Soft gluons and rapidity evolution in double parton scattering

M. Diehl

Deutsches Elektronen-Synchroton DESY

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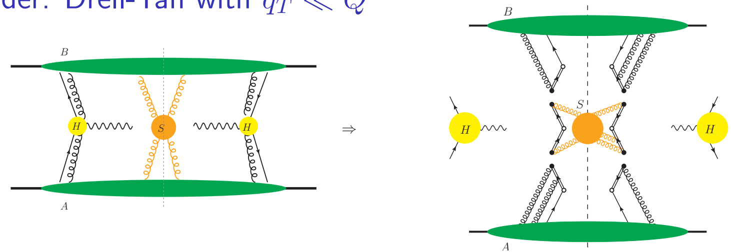
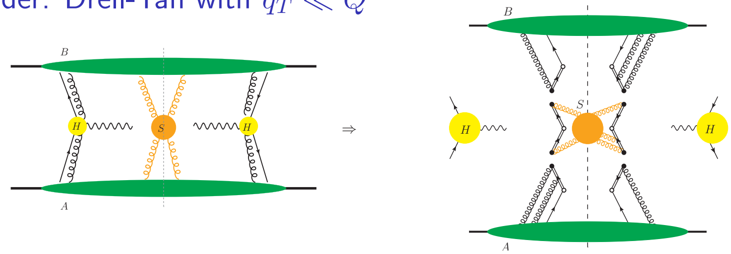
Reminder: Drell-Yan with $q_T \ll Q$



- fast-moving gluons coupling to hard scatter
  - include in Wilson lines in parton density
- soft gluon exchange between left- and right-moving partons
  - include in soft factors = vevs of Wilson lines
    needs: eikonal approximation, Ward identities, Glauber
  - essential for establishing factorisation
  - allows resummation of Sudakov logarithms
    TMD factorisation Collins, Soper, Sterman 1980s; Collins 2011

now give very simplified recap

Reminder: Drell-Yan with $q_T \ll Q$



- absorb soft factor into parton densities

$$\sigma = |H|^2\, BSA = |H|^2\, (B\sqrt{S})\,(\sqrt{S}A) = |H|^2\, f_B\, f_A$$

- S requires a rapidity cutoff for the gluons:

  right-moving gluons $\rightsquigarrow f_A$, left-moving ones $\rightsquigarrow f_B$

- separation at central rapidity Y

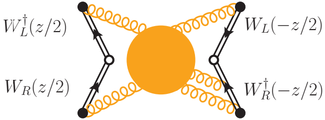
$$\zeta = 2(x p_A^+ e^{-Y})^2 \qquad \bar{\zeta} = 2(\bar{x} p_B^- e^{+Y})^2 \qquad \zeta\bar{\zeta} = Q^4$$

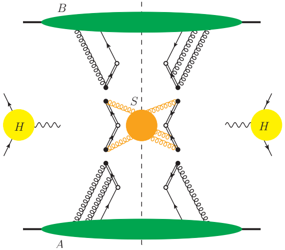
- resum Sudakov logarithms $\log(q_T/Q)$ via evolution equations

$$\frac{d}{d\log\zeta}\, f_A(\zeta) \quad \text{and} \quad \frac{d}{d\log\bar{\zeta}}\, f_B(\bar{\zeta})$$

Reminder: Drell-Yan with $q_T \ll Q$

$$W(z) = \mathrm{P}\exp\left[-igt^a \int_{-\infty}^{0} d\lambda\, vA^a(\lambda v + z)\right]$$





- transverse variables
  - $\boldsymbol{z}$ Fourier conjugate to $\boldsymbol{q}$:
    $$d\sigma/d^2\boldsymbol{q} \propto \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\boldsymbol{q}} f_A(x,\boldsymbol{z};\zeta) f_B(\bar{x},\boldsymbol{z};\bar{\zeta})$$
  - soft factor $S = \frac{1}{N_c} \langle 0 | \operatorname{tr} W_L^\dagger(\frac{z}{2})\, W_R(\frac{z}{2})\, W_R^\dagger(-\frac{z}{2})\, W_L(-\frac{z}{2}) | 0 \rangle$
  - collinear factorisation: in $\int d^2\boldsymbol{q}\;\, d\sigma/d^2\boldsymbol{q}$ have $\boldsymbol{z} = \boldsymbol{0}$

    $\Rightarrow \quad S = 1$

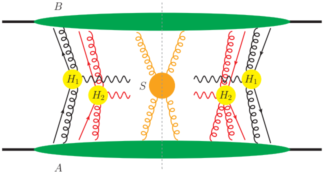
    $\rightsquigarrow$ soft gluon exchanges cancel in sum over all graphs

    $\rightsquigarrow$ no Sudakov logarithms

Double parton scattering

- aim: generalise previous treatment from single to double Drell-Yan and other DPS processes

M Buffing, T Kasemets, MD work in progress



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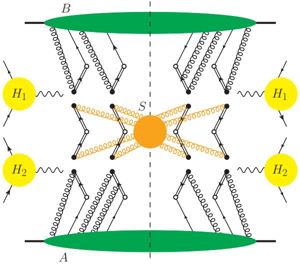
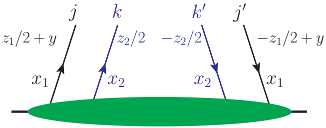


- basic steps can be repeated:
  - collinear gluons ⇝ Wilson lines in DPDs
  - soft gluons ⇝ soft factor

MD, D Ostermeier, A Schäfer 2011; MD, J Gaunt, P Plößl, A Schäfer 2015

Double parton scattering: colour complications

- DPDs have several colour combinations of partons



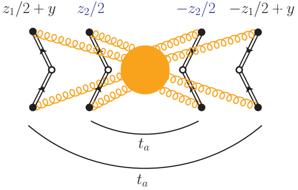
- colour projection operators
- singlet: $P_1^{jj',kk'} = \delta^{jj'}\delta^{kk'}/N_c$ as in usual PDFs
- octet: $P_8^{jj',kk'} = 2t_a^{jj'}\,t_a^{kk'}$
- for gluons: $8^A, 8^S, 10, \overline{10}, 27$

- corresponding combinations in soft factor
  - soft factor $\to$ matrix in colour space
  - for colour octet (and other non-singlets): $W_R t^a W_R^\dagger \neq 1$ when at same position

    $$\Rightarrow \quad S \neq 1$$

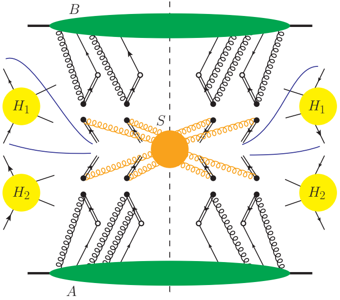
    $\rightsquigarrow$ Sudakov factors even in collinear factoris'n

    M Mekhfi 1988; A Manohar, W Waalewijn 2012



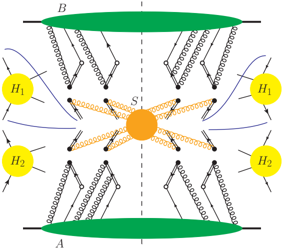
Coloured final states

- processes with coloured final states (jets etc)
  collinear factorisation only
  with measured small q_T no factorisation even for single scattering



- soft factor with more open colour indices
- to be contracted with hard scattering
- for large distance y non-perturbative

- looks grim for phenomenology ...



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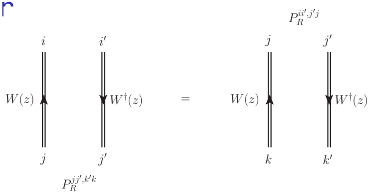
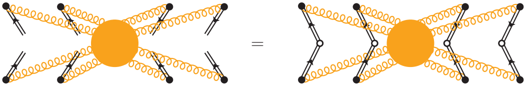
meet the locals: Cañon del Sumidero

Simplification for collinear factorisation



- projector identity for Wilson lines at same position
- also for adjoint Wilson lines (gluons) and mixed case
- use to show
  - S for jet production etc. same as for Drell-Yan:



  - $S(y)$ is diagonal in colour:

$$ {}^{RR'}S(y) \propto \delta^{RR'} \quad \text{with } R = 1, 8, \ldots $$

and octet ${}^{88}S(y)$ is same for quarks and gluons

Collinear factorisation

- in collinear factorisation simple colour structure

$$\sigma_{\text{DPS}} \sim \sum_R |{}^R H_1|^2 \, |{}^R H_2|^2 \int d^2\boldsymbol{y} \; {}^R F_B(\boldsymbol{y}) \, {}^R F_A(\boldsymbol{y})$$

with ${}^R F_A = \sqrt{{}^{RR}S}\,{}^R A$ and ${}^R F_B$ likewise

- evolution of ${}^R F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta)$:

$$\frac{2\partial}{\partial \log \zeta} {}^R F = {}^R J(\boldsymbol{y}; \mu_1, \mu_2) {}^R F \qquad \frac{\partial}{\partial \log \mu_1} {}^R J = - {}^R\gamma_J(\mu_1)$$

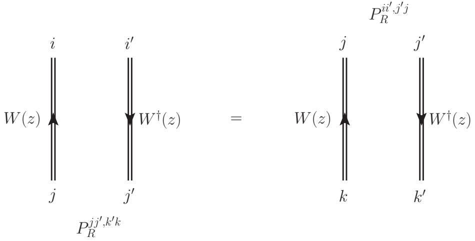
$$\frac{\partial}{2\,\partial \log \mu_1} {}^R F = {}^R P(\mu_1, \zeta) \underset{x_1}{\otimes} {}^R F \qquad \frac{4\partial}{\partial \log \zeta} {}^R P = - {}^R\gamma_J \, \delta(1-x)$$

  - can choose separate factorisation scales μ_1, μ_2 for hard scatters
  - for colour singlet have ${}^1 J = 0$

- solution has form

$${}^R F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta) = e^{-{}^R S(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta)} \; {}^R \widehat{F}(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2)$$

where ${}^R \widehat{F}$ evolves with ${}^R P$ at $\zeta = \mu$; for colour singlet have ${}^1 S = 1$



$\Rightarrow$

another local, Cañon del Sumidero

TMD factorisation

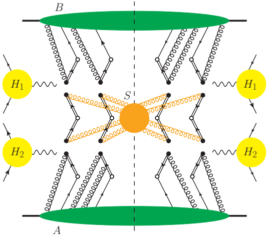
- ${}^{RR'}S = \underline{S}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; Y)$ is matrix in colour space
- rapidity evolution of $\underline{S}$ understood at perturbative two-loop level

A Vladimirov 2016

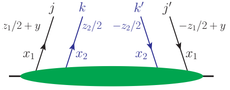
- assume that general structure valid beyond two loops

$$\frac{\partial}{\partial Y}\, \underline{S}(Y) = \underline{\widehat{K}}\, \underline{S}(Y) \quad \text{for} \quad Y \gg 1 \quad \text{and} \quad \underline{S}(Y) > 0$$

- define $\underline{F}_A = \underline{s}\,\underline{A}$ ($\underline{s}$ = matrix equivalent of $\sqrt{S}$)
- cross section $\sigma \propto |H_1|^2\, |H_2|^2 \sum_R {}^{R}F_B\; {}^{R}F_A$



TMD factorisation: evolution



- evolution of ${}^{R}F(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; \mu_1, \mu_2, \zeta)$

$$\frac{\partial}{\partial \log \zeta} \underline{F} = \underline{K}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}; \mu_1, \mu_2)\, \underline{F} \qquad \frac{\partial}{\log \mu_1} \underline{K} = \underline{\mathbb{1}}\, \gamma_K(\mu_1)$$

$$\frac{\partial}{\log \mu_1} \underline{F} = \gamma_F(\mu_1, x_1\zeta/x_2) \qquad \frac{\partial}{\partial \log \zeta} \gamma_F = \gamma_K$$

  - γ_F and γ_K same as for single-parton TMDs
    where have Collins-Soper kernel $K(\boldsymbol{z}, \mu)$
  - write $\underline{K} = \underline{\mathbb{1}}\, [K(\boldsymbol{z}_1, \mu_1) + K(\boldsymbol{z}_2, \mu_2)] + \underline{M} \quad \Rightarrow \quad \underline{M}$ indep't of $\mu_{1,2}$

- solution $\underline{F}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \zeta) = e^{-S(\boldsymbol{z}_1; \mu_1, x_1\zeta/x_2) - S(\boldsymbol{z}_2; \mu_2, x_2\zeta/x_1)}$

$$\times\, e^{\underline{M}(\boldsymbol{z}_i, \boldsymbol{y}) \log(\zeta/\zeta_0)}\, \underline{F}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_0, \mu_0, \zeta_0)$$

  $S(\boldsymbol{z}; \mu, \zeta) =$ Sudakov factor for single-parton TMD
  contains double logarithm, colour independent

- further simplification if $\boldsymbol{z}_{1,2}$ and/or $\boldsymbol{y}$ small
  ⇝ talk by T Kasemets

Summary

- colour structure of DPS is $\gg$ complicated than SPS
  especially due to soft gluon exchange
- significant simplifications for collinear factorisation
  due to projector identity for Wilson lines
  - no cross talk between different colour representations R
  - all R except for colour singlet Sudakov suppressed
- rapidity evolution for TMD factorisation $\rightarrow$ matrix in colour space
  - but: Sudakov double logarithms = two copies of SPS