# Multiparton interactions in QGSJET-II

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Multiple Partonic Interactions at the LHC

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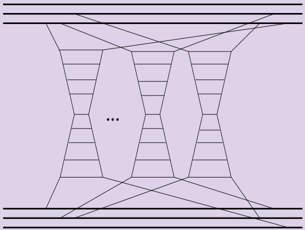
# Multiple scattering & multiparton interactions

- many parton cascades in parallel
- 'real' multiparton interactions – via multiple production of dijets
- also 'soft' (small $p_t$) scattering processes
- virtual (elastic) rescatterings (required by unitarity)
- soft/hard diffraction



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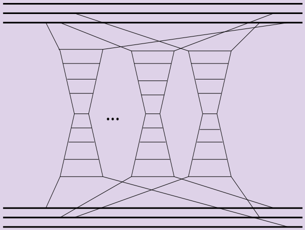
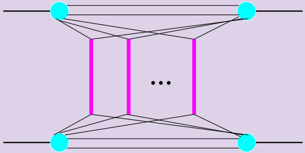



**Basic idea: combined treatment of soft & hard processes in RFT**

- 'elementary' cascades = Pomerons
- requires Pomeron amplitude & Pomeron-hadron vertices



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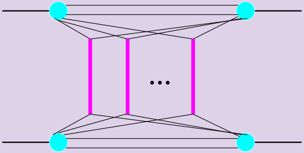
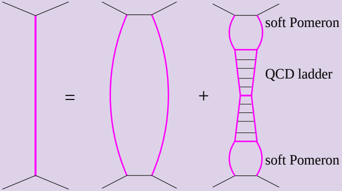



## Hard processes included using 'semihard Pomeron' approach *[Drescher et al., PR350 (2001) 93]*

- soft Pomerons to describe soft (parts of) cascades ($p_t^2 < Q_0^2$)
  - $\Rightarrow$ transverse expansion governed by the Pomeron slope
- DGLAP for hard cascades
- taken together: 'general Pomeron'
- $Q_0$ – just a technical border between the two treatments of a smooth parton evolution



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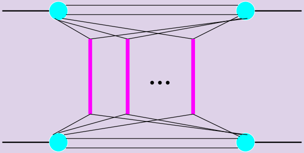

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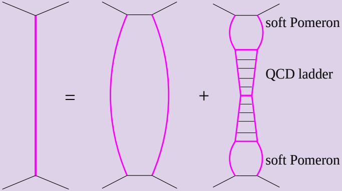

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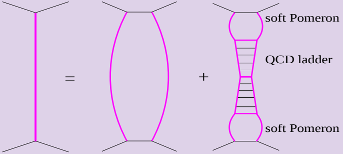
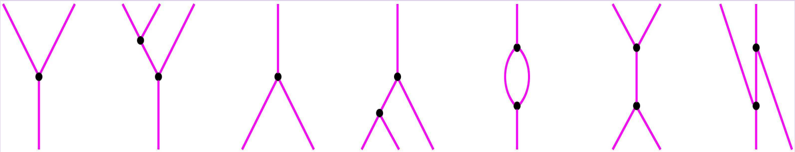



## Nonlinear processes: Pomeron-Pomeron interactions (scattering of intermediate partons off the proj./target hadrons & off each other)



thick lines = Pomerons = 'elementary' parton cascades

- NB: 'soft' $\mathbb{PP}$-coupling assumed
- $\Rightarrow$ missing perturbative parton splitting mechanism

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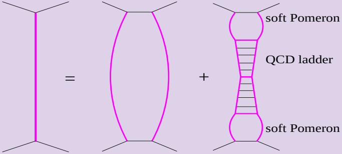


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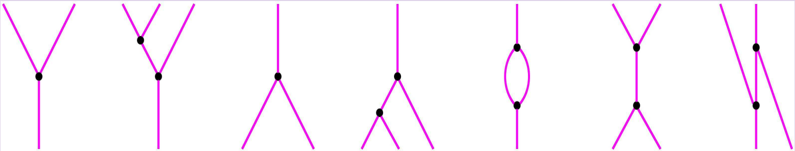

## Hard multiparton interactions (multiple dijets) emerge in two ways:

- from independent parton cascades ('Pomerons')
- from Pomeron-Pomeron interactions (= 'soft' parton splitting)

# Multiple scattering & multiparton interactions

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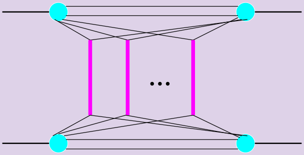


## Good-Walker-like scheme used for low mass diffraction

- $|p\rangle = \sum_i \sqrt{C_i}|i\rangle$, $C_i$ - partial weight for el. scatt. eigenstate $|i\rangle$
- two eigenstates: i) large & dilute (low parton density, large radius), ii) small & dense (high parton density, small radius)
- all multi-Pomeron contributions averaged over the eigenstates
- small size eigenstates: sampled more rarely (small area) but have stronger multiple scattering (higher parton density)
- NB: high mass diffraction – from (cut) enhanced diagrams

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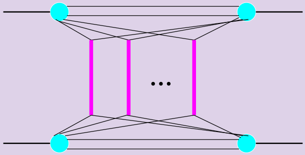


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# Low and high mass diffraction within the same formalism?

**More general Reggeon calculus – based on Pomerons & Reggeons?**

- generally much more challenging
- also: would involve many more parameters

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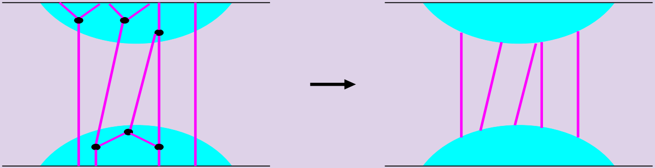
## Treat both LMD & HMD within the Good-Walker framework?



- $\Rightarrow$ hide all the nontrivial dynamics inside the GW eigenstates
- $\Rightarrow$ the structure of the eigenstates would depend nontrivially on the interaction kinematics
  - factorization not possible
  - $\Rightarrow$ complicated parametrizations required
- NB: also the hadronization of the hadron 'remnant' states would depend nontrivially on the kinematics

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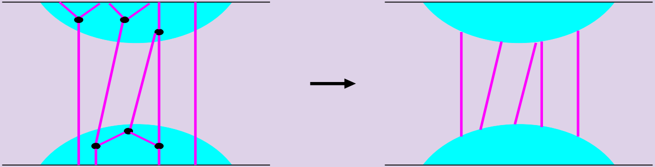

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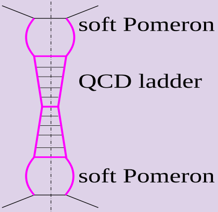
# Structure of constituent parton Fock states

## Initial state emission (ISE) of partons doesn't stop at the $Q_0$-cutoff

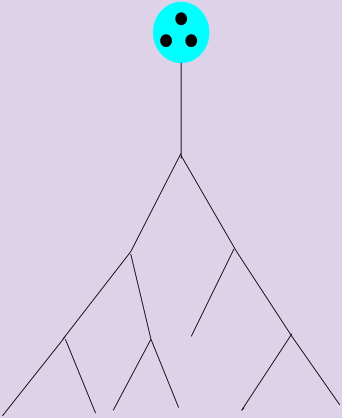
- it is extended into nonperturbative region by the soft Pomeron
- this changes the structure of constituent parton Fock states (represented by end-point partons in ISE)
    - in QGSJET(-II): described by Reggeon asymptotics ($\propto x^{-\alpha_{\mathbb{R}}(0)}$, $\alpha_{\mathbb{R}}(0) \simeq 0.5$)
- observables consequences, compared to the usual treatment?



# Structure of constituent parton Fock states

**Usually: one (implicitely) starts from the same nonperturbative Fock state (typical for models used at colliders, also SIBYLL)**

- multiple scattering has small impact on forward spectra
  - new branches emerge at small $x$ ($G(x,q^2) \propto 1/x$)
- $\Rightarrow$ Feynman scaling & limiting fragm. for forward production
- higher $\sqrt{s} \Rightarrow$ more abundant central particle production only
- forward & central production – decoupled from each other
  - (descreasing number of cascade branches for increasing $x$)



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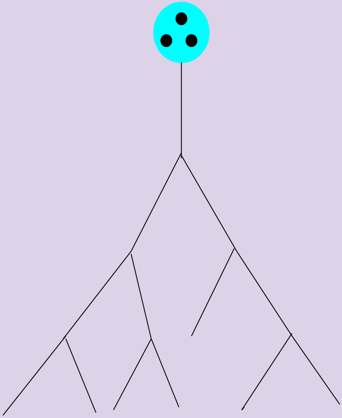
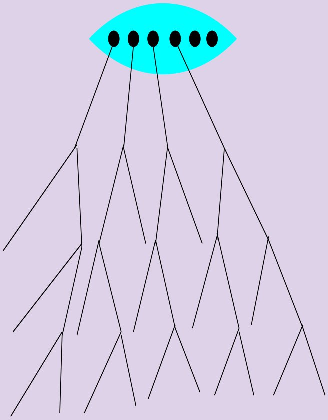


# Structure of constituent parton Fock states

## EPOS & QGSJET(-II): $p = \sum$ of multi-parton Fock states

- many cascades develop in parallel (already at nonperturbative stage)
  - $\Rightarrow$ flatter $dN^{\rm ch}_{pp}/d\eta$ at large $\eta$
- higher $\sqrt{s} \Rightarrow$ larger Fock states come into play: $|qqq\rangle \rightarrow |qqq\bar{q}q\rangle \rightarrow ... |qqq\bar{q}q...\bar{q}q\rangle$
  - $\Rightarrow$ softer forward spectra (energy sharing between constituent partons)
- forward & central particle production - strongly correlated
  - e.g. more activity in central detectors $\Rightarrow$ larger Fock states $\Rightarrow$ softer forward spectra



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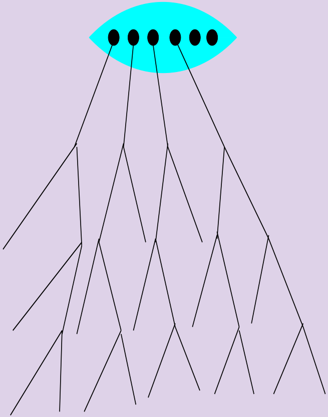

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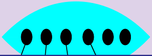
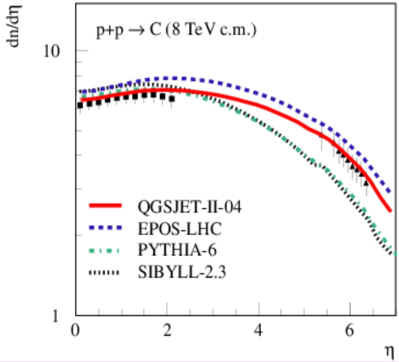


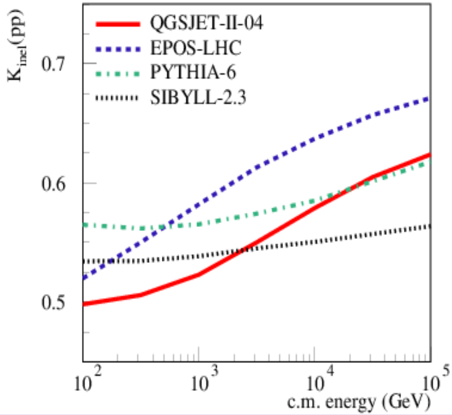
## Example: comparison to combined CMS-TOTEM data on $dN_{ch}/d\eta$




- flatter $dN_{ch}/d\eta$ of EPOS & QGSJET-II agrees with data

# Structure of constituent parton Fock states

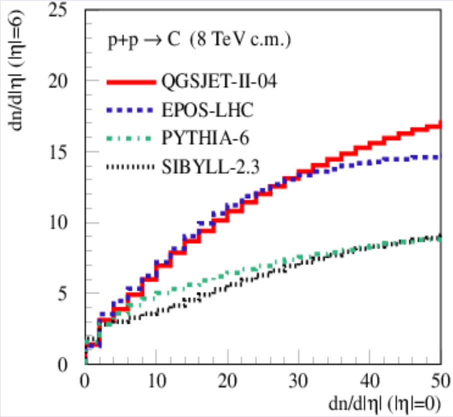
## Of importance for cosmic ray studies: $\sqrt{s}$-dependence of $K_{pp}^{\rm inel}$




- SIBYLL & PYTHIA: weak energy dependence of the nucleon 'inelasticity'
  - for increasing $\sqrt{s}$: mostly central production enhanced
- smaller $K^{\rm inel}$ $\Rightarrow$ stronger 'leading particle' effect
- $\Rightarrow$ slower development of CR-induced air showers

# Structure of constituent parton Fock states

## Crucial test: cross-correlation of $dN^{\rm ch}_{pp}/d|\eta|$ at $\eta = 0$ and $\eta = 6$

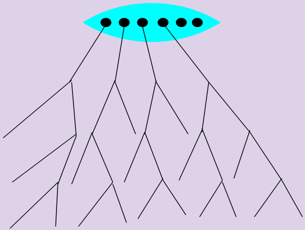



- strong correlation for QGSJET-II & EPOS (apart from the tails of the $N^{\rm ch}$ distributions)
- twice weaker correlation for SIBYLL & PYTHIA

# Comment of the Monash tune of PYTHIA

## ISE in PYTHIA is traced back for all hard scatterings individually

- should it then correspond to the case of multiparton Fock states?
- usually not: the end-point partons (mostly gluons) are sampled predominantly as $\propto 1/x$



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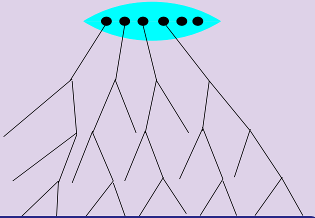
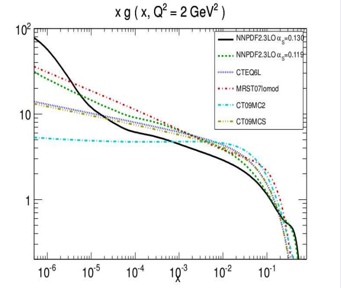


## Not so much in the Monash tune for 2 reasons

- the $p_t$-cutoff scale is relatively low (2 GeV) and $\sqrt{s}$-independent
- the PDFs employed contain 'valence gluon' component
- backward evolution proceeds in $q^2$ & $x$ simultaneously $\Rightarrow$ the (small) 'valence gluon' impacts noticeably the $x$ of the end-point partons



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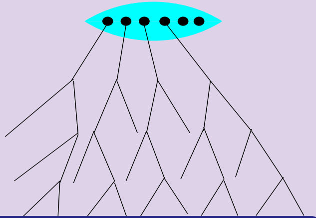

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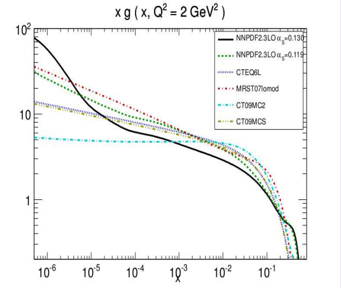

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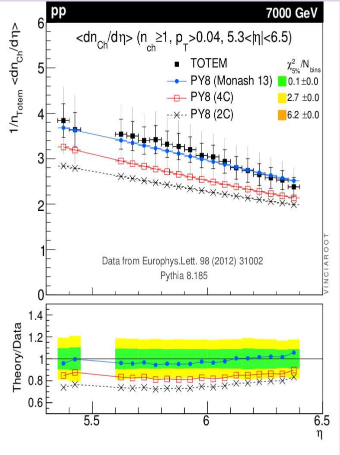
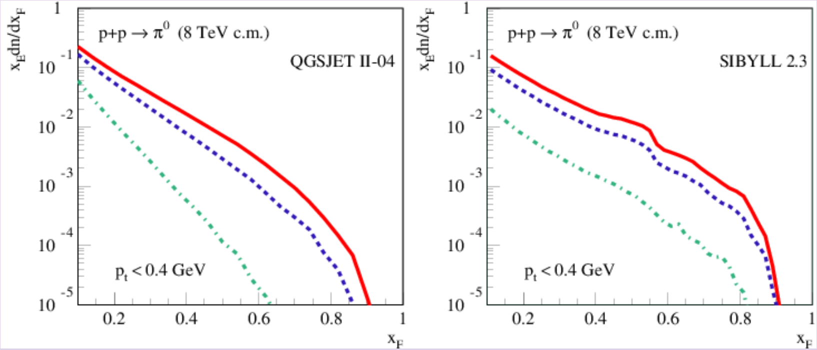
- the PDFs employed contain 'valence gluon' component
- $\Rightarrow$ flatter $dN_{pp}^{\rm ch}/d\eta$
  - crucial: all end-point partons become harder
- probably also the cross-correlation of $dN_{pp}^{\rm ch}/d|\eta|$ at $\eta = 0$ and 6 will move closer to EPOS/QGSJET-II
- though the effect on $K_{pp}^{\rm inel}$ is likely to be weak



# Further discrimination: forward hadrons by LHCf & ATLAS

**Forward $\pi^0$ spectra in LHCf for different ATLAS triggers ($\geq 1$, 6, 20 charged hadrons of $p_t > 0.5$ GeV & $|\eta| < 2.5$)**



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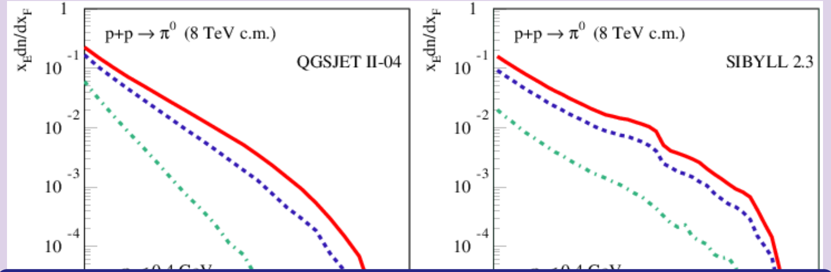


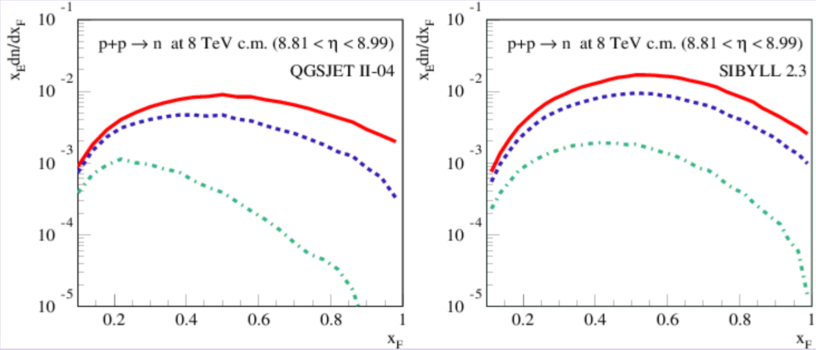

## Compare QGSJET-II-04 (left) to SIBYLL 2.3 (right)

- enhanced multiple scattering $\Rightarrow$ softer pion spectra
- $\Rightarrow$ violation of limiting fragmentation (energy sharing between constituent partons)

- nearly same spectral shape for all the triggers
- $\Rightarrow$ perfect limiting fragmentation (central production decoupled)

# Further discrimination: forward hadrons by LHCf & ATLAS

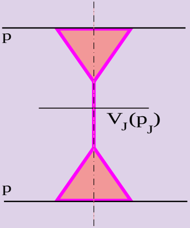
## Neutron spectra in LHCf ($8.99 < \eta < 9.22$) for same triggers




- remarkably universal spectral shape in SIBYLL-2.3 (decoupling of central production)
  - closely related to the small 'inelasticity' of the model
- strong suppression of forward neutrons in QGSJET-II-04
  - higher central activity ⇒ more constituent partons involved ⇒ less energy left for the proton 'remnant'

# Inclusive jet production - a closer look

- described in RFT by Kancheli-Mueller graphs
- projectile & target 'triangles' generally contain absorptive corrections



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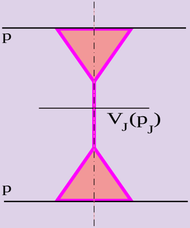
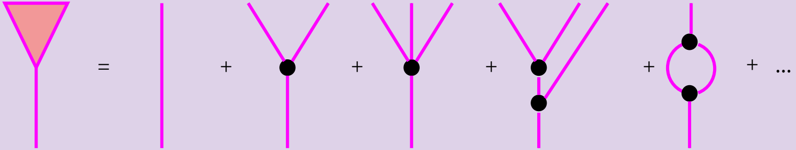



## Examples of graphs hidden in the 'triangles'



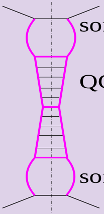
# Inclusive jet production - a closer look

## Dijet cross section (neglecting absorption)

$$\sigma_{pp}^{2\text{jet(no abs)}}(s, p_t^{\text{cut}}) = \sum_{i,j} C_i\, C_j \int d^2b'\, d^2b''$$

$$\times \sum_{I,J} \int \frac{dx^+}{x^+} \frac{dx^-}{x^-}\, \sigma_{IJ}^{\text{QCD}}(x^+x^-s, Q_0^2, p_t^{\text{cut}})$$

$$\times \chi_{(i)I}^{\mathbb{P}_{\text{soft}}}(s_0/x^+, b')\, \chi_{(j)J}^{\mathbb{P}_{\text{soft}}}(s_0/x^-, b'')$$




- $\sigma_{IJ}^{\text{QCD}}$ - contribution of DGLAP ladder with leg parton virtualities $Q_0^2$
- $\chi_{(i)I}^{\mathbb{P}_{\text{soft}}}$ - eikonal for soft Pomeron coupled to eigenstate $|i\rangle$ of the proton & parton $I$

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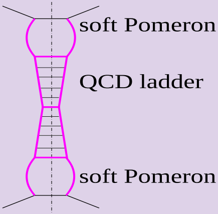
$$\times \sum_{I,J} \int \frac{dx^+}{x^+} \frac{dx^-}{x^-}\, \sigma_{IJ}^{\mathrm{QCD}}(x^+x^-s, Q_0^2, p_{\mathrm{t}}^{\mathrm{cut}})$$

$$\times \chi_{(i)I}^{\mathbb{P}_{\mathrm{soft}}}(s_0/x^+, b')\, \chi_{(j)J}^{\mathbb{P}_{\mathrm{soft}}}(s_0/x^-, b'')$$




## Including absorption $\chi_{(i)I}^{\mathbb{P}_{\mathrm{soft}}}(s_0/x,b)$ is replaced by the solution of 'fan' diagram equation, $x\tilde{f}_I^{(i)}(x,b)$

- $\tilde{f}_I^{(i)}(x,b)$ may be interpreted as GPDs $G_I^{(i)}(x,Q_0^2,b)$ at the virtuality scale $Q_0^2$; higher scales - DGLAP-evolved:

$$G_I^{(i)}(x,Q^2,b) = \sum_{I'} \int_x^1 \frac{dz}{z}\, E_{I'\to I}^{\mathrm{DGLAP}}(z,Q_0^2,Q^2)\, \tilde{f}_{I'}^{(i)}(x/z,b)$$

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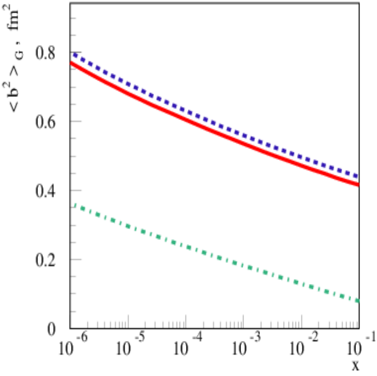
## Impact of transverse diffusion on $\langle b^2 \rangle$ of gluons at $Q_0^2 = 3$ GeV$^2$

- $\langle b^2 \rangle$ - dominated by the largest size Fock state
- quick spread with energy
- $\langle b^2 \rangle$ - slightly larger than in *[Frankfurt, Strikman & Weiss, PRD 69 (2004) 114010*



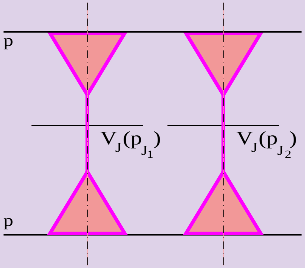
# DPS production of 2 dijets

## Production of 2 dijets by independent parton cascades ('2v2')

$$\sigma_{pp}^{4\text{jet}(2\text{v}2)}(s, p_{\text{t}}^{\text{cut}}) = \frac{1}{2}\sum_{i,j} C_i C_j \int d^2b$$

$$\times \left[ \int dx^+ dx^- \sum_{I,J} \sigma_{IJ}^{\text{QCD}}(x^+x^-s, Q_0^2, p_{\text{t}}^{\text{cut}}) \right.$$

$$\left. \times \int d^2b' \tilde{f}_I^{(i)}(x^+, b') \tilde{f}_J^{(j)}(x^-, |\vec{b} - \vec{b}'|) \right]^2$$




- NB: two dijet processes for the same $b$ & eigenstates $|i\rangle$, $|j\rangle$

# DPS production of 2 dijets

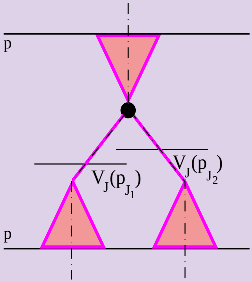
## 'Soft parton splitting' ('2v1s')

$$\sigma_{pp}^{4\mathrm{jet}(2\mathrm{v}1)_s}(s,p_t^{\mathrm{cut}}) = \frac{1}{2}\sum_{i,j} C_i\, C_j$$

$$\times G_{3\mathbb{P}} \int d^2b' \int \frac{dx'}{x'} \left[1 - e^{-\chi^{\mathrm{fan}}_{(i)}(s_0/x',b')}\right]$$

$$\times \int d^2b \left[\int \frac{dx^+}{x^+} \int dx^- \sum_{I,J} \sigma_{IJ}^{\mathrm{QCD}}(x^+x^-s, Q_0^2, p_t^{\mathrm{cut}})\right.$$

$$\left.\times \int d^2b''\, \chi_{\mathbb{P}I}^{\mathbb{P}_{\mathrm{soft}}}(s_0\, x'/x^+, b'')\, \tilde{f}_J^{(j)}(x^-, |\vec{b}-\vec{b}''|)\right]^2$$




- small $\alpha'_{\mathbb{P}}$ $\Rightarrow$ two hard processes are closeby in $b$-space
- involves triple-Pomeron coupling $r_{3\mathbb{P}}$ ($G_{3\mathbb{P}} \propto r_{3\mathbb{P}}$)
- neglecting absorptive corrections $\rightarrow$ triple-Pomeron graph

# DPS production of 2 dijets

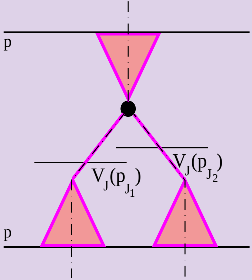
## 'Soft parton splitting' ('2v1s')

$$\sigma_{pp}^{4\mathrm{jet}(2\mathrm{v}1)_\mathrm{s}}(s,p_\mathrm{t}^{\mathrm{cut}}) = \frac{1}{2}\sum_{i,j} C_i\, C_j$$

$$\times G_{3\mathbb{P}} \int d^2b' \int \frac{dx'}{x'} \left[1 - e^{-\chi_{(i)}^{\mathrm{fan}}(s_0/x',b')}\right]$$

$$\times \int d^2b \left[ \int \frac{dx^+}{x^+} \int dx^- \sum_{I,J} \sigma_{IJ}^{\mathrm{QCD}}(x^+x^-s, Q_0^2, p_\mathrm{t}^{\mathrm{cut}}) \right.$$

$$\left. \times \int d^2b''\, \chi_{\mathbb{P}I}^{\mathbb{P}_{\mathrm{soft}}}(s_0\, x'/x^+, b'')\, \tilde{f}_J^{(j)}(x^-, |\vec{b}-\vec{b}''|) \right]^2$$




## We may compare this with the standard DPS formula

$$\sigma_{pp}^{4\mathrm{jet}(\mathrm{DPS})}(s,p_\mathrm{t}^{\mathrm{cut}}) = \frac{1}{2}\int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{\mathrm{t}_1},p_{\mathrm{t}_2}>p_\mathrm{t}^{\mathrm{cut}}} dp_{\mathrm{t}_1}^2\, dp_{\mathrm{t}_2}^2 \sum_{I_1,I_2,J_1,J_2}$$

$$\times \frac{d\sigma_{I_1J_1}^{2\to2}}{dp_{\mathrm{t}_1}^2} \frac{d\sigma_{I_2J_2}^{2\to2}}{dp_{\mathrm{t}_2}^2} \int d^2\Delta b\, F_{I_1I_2}^{(2)}(x_1^+, x_2^+, M_{\mathrm{F}_1}^2, M_{\mathrm{F}_2}^2, \Delta b)\, F_{J_1J_2}^{(2)}(x_1^-, x_2^-, M_{\mathrm{F}_1}^2, M_{\mathrm{F}_2}^2, \Delta b)$$

# DPS production of 2 dijets

## We may compare this with the standard DPS formula

$$\sigma_{pp}^{4\text{jet(DPS)}}(s, p_{\text{t}}^{\text{cut}}) = \frac{1}{2} \int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{\text{t}_1}, p_{\text{t}_2} > p_{\text{t}}^{\text{cut}}} dp_{\text{t}_1}^2 \, dp_{\text{t}_2}^2 \sum_{I_1, I_2, J_1, J_2}$$

$$\times \frac{d\sigma_{I_1 J_1}^{2\to 2}}{dp_{\text{t}_1}^2} \frac{d\sigma_{I_2 J_2}^{2\to 2}}{dp_{\text{t}_2}^2} \int d^2\Delta b \, F_{I_1 I_2}^{(2)}(x_1^+, x_2^+, M_{\text{F}_1}^2, M_{\text{F}_2}^2, \Delta b) \, F_{J_1 J_2}^{(2)}(x_1^-, x_2^-, M_{\text{F}_1}^2, M_{\text{F}_2}^2, \Delta b)$$

## The two contributions (2v2 & 2v1s) correspond to ${}_2$GPDs

$$F_{I_1 I_2}^{(2)}(x_1, x_2, Q_0^2, Q_0^2, \Delta b) = \sum_i C_i \int d^2 b' \left\{ \tilde{f}_{I_1}^{(i)}(x_1, b') \tilde{f}_{I_2}^{(i)}(x_2, |\vec{b}' - \vec{\Delta b}|) \right.$$

$$\left. + \frac{G_{3\mathbb{P}}}{x_1 x_2} \int \frac{dx'}{x'} \left[ 1 - e^{-\chi_{(i)}^{\text{fan}}(s_0/x', b')} \right] \int d^2 b'' \, \chi_{\mathbb{P} I_1}^{\mathbb{P}_{\text{soft}}}\left(\frac{s_0 x'}{x_1}, b''\right) \chi_{\mathbb{P} I_2}^{\mathbb{P}_{\text{soft}}}\left(\frac{s_0 x'}{x_2}, |\vec{b}'' - \vec{\Delta b}|\right) \right\}$$

- 2nd term generates short range two-parton correlations in $b$-space

# DPS production of 2 dijets

## One has to add the hard parton splitting (missing in QGSJET-II)

$$\sigma_{pp}^{4\mathrm{jet}(2\mathrm{v}1)_{\mathrm{h}}}(s,p_{\mathrm{t}}^{\mathrm{cut}}) = \frac{1}{2}\sum_{i,j} C_i\, C_j \int_{q^2>Q_0^2} \frac{dq^2}{q^2} \int \frac{dx}{x^2} \sum_L \left[\int d^2b'\, G_L^{(i)}(x,q^2,b')\right]$$
$$\times \int \frac{dz}{z(1-z)}\, \frac{\alpha_s}{2\pi} \sum_K P^{\mathrm{AP}}_{L\to K(K')}(z) \int dx_1^+ dx_2^+ dx_1^- dx_2^- \int_{p_{\mathrm{t}_1},p_{\mathrm{t}_2}>p_{\mathrm{t}}^{\mathrm{cut}}} dp_{\mathrm{t}_1}^2\, dp_{\mathrm{t}_2}^2$$
$$\times \sum_{I_1,I_2,J_1,J_2} E^{\mathrm{DGLAP}}_{K\to I_1}(x_1^+/x/z,q^2,M_{\mathrm{F}_1}^2)\, E^{\mathrm{DGLAP}}_{K'\to I_2}(x_2^+/x/(1-z),q^2,M_{\mathrm{F}_2}^2)$$
$$\times \frac{d\sigma^{2\to 2}_{I_1J_1}}{dp_{\mathrm{t}_1}^2}\, \frac{d\sigma^{2\to 2}_{I_2J_2}}{dp_{\mathrm{t}_2}^2} \int d^2b\, G_{J_1}^{(j)}(x_1^-,M_{\mathrm{F}_1}^2,b)\, G_{J_2}^{(j)}(x_2^-,M_{\mathrm{F}_2}^2,b)$$

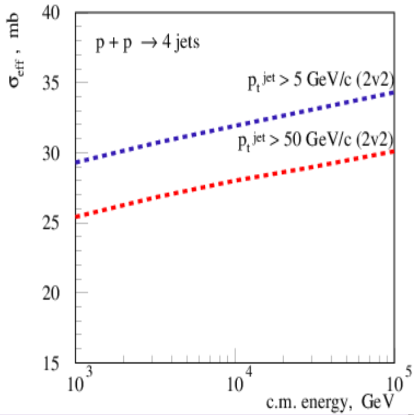
## Calculations are done using the default parameters of QGSJET-II

- tuned to collider data on $\sigma_{pp}^{\mathrm{tot/el/diffr}}$, $d\sigma_{pp}^{\mathrm{el}}/dt$, $F_2$, $F_2^{\mathrm{D}(3)}$
- e.g. $Q_0^2 = 3$ GeV$^2$, $\alpha_{\mathbb{P}} = 1.17$, $\alpha'_{\mathbb{P}} = 0.14\,\mathrm{GeV}^{-2}$, $r_{3\mathbb{P}} = 0.1$ GeV

# Energy dependence of $\sigma_{pp}^{\rm eff}(s,p_{\rm t}^{\rm cut}) = \frac{1}{2}\frac{\left[\sigma_{pp}^{\rm 2jet}(s,p_{\rm t}^{\rm cut})\right]^2}{\sigma_{pp}^{\rm 4jet(DPS)}(s,p_{\rm t}^{\rm cut})}$

## $\sigma_{pp}^{\rm eff}$ for 2 independent parton cascades

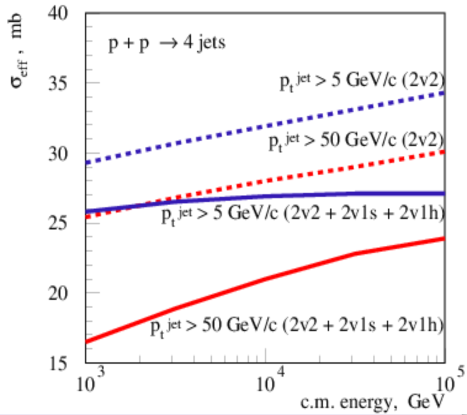
- strong energy rise of $\sigma_{pp}^{\rm eff(2v2)}$ due to parton diffusion
  - slower for higher $p_{\rm t}^{\rm cut}$
- easy to understand; e.g. consider $G_I(x,q^2,b) = f_I(x,q^2)\, e^{-b^2/R_p^2(s)}/\pi/R_p^2(s)$
  - $\Rightarrow \sigma_{pp}^{\rm eff(2v2)} = 4\pi R_p^2(s) \propto {\rm const} + \alpha'_{\mathbb{P}} \ln s$



# Energy dependence of $\sigma_{pp}^{\rm eff}(s, p_{\rm t}^{\rm cut}) = \frac{1}{2}\frac{\left[\sigma_{pp}^{\rm 2jet}(s,p_{\rm t}^{\rm cut})\right]^2}{\sigma_{pp}^{\rm 4jet(DPS)}(s,p_{\rm t}^{\rm cut})}$

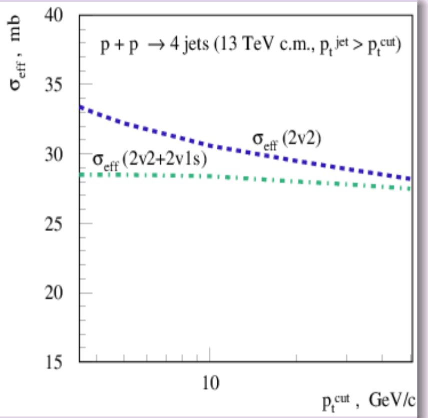
## Including soft & hard parton splitting

- brings $\sigma_{pp}^{\rm eff}$ down to the measured values
- flattens $\sqrt{s}$-dependence for small $p_{\rm t}^{\rm cut}$



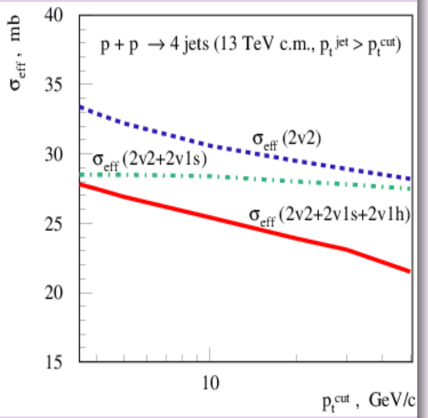
# $p_t^{cut}$-dependence of $\sigma_{pp}^{eff}$ at $\sqrt{s} = 13$ TeV

- $\sigma_{pp}^{eff(2v2)}$ decreases with $p_t^{cut}$ (narrower transverse profile for high $p_t$ partons)
- 'soft splitting': large correction for small $p_t^{cut}$
  - small for high $p_t^{cut}$
  - $\Rightarrow$ flattens $p_t^{cut}$-dependence



# $p_t^{cut}$-dependence of $\sigma_{pp}^{eff}$ at $\sqrt{s} = 13$ TeV

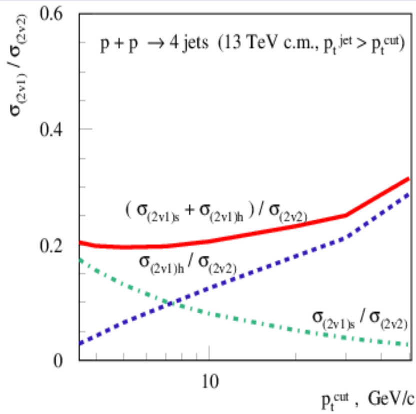
- 'soft splitting': large correction for small $p_t^{cut}$
  - small for high $p_t^{jet}$
  - $\Rightarrow$ flattens $p_t^{cut}$-dependence
- hard splitting: dominant for high $p_t^{cut}$
  - vanishes for $p_t^{cut} \to Q_0$
  - $\Rightarrow$ opposite effect on $\sigma_{pp}^{eff}$
  - irrelevant for minimum bias events



# $p_t^{cut}$-dependence of $\sigma_{pp}^{eff}$ at $\sqrt{s} = 13$ TeV

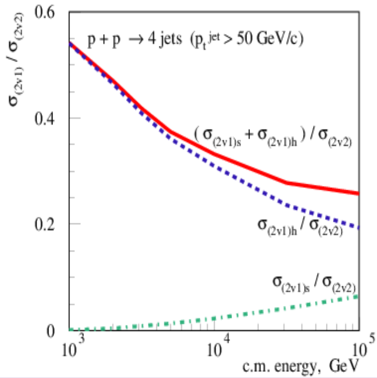
## Relative importance of the soft and hard parton splittings

- same conclusions as above
- combined effect of the soft & hard splittings $\Rightarrow$ weak $p_t^{cut}$ dependence of $R_{(2v1)} = \sigma_{pp}^{4jet(2v1)} / \sigma_{pp}^{4jet(2v2)}$
  - NB: precise shape depends on $\sqrt{s}$



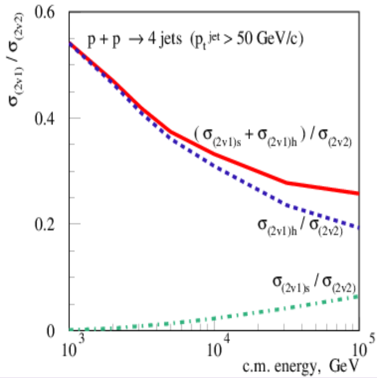
# Ratio of (2v1) to (2v2) contributions: energy dependence

- $\sigma_{pp}^{4jet(2v1)_s}/\sigma_{pp}^{4jet(2v2)}$ rises with $\sqrt{s}$ (larger kinematic range for parton splitting)



# Ratio of (2v1) to (2v2) contributions: energy dependence

- $\sigma_{pp}^{4\text{jet}(2v1)_s}/\sigma_{pp}^{4\text{jet}(2v2)}$ rises with $\sqrt{s}$ (larger kinematic range for parton splitting)
- $\sigma_{pp}^{4\text{jet}(2v1)_h}/\sigma_{pp}^{4\text{jet}(2v2)}$ decreases
  - main reason: lacks one $\ln x$ wrt (2v2) contribution
  - in addition: effect of color fluctuations & diffusion



# Summary

1. QGSJET-II offers a phenomenological RFT-based description of soft & hard processes
2. Extending ISR into soft domain $\rightarrow$ different structure of constituent parton Fock states
   - $\Rightarrow$ strong long-range correlations between central & forward hadron production
3. enhanced Pomeron diagrams generate the 'soft splitting' contribution to DPS
4. $\sigma_{pp}^{\mathrm{eff}}$ obtained using the default parameters of QGSJET-II agrees with the measured values - if the hard parton splitting is taken into account
5. hard splitting has a minor influence on minimum bias events

# Extra slides