# Automated calculations for MPI

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Former Convent of San Agustin, San Cristóbal de las Casas, Chiapas, Mexico

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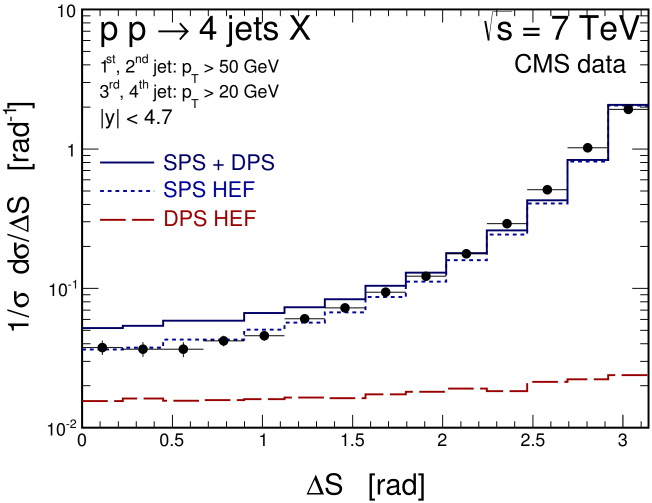
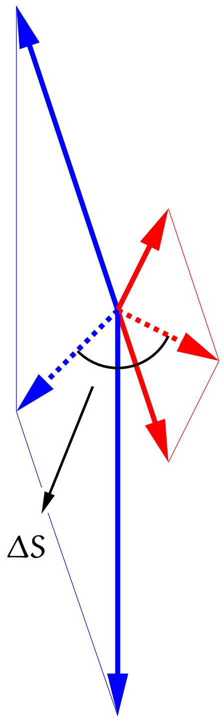
# Outline

- Factorized cross section calculation
- Off-shell amplitudes
- KaTie: for parton-level event generation with $k_T$-dependent initial states

# Four jets with $k_T$-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016




- $\Delta S$ is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- $k_T$-factorization allows for the necessary momentum inbalance.

# Why $k_T$-factorization?

- $\Delta S$ is an example of an observable whos distribution is not calculable at LO in collinear factorization.
- this happens for any angular observable that separates the final-state momenta into $2$ groups.
- $2 \to 2$ processes in particular often need higher order corrections.
- changing the kinematics seems to go beyond what one would expect from *perturbative* corrections.
- $k_T$-factorization provides already at LO a momentum inbalance to the final state.
- $k_T$-dependent pdfs (TMDs,updfs) can provide resummation corrections.

## Disadvantages

- few actual factorization theorems exist.
- required off-shell matrix elements more complicated to calculate.

# Factorization for hadron scattering

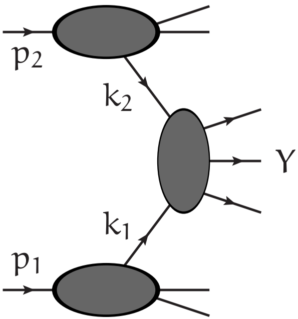
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x}\, f_{i,a}(x,\mu)\, \delta^4(k - x\,p_i)$

$k_T$-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x}\, \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\,p_i - k_T)$

- The *parton level* cross section $d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$ can be calculated within perturbative QCD.
- The *parton distribution functions* $f_{i,a}$ and $\mathcal{F}_{i,a}$ must be modelled and fit against data.
- Unphysical scale $\mu$ is a price to pay, but its dependence is calculable within perturbative QCD via *evolution equations*.



# Factorization for hadron scattering

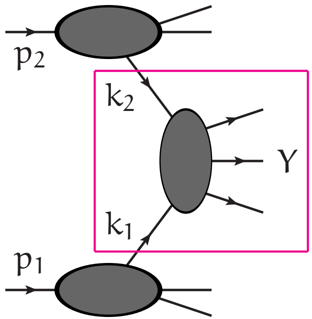
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x} f_{i,a}(x,\mu)\, \delta^4(k - x\, p_i)$

$k_T$-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x} \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\, p_i - k_T)$

$$\hat{\sigma} = \int d\Phi(1,2 \to 3,4,\ldots,n)\, \big|\mathcal{M}(1,2,\ldots,n)\big|^2\, \mathcal{O}(p_3,p_4,\ldots,p_n)$$

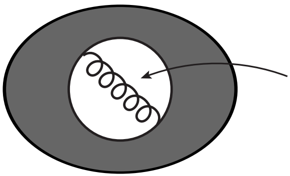



phase space includes summation over color and spin
squared amplitude calculated perturbatively
observable includes phase space cuts, or jet algorithm

# Gauge invariance

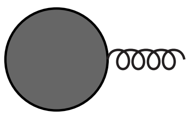
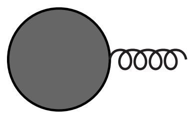
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:



$$\left\{ \begin{array}{l} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2 + \xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{array} \right.$$

Ward identity:

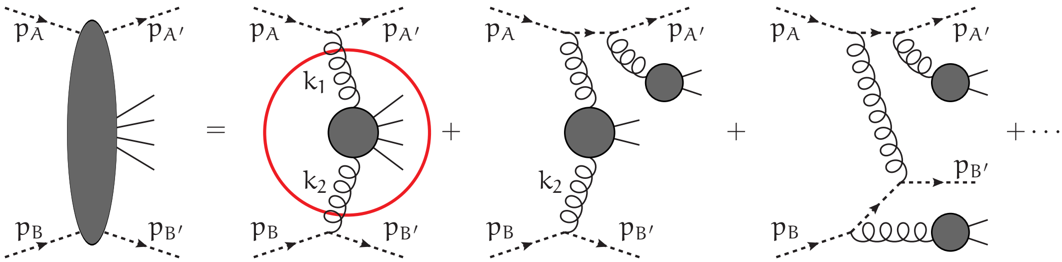
 $_\mu \varepsilon^\mu(k) \quad \rightarrow \quad$  $_\mu k^\mu \; = \; 0$

- Only holds if all external particles are on-shell.
- $k_T$-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- How to define amplitudes with off-shell intial-state momenta?

# Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:

Embed the process in an on-shell process with auxiliary partons




$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$

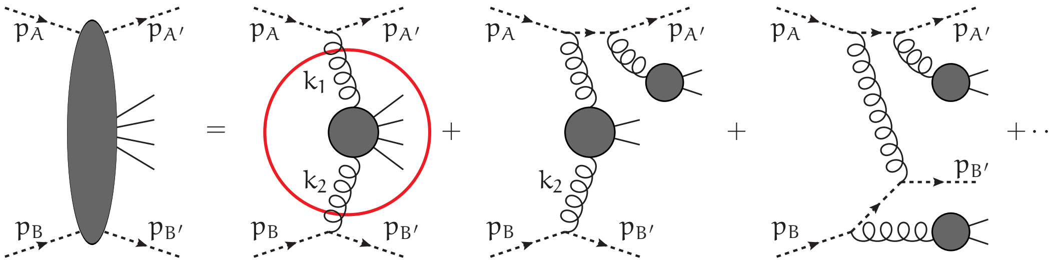
$$p_{A'}^\mu = -(\Lambda - x) p^\mu - \alpha q^\mu + (1-\beta) k_T^\mu$$

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda (p+q)^2} \quad , \quad \beta = \frac{1}{1 + \sqrt{1 - x/\Lambda}}$$

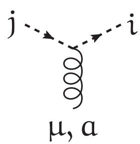
$$p_A^2 = p_{A'}^2 = 0 \quad , \quad p_A^\mu + p_{A'}^\mu = x p^\mu + k_T^\mu$$

# Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.




$$\left.\begin{array}{c} p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \\ p_{A'}^\mu = -(\Lambda - x)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu \\ \Lambda \to \infty \end{array}\right\}$$



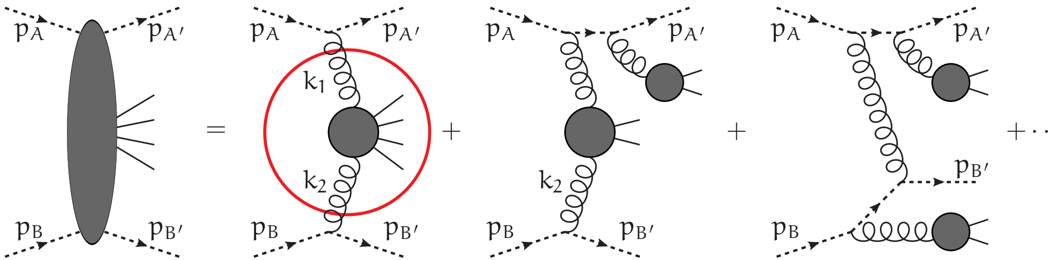

$= -i\, T^a_{i,j}\, p_1^\mu$

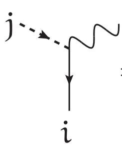
$$j \xrightarrow{K} i = \delta_{i,j}\frac{i}{p_1 \cdot K}$$

# Amplitudes with off-shell partons

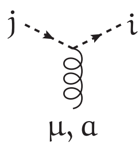
AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

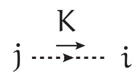


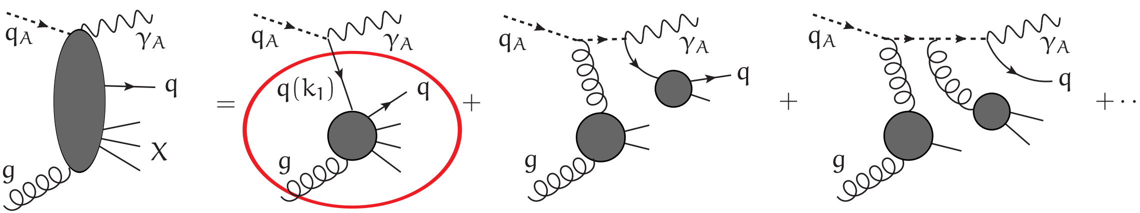

$= -i\,\delta_{i,j}\,u(p_1)$




$= -i\,T^a_{i,j}\,p_1^\mu$




$= \delta_{i,j}\,\frac{i}{p_1\cdot K}$



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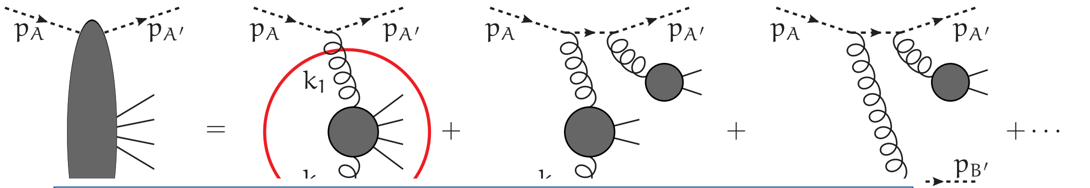
AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.

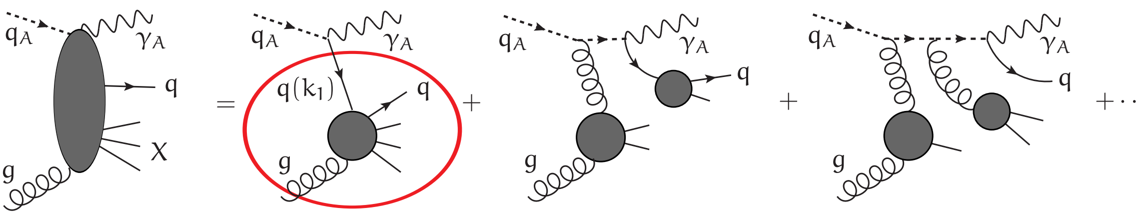



$p_B$

j

In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014

$i \quad = \delta_{i,j}\,\frac{i}{p_1\cdot K}$

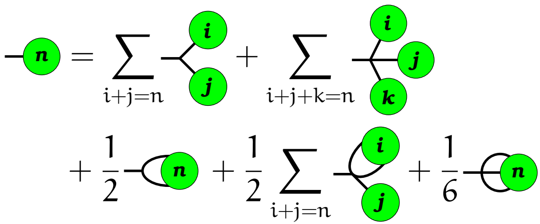


# Tree-level amplitudes with off-shell recursion

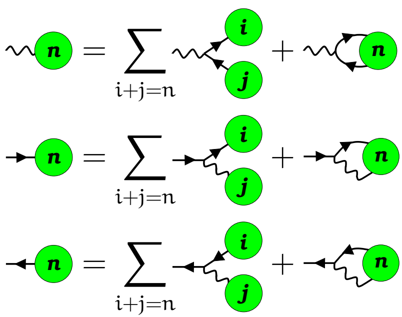
Off-shell currents, or Green functions with all external particles on-shell, satisfy the recursive

*Dyson-Schwinger equations*

Theories with four-point vertices:




Theories with more types of currents:




- Sums are over partitions of on-shell particles over the blobs, and over possible flavors for virtual particles.
- Current with $n = \#\text{externalparticles} - 1$ is completely on-shell and gives the amplitude.
- Solution can be represented as a sum of Feynman graphs,
- but recursion can also be used to construct amplitude directly.
- ideal for efficient and automated numerical evaluation of tree-level amplitudes
- used in Alpgen, Helac, O'mega, Comix, ...

# KaTie https://bitbucket.org/hameren/katie

- parton level event generator, like Alpgen, Helac, MadGraph, etc.
- arbitrary processes within the standard model (including effective $Hg$) with several final-state particles.
- $0$, $1$, or $2$ off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids.
- a calculation is steered by a single input file.
- employs an optimization phase in which the pre-samplers for all channels are optimized.
- during the generation phase several event files can be created in parallel.
- can generate (naively factorized) MPI events.
- event files can be processed further by parton-shower program like CASCADE (talk by Mirko Serino).

```
Ngroup = 1
Nfinst = 4
process = g g  -> g g  g g   factor = 1          groups = 1  pNonQCD = 0 0 0
process = g g  -> g g  q q~  factor = Nf         groups = 1  pNonQCD = 0 0 0
process = g g  -> q q~ q q~  factor = Nf         groups = 1  pNonQCD = 0 0 0
process = g g  -> q q~ r r~  factor = Nf*(Nf-1)  groups = 1  pNonQCD = 0 0 0
process = g q  -> g g  g q   factor = 1          groups = 1  pNonQCD = 0 0 0
process = g q  -> g q  q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
process = g q  -> g q  r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q g  -> g g  g q   factor = 1          groups = 1  pNonQCD = 0 0 0
process = q g  -> g q  q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
process = q g  -> g q  r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q q~ -> g g  g g   factor = 1          groups = 1  pNonQCD = 0 0 0
process = q q~ -> g g  q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
process = q q~ -> g g  r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q q~ -> q q~ q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
process = q q~ -> q q~ r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q q~ -> r r~ r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q q  -> g g  q q   factor = 1          groups = 1  pNonQCD = 0 0 0
process = q q  -> q q  q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
process = q q  -> q q  r r~  factor = Nf-1       groups = 1  pNonQCD = 0 0 0
process = q r  -> g g  q r   factor = 1          groups = 1  pNonQCD = 0 0 0
process = q r  -> q r  q q~  factor = 1          groups = 1  pNonQCD = 0 0 0
lhaSet = MSTW2008nlo68cl
offshell = 1 1
tmdTableDir = /home/user0/kTfac/tables/krzysztof02/
tmdpdf = g  KMR_gluon.dat
tmdpdf = u  KMR_u.dat
tmdpdf = u~ KMR_ubar.dat
tmdpdf = d  KMR_d.dat
tmdpdf = d~ KMR_dbar.dat
tmdpdf = s  KMR_s.dat
tmdpdf = s~ KMR_sbar.dat
tmdpdf = c  KMR_c.dat
tmdpdf = c~ KMR_cbar.dat
tmdpdf = b  KMR_b.dat
tmdpdf = b~ KMR_bbar.dat
```

$pp \to 4j$ SPS

```
Nflavors = 5
helicity = sampling
Noptim = 100,000
Ecm = 7000
Esoft = 20
cut = {deltaR|1,2|} > 0.5
cut = {deltaR|1,3|} > 0.5
cut = {deltaR|1,4|} > 0.5
cut = {deltaR|2,3|} > 0.5
cut = {deltaR|2,4|} > 0.5
cut = {deltaR|3,4|} > 0.5
cut = {pT|1|1,2,3,4} > 50
cut = {pT|2|1,2,3,4} > 50
cut = {pT|3|1,2,3,4} > 20
cut = {pT|4|1,2,3,4} > 20
cut = {rapidity|1|} > -4.7
cut = {rapidity|2|} > -4.7
cut = {rapidity|3|} > -4.7
cut = {rapidity|4|} > -4.7
cut = {rapidity|1|} <  4.7
cut = {rapidity|2|} <  4.7
cut = {rapidity|3|} <  4.7
cut = {rapidity|4|} <  4.7
scale = ({pT|1|}+{pT|2|}+{pT|3|}+{pT|4|})/2
mass = Z   91.1882  2.4952
mass = W   80.419   2.21
mass = H  125.0     0.00429
mass = t  173.5
switch = withQCD   Yes
switch = withQED   No
switch = withWeak  No
switch = withHiggs No
switch = withHG    No
coupling = Gfermi 1.16639d-5
```

$pp \longrightarrow 4j$ SPS

```
Ngroup = 2
Nfinst = 2 2
process = g g  -> g g   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = g g  -> q q~  factor = Nf    groups = 1 2  pNonQCD = 0 0 0
process = g q  -> q g   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q g  -> q g   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q q  -> q q   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q r  -> q r   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q q~ -> g g   factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q q~ -> q q~  factor = 1     groups = 1 2  pNonQCD = 0 0 0
process = q q~ -> r r~  factor = Nf-1  groups = 1 2  pNonQCD = 0 0 0
lhaSet = MSTW2008nlo68cl
offshell = 1 1  # eg.  g* g* -> ...
tmdTableDir = /home/user0/kTfac/tables/krzysztof02/
tmdpdf = g  KMR_gluon.dat
tmdpdf = u  KMR_u.dat
tmdpdf = u~ KMR_ubar.dat
tmdpdf = d  KMR_d.dat
tmdpdf = d~ KMR_dbar.dat
tmdpdf = s  KMR_s.dat
tmdpdf = s~ KMR_sbar.dat
tmdpdf = c  KMR_c.dat
tmdpdf = c~ KMR_cbar.dat
tmdpdf = b  KMR_b.dat
tmdpdf = b~ KMR_bbar.dat
sigma_eff = 15d6
Nflavors = 5
helicity = sum
Noptim = 100,000
Ecm = 7000
Esoft = 10
```

$pp \to 4j$ DPS

```
cut = {deltaR|1,2|} > 0.4
cut = {deltaR|1,3|} > 0.4
cut = {deltaR|1,4|} > 0.4
cut = {deltaR|2,3|} > 0.4
cut = {deltaR|2,4|} > 0.4
cut = {deltaR|3,4|} > 0.4
cut = {pT|1|1,2,3,4} > 40
cut = {pT|2|1,2,3,4} > 30
cut = {pT|3|1,2,3,4} > 20
cut = {pT|4|1,2,3,4} > 10
cut = {rapidity|1|} > -2.1
cut = {rapidity|2|} > -2.1
cut = {rapidity|3|} > -2.1
cut = {rapidity|4|} > -2.1
cut = {rapidity|1|} <  2.1
cut = {rapidity|2|} <  2.1
cut = {rapidity|3|} <  2.1
cut = {rapidity|4|} <  2.1
scale = entry 1 ({pT|1|}+{pT|2|})/2
scale = entry 2 ({pT|3|}+{pT|4|})/2
cut = group 1 {deltaR|1,2|} > 0.4
cut = group 1 {pT|1|} > 10
cut = group 1 {pT|2|} > 10
cut = group 1 {rapidity|1|} > -2.1
cut = group 1 {rapidity|2|} > -2.1
cut = group 1 {rapidity|1|} <  2.1
cut = group 1 {rapidity|2|} <  2.1
scale = group 1 ({pT|1|}+{pT|2|})/2
cut = group 2 {deltaR|1,2|} > 0.4
cut = group 2 {pT|1|} > 10
cut = group 2 {pT|2|} > 10
cut = group 2 {rapidity|1|} > -2.1
cut = group 2 {rapidity|2|} > -2.1
cut = group 2 {rapidity|1|} <  2.1
cut = group 2 {rapidity|2|} <  2.1
scale = group 2 ({pT|1|}+{pT|2|})/2
```

```
mass = Z   91.1882  2.4952
mass = W   80.419   2.21
mass = H  125.0     0.00429
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switch = withQCD   Yes
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```

# $pp \to 4j$ DPS

# Conclusions

- $k_T$-factorization allows for the parton-level description of kinematical situations inaccessible with LO collinear factorization, eg. $\Delta S$ for four jets.
- Factorization prescriptions with explicit $k_T$ dependence in the pdfs ask for hard matrix elements with off-shell initial-state partons.
- The necessary amplitudes can be defined in a manifestly gauge invariang manner that allows for *e.g.* Dyson-Schwinger recursion, both for off-shell gluons and off-shell quarks.
- KaTie generates parton-level events with $k_T$-dependent initial states, both for SPS and DPS.

# Backup

# Amplitudes with off-shell gluons

$n$-parton amplitude is a function of $n$ momenta $k_1, k_2, \ldots, k_n$
and $n$ *directions* $p_1, p_2, \ldots, p_n$

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and $n$ *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$

$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

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$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

With the help of an auxiliary four-vector $q^\mu$ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

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$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

With the help of an auxiliary four-vector $q^\mu$ with $q^2 = 0$, we define

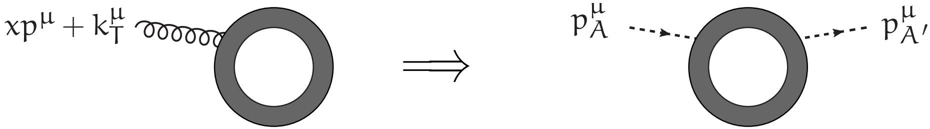
$$k_T^\mu(q) = k^\mu - x(q)p^\mu \quad \text{with} \quad x(q) \equiv \frac{q\cdot k}{q\cdot p}$$

Construct $k_T^\mu$ explicitly in terms of $p^\mu$ and $q^\mu$:

$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\[2ex] \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of $q^\mu$, but also individually $\kappa$ and $\kappa^*$ are independent of $q^\mu$.

# Off-shell one-loop amplitudes




$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \quad , \quad p_{A'}^\mu = -(\Lambda - x)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu \ ,$$

where $p, q$ are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \ .$$

With this choice, the momenta $p_A, p_{A'}$ satisfy the relations

$$p_A^2 = p_{A'}^2 = 0 \quad , \quad p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu$$

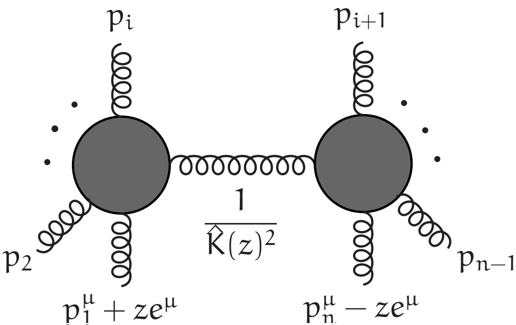
for any value of the parameter $\Lambda$. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$.

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}\big(\Lambda^{-1}\big)$$

Taking this limit after loop integration will lead to singularities $\log\Lambda$.

# BCFW recursion for on-shell amplitudes

Gives compact expression through recursion of *on-shell* amplitudes.




$$\hat{K}^\mu(z) = p_1^\mu + \cdots + p_i^\mu + ze^\mu$$
$$= -p_{i+1}^\mu - \cdots - p_n^\mu + ze^\mu$$

$$e^\mu = \tfrac{1}{2}\langle p_1|\gamma^\mu|p_n]$$

$$\hat{K}(z)^2 = 0 \quad \Leftrightarrow \quad z = -\frac{(p_1 + \cdots + p_i)^2}{2(p_2 + \cdots + p_i)\cdot e}$$

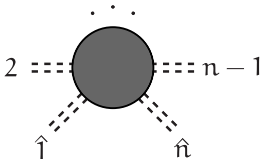
$$\mathcal{A}(1^+, 2, \ldots, n-1, n^-) = \sum_{i=2}^{n-1} \sum_{h=+,-} \mathcal{A}(\hat{1}^+, 2, \ldots, i, -\hat{K}_{1,i}^h)\, \frac{1}{K_{1,i}^2}\, \mathcal{A}(\hat{K}_{1,i}^{-h}, i+1, \ldots, n-1, \hat{n}^-)$$

$$\mathcal{A}(1^+, 2^-, 3^-) = \frac{\langle 23\rangle^3}{\langle 31\rangle\langle 12\rangle} \quad , \quad \mathcal{A}(1^-, 2^+, 3^+) = \frac{[32]^3}{[21][13]}$$

# BCFW recursion for off-shell amplitudes

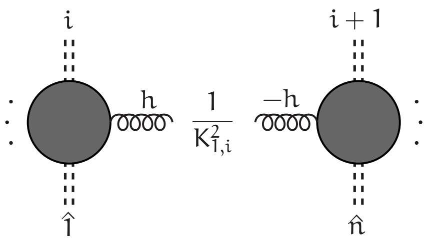
AvH 2014

The BCFW recursion formula becomes

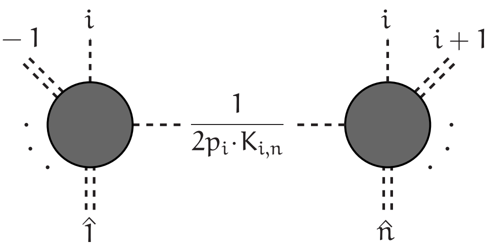



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

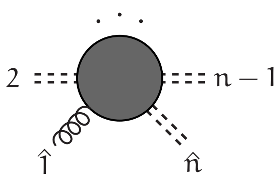
$A_{i,h} =$



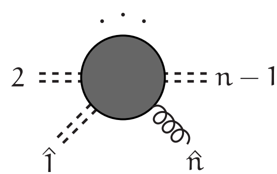

$B_i =$




$C = \frac{1}{\kappa_1}$




$D = \frac{1}{\kappa_1^*}$

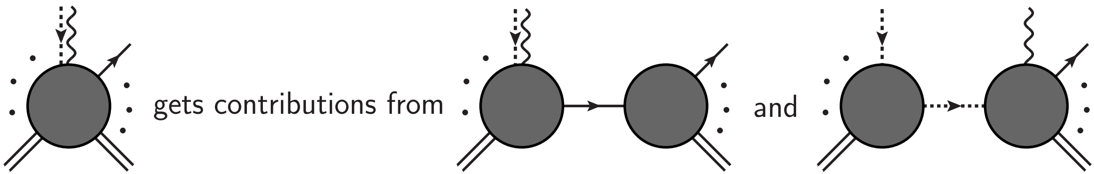



The hatted numbers label the shifted external gluons.

# BCFW recursion with (off-shell) quarks



- on-shell case treated in Luo, Wen 2005
- any off-shell parton can be shifted: propagators of "external" off-shell partons give the correct power of $z$ in order to vanish at infinity
- different kinds of contributions in the recursion




- many of the MHV amplitudes come out as expected
- some more-than-MHV amplitudes do not vanish, but are sub-leading in $k_T$

$$\mathcal{A}(1^+,2^+,\ldots,n^+,\bar{q}^*,q^-) = \frac{-\langle\bar{q}q\rangle^3}{\langle 12\rangle\langle 23\rangle\cdots\langle n\bar{q}\rangle\langle\bar{q}q\rangle\langle q1\rangle}$$

- off-shell quarks have helicity

$$\mathcal{A}(1,2,\ldots,n,\bar{q}^{*(+)},q^{*(-)}) \neq \mathcal{A}(1,2,\ldots,n,\bar{q}^{*(-)},q^{*(+)})$$