# Parton correlations effects in double parton distribution functions


**Matteo Rinaldi**[1]

In collaboration with :

**Federico Alberto Ceccopieri**[2], **Sergio Scopetta**[2], **Marco Traini**[3], **Vicente Vento**[1]

[1]Dep. of Theor. Physics, Valencia University and IFIC, Valencia, Spain

[2]Dep. of Physics and Geology, Perugia University and INFN, Perugia, Italy

[3]Dep. of Physics Trento University and INFN-TIFPA, Italy


IFIC INSTITUT DE FÍSICA CORPUSCULAR

CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

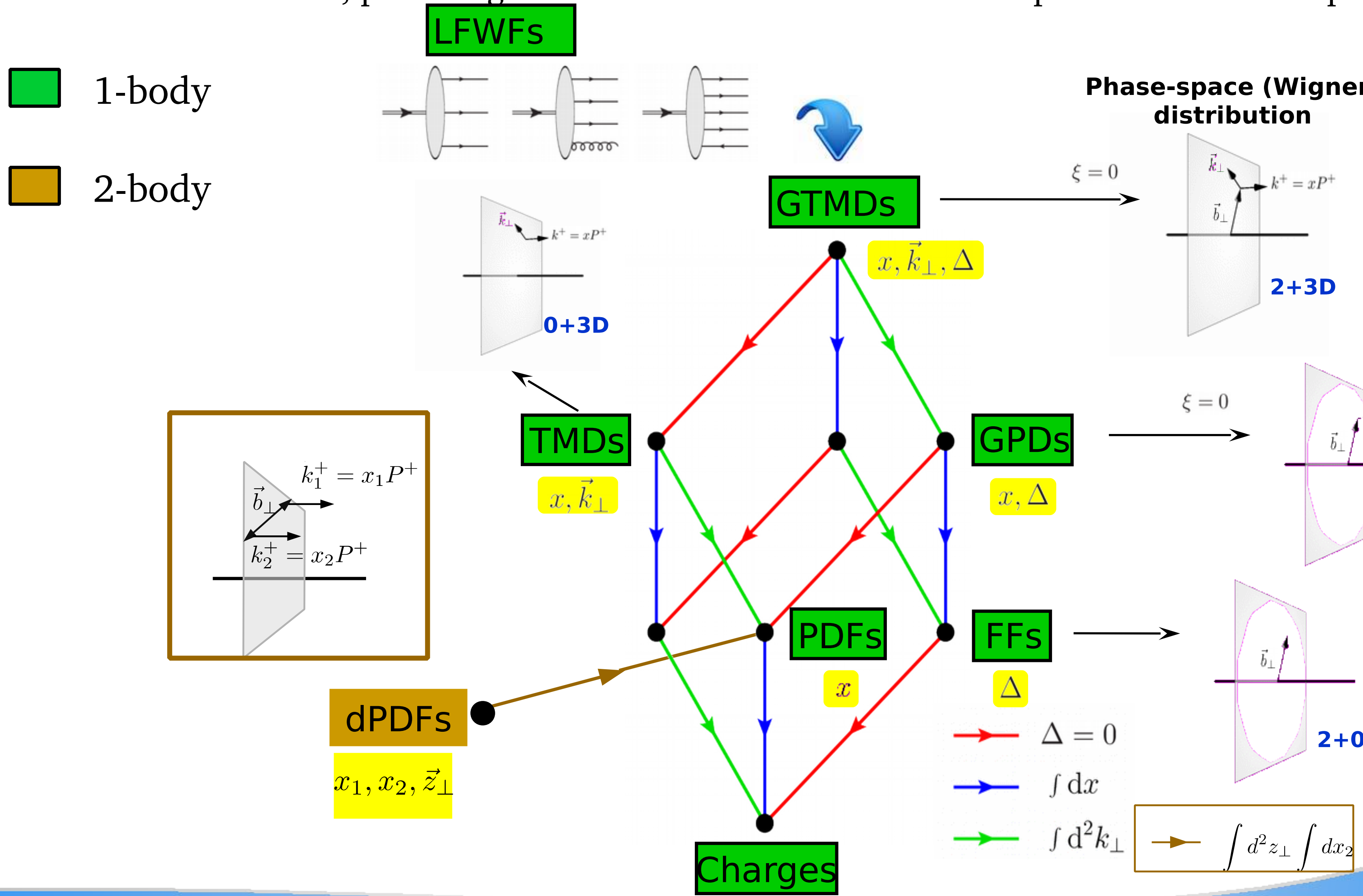
VNIVERSITAT DE VALÈNCIA

# Outlook

- Introduction
- The 3D proton structure in single & double parton scatterings (DPS)
- Double parton scattering and **double parton distribution functions** (dPDFs)
- Double parton correlations (DPCs) in double parton distribution functions
- dPDFs in constituent quark models and first proton "imaging" from DPS
  **M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013)**
  **M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)**
  **M. R., F. A. Ceccopieri, arXiv: 1611.04793, submitted**
- Calculation of the "effective X-section"
  **M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)**
  **M. Traini, S. Scopetta, M. R. , arXiv:1609.07242, submitted**
- Analyses of perturbative e non perturbative correlations
  **M. R., S. Scopetta, M. Traini and V.Vento, JHEP 10, 063 (2016)**
- Conclusions

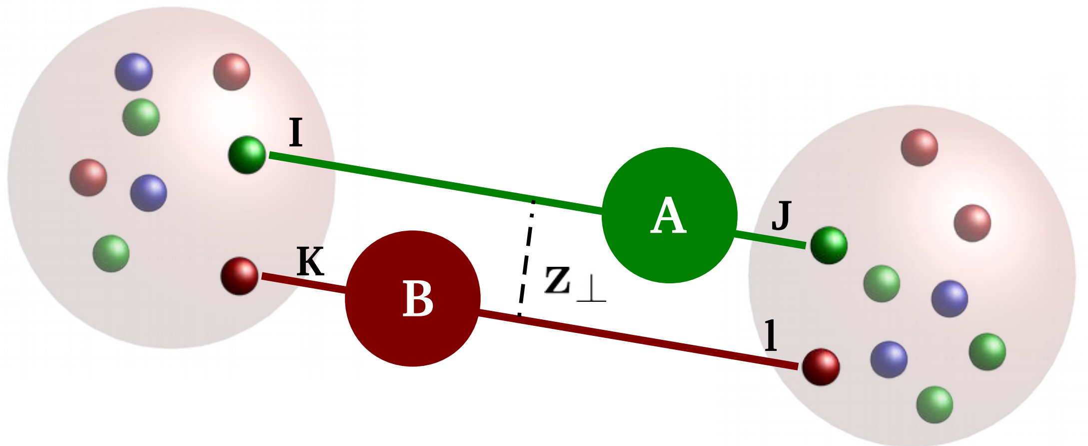
# How 3-Dimensional structure of a hadron can be investigated?

The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS, double parton sattering ...), measuring different kind of parton distributions, providing different kind of information. The parton distribution puzzle is:



# DPS and dPDFs from multi parton interactions

Multi parton interaction (MPI) can contribute to the, *pp* and *pA,* cross section @ the LHC:




The cross section for a DPS event can be written in the following way:
**(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))**

dPDF

Momentum scale

$$d\sigma = \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(x_1,x_3,\mu_A)\hat{\sigma}_{kl}(x_2,x_4,\mu_B)\int d\tilde{z}_\perp F_{ik}(x_1,x_2,z_\perp,\mu_A,\mu_B)F_{jl}(x_3,\boxed{x_4},z_\perp,\mu_A,\mu_B)$$

Momentum fraction carried by the parton inside the hadron

Transverse distance between the two partons

**DPS processes are important for fundamental studies, *e.g.* the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON**

# Parton correlations and dPDFs

@ LHC kinematics it is often used a factorized form of the dPDFs:$(\mathbf{x_1}, \mathbf{x_2}) - \mathbf{z}_\perp$ factorization:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

* Here and in the following: $\mu = \mu_A = \mu_B$

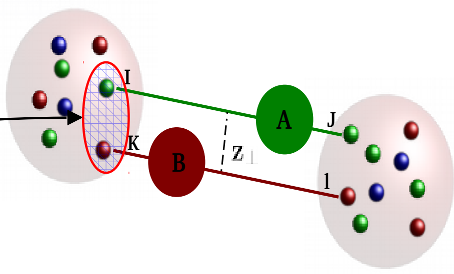
and $\mathbf{x_1}, \mathbf{x_2}$ factorization:

$$\overbrace{F_{ij}(x_1, x_2, \mu)}^{\text{dPDF (2-Body)}} = \overbrace{q_i(x_1, \mu)}^{\text{PDF (1-Body)}} \boxed{q_j(x_2, \mu)} \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

Unknown

Data available

NO CORRELATION ANSATZ




**In this scenario, parton correlations inside the proton are neglected.**

**NO NEW INFORMATION**

- In principle, they are present!
- dPDFs are non-perturbative quantities ⟺ DPCs not calculated directly from QCD

HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION?

WHAT CAN WE LEARN ABOUT dPDFs?

# DPCs in constituent quark models (CQM)

- Main features:
  - potential model
  - effective particles
  - particles are strongly bound and correlated

- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region at low energy scale, while LHC data are available at small $x$

- At very low $x$ , due to the large population of partons, the role of correlations may be less relevant BUT theoretical microscopic estimates are necessary

**pQCD evolution of the calculated dPDFs is necessary to move towards the experimental kinematics!**

*i)* dPDF evaluated at the initial scale of the model $\xrightarrow{\text{pQCD}}$ *ii)* dPDF evaluated at high generic scale

CQM calculations are able to reproduce the gross-feature of experimental PDFs in the valence region. CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

# The Light-Front approach

Relativity can be implemented, for a CQM, by using a Light-Front (LF) approach yielding, among other good features, the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

$\boxed{a^{\pm} = a_0 \pm a_3}$

RHD
- Instant Form: $t_0 = 0$
  Evolution Operator: $P^0 = E$
- **Front Form (LF):**
  $x^+ = t_0 + z = 0$
  **Evolution Operator:** $P^-$

- Full Poincaré covariance
- fixed number of on-mass-shell particles

Among the 3 possibles forms of RHD we have chosen the LF one since there are several advantages. The most relevant are the following:

- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\mathbf{P}^+$, $\mathbf{P}_\perp$, iii) Rotation around z.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- In a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- The IMF (Infinite Momentum Frame) description of DIS is easily included.

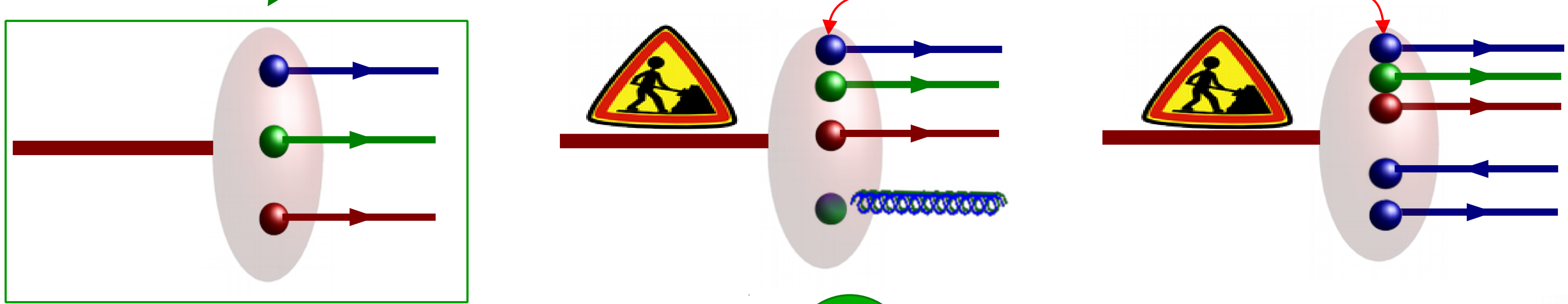
The LF approach is extensively used for hadronic studies ( e.m. form factors, PDFs, GPDs, TMDs........)

# A Light-Front wave function representation

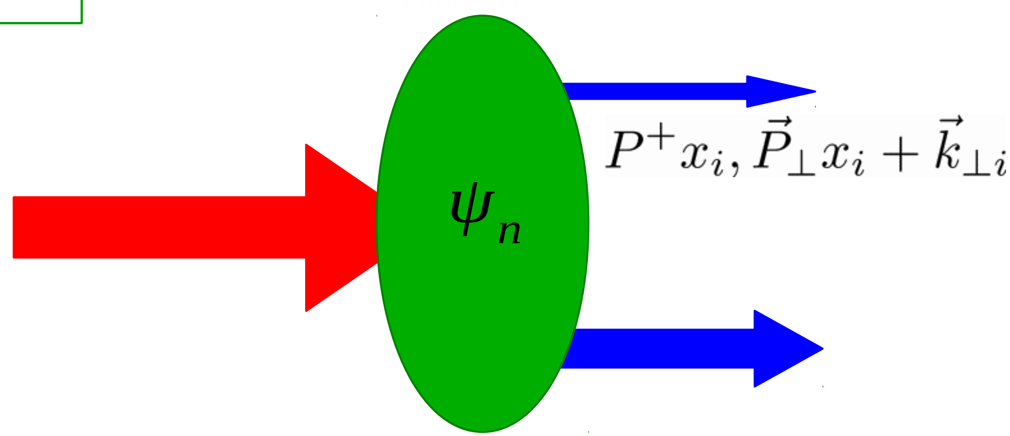
The proton wave function can be represented in the following way:

see *e.g.*: **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+ \ \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqq\ g}|qqq\ g\rangle + \psi_{qqq\ q\bar{q}}|qqq\ q\bar{q}\rangle$$



p: $P^+ \ \vec{P}_\perp$




$$\begin{cases} x_i = \dfrac{k_i^+}{P^+} \\ \displaystyle\sum_i^n x_i = 1 \\ \displaystyle\sum_i^n \vec{k}_{\perp i} = 0 \end{cases}$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i) \longleftrightarrow$ Invariant under LF boosts!

# First look at two partons inside the proton

M.R., F. A. Ceccopieri, arXiv: 1611.04793, submitted

$$F_{u_v d_v}(x_1, x_2, \vec{b}_\perp, \mu_0^2) = \int d\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{b}_\perp} F_{u_v d_v}(x_1, x_2, \vec{k}_\perp, \mu_0^2)$$

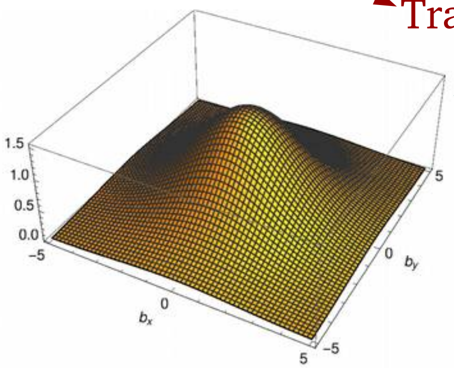
Transverse distance

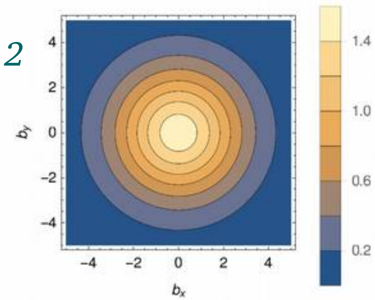
The distribution has been calculated within different CQM models.
The harmonic oscillator and the ones of Refs.:

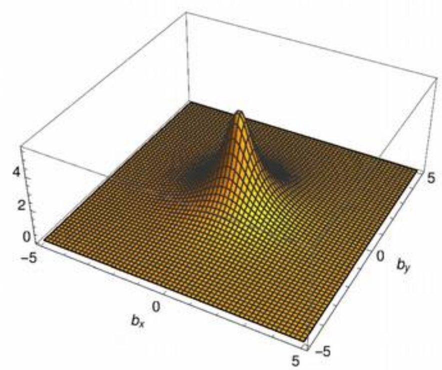
**P. Faccioli *et al*, Nucl. Phys. A 656, 400-420 (1999)**

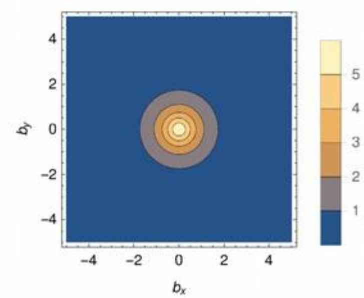
**E. Santopinto *et al*, PLB 364 (1995)**

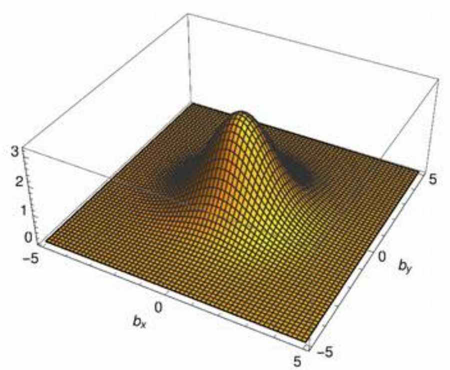
$x_1 = 0.3 \quad x_2 = 0.2$

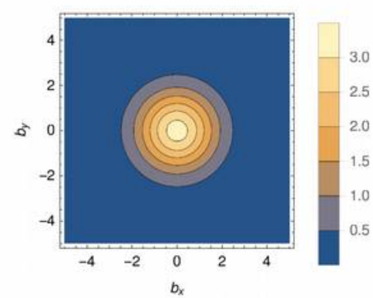






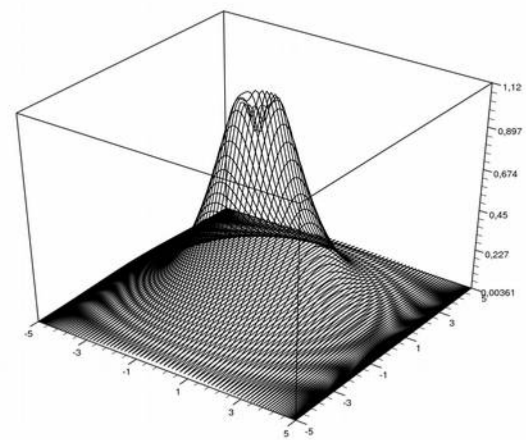






D I F F E R E N T M O D E L S

$x_1 = 0.3 \quad x_2 = 0.04$



**E.g., in our model, quarks with similar longitudinal momentum fraction “prefer” to be close to each other!**

Results on distributions with longitudinally and transversely polarized quarks are coming!

# The Effective X-section

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section": $\sigma_{eff}$

This object can be defined through a "pocket formula":

***Sensitive to correlations***

$$\sigma_{eff} = \boxed{\frac{m}{2}} \frac{\sigma_A^{pp'} \boxed{\sigma_B^{pp'}}}{\sigma_{double}^{pp}}$$

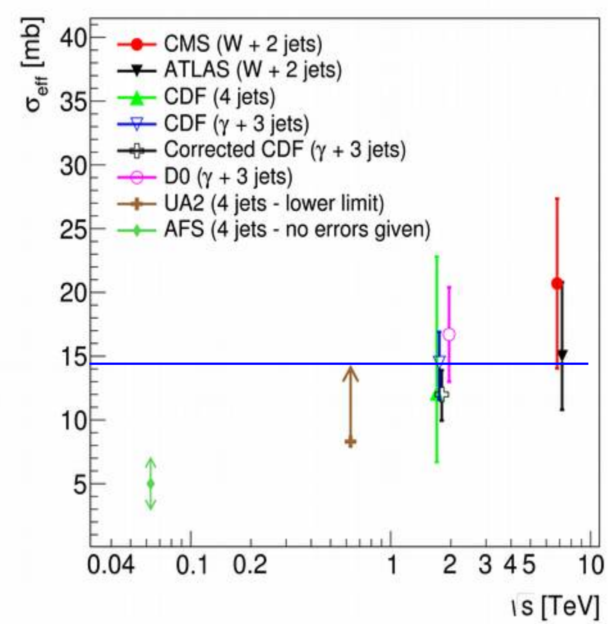
Differential cross section for the process: pp' $\rightarrow A(B) + X$

Combinatorial factor

Differential cross section for a DPS event: pp' $\rightarrow A + B + X$

**....EXPERIMENTAL STATUS:**

- Difficult extraction, approved analysis for the production of same sign *WW* @LHC (RUN 2)
- the model dependent extraction of $\sigma_{eff}$ from data is consistent with a "constant" , nevertheless there are large errorbars (uncorrelated ansatz assumed!)
- different ranges in $x_i$ accessed in different experiments!




High $x$ for hard jets (heavy particles detected, large partonic *s*):

AFS $\longrightarrow$ $y \sim 0;\ x_1 \sim x_2;\ 0.2 < x_{1,2} < 0.4$

CDF $\longrightarrow$ $0.02 < x_{1,2,3,4} < 0.4$

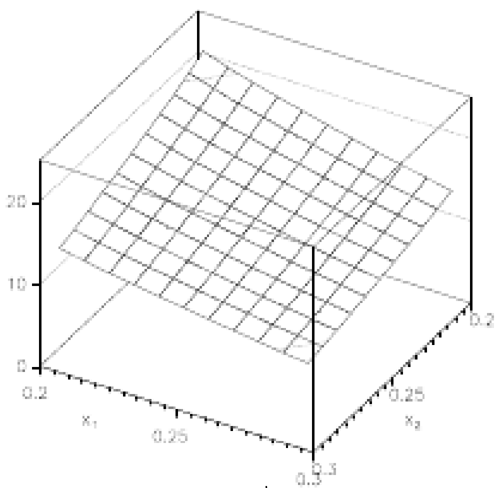
**valence region included!**

# Numerical results I

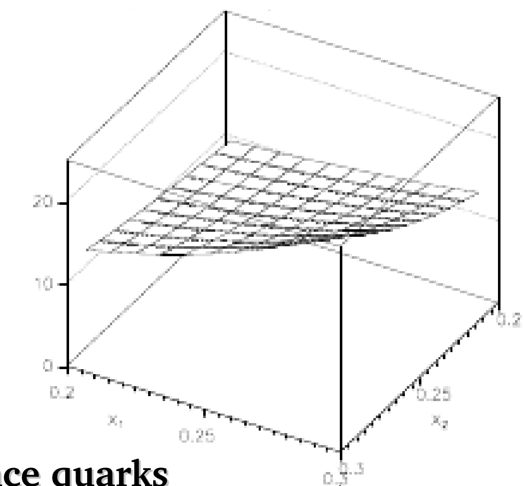
M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

Our predictions of $\sigma_{eff}$ in the valence region at different energy scales:

$\sigma_{eff}(x_1, x_2, \mu_0^2)$



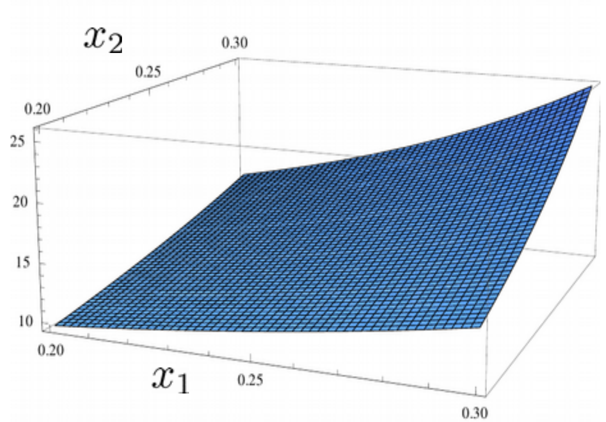
$\sigma_{eff}(x_1, x_2, Q^2 = 250\ \mathrm{GeV}^2)$



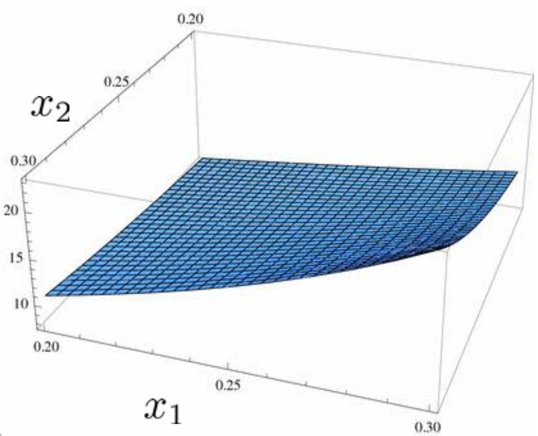
**Valence quarks**

$\boxed{\overline{\sigma_{eff}} \sim 11\ \mathrm{mb}}$

$\sigma_{eff}(x_1, x_2, Q^2 = 250\ \mathrm{GeV}^2)$




**Valence quark ⊗ Gluon**



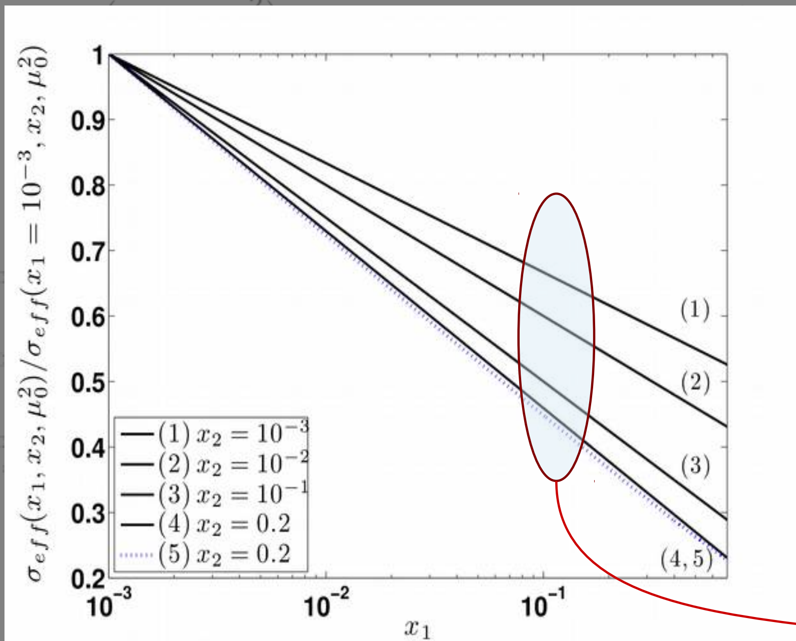

**Valence quark ⊗ Sea quark**
Partons involved in, e.g., same sign WW production.

**The old data lie in the obtained range of $\sigma_{eff}$**

# Numerical results II

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)
M. R., S. Scopetta, M. Traini and V.Vento, arXiv: 1609.07242, submitted

Our predictions of $\sigma_{eff}$ in the valence region at different energy scales:




$\sigma_{eff} \sim 11$ mb

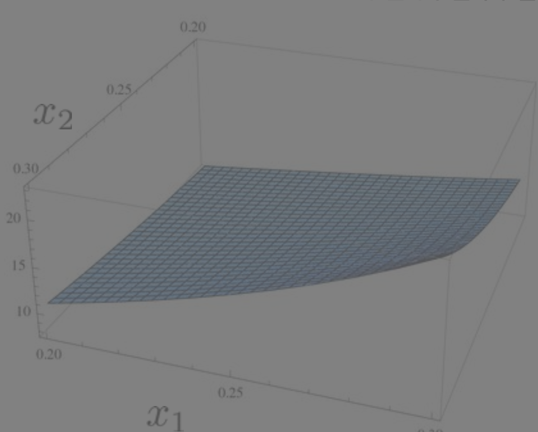
Ratio of $\sigma_{eff}$ calculated by using:

$$F_{12}(x_1, x_2, \vec{z}_\perp) \sim \int d\vec{b} f(x_1, 0, \vec{b} + \vec{z}_\perp) \boxed{f(x_2, 0, \vec{b})}$$

GPDs calculated within ADS/QCD soft wall model

**Valence quark ⊗ Gluon**

Also in this case strong *x* dependence is found!




**Valence quark ⊗ Sea quark**
Partons involved in, e.g., same sign WW production.

**The old data lie in the obtained range of $\sigma_{eff}$**

# Effects of evolution and correlations I

M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

In the analysis of $\sigma_{eff}$, the factorized ansatz for dPDF in terms of PDF, is commonly used. This is consistent with $\dfrac{F_{ab}(x_1, x_2, k_\perp = 0; Q^2)}{a(x_1; Q^2) b(x_2; Q^2)} \sim 1$

***It is worth to notice that the dPDFs and PDFs obey to different pQCD evolution scheme.***

In order to distinguish effects of dynamical correlations, from those arising from the pQCD evolution, we have studied different ratios:

$$r_{ab}^{[1]} = \frac{F_{ab}(x_1, x_2, k_\perp = 0; Q^2)}{a(x_1; Q^2) b(x_2; Q^2)}$$

The numerator, being a dPDF, evolves with the usual pQCD evolution of dPDFs

*The denominator evolves as the product of evolved single PDFs*

***Full Correlations***

$$r_{ab}^{[2]} = \frac{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}{a(x_1; Q^2) b(x_2; Q^2)}$$

The numerator, product of PDFs, evolves with the pQCD evolution equations of dPDFs (PDF x PDF = dPDF )!

*The denominator evolves as the product of evolved single PDFs*

***Perturbative Correlations***

$$r_{ab}^{[3]} = \frac{F_{ab}(x_1, x_2, k_\perp = 0, Q^2)}{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}$$

The numerator, being a dPDF, evolves with the usual pQCD evolution of dPDFs

*The numerator, product of PDFs, evolves with the pQCD evolution equation of dPDFs (PDF x PDF = dPDF )!*

***Non-Perturbative Correlations***

# Effects of evolution and correlations II

M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

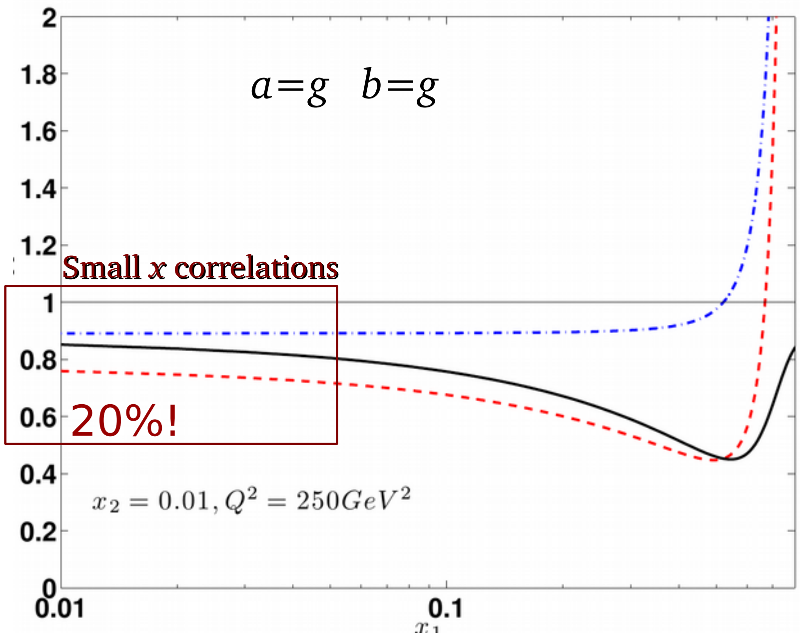
Ratios previously shown are calculated for the following partonic spaces:

$a = u_v \quad b = u_v$ and $a = b = g$

$$r^{[1]}_{ab} = \frac{F_{ab}(x_1, x_2, k_\perp = 0; Q^2)}{a(x_1; Q^2) b(x_2; Q^2)}$$

$$r^{[2]}_{ab} = \frac{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}{a(x_1; Q^2) b(x_2; Q^2)}$$

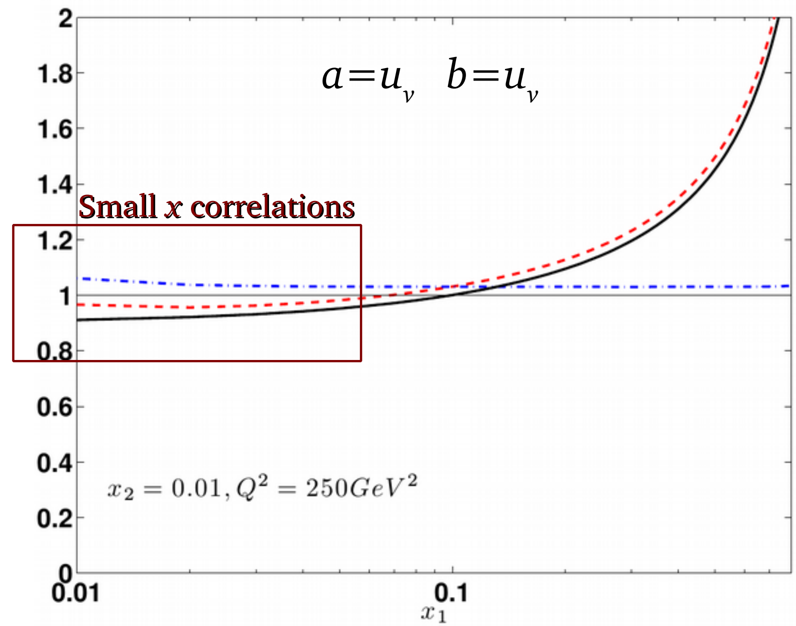
$$r^{[3]}_{ab} = \frac{F_{ab}(x_1, x_2, k_\perp = 0, Q^2)}{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}$$




$r^{[1]}_{ab}$

$r^{[2]}_{ab}$

$r^{[3]}_{ab}$




Let us remark that usually in MC analyses, the effective X-section is estimated consistently with:

$$r^{[1]}_{ab} = r^{[2]}_{ab} \; r^{[3]}_{ab} \sim 1$$

$$r^{[1,2,3]}_{ab} \neq 1$$

**CORRELATIONS**

For $a = u_v \quad b = u_v$, perturbative correlations compensate the non perturbative ones!

For $a = b = g$, perturbative and non-perturbative correlations coherently interfere.

# Introduction of non perturbative sea quarks

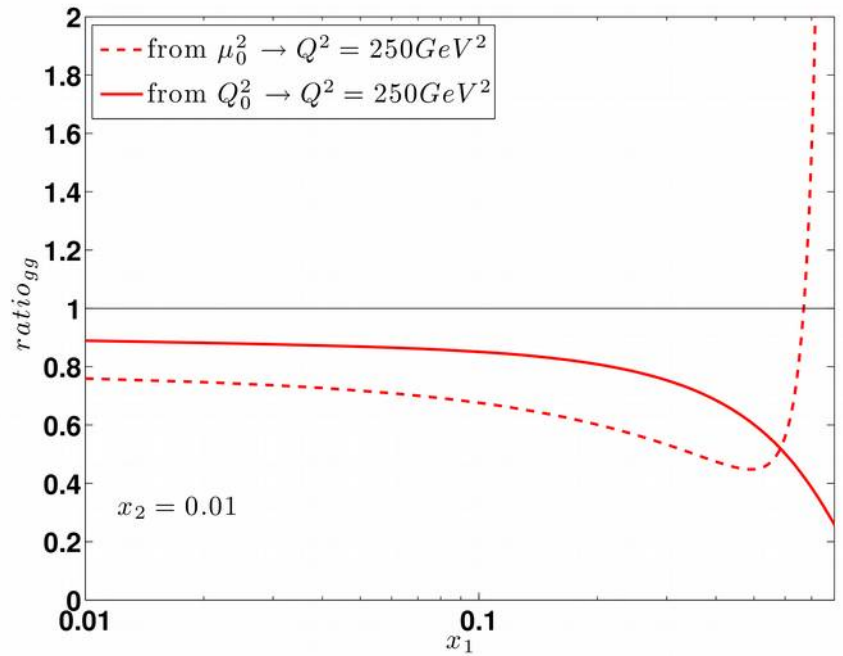
M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

From PDF analyses it is clear the necessity of including non perturbative sea quarks and gluons at the initial scale of the model. In order to face this problem, a simplified approach has been used:

$$F_{uu}(x_1,x_2,k_\perp=0;Q_0^2) \sim \underbrace{F_{u_v u_v}(x_1,x_2,k_\perp=0;Q_0^2)} + \boxed{(1-x_1-x_2)^n\theta(1-x_1-x_2) + u_v(x_1;Q_0^2)\bar{u}(x_2;Q_0^2) + \bar{u}(x_1;Q_0^2)u_v(x_2;Q_0^2)}$$

- Pure valence contribution obtained evolving in pQCD the model calculation of dPDF from the initial scale $\mu_0^2$ to the scale $Q_0^2$

- Non perturbative sea quark contributions (effective high Fock states ) *n=0.2*




**PDF LO MSTW2008**

$\bar{\mathbf{u}}(\mathbf{x};\mathbf{Q_0^2})$ $\mathbf{u_v}(\mathbf{x};\mathbf{Q_0^2})$ $\mathbf{Q_0^2 = 1}$ $\mathbf{GeV^2}$

$$\boxed{ratio_{gg} \neq 1}$$

CORRELATIONS

# LF RELATIVISTIC EFFECTS I

M.R., F. A. Ceccopieri, arXiv: 1611.04793, submitted

The expression of dPDF in the canonical (e.g. NR) and LF forms are quite similar for small values of $x$:

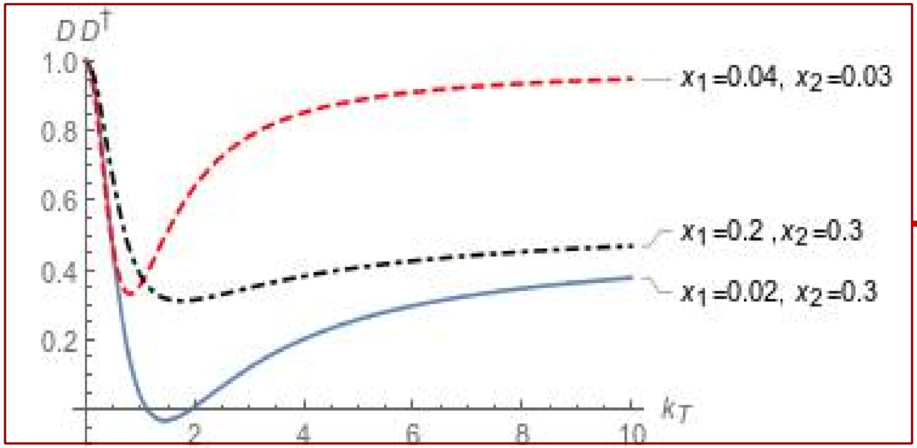
$$F_{[I]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \delta\left(x_1 - \frac{k_1^+}{M_P}\right) \delta\left(x_2 - \frac{k_2^+}{M_P}\right)$$

$$\begin{aligned} F_{[L]}(x_1, x_2, k_\perp) &= \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \langle SPIN | \boxed{O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp}| SPIN \rangle \\ &\times \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right) \end{aligned}$$

Melosh Operators!

$f(\vec{k}_1, \vec{k}_2, k_\perp)$ = product of the canonical proton wave-function

For very small values of $x_1$ and $x_2$, the main difference in the two approaches, in the calculation of dPDF, are Melosh!




$$DD^\dagger = \langle SU(6) | O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp) | SU(6) \rangle$$

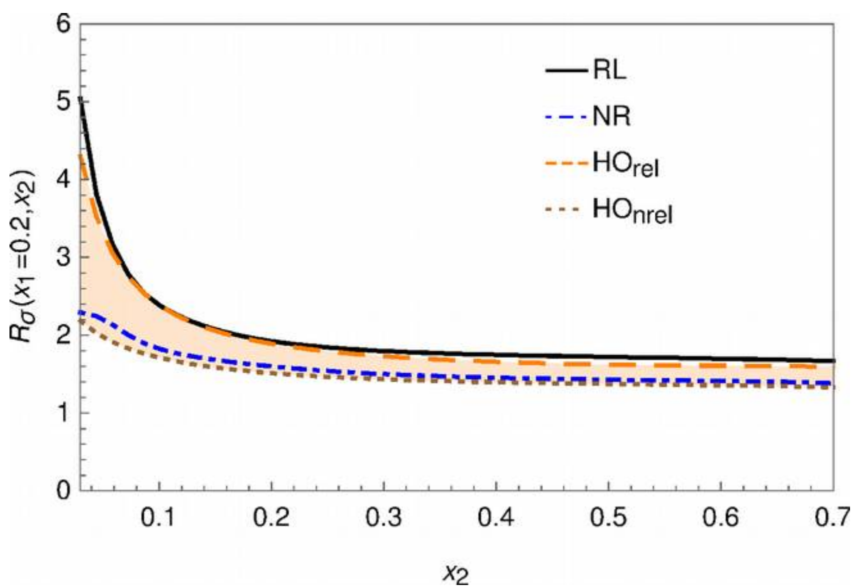
Correlations between $x_i$ and $k_\perp$

# LF RELATIVISTIC EFFECTS II

M.R., F. A. Ceccopieri, arXiv: 1611.04793, submitted

Melosh effetcs in $\sigma_{eff}$ studied by defining such ratio:

$$R_\sigma(x_1,x_2) = \frac{\int d\vec{b}_\perp F(x_1,x_2,b_\perp)^2}{\int d\vec{b}_\perp F_{NM}(x_1,x_2,b_\perp)^2}$$




Relativistic Hyper central Model

NR Hyper central Model

Relativistic Harmonic Oscillator (HO) model $\alpha^2_{rel} = 25\ \mathrm{fm}^{-2}$

$\alpha^2_{nrel} < \alpha^2 < \alpha^2_{rel}$

NR Harmonic Oscillator mode$\alpha^2_{nrel} = 6\ \mathrm{fm}^{-2}$

**Important effect due to Melosh's operators found in this observable!!**

Now the same analysis must be done at high energy scales and considering also gluons and sea quarks!

# Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta in preparation.

- same sign W's production is candidate process for DPS observation at hadron collider;
- W decay in muon channel, muon phase space : $|\eta| < 2.4$, $E_T^\mu > 25$ GeV;
- the differential DPS cross sections reads

$$\frac{d^4\sigma^{pp\to\mu^\pm\mu^\pm X}}{d\eta_1 dE_{T,1} d\eta_2 dE_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2\boldsymbol{b} D_{ij}(x_1, x_2, \boldsymbol{b}, M_W) D_{kl}(x_3, x_4, \boldsymbol{b}, M_W) \frac{d^2\sigma_{ik}^{pp\to\mu^\pm X}}{d\eta_1 dE_{T,1}} \frac{d^2\sigma_{jl}^{pp\to\mu^\pm X}}{d\eta_2 dE_{T,2}}$$

- From LF model, we obtain single and double PDFs, normalised as $D_{uu}^{LF}(x_i, \boldsymbol{b}_\perp, Q_0) = D_{du}^{LF}(x_i, \boldsymbol{b}_\perp, Q_0) = D_{ud}^{LF}(x_i, \boldsymbol{b}_\perp, Q_0)$
- dPDFs are evolved from $Q_0$ up to $M_W$ with hom. evol. eqs at fixed $|\boldsymbol{b}_\perp|$

$\Rightarrow$ Preliminar W-charge inclusive $\sigma(WW) \sim 1$ fb (in dimuon channel)

$\Rightarrow$ Extract $\hat{\sigma}_{eff}^{LF} \simeq$ 15 - 20 mb (LF model reproduces the magnitudo of transverse correlations obtained in experimental analyses)

- Presently under investigation : th. syst. errors and IR sensitivity to the choice of $Q_0$.

Slide by Federico A. Ceccopieri

# Conclusions

- **A CQM calculation of the dPDFs with a fully covariant approach**
  **M. R., S. Scopetta and V.Vento, PRD 87, 114021 (2013)**
  - Calculation of dPDFs within a NR CQM model. Strong *x1-x2* correlations are found
- **A CQM calculation of the dPDFs with a fully covariant approach**
  **M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)**
  - symmetry in the exchange of two partons in the dPDFs correctly restored
  - violations of both the $(x_1, x_2) - k_\perp$ and $x_1, x_2$ factorizations for the polarized and unpolarized $_2$GPDs
  - Analysis of effects of perturbative and non perturbative correlations: for some partonic species, sizable correlations are found also at small *x*
    **M. R., S. Scopetta, M. Traini and V.Vento, JHEP 10, 063 (2016)**
- **Calculation of the effective X-section**
  **M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2015)**
  **M. R., S. Scopetta, M. Traini and V.Vento, arXiv: 1609.07242, submitted**
  - Calculation of the effective X-section at the hadronic and at high energy scales within different models
  - **x-dependent quantity obtained!** Qualitatively in agreement with data
  - The x-dependence of the "effective X-section" could give information on the **3d structure of the proton!**
- **What are we working on**
  **M. R., F. A. Ceccopieri, arXiv: 1611.04793, submitted**
  **M.R., F. A. Ceccopieri, S. Scopetta and M. Traini, in preparation**
  - First model analysis of the 3D structure of the proton through dPDF and study of relativistic effects
  - analysis of the inhomogeneous contribution in the pQCD evolution and calculation of DPS cross section

# The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\boxed{\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}}$$

This quantity can be written in terms of PDFs and dPDFs ($_2$GPDs)!

In terms of PARTON DISTRIBUTIONS, $\sigma_{A(B)}^{pp'}$ and $\sigma_{double}^{pp}$ can be written as follows:

$$\sigma_{A(B)}^{pp'}(x_1, x_1', \mu_1) = \sum_{\boxed{i,k}} F_i^p(x_1,\mu_1)\boxed{F_k^{p'}(x_{1'},\mu_1)}\,\hat{\sigma}_{ik}^{A(B)}(x_1, x_{1'}, \mu_1)$$

$$\boxed{i,k = \{q, \bar{q}, g\}}$$

Standard PDF

Proportional to colour coefficient and universal function:

$C_{ij}\bar{\sigma}(x, x')$

$$\sigma_{double}^{pp}(x_1, x_1', x_2, x_2', \mu) = \frac{m}{2}\sum_{i,j,k,l}\hat{\sigma}_{ik}^{A}(x_1, x_1', \mu)\hat{\sigma}_{jl}^{B}(x_2, x_2', \mu) \times \int \frac{d\vec{k}_\perp}{(2\pi)^2} F_{ij}(x_1, x_2, k_\perp, \mu)\boxed{F_{kl}(x_1', x_2', -k_\perp, \mu)}$$

$_2$GPDs

Finally, combining the previous equations in the “pocket formula”, one obtains:

Here the scale is omitted

$$\boxed{\sigma_{\mathbf{eff}}(\mathbf{x_1}, \mathbf{x_1'}, \mathbf{x_2}, \mathbf{x_2'}) = \frac{\sum_{\mathbf{i,k,j,l}} \mathbf{F_i(x_1)F_k(x_1')F_j(x_2)F_l(x_2')C_{ik}C_{jl}}}{\sum_{\mathbf{i,j,k,l}} \mathbf{C_{ik}C_{jl}} \int \mathbf{F_{ij}(x_1, x_2; k_\perp)F_{kl}(x_1', x_2'; -k_\perp)\frac{dk_\perp}{(2\pi)^2}}}}$$

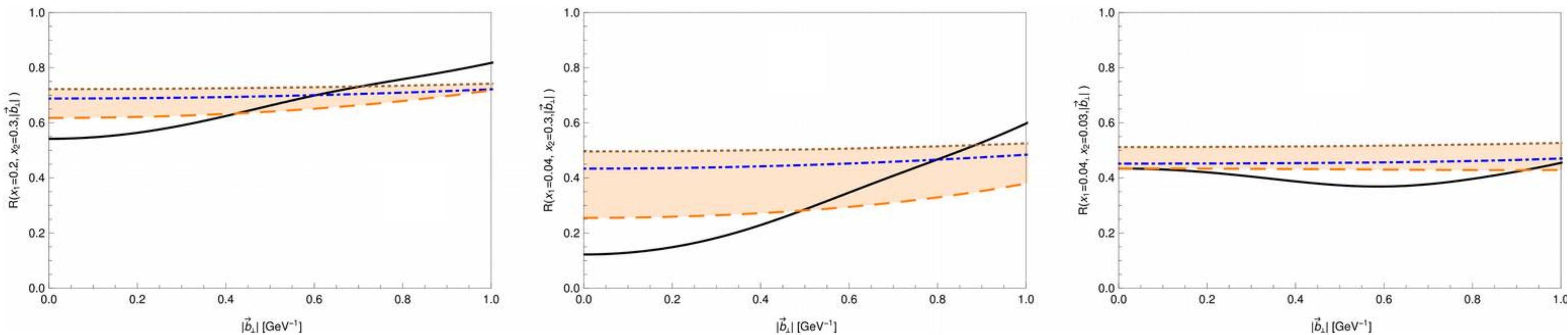
**Non trivial x-dependence**

# LF RELATIVISTIC EFFECTS II

M.R., F. A. Ceccopieri, arXiv: 1611.04793, submitted

We can estimate Melosh effetcs in dPDF studyng such ratio:

$$\boxed{R(x_1,x_2,b_\perp)=\frac{F(x_1,x_2,b_\perp)}{F_{NM}(x_1,x_2,b_\perp)}}$$




In these plots we can still appreciate correlations between $x$ and $b_\perp$ . Moreover the calculation has been performed using different quark models in order to show model independent effects!

Relativistic Hyper central Model

NR Hyper central Model

Relativistic Harmonic Oscillator (HO) model $\alpha^2_{rel}=25\ \text{fm}^{-2}$

$\alpha^2_{nrel}<\alpha^2<\alpha^2_{rel}$

NR Harmonic Oscillator model $\alpha^2_{nrel}=6\ \text{fm}^{-2}$