Double parton scattering
theory introduction

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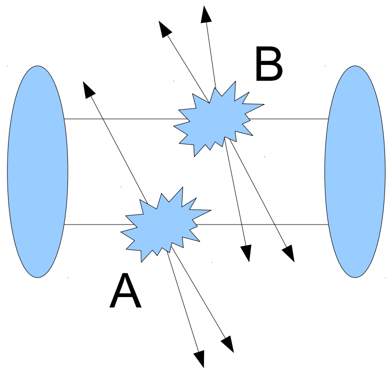
Why study DPS?



Double Parton Scattering (DPS) = when you have two separate hard interactions in a single proton-proton collision

In terms of total cross section for production of AB, DPS is power suppressed with respect to single parton scattering (SPS) mechanism: $\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \frac{\Lambda^2}{Q^2}$

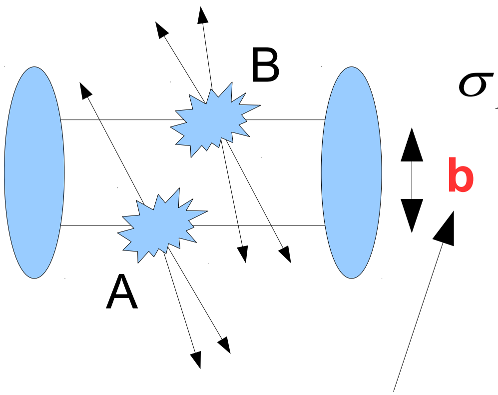
Why then should we study DPS?

1. DPS can compete with SPS if SPS process is suppressed by small/multiple coupling constants (same sign WW, H+W production). JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65
Del Fabbro, Treleani, Phys. Rev. D61 (2000) 077502
Bandurin, Golovanov, Skachkov, JHEP 1104 (2011) 054
2. DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.
3. DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller *x* values where there is a larger density of partons.
4. DPS reveals new information about the structure of the proton – in particular, correlations between partons in the proton.

Inclusive cross section for DPS

We know that in order to make a prediction for any process at the LHC, we need a factorisation formula (always hadrons/low energy QCD involved).

It's the same for double parton scattering. Postulated form for integrated double parton scattering cross section based on analysis of lowest order Feynman diagrams / parton model considerations:



$$\sigma_D^{(A,B)} = \overset{\text{Symmetry factor}}{\frac{m}{2}} \sum_{i,j,k,l} \int \overbrace{\Gamma_h^{ik}(x_1, x_2, \mathbf{b}; Q_A, Q_B)\Gamma_h^{jl}(x_1', x_2', \mathbf{b}; Q_A, Q_B)}^{\text{Collinear double parton distribution (DPD)}}$$
$$\times \underbrace{\hat{\sigma}_{ij}^{A}(x_1, x_1')\hat{\sigma}_{kl}^{B}(x_2, x_2')}_{\text{Parton level cross sections}} \mathrm{d}x_1\, \mathrm{d}x_1' \mathrm{d}x_2\, \mathrm{d}x_2' \mathrm{d}^2\mathbf{b}$$

N. Paver, D. Treleani, Nuovo Cim. A70 (1982) 215.
M. Mekhfi, Phys. Rev. D32 (1985) 2371.
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

b = separation in transverse space between the two partons

(n.b. $\Gamma = D = F$ in the following)

Simplifying assumptions often used

If one ignores correlations between partons in the proton:

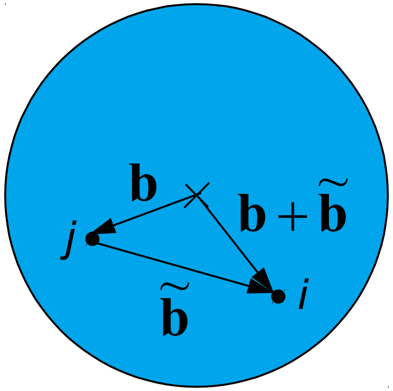
$$D_p^{ij}(x_1,x_2;\mathbf{b}) = \int d^2\tilde{\mathbf{b}}\, D_p^i(x_1;\tilde{\mathbf{b}}+\mathbf{b})\, D_p^j(x_2;\tilde{\mathbf{b}})$$

Impact parameter dependent PDFs

or equivalently

$$D_p^{ij}(x_1,x_2;\mathbf{\Delta}) \approx D_p^i(x_1;\mathbf{\Delta})\, D_p^j(x_2;-\mathbf{\Delta})$$

'GPD'



Common 'lore': approximately valid at low *x*, due to the large population of partons at such x values.

Further approximation that is often made: $D_p^i(x_1;\tilde{\mathbf{b}}) = D_p^i(x_1)\,\rho(\tilde{\mathbf{b}})$

$$\Rightarrow\; D_p^{ij}(x_1,x_2;\mathbf{b}) = D_p^i(x_1) D_p^j(x_1) \int d^2\tilde{\mathbf{b}}\, \rho(\tilde{\mathbf{b}}+\mathbf{b})\, \rho(\tilde{\mathbf{b}})$$

Several MCs (PYTHIA, HERWIG) use these approximations to model MPI

$$\Rightarrow\; \sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\sigma_S^{(B)}}{\sigma_{eff}}$$

Some refinements – e.g. *x* dependent proton size: Corke, Sjöstrand, JHEP 05 (2011) 009)

Many phenomenological estimates of DPS use this equation

Transverse-momentum-differential cross section

For region of small $\mathbf{q}_i$ << Q where DPS is enhanced, we actually need cross section differential in the $\mathbf{q}_i$

Differential DPS cross section from lowest order Feynman diagrams:

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))

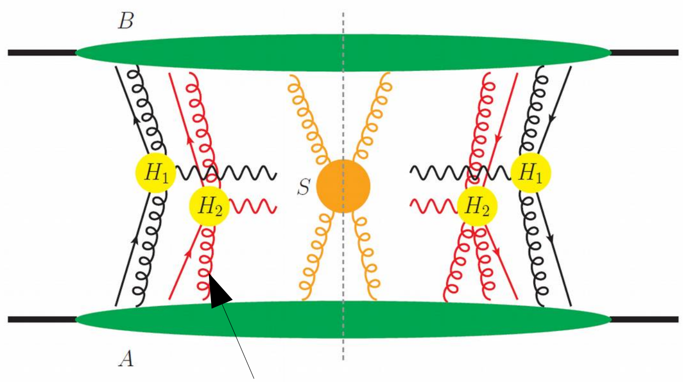
DTMD (double transverse-momentum-dependent PDF)

$$\frac{d\sigma_D^{(A,B)}}{d^2\mathbf{q}_1 d^2\mathbf{q}_2} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_h^{ik}(x_1,x_2,\mathbf{k}_1,\mathbf{k}_2,\mathbf{b})\Gamma_h^{jl}(x_1',x_2',\overline{\mathbf{k}}_1,\overline{\mathbf{k}}_2,\mathbf{b})$$
$$\times\hat{\sigma}_{ij}^A(x_1,x_1')\hat{\sigma}_{kl}^B(x_2,x_2')\mathrm{d}x_1\,\mathrm{d}x_1'\mathrm{d}x_2\,\mathrm{d}x_2'\mathrm{d}^2\mathbf{b}$$
$$\times\prod_{i=1,2}\int d^2\mathbf{k}_i d^2\overline{\mathbf{k}}_i\delta(\mathbf{k}_i+\overline{\mathbf{k}}_i-\mathbf{q}_i)$$

Complications: soft and collinear

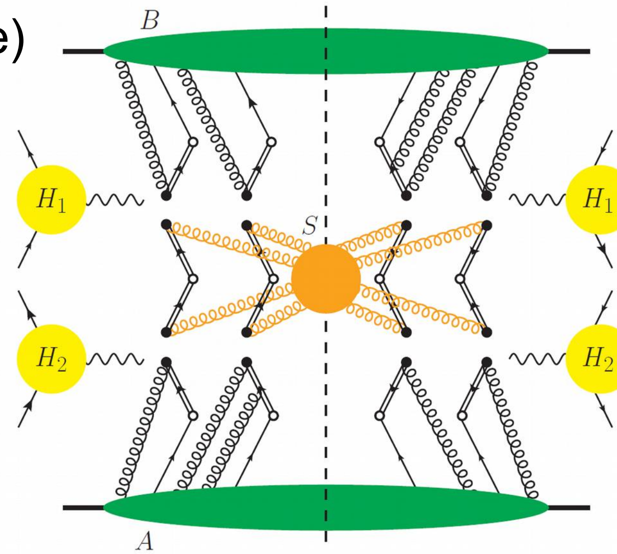
Getting parton-model style factorisation formula in full QCD is extremely nontrivial, and not guaranteed!

Leading contributions to double DY (TMD case)



Longitudinally polarised gluons

$\Rightarrow$



Ward identities, cancellation of Glauber modes

Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Diehl, Gaunt, Ostermeier, Plößl, Schafer (JHEP 1601 (2016) 076)

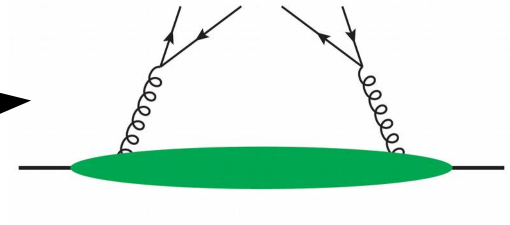
Split soft function in 'half', absorb half into each collinear factor

$$\sigma \propto |H_1|^2 \, |H_2|^2 \sum_R {}^R F_B \; {}^R F_A$$

See talk by M. Diehl

Complications: DPS/SPS overlap

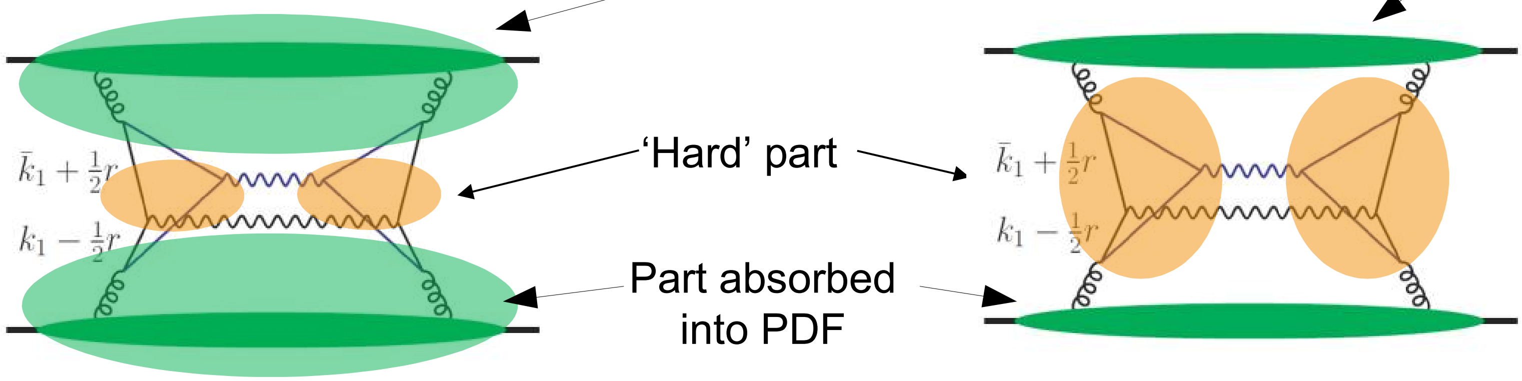
Parton pair entering DPS process can arise as a result of perturbative splitting



Naive addition of perturbative splitting piece to DPDs yields power divergence associated with diagrams where perturbative splittings occur in both hadrons:

$$\int \frac{d^2y}{y^4} = ?$$

This is related to the fact that such graphs can also be regarded as an SPS loop correction



$$\frac{\Lambda^2}{Q^4} \longrightarrow \frac{1}{Q^2}$$

Power divergence!

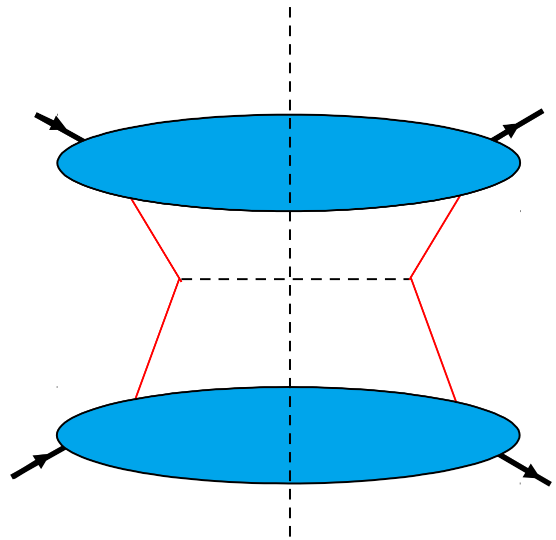
See talk by J. Gaunt

Complications: interference distributions

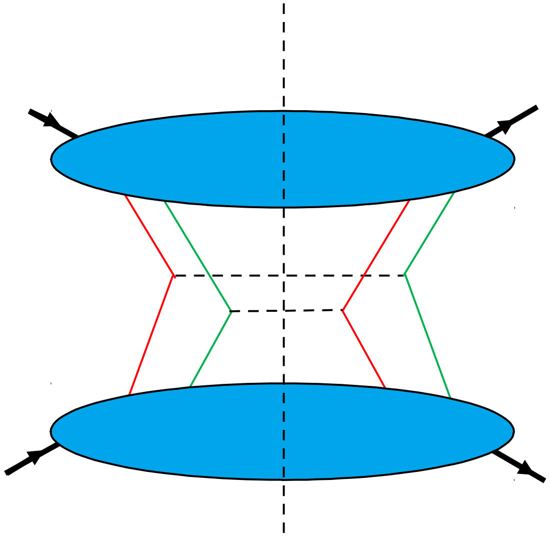
SPS: One parton per proton ‘leaves’, interacts and ‘returns’.

To reform proton, parton must return with same quantum numbers.

No interference contributions to SPS cross section.



Here we have two partons per proton interacting.



Interference contributions to total cross section in which quantum numbers are swapped between parton legs. Complementary swap is required in other proton.

Can get interference contributions in colour, spin, flavour, and quark number.

See talks by J. Gaunt, M. Diehl

Complications: correlation distributions

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g.

$$\Delta q_1 \Delta q_2 = \underbrace{q_1\!\uparrow q_2\!\uparrow + q_1\!\downarrow q_2\!\downarrow}_{\text{Same spin}} - \underbrace{q_1\!\uparrow q_2\!\downarrow - q_1\!\downarrow q_2\!\uparrow}_{\text{Opposing spin}}$$

When considering the differential DPS cross section and TMD DPDs, there are even more distributions to consider:

e.g. $D_p^{q\Delta q}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, \mathbf{b})$

Measures correlation between one of the quark spins and $\mathbf{k}_1$ x $\mathbf{k}_2$ (for example).

See talk by J. Gaunt

Simplifications in differential cross section: perturbative **q**

SPS:

If |**q**| >> Λ (but still << Q), then TMD can be written in terms of collinear PDFs and a perturbative factor.

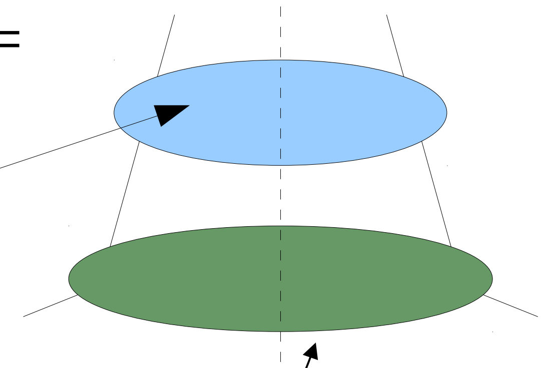
Collins, Soper, Sterman , Nucl.Phys. B250 (1985) 199
Collins, pQCD book, Ch. 13

$$F_h(x,\mathbf{k}) =$$

$$T(x,\mathbf{k})$$

$$\otimes$$

$$D_h(x, \mu^2 = \mathbf{k}^2)$$



Collinear (single) PDF

Indeed, at double leading logarithmic order, we obtain the DDT formula for the differential SPS cross section for |**q**| >> Λ :

$$\frac{d\sigma}{dq^2\, dq_\perp^2} = \frac{d\sigma_{\rm tot}}{dq^2} \times \frac{\partial}{\partial q_\perp^2} \left\{ D_a^q\left(x_1, q_\perp^2\right) D_b^{\bar{q}}\left(x_2, q_\perp^2\right) S_q^2\left(q^2, q_\perp^2\right) \right\}.$$

Sudakov factor

We expect there to be a similar relation between DPDs and TMD DPDs at perturbative transverse momenta. Various added complexities (interplay with argument **y**, colour...)

See talk by T. Kasemets

(Studies in double leading logarithmic order in Blok et al. Eur.Phys.J. C72 (2012) 1963)

DPD modelling

Even with a factorisation formula + perturbative evolution equations, one still needs low scale input double parton distributions (DPDs) to make predictions.

Much recent work using constituent quark models to study valence DPDs – detailed studies of various possible correlations using these models.

See talk by M. Rinaldi

Sum rules would provide valuable constraint in modelling DPDs. A set of sum rules was proposed by Gaunt & Stirling, but this predated a lot of the recent developments in DPS theory. Worthwhile to revisit sum rules and ask if and how exactly they apply to DPDs, and if relations survive renormalisation.

See talk by P. Plößl

DPS phenomenology

Various processes of interest to study DPS:

- Processes with heavy flavour (DPS and SPS total xsec comparable)
  - e.g. $c\bar{c}c\bar{c}$, $c\bar{c}$ jet jet

See talks by A. Szczurek, M. Strikman

- W/Z + jets (clean process + process with large rate)
- Jets (much larger rate, DPS extraction tough)

See talk by M. Serino

- Same sign WW (clean, rare)