Double parton scattering in the ultraviolet:
addressing the double counting problem

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European Research Council
Established by the European Commission
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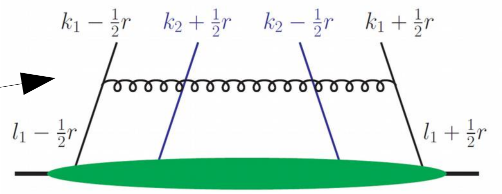
MPI@LHC 2016, 29/11/16

Based on work with Markus Diehl and Kay Schoenwald

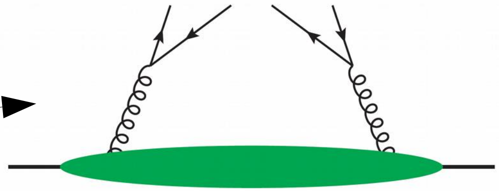
QCD evolution effects

Consider effects of QCD evolution in DPS, going backwards from the hard interaction.

Some effects are similar to those encountered in SPS – i.e. (diagonal) emission from one of the parton legs. These can be treated in same way as for SPS.



However, there is a new effect possible here – when we go backwards from the hard interaction, we can discover that the two partons arose from the perturbative '1 → 2' splitting of a single parton.



This 'perturbative splitting' yields a contribution to the DPD of the following form:

Single PDF

$$F(x_1, x_2, y) \propto \alpha_s \frac{f(x_1 + x_2)}{x_1 + x_2} P\left(\frac{x_1}{x_1 + x_2}\right) \frac{1}{y^2}$$

Dimensionful part

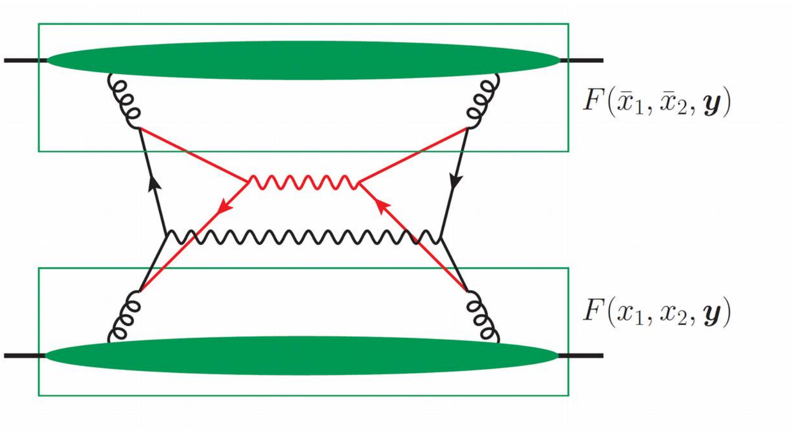
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Perturbative splitting kernel

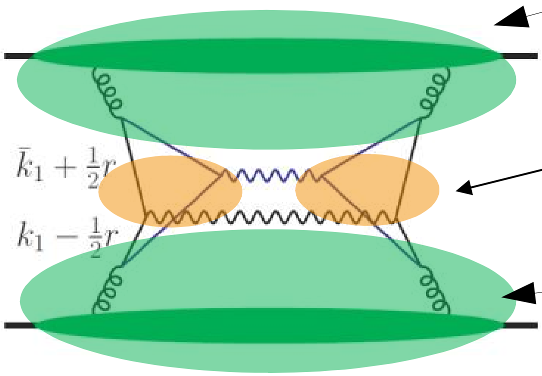
Problems...

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$

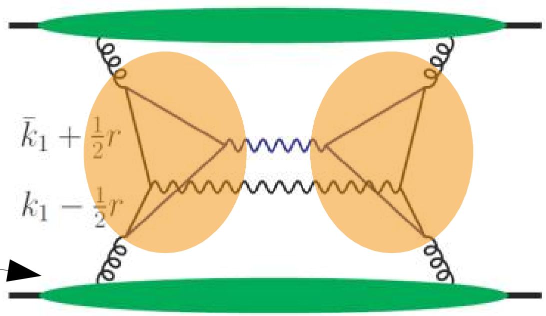


This is related to the fact that this graph can also be regarded as an SPS loop correction



'Hard' part

Part absorbed into PDF



$$\frac{\Lambda^2}{Q^4} \xrightarrow{\hspace{2cm}} \frac{1}{Q^2}$$

Power divergence!

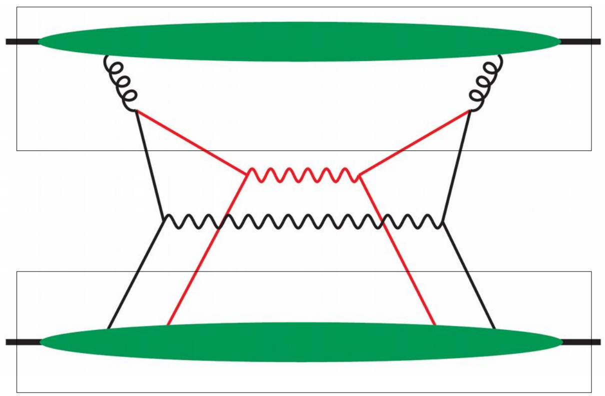
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn Phys.Lett. 713 (2012) 196
JG and Stirling, JHEP 1106 048 (2011)
Blok et al. Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

Single perturbative splitting graphs

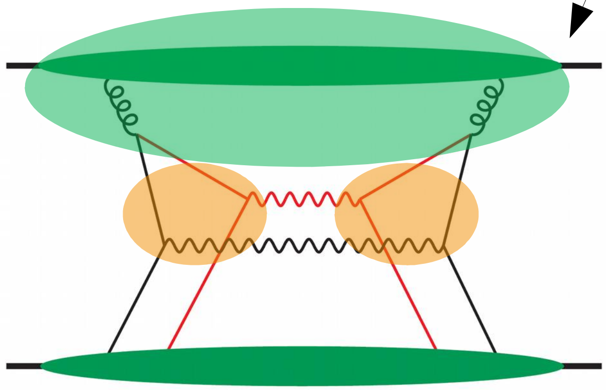
Also have graphs with perturbative 1→2 splitting in one proton only (2v1 graph).

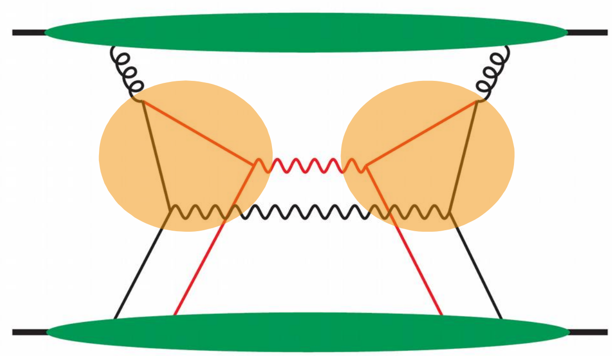
This has a log divergence:

$$\int d^2y/y^2 F_{\rm intr}(x_1,x_2;y)$$



Related to the fact that this graph can also be thought of as a twist 4 x twist 2 contribution to AB cross section





$$\frac{\Lambda^2}{Q^4} \xrightarrow{\text{Logarithmic divergence}} \frac{\Lambda^2}{Q^4}$$

Blok et al., Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011
JG, JHEP 1301 (2013) 042

Desirable features of a solution to these issues

- Render DPS contribution finite, with no double counting between DPS and SPS.
- Retain concept of the DPD for an individual hadron, with a field theoretic definition. This allows us to investigate these functions using nonperturbative methods such as lattice calculations.
- Should resum DGLAP logarithms in all types of diagram (1v1, 2v1, 2v2) where appropriate.
- Should permit a formulation at higher orders in perturbation theory (that is not too complicated in practice).

No existing solution satisfies all of these!

Our solution

Diehl, JG, Schoenwald, work in progress

[Focus for the moment only on the double perturbative splitting issue]

Insert a regulating function into DPS cross section formula:

$$\sigma_{\mathrm{DPS}} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

Requirements: $\Phi(u) \to 0 \text{ as } u \to 0$

$\Phi(u) \to 1 \text{ for } u \gg 1$ e.g. $\Phi(u) = \theta(u-1)$

In this way, we cut contributions with 1/y much bigger than the scale ν out of what we define to be DPS, and regulate the power divergence.

Note that the *F*s here contain both perturbative and nonperturbative splittings.

Our solution

Now we have introduced some double counting between SPS and DPS – we fix this by including a double counting subtraction:

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$$

The subtraction term is given by the DPS cross section with both DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

Subtraction term constructed along the lines of general subtraction formalism discussed in Collins pQCD book

Note: computation of subtraction term much easier than full SPS X sec

Straightforward extension of formalism to include twist 4 x twist 2 contribution and remove double counting with 2v1 DPS:

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub (1v1 + 2v1)}} + \sigma_{\text{tw4}\times\text{tw2}}$$

Tw2 x tw 4 piece with hard part computed according to fixed order DPS expression

How the subtraction works

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$$

For small y (of order 1/Q) the dominant contribution to σ_{DPS} comes from the (fixed order) perturbative expression $\Longrightarrow \quad \sigma_{\text{DPS}} \simeq \sigma_{\text{sub}}$

& $\quad \sigma_{\text{tot}} \simeq \sigma_{\text{SPS}}$ (as desired)

(dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub})

For large y (much larger than 1/Q) the dominant contribution to σ_{SPS} is the region of the 'double splitting' loop where DPS approximations are valid

$\Longrightarrow \quad \sigma_{\text{SPS}} \simeq \sigma_{\text{sub}}$

& $\quad \sigma_{\text{tot}} \simeq \sigma_{\text{DPS}}$ (as desired)

(similar considerations hold for 2v1 part of DPS and tw4xtw2 contribution)

Numerical illustration

That's the formalism – also useful to look at quantitative numerical illustrations, to get an idea of relative contributions of various pieces under different conditions.

Here: look mainly at DPS piece (from this alone can already get information about when SPS and subtraction will be large/needed)

In particular will mainly focus on the DPS luminosity:

$$\mathcal{L}_{ijkl}(x_i, \bar{x}_i, \mu_i, \nu) = \int d^2y \, \Phi^2(y\nu) \, F_{ij}(x_i, y; \mu_i) \, F_{kl}(\bar{x}_i, y; \mu_i) \, ,$$

For cut-off function we use $\Phi(u) = \theta(u - b_0) \qquad b_0 = 2e^{-\gamma_E} = 1.1229...$

Modelling of DPD

For modelling, we write DPD as the sum of two terms:

$$F^{ij}(x_1,x_2,y,\mu) = F^{ij}_{\text{spl}}(x_1,x_2,y,\mu) + F^{ij}_{\text{int}}(x_1,x_2,y,\mu)$$

Initialise at low scale

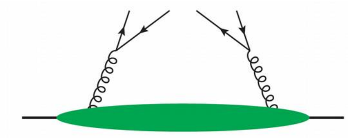
$\mu_0 = 1$ GeV

$$F^{ij}_{\text{int}}(x_1,x_2,y,\mu_0) = \frac{1}{4\pi h_{ij}}\, e^{-\frac{y^2}{4h_{ij}}}\, f_i(x_1,\mu_0)\, f_j(x_2,\mu_0)(1-x_1-x_2)^2(1-x_1)^{-2}(1-x_2)^{-2}$$

Smooth transverse y profile, radius ~ R_p

'Usual' product of PDFs

Factor to suppress DPD near phase space limit $x_1 + x_2 = 1$



Initialise at scale $\mu_y \sim \frac{1}{y}$

$$F^{ij}_{\text{spl}}(x_1,x_2,y,\mu_y) = e^{-\frac{y^2}{4h_{ij}}} \frac{1}{\pi y^2} \frac{\alpha_s(\mu_y)}{2\pi} \sum_k \frac{f_k(x_1+x_2,\mu_y)}{x_1+x_2} P_{k\to i}\left(\frac{x_1}{x_1+x_2}\right)$$

Gaussian suppression at large y

Perturbative splitting expression

Evolve both to scale μ using homogeneous double DGLAP $\frac{d}{d\log\mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$

Parton luminosities

Plot luminosity against rapidity of A with B central for $Q_A = Q_B = M_W$ and $\sqrt{s} = 14 \text{ TeV}$

Split luminosity into 2v2 $(F_{\text{int}} \otimes F_{\text{int}})$ 2v1 $(F_{\text{spl}} \otimes F_{\text{int}} + F_{\text{int}} \otimes F_{\text{spl}})$ 1v1 $(F_{\text{spl}} \otimes F_{\text{spl}})$

Bands in these plots are produced by varying ν only by a factor of 2 around 80 GeV, to illustrate dependence on this cutoff.

Naive expectations ignoring evolution: 1v1 $\displaystyle\int_{b_0^2/\nu^2} \frac{dy^2}{y^4} \sim \nu^2$

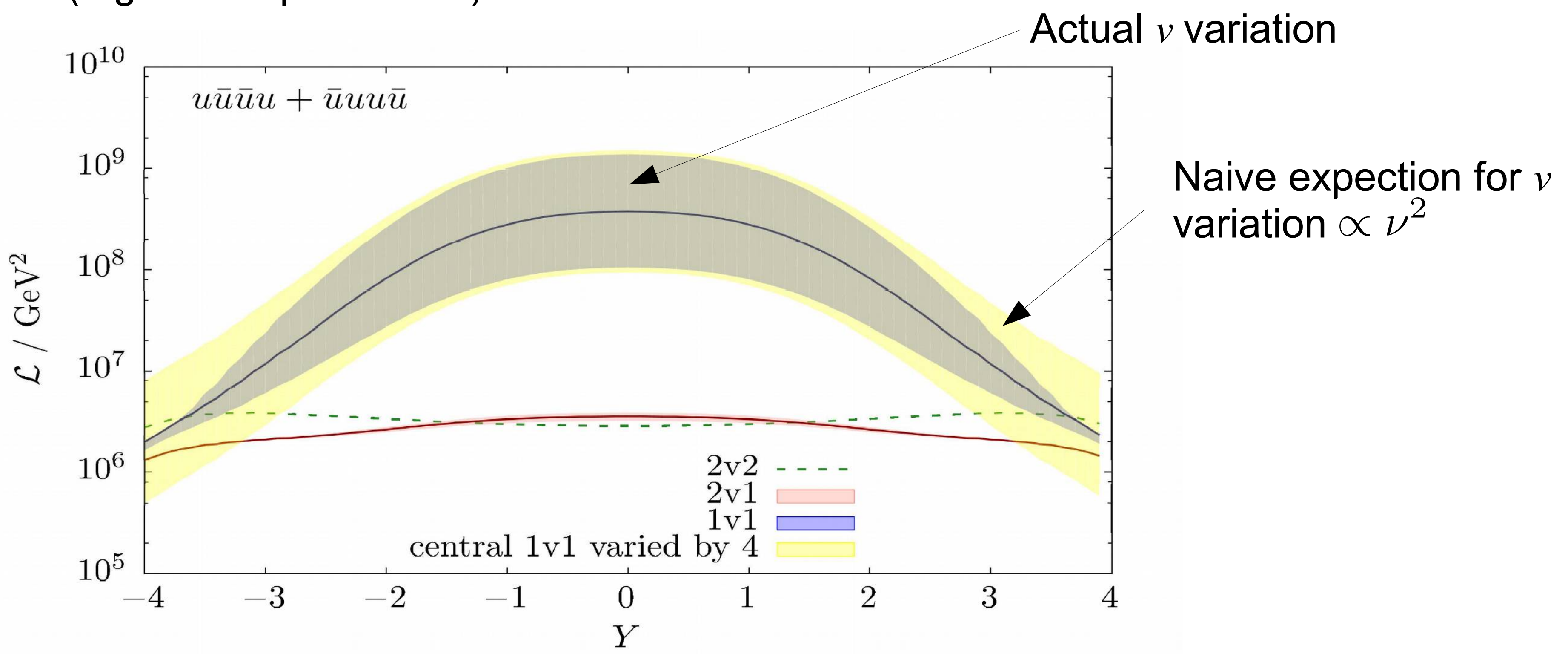
2v1 $\displaystyle\int_{b_0^2/\nu^2} \frac{dy^2}{y^2} \sim \log(\nu)$

2v2 $\displaystyle\sim \frac{\Lambda^2}{\nu^2}$

Note that at leading logarithmic level, our predictions for 2v1 agree with those put forward by Blok et al., Eur.Phys.J. C72 (2012) 1963, Ryskin, Snigirev, Phys.Rev.D83:114047,2011, JG, JHEP 1301 (2013) 042

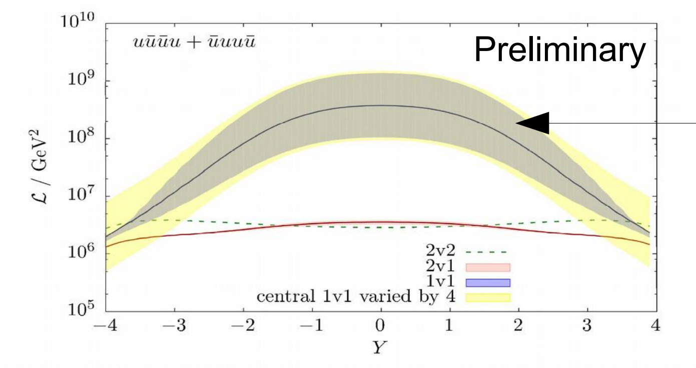
Parton luminosities - $u\bar{u}$

(e.g. for ZZ production)



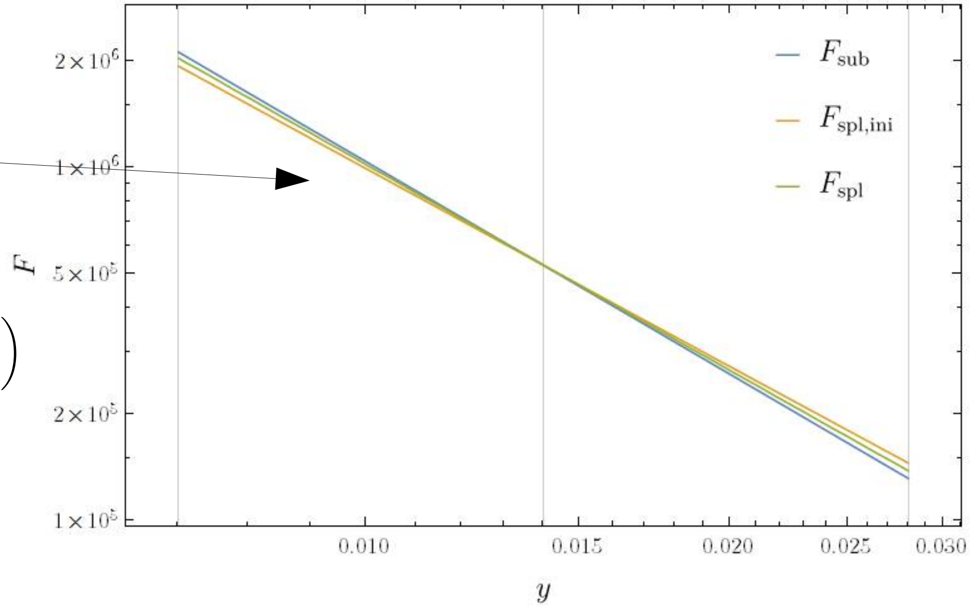
Very large 1v1, with large ν variation – need to include SPS with subtraction.

Parton luminosities - $u\bar{u}$



Actual and naive ν variation bands lie very close – effect of evolution is numerically very small here!

Can illustrate at level of DPD vs y – evolved DPD close to initial conditions!



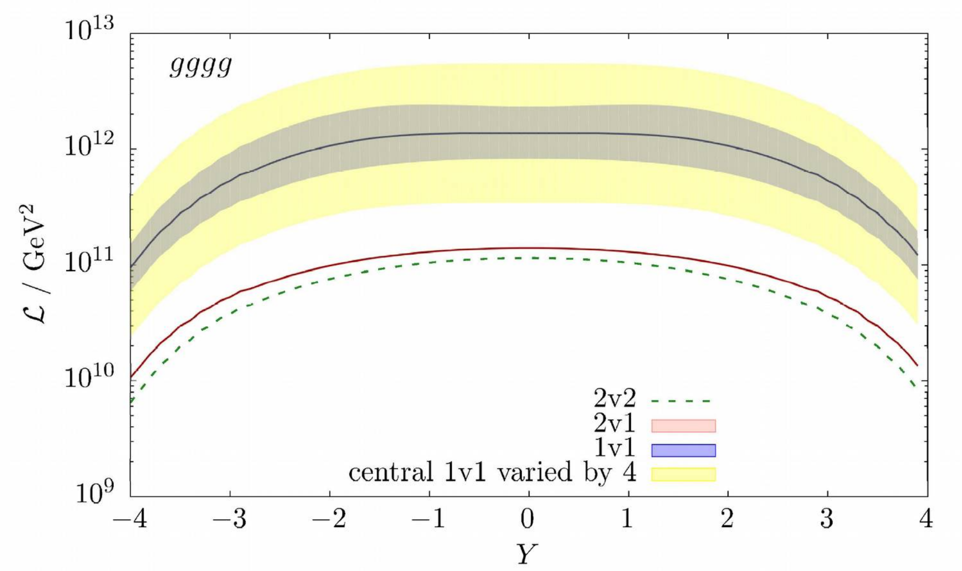
n.b.

$$F^{ij}_{\text{sub}}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \frac{\alpha_s(\mu)}{2\pi} \sum_k \frac{f_k(x_1 + x_2, \mu)}{x_1 + x_2} P_{k \to i}\left(\frac{x_1}{x_1 + x_2}\right)$$

Caused by small ug distribution → not enough feed into uu.

Parton luminosities - gg

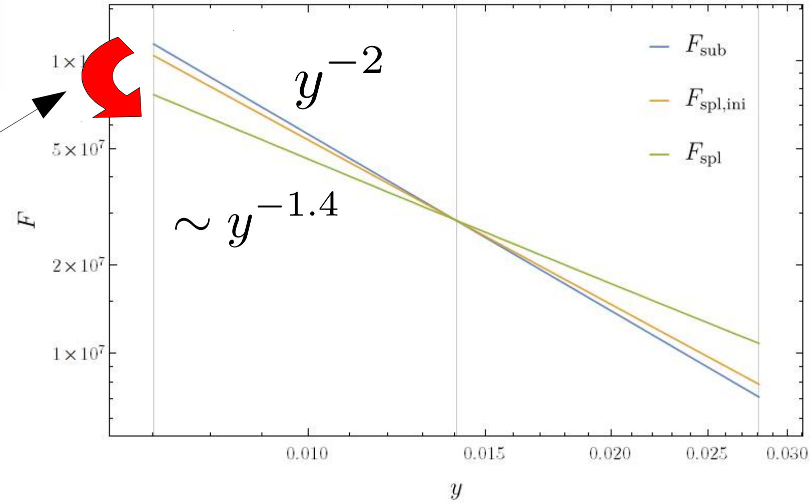
(e.g. for 4j production)



Again 1v1 much larger than 2v1, 2v2

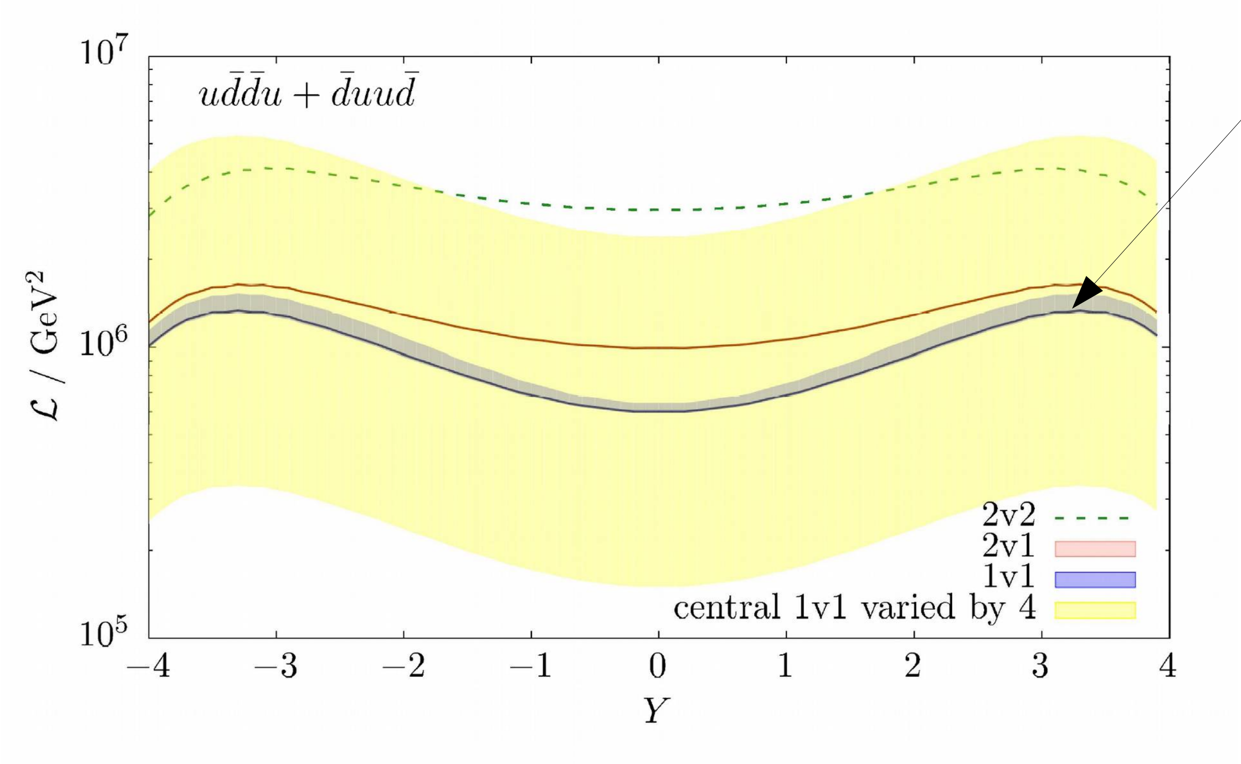
Actual ν variation much smaller than naive expectation – significant evolution effect

Evolution causes significant change of DPD slope vs y

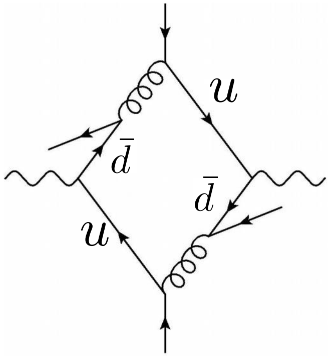


Parton luminosities - $u\bar{d}$

(e.g. for W^+W^+ production)



Small 1v1 contribution, as no direct splitting yielding $u\bar{d}$



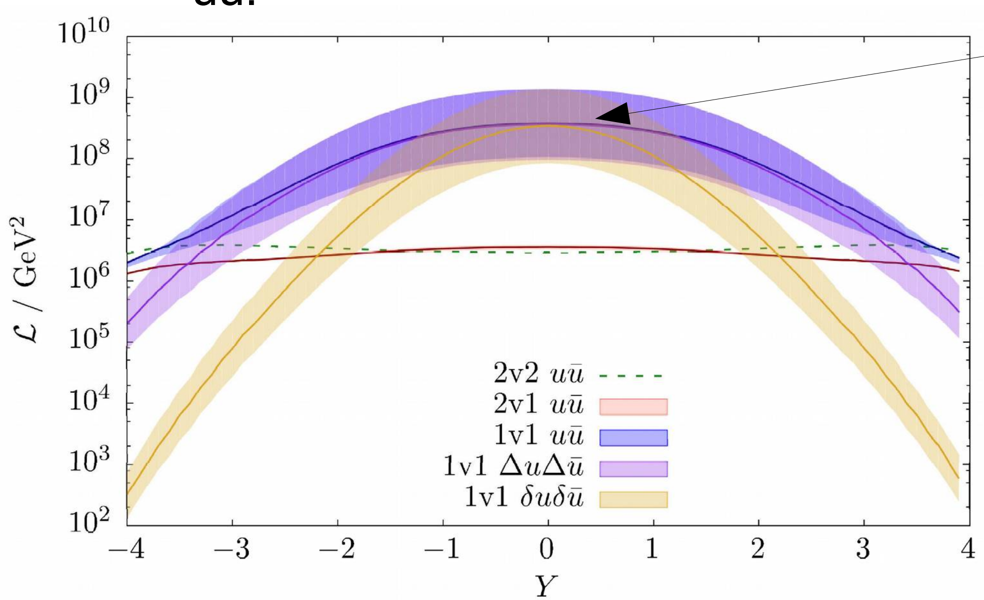
Polarised contributions

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g. $$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

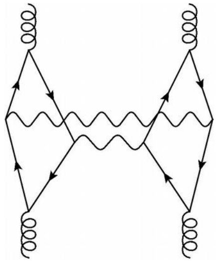
Can use same scheme to handle SPS/DPS double counting for polarised distributions

$u\bar{u}$:



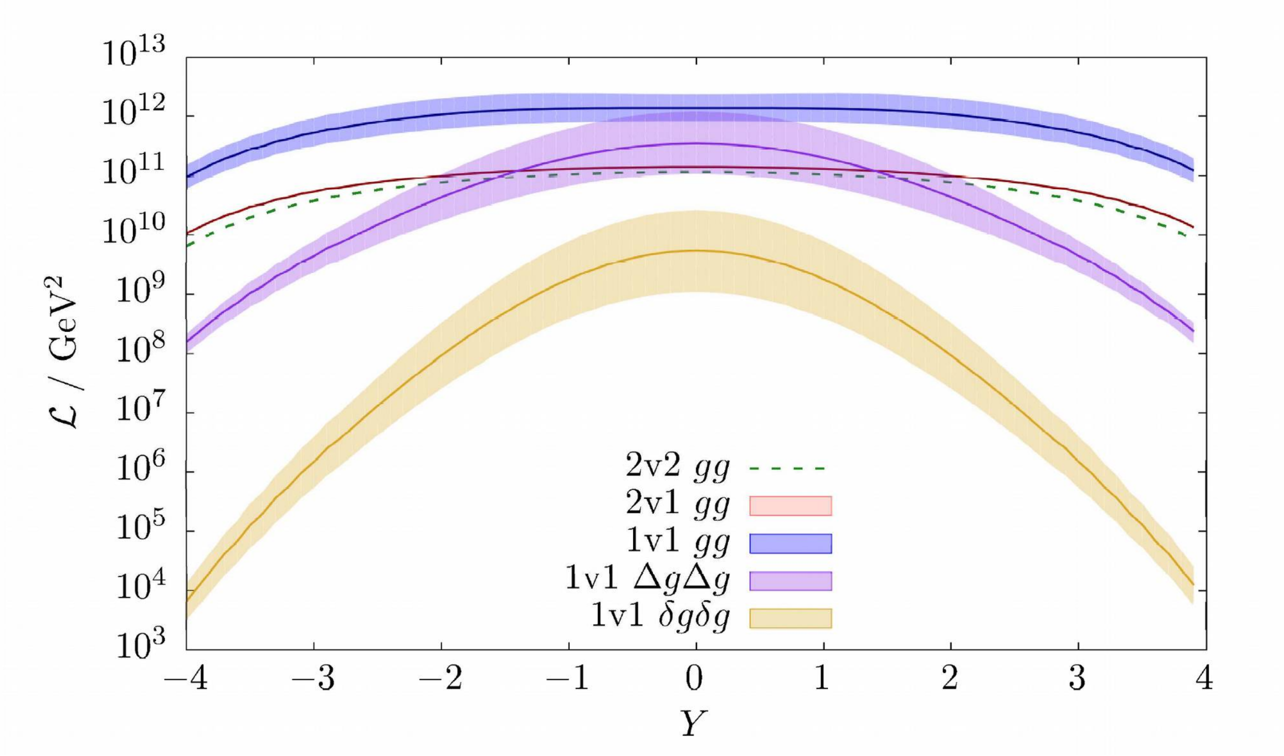
1v1 for all polarised and unpolarised contributions are large with large scale dependence (~same for all). Need to add SPS with subtractions.

Note that the SPS computation automatically contains spin correlations at fixed order – in box they are very large



Polarised contributions

gg:



Some differences in luminosity for gg – mainly driven by differences in initial conditions.

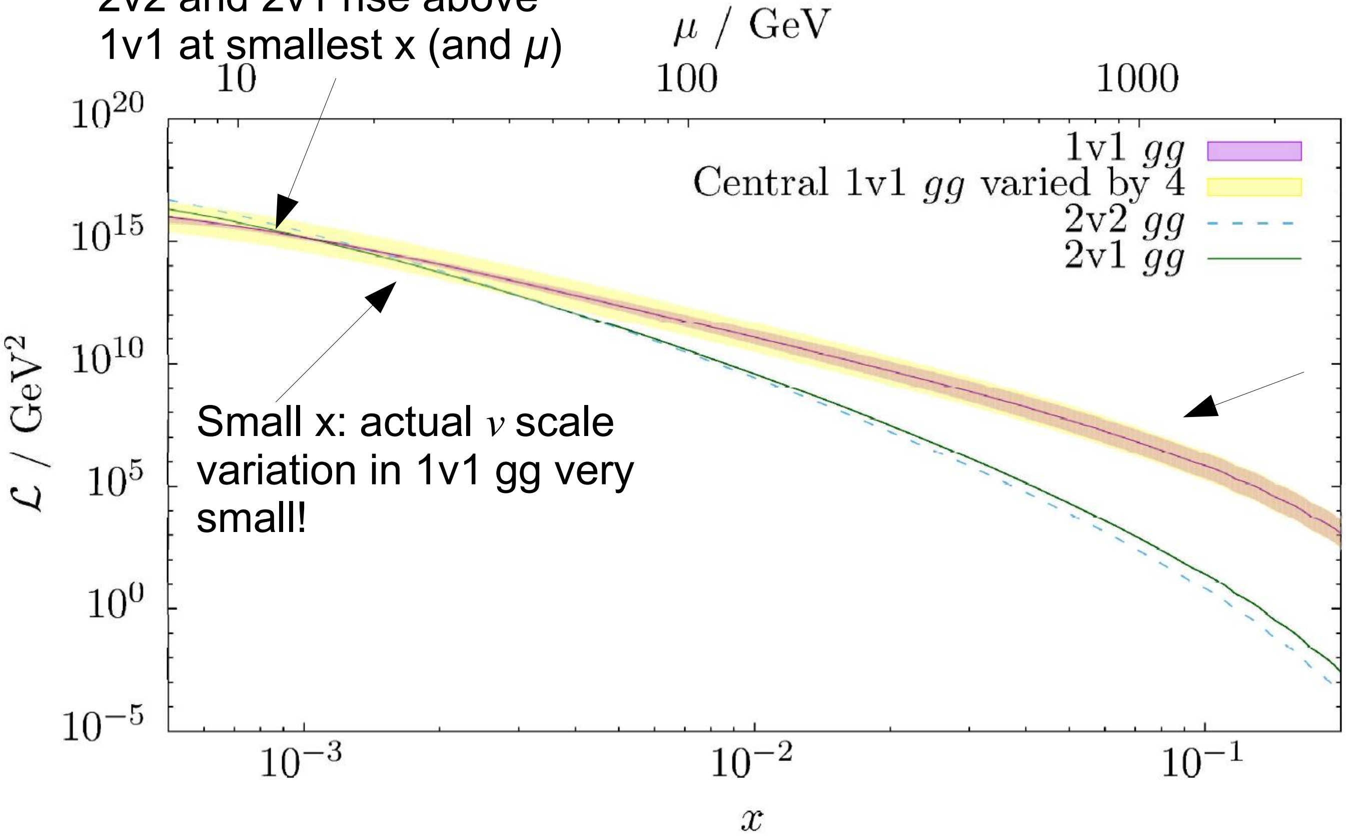
Gluon-gluon luminosities at small *x*

Expect greater numerical impact of evolution effects as x decreases – in particular in gg channel, expect greater modification of DPD y slope, leading to smaller ν variation in luminosity, as *x* decreases.

Ryskin, Snigirev, Phys.Rev.D83 (2011) 114047, Phys.Rev. D86 (2012) 014018

Investigate this numerically: fix √s, set all *x* values equal (central rapidity), and vary *x*

2v2 and 2v1 rise above 1v1 at smallest x (and μ)



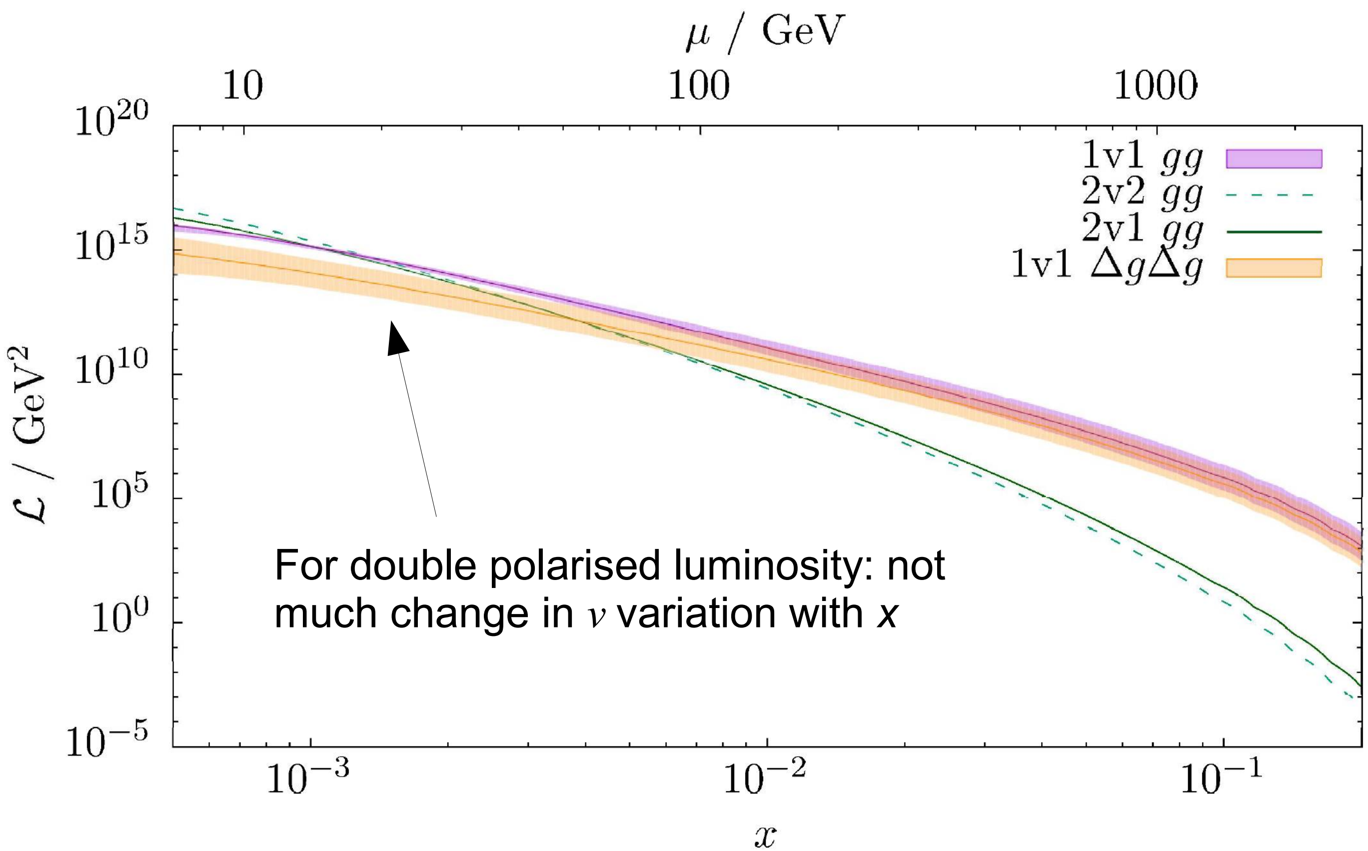
Small x: actual ν scale variation in 1v1 gg very small!

Large x: actual ν scale variation in 1v1 gg close to naive ν^2 expectation

Gluon-gluon luminosities at small *x*

Expect greater numerical impact of evolution effects as x decreases – in particular in gg channel, expect greater modification of DPD y slope, leading to smaller ν variation in luminosity, as *x* decreases.

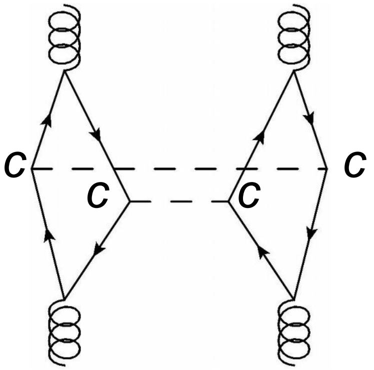
Investigate this numerically: fix √s, set all *x* values equal (central rapidity), and vary *x*



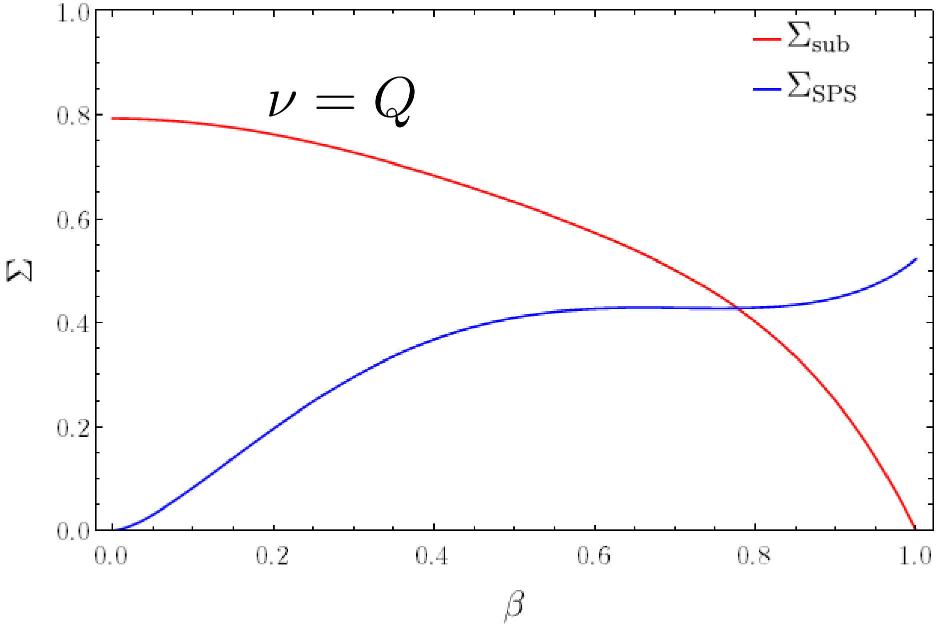
At sufficiently small *x*, possibility of achieving predictions with acceptably small ν uncertainties without having to compute the SPS term up to the order that contains the first nonzero DPS-type loop.

How do the subtraction and SPS terms compare?

Interesting to compare subtraction term to order of SPS containing DPS-type box graphs – are they comparable?



Check for a particular process – production of a pair of massive scalar bosons ϕ with constant coupling *c* to light quarks – artificial process, but simplest to compute



Compare subtraction and *gg*-initiated part of SPS (all boxes, gauge-invariant). For comparison use:

$$\Sigma(\beta) = \frac{d\sigma}{dY\,d\beta}\,\frac{x\bar{x}}{g(x)g(\bar{x})}\,\frac{128\pi Q^2(N_c^2-1)}{c^2\alpha_s^2}$$

$$\beta = \sqrt{1 - 4Q^2/\hat{s}}$$

(Surprisingly) good agreement in overall order of magnitude between the two pieces – worsens towards $\beta \to 0$ (threshold) and $\beta \to 1$ (high energy).

Summary

- Power divergence in naive treatment of DPS including perturbative splittings (= 'leaking' of DPS into leading power SPS region).
- We have proposed a solution that retains the concept of a DPD for an individual hadron, and avoids double counting. Involves introduction of a regulator at the DPS cross section level, + a subtraction to remove double counting overlap between SPS and DPS.
- DPS luminosities: generically very large 1v1 with large uncertainty – have to compute SPS up to two-loop and subtraction. Possibility to avoid this for certain processes/regions (same sign WW, processes at small x).
- Study of ϕ pair production: indicates that subtraction gives a suitable order-of-magnitude estimate of SPS order containing DPS-like boxes

Summing DGLAP logarithms

DPDs are a matrix element of a product of twist 2 operators:

$$F(x_1, x_2, \boldsymbol{y}, \mu_1, \mu_2) = \langle p| \, \mathcal{O}_1(\mathbf{0}, \mu_1) \mathcal{O}_2(\boldsymbol{y}, \mu_2) \, |p\rangle \qquad \left[f(x,\mu) = \langle p| \, \mathcal{O}(\mathbf{0}, \mu) \, |p\rangle \right]$$

⇨ Separate DGLAP evolution for partons 1 and 2 $\frac{d}{d \log \mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$

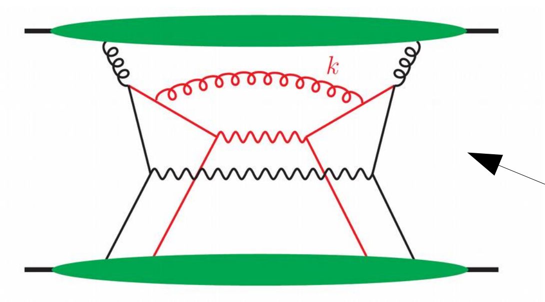
(same as for single PDF evolution)

Appropriate initial conditions for DPD are something like $F = F_{\text{split}} + F_{\text{intr}}$

$$F_{\text{split}}(x_1, x_2, \boldsymbol{y}; 1/y^{*}, 1/y^{*}) = F_{\text{perturb.}}(y^{*}) \, e^{-y^2 \Lambda^2} \quad \text{with} \quad 1/y^{*2} = 1/y^2 + 1/y_{\text{max}}^2$$

$F_{\text{intr}}(x_1, x_1, \boldsymbol{y}, \mu_0, \mu_0)$ = NP piece, something with smooth y dependence over scales of order proton radius

(for modelling we use $f(x_1; \mu_0) f(x_2; \mu_0) \, \Lambda^2 e^{-y^2 \Lambda^2} / \pi$)



Putting this information in and choosing μ_i, ν appropriately, we can sum up DGLAP logs appropriately in various scenarios

e.g. our DPS cross section contains the correct log²(Q/Λ) corresponding to this 2v1 diagram if we take $\mu_1 \sim \mu_2 \sim \nu \sim Q$

Extension to measured transverse momenta

So far just discussed DPS at the total cross section level.

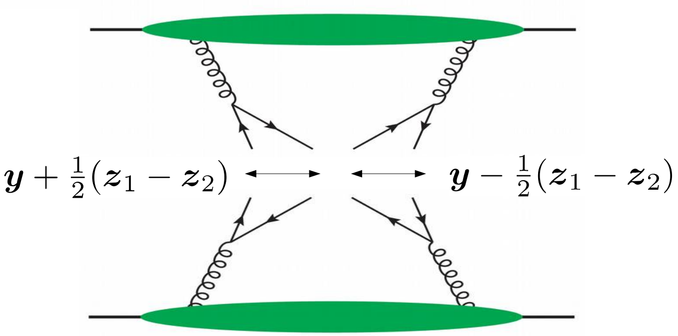
However, since DPS preferentially populates the small $\mathbf{q}_A$, $\mathbf{q}_B$ region, the transverse-momentum-differential cross section for the production of AB for small $\mathbf{q}_A$, $\mathbf{q}_B$ is also of significant interest. Need to adapt SPS TMD formalism to double scattering case.

Our scheme can be readily adapted to solve double counting issues in this case. DPS cross section involves the following regularised integral:

$$\int d^2\boldsymbol{y}\, d^2\boldsymbol{z}_1\, d^2\boldsymbol{z}_2\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, \Phi(\nu y_+)\, \Phi(\nu y_-)\, F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y})$$

Regulate (logarithmic) singularities in double perturbative splitting mechanism at the points

$$y_\pm \equiv \left|\boldsymbol{y} \pm \tfrac{1}{2}(\boldsymbol{z}_1 - \boldsymbol{z}_2)\right| = 0$$



Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

Previous attempts to handle these issues

Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.
JG and Stirling, JHEP 1106 048 (2011)
Blok et al. Eur.Phys.J. C72 (2012) 1963

Most popular suggestion previously:

- Completely remove 1v1 graphs from DPS cross section, and consider these as pure SPS (no natural part of these graphs to separate off as DPS).

- Put (part of) 2v1 graphs in DPS – sum logs of 1→2 splitting + DGLAP emissions in this contribution.

This scheme comes out if one chooses to regulate y integral using dim reg:

$$\int d^2y/y^4 \rightarrow \int d^{2-2\epsilon}y/y^4 = 0$$

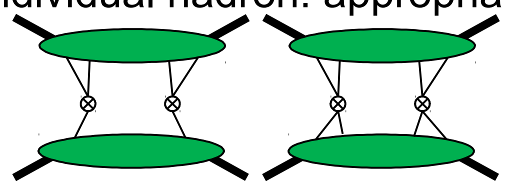
Manohar, Waalewijn Phys.Lett. 713 (2012) 196–201.

Drawback of this approach: The cross section can no longer be written as parton level cross sections convolved with overall DPD factors for each hadron.

$$\sigma^{DPS} = \cancel{\int d^2y F(y) F(y)} \rightarrow \text{2v2} \quad + \quad \text{2v1} \quad + \quad \text{1v2}$$

$$(A+B)^2 \quad \neq \quad A^2 + AB + BA$$

No concept of the DPD for an individual hadron: appropriate hadronic operators in DPS involve both hadrons at once!



Previous attempts to handle these issues

An alternative suggestion – just add a cut-off to the y integral at y values of order 1/Q

Ryskin, Snigirev, Phys.Rev.D83:114047,2011

$$\int \frac{d^2y}{y^4} \to \int_{|y|>1/Q} \frac{d^2y}{y^4}$$

(note that technically Ryskin, Snigirev impose the cutoff in the Fourier conjugate space, but the principle is the same)

This regulates the power divergence, but:

- there is now some double counting between DPS and SPS cross sections
- in general, a sizeable contribution to the 'double perturbative splitting' part of the DPS cross section comes from y values of order 1/Q, where the DPS picture is not valid.
- strong (quadratic) dependence of result on cut-off – why take cut-off of 1/Q rather than 1/(2Q) or 2/Q?