\left[\sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x)\right]\right\} W\left(x, x-\frac{z}{2}\right)$$

For constant and homogenous magnetic field along the 3-axis in the Landau gauge we have $\mathcal{A}_\mu = B\, x_1\, \delta_{\mu 2}$. We work in MFA assuming that σ(x) has a nontrivial translational invariant MF value $\bar{\sigma}$, while $\bar{\pi}_i = 0$. Then,

$$\boxed{\mathcal{D}^{MFA}(x,x') = \delta^{(4)}(x-x')\left(-i\,\partial\!\!\!/ - \hat{Q}Bx_1\gamma_2 + m_c\right) + \bar{\sigma}\,\mathcal{G}(x-x')\exp\left[\frac{i}{2}\hat{Q}B(x_2 - x_{2'})(x_1 + x_{1'})\right]}$$

To deal with this operator we introduced its Ritus transform

$$\boxed{\mathcal{D}^{MFA}_{p,p'} = \int d^4x\, d^4x'\; \bar{\mathbb{E}}_p(x)\, \mathcal{D}^{MFA}(x,x')\; \mathbb{E}_{p'}(x')}$$

where $\mathbb{E}_p(x)$ are the usual Ritus matrices, with $p = (k, p_2, p_3, p_4)$.

After some calculation we find that is diagonal not only in flavor space but also in p-space. Thus, the corresponding "ln Det" can be readily determined. In this way we obtain

$$\boxed{\begin{aligned}\frac{S^{\rm MFA}_{\rm bos}}{V^{(4)}} = \frac{\bar\sigma^2}{2G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{d^2 p_\parallel}{(2\pi)^2} \Bigg\{ \ln\left[p_\parallel^2 + \left(M^{s_f,f}_{0,p_\parallel} \right)^2 \right] \\ + \sum_{k=1}^{\infty} \ln\left[\left(2k|q_f B| + p_\parallel^2 + M^{-,f}_{k,p_\parallel} M^{+,f}_{k,p_\parallel} \right)^2 + p_\parallel^2 \left(M^{+,f}_{k,p_\parallel} - M^{-,f}_{k,p_\parallel} \right)^2 \right] \Bigg\}\end{aligned}}$$

where $$\boxed{M^{\lambda,f}_{k,p_\parallel} = \frac{4\pi}{|q_f B|} (-1)^{k_\lambda} \int \frac{d^2 p_\perp}{(2\pi)^2} \exp\left[-p_\perp^2 / |q_f B| \right] \left(m_c + \bar\sigma\, g(p_\perp^2 + p_\parallel^2) \right) L_{k_\lambda}(2 p_\perp^2 / |q_f B|)}$$

Here, $p_\perp = (p_1, p_2)$, $p_\parallel = (p_3, p_4)$, $s_f = \mathrm{sign}(q_f B)$, $k_\lambda = k - \frac{1}{2} \pm \frac{s_f}{2}$

$L_n(x)$: Laguerre polynomials

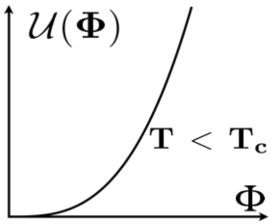
In the extension to finite temperature with consider the coupling of the quarks to the Polyakov loop (order parameter for deconfinement)

- Polyakov loop $\Phi(\vec{x}) = \frac{1}{N_c} Tr\left[i\int_0^{1/T} d\tau\, A_4(\vec{x},\tau) \right]$

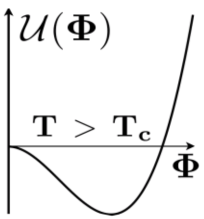
Polyakov, PLB (78)

pure gauge Z(3) symmetry

$\mathcal{U}(\boldsymbol{\Phi})$ Effective potential



confinement:
Z(3) symmetry
not broken
$\langle \boldsymbol{\Phi} \rangle = 0$



deconfinement:
Z(3) symmetry
spontaneously
broken
$\langle \boldsymbol{\Phi} \rangle \neq 0$

Fukushima (03), Megias,Ruiz Arriola, Salcedo (06), Ratti, Thaler, Weise (06),…

For the Polyakov Loop effective potential we take (Ratti, Thaler, Weise (06))

$$\boxed{\frac{\mathcal{U}(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{3}\Phi^3 + \frac{b_4}{4}\Phi^4 \;,}$$

where $b_2(T) = a_0 + a_1\, t + a_2\, t^2 + a_3\, t^3 \;\; ; \quad t = T_0 / T; \quad \Phi = \left[1 + 2\cos(\phi_3 / T)\right]/2$

and a_i, b_i chosen to fit Quenched LQCD results. Due finite quark mass effects we consider T_0= 210 MeV (Schaefer, Pawlowski, Wambach (07))

To obtain the thermodynamical potential Ω_{MFA} at finite temperature we use the Matsubara formalism in the quark sector

$$N_c \int \frac{d^2 p_{\|}}{(2\pi)^2} F(p_{\|}^2) \rightarrow T \sum_{c=r,g,b} \sum_{n=-\infty}^{\infty} \int \frac{dp_3}{2\pi} F(p_{\|nc}^2)$$

$$p_{\|nc} = \left(p_3, (2n+1)\pi T + \phi_c \right)$$

$$\phi_r = -\phi_g = \phi_3 \, ; \, \phi_b = 0$$

and include the contribution of the Polyakov loop potencial

Given Ω_{MFA} we obtain the gap equations as

$$\partial\Omega_{\text{MFA}} / \partial\bar{\sigma} = 0 \quad ; \quad \partial\Omega_{\text{MFA}} / \partial\Phi = 0$$

To be solved numerically

and the quark condensates $< \bar{q}_f q_f >_{B,T} = \partial\Omega_{\text{MFA}} / \partial m_c^f$

To compare with LQCD calculations of Bali et al we also define

$$\Sigma_{B,T}^f = \frac{2m_c}{S^4} \left[\langle \bar{q}_f q_f \rangle_{B,T} - \langle \bar{q}_f q_f \rangle_{0,0} \right] + 1 \qquad S = (135 \times 86)^{1/2} \, \text{MeV}$$

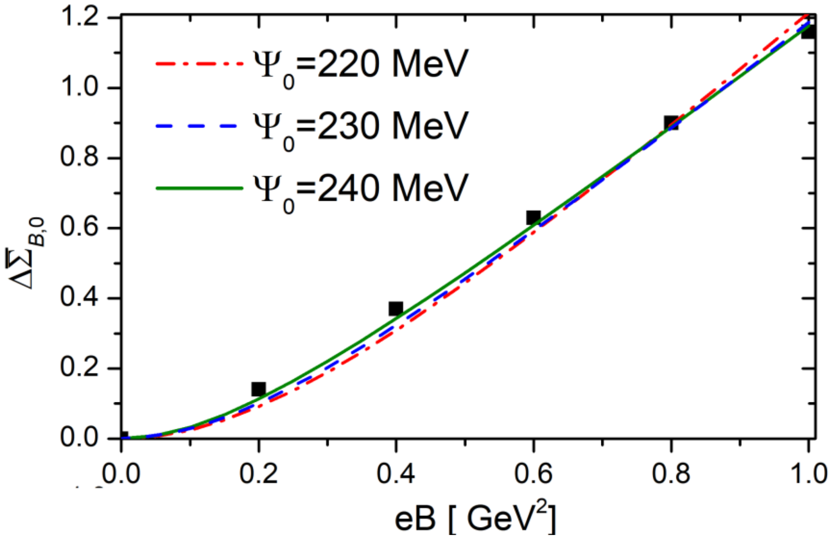
and

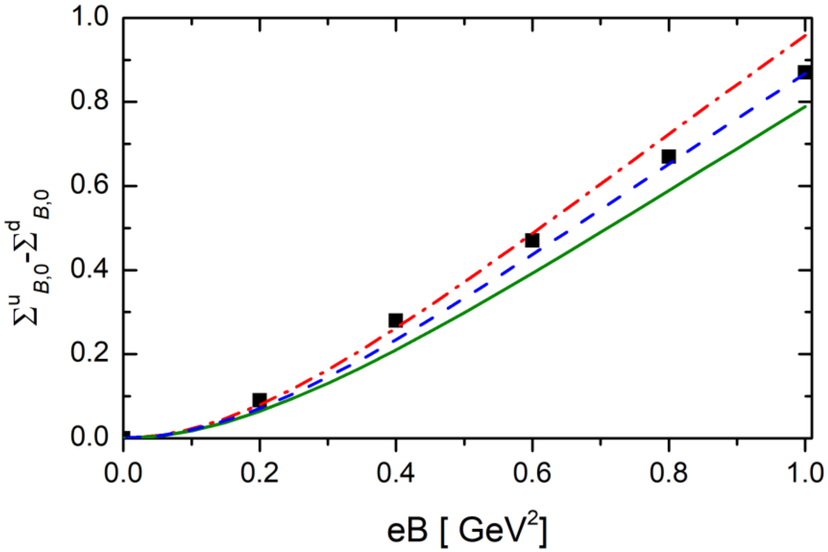
$$\Delta\Sigma_{B,T}^f = \Sigma_{B,T}^f - \Sigma_{0,T}^f$$ and $$\Delta\bar{\Sigma}_{B,T} = (\Delta\Sigma_{B,T}^u + \Delta\Sigma_{B,T}^d) / 2.$$

Results

In our numerical calculations we use Gaussian (GFF) and 5-Lorenztian (5LFF) form factor and fix model parameters by fitting the vacuum empirical values of m_π and f_π, and a given value of $\Psi_0 = \left(-<\bar{q}_f q_f>_{0,0}\right)^{1/3}$

Behavior of the T=0 condensates as functions of B for GFF and several parameterizations compared with LQCD of Bali et al. (black squares)

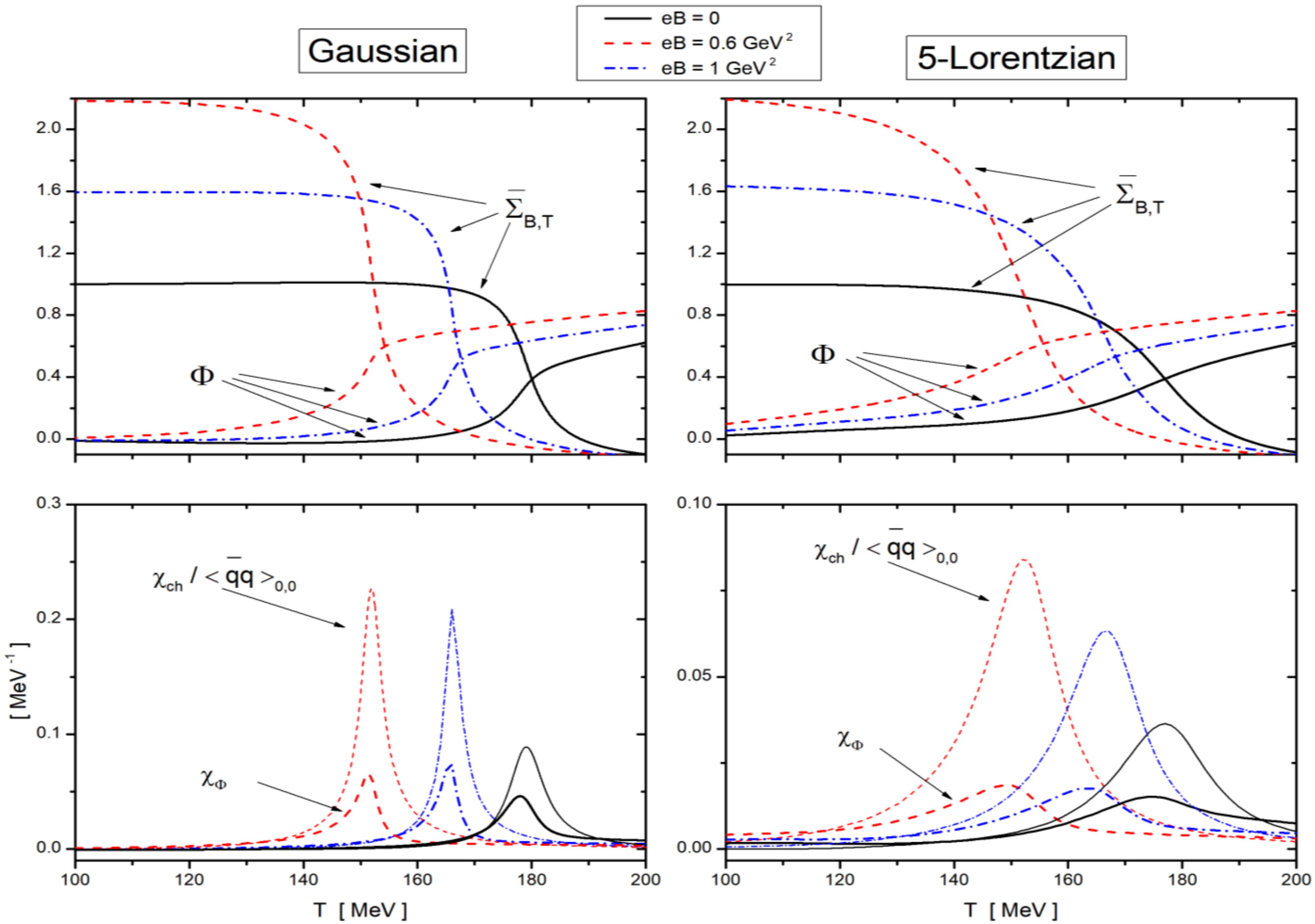




Good agreement with LQCD results

Very similar results for 5LFF

Behavior of the condensates vs temperature for given values of eB



Results for
Ψ_0=230 MeV

T_c 's decreases as eB increases

For all eB both transitions occur at aprox. the same T_c

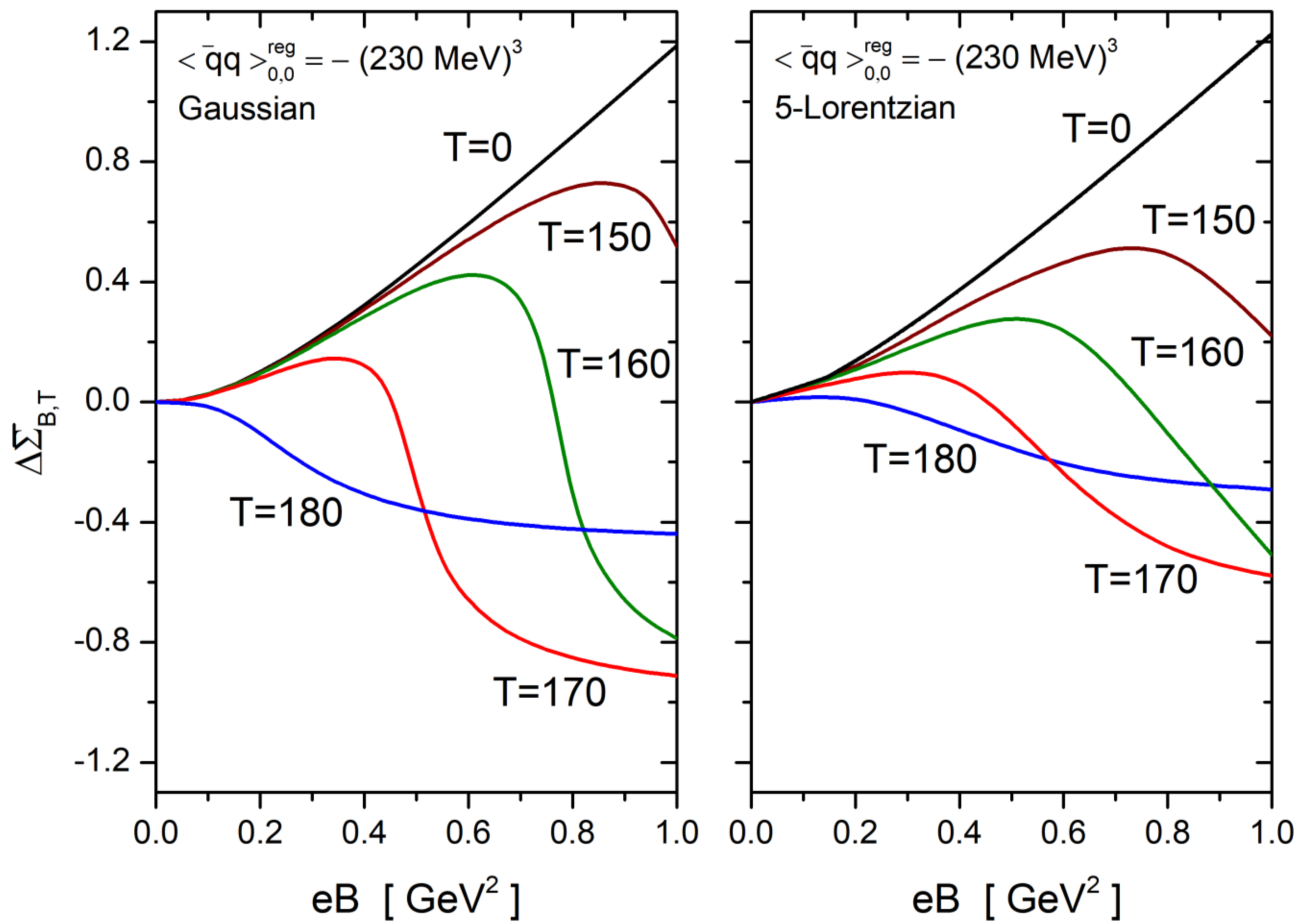
$$\chi_{ch} = d(-\langle\overline{\bar{q}_f q_f}\rangle^{reg}_{B,T})/dT$$

$$\chi_\Phi = d\Phi/dT$$

T_c for eB=0		Gaussian			5-Lorenztian		
$\Psi_0 = -<\bar{q}q>_{0,0}^{1/3}$	MeV	220	230	240	220	230	240
Chiral T_c	MeV	182	179	177	177	177	178
Deconf T_c	MeV	182	178	176	175	175	176

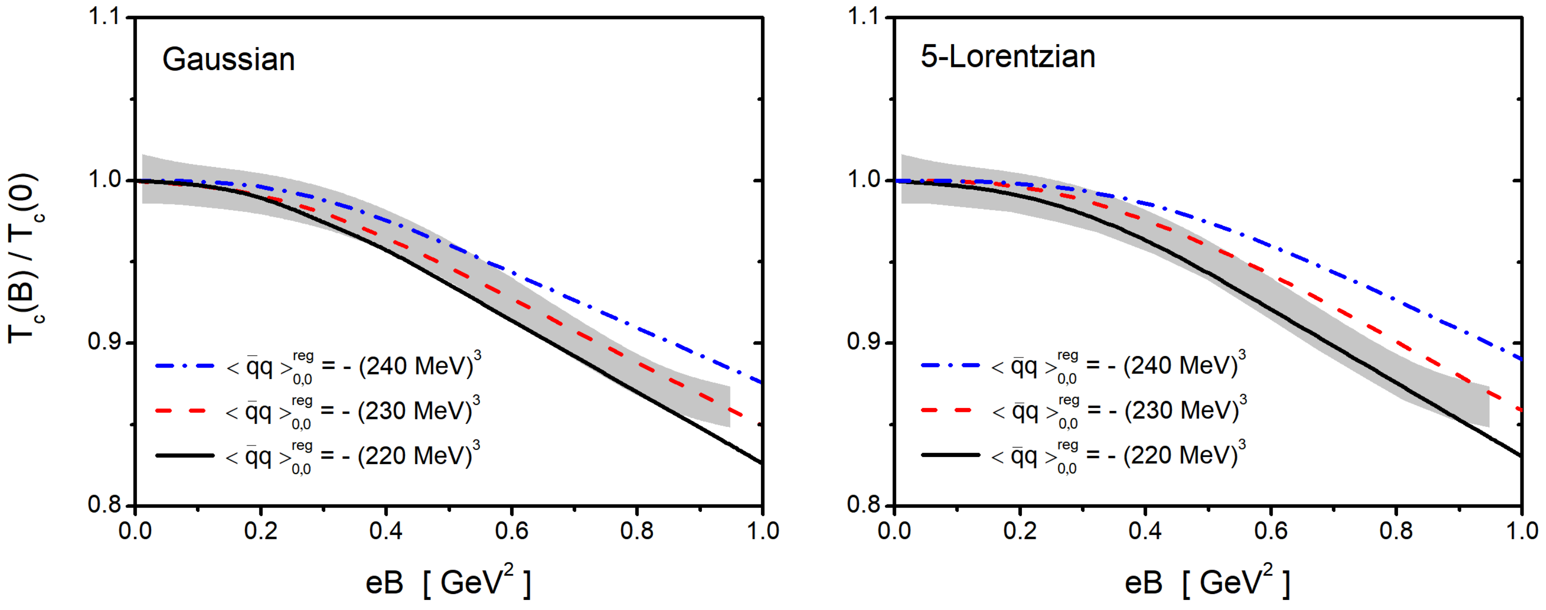
LQCD
T_c ~160-170 MeV

Behavior of the condensates vs eB for given values of temperature



At T=0 magnetic catalysis. Close to T_c non-monotonic behavior

Behavior of the critical temperature for chiral restoration and deconfinement as a function of B for various model parameterizations as compared to LQCD results (grey band)



Good agreement with LQCD results. No need for extra ad-hoc parameters

Summary and Outlook

- We have studied the behavior of quark matter under strong magnetic fields in the framework on non local Polyakov quiral quark models.

- We have that at T=0 the quark condensates increase with the magnetic field in agreement with the expected «Magnetic catalysis» phenomenon. Our results are in good quantitative agreement with the LQCD ones.

- For T's close to those for chiral restoration our results for the quark condensates exhibit a non-monotonic behavior as functions of B, which results in a decrease of the transition temperature when the magnetic field is increased. Namely, non-local models naturally lead to IMC.

- The model predicts the "entaglement" of the chiral and deconfinement transitions in a natural way.

- Future work: use of form factors extracted from LQCD, extension to finite density, study of meson properties at finite B and T, etc