Matching and resummation in double parton scattering

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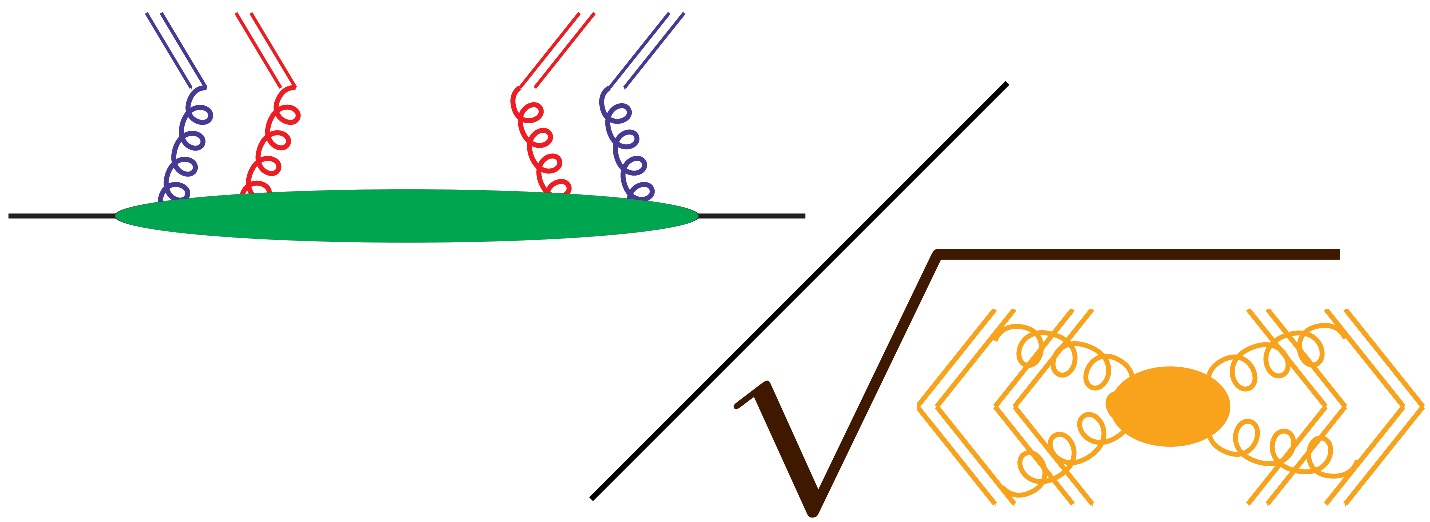
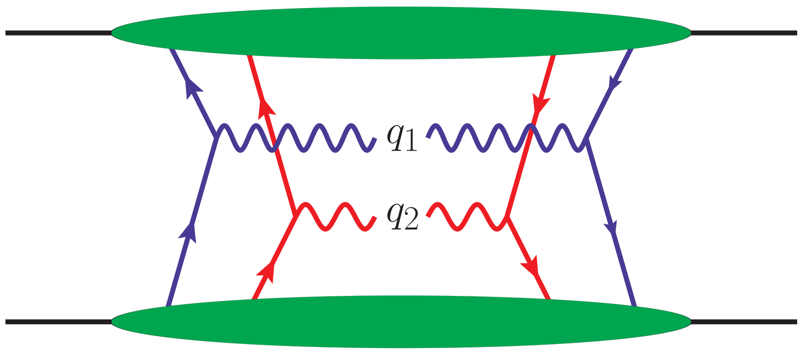
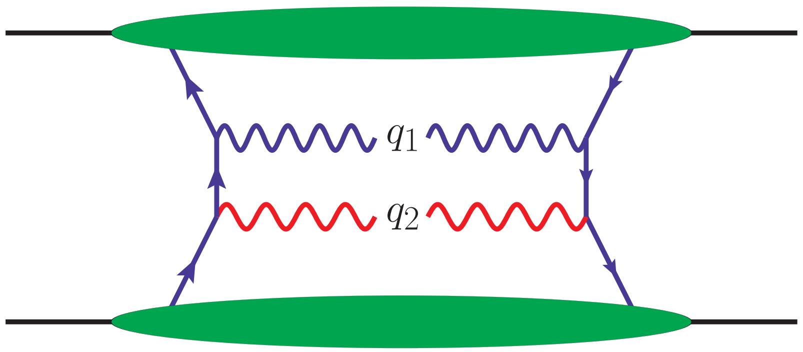


Based on work with Maarten Buffing and Markus Diehl

MPI@LHC - San Cristóbal de las Casas, November 29, 2016

DPS differential in transverse momenta





- Total cross section

$$\sigma_{\mathrm{DPS}}/\sigma_{\mathrm{SPS}} \sim \frac{\Lambda^2}{Q^2}$$

- DPS populates final state phase space in a different way than SPS

$$|\boldsymbol{q}_1|,\ |\boldsymbol{q}_2| \sim \Lambda << Q: \quad \frac{d\sigma_{SPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{DPS}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\Lambda^2}$$

  DPS same power as SPS

- Makes small transverse momentum region a very interesting region for DPS
- Any factorization theorem for this region, must include both single and double parton scattering

Lessons from TMD factorization and pT resummation

- TMD Drell-Yan cross section (unpolarized)

$$\frac{d\sigma}{dx d\bar{x} d^2\boldsymbol{q}} = \sum_q \hat{\sigma}_{q\bar{q}}(q^2,\mu^2) \int \frac{d^2\boldsymbol{z}}{(2\pi)^2} e^{-i\boldsymbol{q}\boldsymbol{z}}\, W_{q\bar{q}}(x,\bar{x},\boldsymbol{z};\mu)$$

  with

  TMDs: $W_{q\bar{q}} = f_q(x,\boldsymbol{z};\mu,\zeta) f_{\bar{q}}(\bar{x},\boldsymbol{z};\mu,\bar{\zeta})$

  and born x hard-matching $\hat{\sigma}_{q\bar{q}}(q^2,\mu^2) = \hat{\sigma}^0_{q\bar{q}} C_H(q^2,\mu^2)$

- TMDs defined as combination of soft and collinear to cancel rapidity divergencies

  Collins, 2011; Echevarria, Idilbi, Scimemi, 2011; Echevarria, TK, Mulders, Pisano, 2015

- Depends on two scales, UV and rapidity regularization.

$$\frac{\partial}{\partial \log\mu} f_a(x,\boldsymbol{z};\mu,\zeta) = \gamma_{F,a}(\mu,\zeta)\, f_a(x,\boldsymbol{z};\mu,\zeta) \qquad \frac{\partial}{\partial \log\mu} K_a(\boldsymbol{z};\mu) = -\gamma_{K,a}(\mu)$$

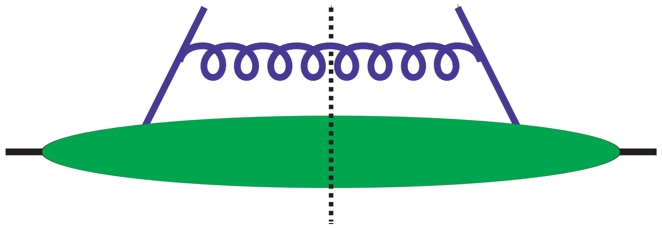
$$\frac{\partial}{\partial \log\zeta} f_q(x,\boldsymbol{z};\zeta,\mu) = \frac{1}{2} K_q(\boldsymbol{z})\, f_q(x,\boldsymbol{z};\zeta,\mu)\,. \qquad \frac{\partial}{\partial \log\zeta} \gamma_{F,a}(\mu,\zeta) = -\frac{1}{2}\gamma_{K,a}(\mu)\,.$$

$$\gamma_K = \Gamma_{\text{cusp}}$$

TMD factorization and pT resummation

- For perturbatively small $\boldsymbol{z}$, can match TMDs onto PDFs

$$f_a(x,\boldsymbol{z};\zeta,\mu) = \sum_b C_{ab}(x',\boldsymbol{z};\zeta,\mu) \underset{x}{\otimes} f_b(x';\mu)\,,$$

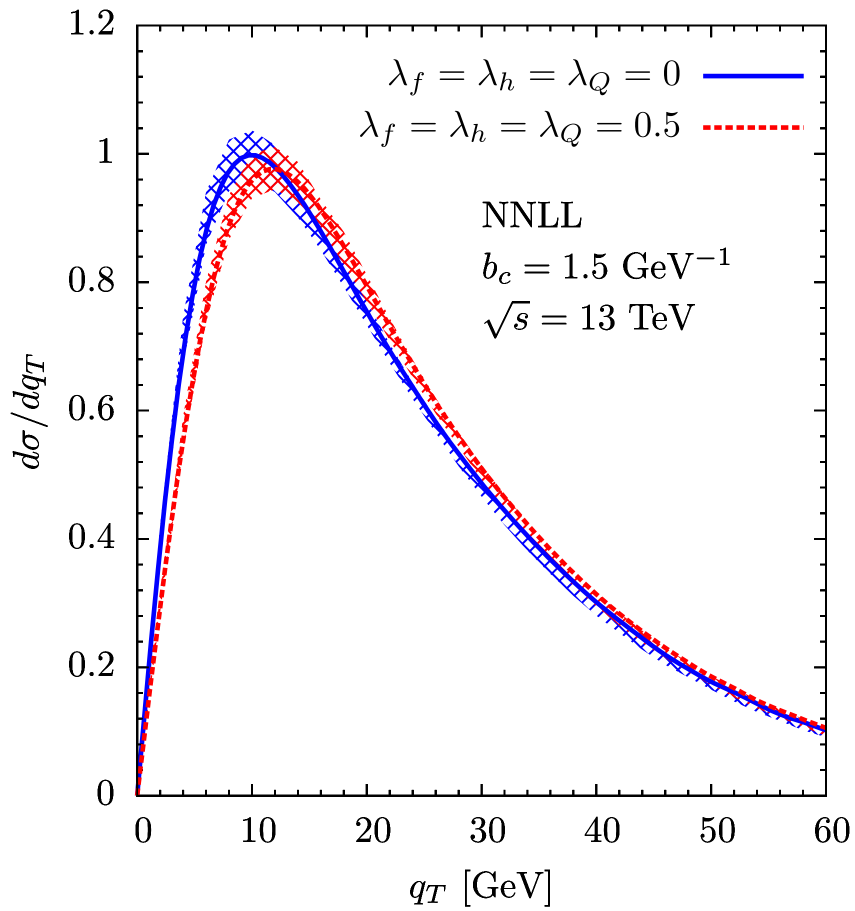


- Solving the evolution equations gives the evolved TMDs

$$f_a(x,\boldsymbol{z};\mu,\zeta) = \sum_b C_{ab}(x,\boldsymbol{z};\mu_0,\mu_0^2) \underset{x}{\otimes} f_b(x';\mu_0,\zeta_0)$$

$$\times \exp\left\{\int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu}\left[\gamma_{F,a}(\mu,\mu^2) - \gamma_{K,a}(\mu)\log\frac{\sqrt{\zeta}}{\mu}\right] + {}^1K_a(\boldsymbol{z},\mu_0)\log\frac{\sqrt{\zeta}}{\mu_0}\right\}$$

- Can be supplemented by non-perturbative transverse momentum dependence etc.
- Perturbative input alone gives pT resummed cross section
- High scale processes, e.g. Higgs: small dependence on non-pert. input



Echevarria, TK, Mulders, Pisano, 2015

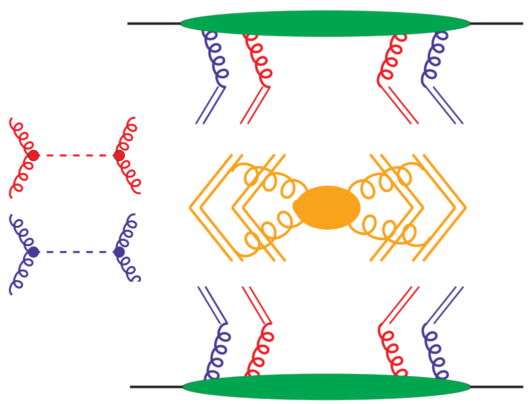
Goal of project:

- Set up the theoretical (DTMD) framework, within QCD
  - As few assumptions as possible
  - As much perturbative input as possible, to enhance predictive power
- Provide the basis, correctly including and treating the different effects.
  - Once set up in place, can introduce modeling and approximations to connect with experiments
- Additional difficulties compared to TMDs for SPS
  - Different regions which require different matchings
  - Color (and polarization) structure
  - etc.

talk by Markus Diehl

- Compared to the pocket formula, it represents the other end of DPS research

Soft and collinear functions

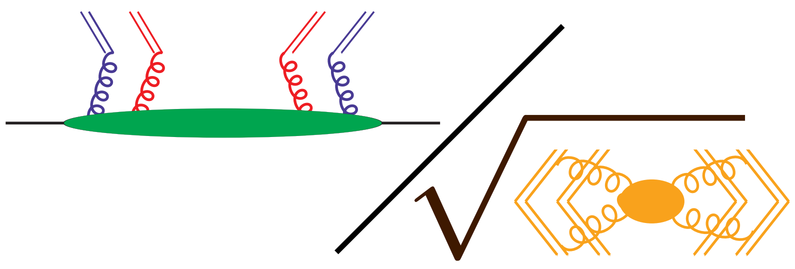


- DPS cross section proportional to

$$F^T_{\mathrm{us},gg}(Y_R)\, s^{T-1}(Y_R - Y_C)\, s^{-1}(Y_C - Y_L)\, F_{\mathrm{us},gg}(Y_L) = F^T_{gg}(Y_C)\, F_{gg}(Y_C)$$

- We define rapidity divergency free DTMDs as

$$F_{gg}(Y_C) = \lim_{Y_L \to -\infty} s^{-1}(Y_C - Y_L)\, F_{\mathrm{us},gg}(Y_L)\,,$$



- Collinear matrix element

$$F_{\mathrm{us},gg}(x_1, x_2, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) \sim \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi}\, dy^-\, e^{-i(x_1 z_1^- + x_2 z_2^-)p^+} \times \langle p|\, \mathcal{O}_g(0, z_2)\, \mathcal{O}_g(y, z_1)\, |p\rangle\,,$$

Diehl, Schäfer, Ostermeier, 2011

operators dressed by Wilson lines (adjoint rep.)

$$\mathcal{O}_{g_i}(y, z_i) = g_{T\mu\nu}\, \mathcal{W}^\dagger G^{+\nu}\, \mathcal{W} G^{+\mu}\Big|_{z_i^+ = y^+ = 0},$$

- Soft function, matrix in color space

$$S \sim \langle 0\,|\mathcal{W}\mathcal{W}^\dagger\, \mathcal{W}\mathcal{W}^\dagger \mathcal{W}\mathcal{W}^\dagger\, \mathcal{W}\mathcal{W}^\dagger|\,0\rangle$$

talk by Markus Diehl

perturbative calculation at NNLO

Vladimirov, 2016

DTMD cross section

- For color singlet production (photon, z, Higgs etc.) at $|\boldsymbol{q}_{1,2}| \sim q_T \ll Q$

$$\frac{d\sigma_{\mathrm{DPS}}}{dx_1\, dx_2\, d\bar{x}_1\, d\bar{x}_2\, d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} = \frac{1}{C} \sum_{a_1,a_2,b_1,b_2} \hat{\sigma}_{a_1 b_1}(q_1^2, \mu_1^2)\, \hat{\sigma}_{a_2 b_2}(q_2^2, \mu_2^2)$$

$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, d^2\boldsymbol{y}\, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2}\, W_{a_1a_2b_1b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu)$$

with:
$$W = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \sum_R {}^R F_{b_1b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta})\, {}^R F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$$

- $\Phi(\nu \boldsymbol{y}_\pm)$ removes UV region $\boldsymbol{y}_\pm \ll 1/\nu$. Choose $\nu \sim Q$. $\boldsymbol{y}_\pm = \boldsymbol{y} \pm \frac{1}{2}(\boldsymbol{z}_1 - \boldsymbol{z}_2)$
  Φ dependence cancelled by subtraction talk by Jo Gaunt

- Double TMDs (DTMDs) ${}^R F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta)$ depend on:
  $R = 1, 8, \ldots$ color label, $a_{1,2}$, $b_{1,2}$ = parton and polarization label
  $x_{1,2}$ = momentum fractions, $\boldsymbol{y}$, $\boldsymbol{z}_{1,2}$ = transverse distances
  $\mu_{1,2}$ = UV renormalization scales, ζ = rapidity regularization scale, $\zeta\bar{\zeta} = Q_1^2 Q_2^2$

Scale evolution

- UV and rapidity scale

$$\frac{\partial}{\partial \log \mu_1} \, {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \gamma_{F,a_1}(\mu_1, x_1\zeta/x_2) \, {}^{R}F_{a_1 a_2}$$

$$\frac{\partial}{\partial \log \zeta} \, {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) = \frac{1}{2} \, {}^{RR'}K_{a_1 a_2}(\boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{y}) \, {}^{R'}F_{a_1 a_2}$$

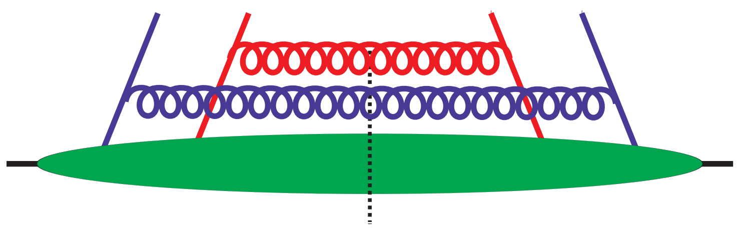
- Complicated functions (3 transverse vectors!), little predictive power
- When $\Lambda \ll q_T \ll Q$: $|\boldsymbol{q}_1| \sim |\boldsymbol{q}_2| \sim |\boldsymbol{q}_1 \pm \boldsymbol{q}_2| \sim q_T$

$$\int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2} \, d^2\boldsymbol{y} \, e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2} \, W_{a_1 a_2 b_1 b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_1, \mu_2, \nu)$$

  then region of perturbative $|\boldsymbol{z}_i| \sim 1/q_T$ dominates result

- But what about the size of $\boldsymbol{y}$
  — can be either small $|\boldsymbol{y}| \sim 1/q_T$ or large $|\boldsymbol{y}| \sim 1/\Lambda$

Region of large $\boldsymbol{y}$



- Scalings $|\boldsymbol{z}_i| \sim \dfrac{1}{q_T}, |\boldsymbol{y}| \sim \dfrac{1}{\Lambda}$ $\qquad \Lambda \ll q_T \ll Q$
- Match DTMDs onto the DPDFs

$$ {}^{R}F_{a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = \sum_{b_1 b_2} {}^{R}C_{f,a_1 b_1}(x_1', \boldsymbol{z}_1) \underset{x_1}{\otimes} {}^{R}C_{f,a_2 b_2}(x_2', \boldsymbol{z}_2) \underset{x_2}{\otimes} {}^{R}F_{b_1 b_2}(x_i', \boldsymbol{y}) $$

  - Mixing between quark and gluon distributions
  - Combine ${}^{RR}S_{qq}$ and ${}^{R}F_{\mathrm{us},a_1 b_1}$ into subtracted DTMD possible since ${}^{RR}S_{qq}(\boldsymbol{y}) = {}^{RR}S_{gg}(\boldsymbol{y})$ (independent of parton type)

- We calculate soft function and matching coefficients at one-loop order (all parton types, polarizations and color representations, CSS and SCET)
  - Coefficients equal to TMDs — PDFs matching coeffs. appart from:
    1) Color factors for non-singlet
    2) Different vector dependence, since DTMDs and DPDs are parametrized in terms of same distance between partons
    3) additional polarizations possible

Region of large $\boldsymbol{y}$

- Rapidity evolution kernel simplifies considerably

$$^{RR'}K_{a_1a_2}(\boldsymbol{z}_i, \boldsymbol{y}; \mu_i) = \delta_{RR'} \left[{}^{R}K_{a_1}(\boldsymbol{z}_1; \mu_1) + {}^{R}K_{a_2}(\boldsymbol{z}_2; \mu_2) + {}^{R}J(\boldsymbol{y}; \mu_i) \right]$$

  - Diagonal in color, distance dependence separated
    ${}^{1}K_{a_1}(\boldsymbol{z}_1; \mu_1)$ usual Collins-Soper kernel
  - ${}^{R}J(\boldsymbol{y}; \mu_i)$ remains for DPDFs (rapidity scale evolution for collinear func.)

- Solution to evolution equations:

$$\begin{aligned}
&{}^{R}F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta) \\
&\quad = \sum_{b_1 b_2} {}^{R}\Big[C_{a_1b_1}(\boldsymbol{z}_1) \underset{x_1}{\otimes} C_{a_2b_2}(\boldsymbol{z}_2) \underset{x_2}{\otimes} F_{b_1b_2}(x_i', \boldsymbol{y}; \mu_{0i}, \zeta_0) \Big] \\
&\qquad \times \exp\Bigg\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1} - \gamma_{K,a_1} \log \frac{\sqrt{x_1\zeta/x_2}}{\mu} \right] + {}^{R}K_{a_1}(\boldsymbol{z}_1) \log \frac{\sqrt{x_1\zeta/x_2}}{\mu_{01}} \\
&\qquad\qquad + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2} - \gamma_{K,a_2} \log \frac{\sqrt{x_2\zeta/x_1}}{\mu} \right] + {}^{R}K_{a_2}(\boldsymbol{z}_2) \log \frac{\sqrt{x_2\zeta/x_1}}{\mu_{02}} \\
&\qquad\qquad + {}^{R}J(\boldsymbol{y}) \log \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \Bigg\}
\end{aligned}$$

Region of large $\boldsymbol{y}$

- Cross section for large y:

$$W_{\text{large } \boldsymbol{y}} = \sum_{c_1 c_2 d_1 d_2,\, R} \big[\Phi(\nu \boldsymbol{y})\big]^2 \exp\left[{}^{R}J(\boldsymbol{y}, \mu_{0i}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right]$$

$$\times \exp\left\{ \int_{\mu_{01}}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1}(\mu, \mu^2) - \gamma_{K,a_1}(\mu) \log \frac{q_1^2}{\mu^2} \right] + {}^{R}K_{a_1}(\boldsymbol{z}_1, \mu_{01}) \log \frac{q_1^2}{\mu_{01}^2} \right.$$

$$\left. + \int_{\mu_{02}}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2}(\mu, \mu^2) - \gamma_{K,a_2}(\mu) \log \frac{q_2^2}{\mu^2} \right] + {}^{R}K_{a_2}(\boldsymbol{z}_2, \mu_{02}) \log \frac{q_2^2}{\mu_{02}^2} \right\}$$

$$\times {}^{R}\Big[C_{b_1 d_1}(\boldsymbol{z}_1) \underset{\bar{x}_1}{\otimes} C_{b_2 d_2}(\boldsymbol{z}_2) \underset{\bar{x}_2}{\otimes} F_{c_1 c_2}(x_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \Big]$$

$$\times {}^{R}\Big[C_{a_1 c_1}(\boldsymbol{z}_1) \underset{x_1}{\otimes} C_{a_2 c_2}(\boldsymbol{z}_2) \underset{x_2}{\otimes} F_{d_1 d_2}(\bar{x}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \Big]$$

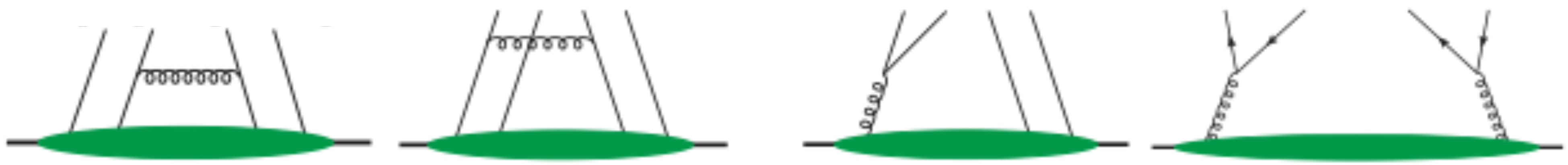
- Non-perturbative input: — collinear DPDFs, only one transverse distance and several model calculations available

talk by Matteo Rinaldi

  - at large scales, color singlet distributions dominate
  - ideal future, measured distributions — still a long way to go

Region of small $\boldsymbol{y}$

- Scaling: $\boldsymbol{y} \sim 1/q_T \sim \boldsymbol{z}_i$
- Soft function perturbatively calculable ${}^{RR'}S_{a_1a_2}(\boldsymbol{z}_i, \boldsymbol{y}) = {}^{RR'}C_{s,a_1a_2}(\boldsymbol{z}_i, \boldsymbol{y})$
- Expand on collinear distributions (all fields at same position)



$$F_{\text{intr}} = G + C \otimes G \sim \Lambda^2,\ G = \text{twist 4},\ C \propto \alpha_s$$

$$F_{\text{tw3}},\quad \text{only chiral odd, discard}$$

$$F_{\text{spl}} \sim \frac{\boldsymbol{y}_+}{\boldsymbol{y}_+^2}\frac{\boldsymbol{y}_-}{\boldsymbol{y}_-^2} T \cdot f(x_1 + x_2) \sim q_T^2,\ f = \text{PDF},\ T \propto \alpha_s$$

- ${}^{R}F = {}^{R}F_{\text{split}} + {}^{R}F_{\text{intr}}$

- Size of the contributions

$$\int d^2\boldsymbol{y} W(\boldsymbol{z}_i, \boldsymbol{y})\Big|_{\text{small } \boldsymbol{y}} \sim \begin{cases} \alpha_s^2 q_T^2 & \text{from } F_{\text{split}} \times F_{\text{split}}\ (1\text{vs}1) \\ \alpha_s \Lambda^2 & \text{from } F_{\text{split}} \times F_{\text{intr}}\ (1\text{vs}2) \\ \Lambda^4/q_T^2 & \text{from } F_{\text{intr}} \times F_{\text{intr}}\ (2\text{vs}2) \end{cases}$$

Region of small $\boldsymbol{y}$

- DPS cross section contribution

$$W_{\text{small } \boldsymbol{y}} = \exp\left\{ \int_{\mu_0}^{\mu_1} \frac{d\mu}{\mu} \left[\gamma_{F,a_1} - \gamma_{K,a_1} \log \frac{q_1^2}{\mu^2}\right] + {}^{1}K_{a_1}(\boldsymbol{z}_1, \mu_0) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right.$$
$$\left. + \int_{\mu_0}^{\mu_2} \frac{d\mu}{\mu} \left[\gamma_{F,a_2} - \gamma_{K,a_2} \log \frac{q_2^2}{\mu^2}\right] + {}^{1}K_{a_2}(\boldsymbol{z}_2, \mu_0) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right\}$$
$$\times \sum_{RR'} \left[{}^{R}F_{\text{spl+int},\, b_1 b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]^{RR'} \exp\left[M_{a_1 a_2}(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0}\right]$$
$$\times \left[{}^{R'}F_{\text{spl+int},\, a_1 a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]$$

- Non-perturbative input: PDF
  and twist-four collinear (all color representations)
  - Twist-four contribution can (at leading order) be modeled through DPDFs (part of both DTMDs and DPDFs in this region can be matched onto twist four distributions)

Combine regions

- Contributions from the two regions:

$$W_{\text{large } \boldsymbol{y}} = \left[\Phi(\nu \boldsymbol{y})\right]^2 \sum_R \exp\left\{ {}^R S(\boldsymbol{z}_1) + {}^R S(\boldsymbol{z}_1) \right\} \exp\left[{}^R J(\boldsymbol{y}, \mu_{0i}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right]$$

$$\times\, {}^R\!\left[C(\boldsymbol{z}_1) \underset{\bar{x}_1}{\otimes} C(\boldsymbol{z}_2) \underset{\bar{x}_2}{\otimes} F(\bar{x}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right] {}^R\!\left[C(\boldsymbol{z}_1) \underset{x_1}{\otimes} C(\boldsymbol{z}_2) \underset{x_2}{\otimes} F(x_i, \boldsymbol{y}; \mu_{0i}, \zeta_0)\right]$$

$$W_{\text{small } \boldsymbol{y}} = \Phi(\nu \boldsymbol{y}_+)\Phi(\nu \boldsymbol{y}_-) \exp\left\{ {}^1 S(\boldsymbol{z}_1) + {}^1 S(\boldsymbol{z}_2) \right\} \sum_{RR'} \left[{}^R F_{\text{spl+int}}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right]$$

$$\times\, {}^{RR'} \exp\left[M(\boldsymbol{z}_i, \boldsymbol{y}) \log \frac{\sqrt{q_1^2 q_2^2}}{\zeta_0} \right] \left[{}^{R'} F_{\text{spl+int}}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_{0i}, \zeta_0) \right]$$

- Combine large and small y:

$$W = W_{\text{large } \boldsymbol{y}} - W_{\text{subt}} + W_{\text{small } \boldsymbol{y}}$$

  - Collins subtraction formalism $W_{\text{subt}} = W_{\text{large } \boldsymbol{y}}\big|_{|\boldsymbol{y}| \ll 1/\Lambda}$ talk by Jo Gaunt
    or $W_{\text{subt}} = W_{\text{small } \boldsymbol{y}}\big|_{|\boldsymbol{z}_i| \ll |\boldsymbol{y}|}$, equal up to differences in scale choice (beyond accuracy for suitable choices)

What can we do with this

- Describe the DPS cross section for double color singlet production differential in the transverse moment of each of the singlets
  - Region where DPS is large and at same power as single parton scattering
- Example processes:
  - Double same-sign W (where DPS is enhanced over SPS)
  - Other diboson produciton, ZZ, HW, etc
  - Color singlet quarkonia
- Perturbative input alone gives transverse momentum resummed cross section (same non-perturbative input as collinear DPS)
- Provides a framework where we know what we neglect in phenomenology, and where we can study the common approximations
- Can serve as input to refine the modeling and improve Monte-Carlo generators. Can ZZ results serve as validation for DPS modeling?

Summary

- Large fraction of DPS at low/intermediate transverse momenta
  - Transverse momentum dependent framework necessary
- Development of DTMD framework: definitions of DTMDs, their evolution and matching in different regimes
- Provide maximal perturbative information: significantly limits the new non-perturbative (unknown) information required compared to integrated cross section.
- A lot of potential to:
  - do phenomenology
  - connect with experiments
  - provide useful input to MC generators
  - suggestions welcome!