AdS/CFT Correspondence, Superconductivity, Ginzburg-Landau and All That...

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Outline of this Talk

The **main objective** of this talk is show how **string theory, through the AdS/CFT correspondence**, can be applied to the study of **high-temperature superconductivity**

- **A Motivation**: High-Temperature Superconductivity and its problems
- How does AdS/CFT can help us understand high-temperature superconductivity = **Holographic Superconductivity**
- **Ginzburg-Landau Approach** to Holographic Superconductivity
- Holographic Superconductivity with **Lifshitz Scaling**

Motivation: High-Temperature Superconductors

- Also known as **Cuprates**: Anti-Ferromagnetic ceramics that become SC after doping
- High-T_c Superconductors have a very rich, **almost universal phase diagram**

- **Superconducting Phase: The Dome**

- $T_c \sim 90K$. Usual SC has $T_c \leq 30K$
- **d-Wave** Order Parameter:

$$\Delta_{\mathbf{k}} \sim \cos k_x - \cos k_y$$

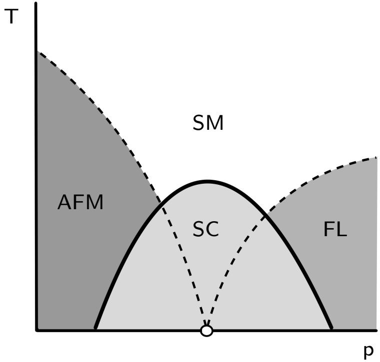
- **Normal Phase: Strange Metal**

- Meaning **Non-Fermi Liquid** behaviour:
- **Linear inverse quasi-particle lifetime**:

$$\frac{1}{\tau} \sim \omega$$

- **Linear Resistivity**:

$$\rho \sim T$$

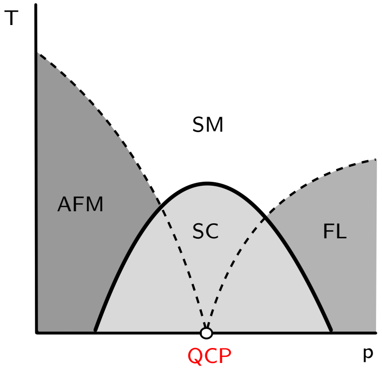


Cuprates: A Hidden Quantum Critical Point

• **Conjecture:** There is a "hidden" **Quantum Critical Point (QCP)** inside the superconducting dome, that would induce strong quantum fluctuations **leading to a NFL behavior**. This is the starting point of many QFT models of the cuprates
• A strong candidate for such a model is the **Spin-Fermion Model**.
-**[Abanov, Chubukov, Schmalian. Adv.Phys. 52]**, **[Melitsky, Sachdev. Phys.Rev.B 82]**
-It proposes electron pairing mediated by **Spin Density Wave** $\mathbf{S_k}$.
-It makes **very successful predictions:** SC Instability, d-wave order parameter, **NFL liquid behaviour** from one loop corrections in the self energy

It has serious limitations:

- It has a **strong coupling**: $\lambda \sim 2$, while for usual Superconductors $\lambda \sim 0.3$
- There is a **quasi-particle picture breakdown** at the hot-spots of the Fermi Surface of the cuprates



- **Standard QFT approaches** result in **strongly coupled** theories, with sometimes a **breakdown of basic interacting many-body theoretical assumptions**
- We need to look for a **different approach to the problem**
- We can try to **apply the AdS/CFT correspondence** using the following motivation...

- **AdS/CFT Correspondence: Rough Statement**

Large-N **Strongly Coupled** Quantum SU(N) Gauge Field Theory in **d Dimensions** **(BOUNDARY THEORY)**	$\longleftrightarrow$	**Classical** Anti-deSitter Gravitational Theory in **d+1 Dimensions** **(BULK THEORY)**

- **What part can it play in SC: Motivation**
  - Using the correspondence, we can look for the **superconducting phase of these particular strongly-coupled QFT's** by studying the **equivalent, tractable classical dynamics of the dual gravitational system**
  - Then, look for **universal phenomena shared by real-world cuprates**
    That is: look for both theories to belong to the same **universality class**

- **The Dictionary: Scalar Fields**
  - AdS$_{d+1}$ Spacetime in the bulk

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}d\vec{x}^2$$

  - The r-coordinate is the **Radial Dimension**

$$r = 0 \ \ \text{(AdS "Horizon")} \qquad\qquad r = \infty \ \ \text{(AdS Boundary)}$$

  - **Massive Bulk Scalar Field** $\Psi(r, x^\mu)$.

$$\Psi(r \to \infty, x^\mu) \approx \frac{\psi_0}{r^{\Delta_-}} + \frac{\mathcal{O}}{r^{\Delta_+}} + \cdots$$

$$\Delta\,(\Delta - d) = L^2 m^2$$

  - **Translation (The Master Equation):**

$$e^{-\Gamma_{\text{CFT}}[\psi_0]} = \left\langle \exp - \int \psi_0 \mathcal{O} \right\rangle_{\text{CFT}} = e^{-S_{\text{Classical Gravity}}[\Psi]}$$

$$\psi_0 : \text{Source} \qquad\qquad \mathcal{O} : \text{VEV, Dimension } \Delta_+$$

AdS/CFT: The Dictionary. Essential Vocabulary

- **The Dictionary: Gauge Fields**
  - AdS_{d+1} Spacetime in the bulk

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}d\vec{x}^2$$

  - The r-coordinate is the **Radial Dimension**.

$$r = 0 \ \text{(AdS ``Horizon'')} \qquad\qquad r = \infty \ \text{(AdS Boundary)}$$

  - **Bulk Gauge Field** $A_\alpha(r, x^\mu)$.

$$A_\alpha(r \to \infty, x^\mu) \approx \mathcal{S}_\alpha + \frac{\mathcal{J}_\alpha}{r^{d-2}} + \cdots$$

$$A = A_\alpha(r, x^\mu) dX^\alpha$$

  - **Translation (The Master Equation):**

$$\left\langle \exp - \int \mathcal{S}_\alpha \mathcal{J}^\alpha \right\rangle_{\text{CFT}} = e^{-S_{\text{Classical Gravitiy}}[A]}$$

$$\mathcal{S}_\alpha \ : \ \text{Source} \qquad\qquad \mathcal{J}_\alpha \ : \ \text{VEV}$$

Inspired on **[Hartnoll, Herzog, Horowitz. JHEP 0812 (2008)]** in D=3+1
The following is based on **[Dector. JHEP 1412 (2015)]** in D=4+1

- **Basic Bulk Model in D=4+1**

$$S_{\text{Bulk}} = \int d^5x\sqrt{g}\left\{R - \frac{1}{4}F^2 + \frac{12}{L^2} - |\partial\Psi - iqA|^2 - m^2\,|\Psi|^2\right\}$$

- **Basic Ansatz**

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2 d\vec{x}^2_{d-1}$$

$$A = \Phi(r)dt\,, \qquad \Psi = \psi(r)/\sqrt{2}$$

- **Scalar field** $\Psi(r)$ is dual to **s-wave SC order parameter**. Mass above **BF Bound** $m^2 \geq -4$
- We will look for asymptotically AdS$_5$ **BH solutions** that introduce **Temperature**
- $\Phi(r \to \infty) \approx \mu - \rho/r^2 + \cdots$. Following the AdS/CFT dictionary, ρ is **charge density**, μ **chemical potential**. We always work in the **canonical ensemble**, ρ fixed to unity
- Action has $U(1)$ local symmetry $\to U(1)$ global symmetry in boundary theory. We assume we can gauge it and have a SC interpretation
- We will work with **full back-reacted solutions** and using the **shooting method**
- **Normal Phase:** $\Psi(r) = 0$, Exact AdS-RN-BH solution

- **The Superconducting Instability**
  - There can be an **scalar field instability** in the near-horizon region of the normal solution, **ridden by the scalar charge** q and leading to **scalar hairy BH solutions**. Finding these hairy solutions translates to **U(1) symmetry breaking** and **condensation** in the dual QFT

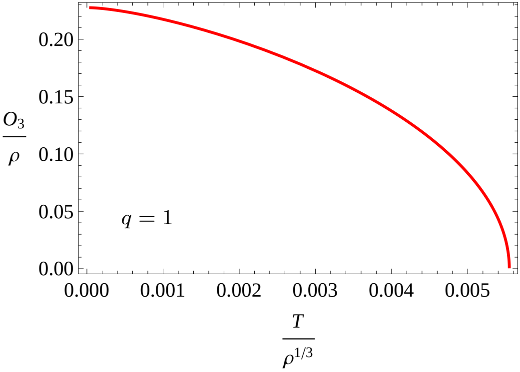
- Set **scalar mass** $m^2 = -3$
- Scalar charge q is **input parameter** and consider different values of q to **probe phenomenology**
- The EOMs give the asymptotics

$$\psi(r \to \infty) \approx \frac{C_1}{r} + \frac{\mathcal{O}_3}{r^3} + \cdots$$

- Solve using boundary condition $C_1 = 0$, meaning **Unsourced Spontaneous Condensation**
- SC Order Parameter is $\mathcal{O}_3$
  Model predicts near-T_c behavior

$$\mathcal{O}_3 \sim (1 - T/T_c)^{1/2}$$

as **real-world superconductors**



- **Field Perturbations: Gauge Field A_μ**
  - We consider a **small perturbation** to the gauge field

$$A = \Phi(r)dt + e^{-i\omega t + iky} A_x(r) dx$$

- The A_x EOM gives the asymptotics

$$A_x(r \to \infty) \approx A_x^{(0)} + \frac{J_x}{r^2} + \cdots$$

$A_x^{(0)}$: **Dual Vector Potential**
J_x: **Its Conjugated Current**

- We can relate these trough the **London Equation**

$$J_x = -q^2 n_s A_x^{(0)}$$

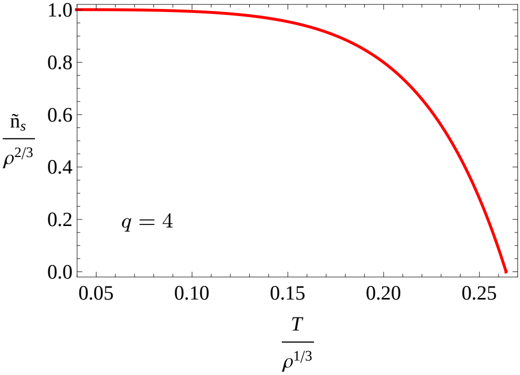
n_s : **SC Number Density**

- Define for simplicity

$$\tilde{n}_s \equiv q^2 n_s = -J_x / A_x^{(0)}$$

- Physical near-T_c behaviour

$$n_s \sim (1 - T/T_c)$$



Bulk Field Perturbations

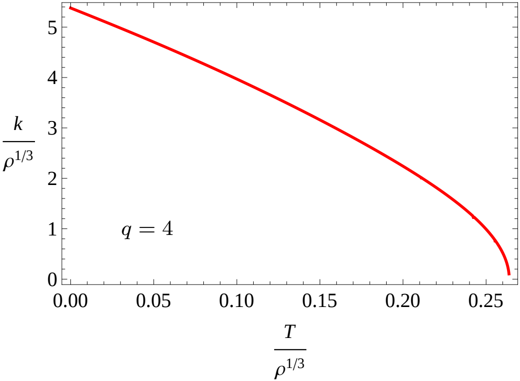
- **Field Perturbations: Scalar Field ψ**
  - Likewise, we consider a small harmonic perturbation to the scalar field

$$\Psi(r, y) = \frac{1}{\sqrt{2}} \left(\psi(r) + e^{iky} \eta(r) \right)$$

- The η EOM can be written as an **eigenvalue equation**

$$\mathcal{L}\{\eta\} = k^2 \eta$$

- We solve for **Permitted Eigenvalues of Wave Number k**



- **Taking a Step Back: What are we doing?**
  - We are following the **Bottom-Up Approach** to Holographic Superconductivity
  - **Very Phenomenological**. Start from simple, hand-made bulk models.
  - One has a **broad input parameter space**: m^2, q, Potential Terms, etc.
  - One can realize **tractable computations and probe the superconducting phenomenology** of the dual QFT theory

- **However, from the context of the AdS/CFT correspondence:**
  - **One cannot "track up" the bulk model back to full Type IIB String Theory origin**
  - **We do not know the details of the dual QFT and the particular condensing operators**

To have some **basic knowledge of the dual field theory**, we can always **follow the same intuition developed by Ginzburg and Landau**...

Enter Ginzburg-Landau Approach

- **Ginzburg-Landau Effective Boundary Action**
  - We propose that our boundary theory can be described **effectively near the critical temperature** by

$$S_{\text{eff}}^{\text{Boundary}} \approx \int d^4x \left\{ \alpha \left|\Psi_{\text{GL}}\right|^2 + \frac{\beta}{2} \left|\Psi_{\text{GL}}\right|^4 + \frac{1}{2} \left|\partial\Psi_{\text{GL}} - iqA\Psi_{\text{GL}}\right|^2 + \cdots \right\}$$

  - Microscopic DOF's are hidden in GL order parameter $|\Psi_{\text{GL}}|$
  - This is the original **phenomenological intuition** followed by GL to explain SC, without knowing the microscopic details of electron pairing

- **Constructing the GL Boundary Action**

To construct the GL boundary action in a **self-consistent manner**, we must then:

- Identify holographically the **GL order parameter** $|\Psi_{\text{GL}}|$
- Compute holographically the **GL coefficients** α, β
- Check with standard **GL Theory predictions**

Determining Ginzburg-Landau: The GL Order Parameter

- **The Ginzburg-Landau Order Parameter $|\Psi_{\mathsf{GL}}|$**
  - Since **GL theory predicts**

    $$|\Psi_{\mathsf{GL}}| \sim (1 - T/T_c)^{1/2}$$

    which has the same critical exponent as $\mathcal{O}_3$. We **match exponents** and simply propose

    $$|\Psi_{\mathsf{GL}}|^2 = N_q \mathcal{O}_3^2$$

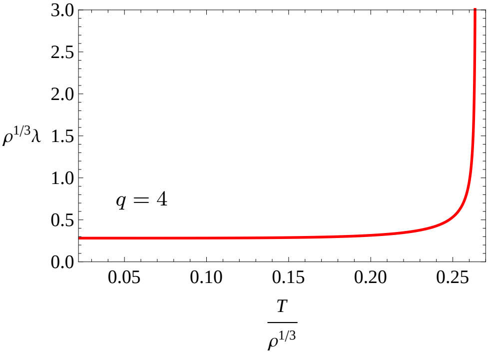
    where we can compute holographically the proportionality constant N_q, given by

    $$N_q = \frac{1}{q\, C_0 T_c(q)}$$

  - Thus, the **GL order parameter is holographically identified**.

- **The Penetration Length λ**
  - Measure exponential decay of magnetic fields inside superconductor
  - It can be holographically computed directly from $\tilde{n}_s$. **According to GL Theory**

$$\lambda = \frac{1}{\sqrt{4\pi\tilde{n}_s}}$$



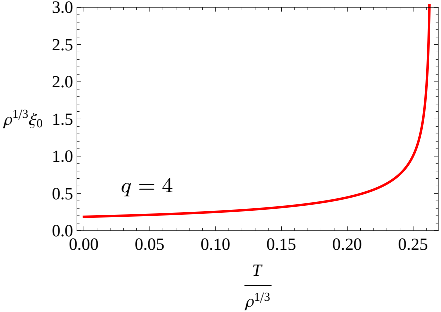
  - Model predicts a near-T_c behaviour

$$\lambda \sim \frac{1}{(1 - T/T_c)^{1/2}}$$

as measured in **Real-World Superconductors**

- **The Coherence Length ξ**
  - Measure of exponential decay of perturbations of the order parameter
  - It can be holographically computed directly from the wave number k. Indeed, the coherence length ξ is the **inverse of the pole of the correlation function** written in Fourier Space

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim \frac{1}{k^2 + 1/\xi^2} \qquad \rightarrow \qquad |\xi| = 1/\,|k|$$

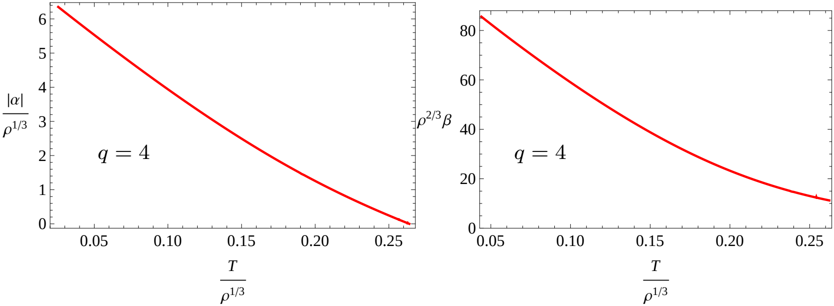


  - Model predicts a near-T_c behaviour as **Real-World Superconductors**

$$\xi \sim \frac{1}{(1 - T/T_c)^{1/2}}$$

- **The Ginzburg-Landau Coefficients α and β**
  - Having computed the characteristic lengths, we now can compute the **GL Coefficients**. The α and β coefficients are then given ultimately by the concise expressions

$$|\alpha| = \frac{1}{4\xi^2} \qquad \beta = \frac{q\,C_0 T_c}{4}\frac{1}{\xi^2 \mathcal{O}_3^2}$$



  - They have the near-T_c behaviour **required by GL**

$$|\alpha| \sim \alpha_1(1 - T/T_c) \qquad \beta \sim \beta_0 + \beta_1(1 - T/T_c)$$

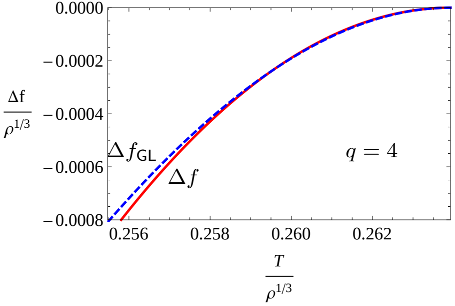
In particular, the β functional relation is **quite non-trivial**, coming from $\beta \sim (\mathcal{O}_3\xi)^{-2}$

- So, having determined holographically the effective GL boundary action, let's **look for some ways to check its consistency**...

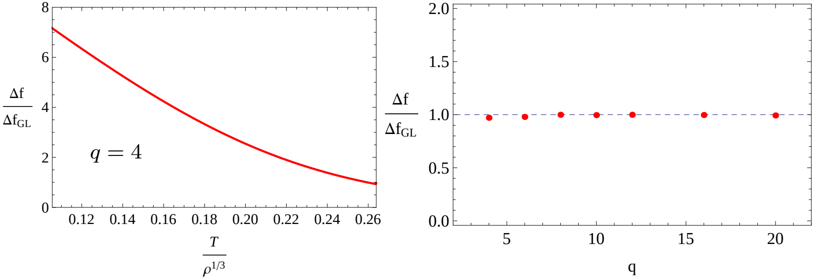
- **The Helmholtz Free Energy**

  - We can calculate the Helmholtz FE difference $\Delta f = f_{\mathsf{sc}} - f_{\mathsf{n}}$. By **standard holographic methods** we must calculate the **on-shell value of the regulated gravitational action at the boundary**. Part of the usual holographic toolkit
  - On the other hand, **according to GL theory**, we approximate this Helmholtz FE as

$$\Delta f_{\mathsf{GL}} \sim \alpha\,|\Psi_{\mathsf{GL}}|^2 + \frac{\beta}{2}\,|\Psi_{\mathsf{GL}}|^4 = -\frac{1}{8\,q\,C_0 T_c}\,\frac{\mathcal{O}_3^2}{\xi^2}$$



- **The Helmholtz Free Energy: A Comparison**
  - We can compare the ratio of both results, and find **Excellent Agreement**



- We can further also with **[Herzog, Kovtun, Son. Phys.Rev.D 79 (2009)]**
- **In a Nutshell:** Computed FE $\longrightarrow$ Fitted to a GL form $\longrightarrow$ Found numerically α and β
- Using **their method**, one finds ($q = 4$)

$$|\alpha| = 4.41(1 - T/T_c) \qquad \beta = 10.95 + 36.75(1 - T/T_c)$$

- Whereas, in our **GL Approach**

$$|\alpha_{\mathsf{GL}}| = 4.45(1 - T/T_c) \qquad \beta_{\mathsf{GL}} = 11.23 + 35.2(1 - T/T_c)$$

- So, again there is **Good Agreement**. However, our method only depends on **simple holographic expressions** obtained by self-consistency: $\alpha \sim \xi^{-2}$, $\beta \sim (\mathcal{O}_3 \xi)^{-2}$

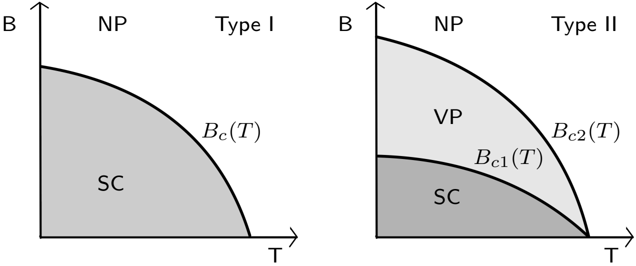
- **Our Minimal D=5 Model: Main Results so Far**

  - We have constructed **holographically** and **self-consistently** and **effective GL action for the boundary theory**
  - By computing **a wide array of physical SC quantities** ($\mathcal{O}_3$, n_s, λ, ξ), our simple model **predicts** a behaviour **in agreement with real-world SC phenomenology**
  - Our holographic computations are **in agreement with non-trivial functional dependencies of GL Theory**: α, β. These are encoded in **simple, concise holographic expressions**
  - The GL Approach computation of the **Free Energy** is **in agreement** with the standard holographic method, and **in agreement with previous research**

- Finally, lets use our approach to **study the magnetic phenomenology** of our Holographic Superconductor...

Small Aside: Magnetic Phenomena in Superconductivity

- **Meissner Effect:** Expulsion of Magnetic fields from the volume of a SC
- However, **increasing the magnitude of the field breaks the SC phase** in two distinct manners
- This provides one of the **main ways to classify a SC**



- **Type I:** Superconducting Phase$\longrightarrow$ Normal Phase at B_c
  First order phase transition
- **Type II:** Superconducting Phase$\longrightarrow$ Abrikosov Vortex Phase at B_{c1} $\longrightarrow$ Normal Phase at B_{c2}
  Second order phase transition

Holographic Superconductor: Type I or II?

- **The Ginzburg-Landau Parameter κ**
  - One of the **great triumphs of Ginzburg-Landau Theory** was to encode the Type I/Type II classification in a single parameter, known as the **Ginzburg-Landau Parameter κ**

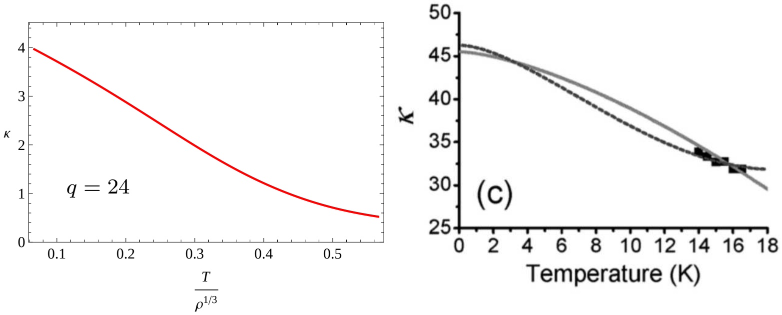
    Quite succinctly, **GL Theory tells** us that a superconductor is:

    - **Type I** if $\kappa < 1/\sqrt{2}$
    - **Type II** if $\kappa > 1/\sqrt{2}$

Holographic Superconductor: Type I or II?

- **The Ginzburg-Landau Parameter κ**
  - With the characteristic lengths, we can also holographically compute the **Ginzburg-Landau Parameter κ**

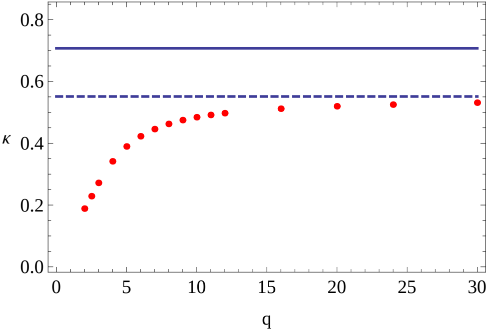
$$\kappa \equiv \frac{\lambda}{\xi} = \sqrt{\frac{1}{8\pi \tilde{n}_s \xi^2}}$$



  - **Very non-trivial**. It follows perfectly the **Summers Empirical Fitting** for Nb_3Sn

$$\kappa(T) = \kappa(0)\left(a_0 - b_0 (T/T_c)^2 (1 - c_0 \log(T/T_c))\right)$$

- **The Ginzburg-Landau Parameter κ. Type I or II?**
  - For each value of q, $\kappa(T_c)$ is **always finite value**, which we take as the **characteristic value** of κ for the SC model at a given q
  - We take this value and see **how it evolves with the scalar charge q**



  - We see κ approaches asymptotically $\kappa \sim 0.55$
  - Since this value is below $1/\sqrt{2} \sim 0.71 \longrightarrow$ **the Superconductor must be Type I**

- **Droplet Solutions**
  - We apply a **constant magnetic field** following the magnetic-brane solution by **[D'Hoker, Krauss. JHEP 1003 (2010)]. Used for the first time done in HSC**
  - We add a **Magnetic Component** to the gauge field ansatz

$$A = \Phi(r)dt + \frac{B}{2}\left(-ydx + xdy\right) \qquad F_{xy}|_{r\to\infty} = B$$

  - Take D'Hoker-Kraus background as fixed and add scalar field
  - The scalar field equation results to be **separable**

$$\Psi(r,u) = \frac{1}{\sqrt{2}}R(r)U(u)$$

    where u is the **radial polar coordinate** in the (x, y) plane
  - U-equation has the following solution

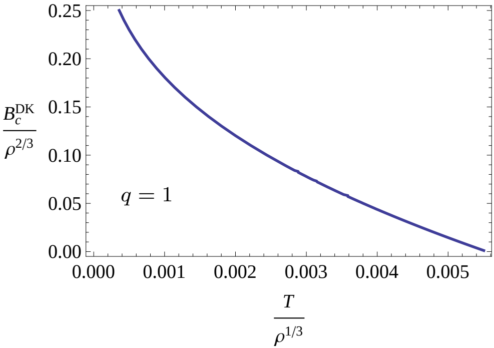
$$U(u) = \exp\left(-\frac{qB}{4}u^2\right)$$

    Thus, we have **Superconducting Droplet Solutions**
  - The R-equation develops an **effective mass:**

$$m^2_{\text{eff}} = m^2 - \frac{q^2\Phi^2}{g} + 2\,q\,B\,e^{-2V}$$

    The Φ contribution lowers the effective mass, making the system unstable. However, the magnetic field reverts the scalar field instability
    $\longrightarrow$ Returns us to **normal phase** at a critical magnetic field B_c

- **Critical Magnetic Field**
  - We solve the R-equation for the value B_c that returns us to the normal phase



  - The model predicts a near-T_c behaviour of B_c as

$$B_c \sim (1 - T/T_c)$$

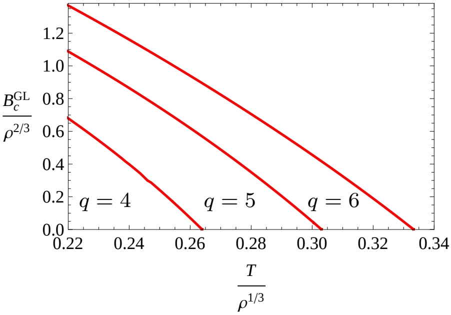
in accordance to the **critical fields measured in real superconductors**

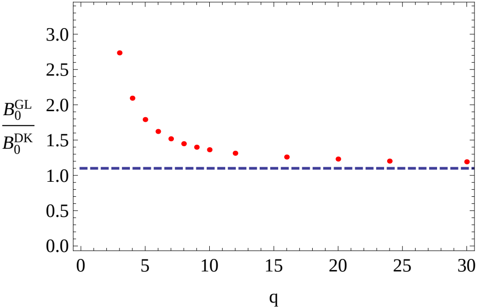
- **Magnetic Comparison**

  - **According to GL Theory**, the critical magnetic field is given by

$$B_c^{\mathsf{GL}} = \sqrt{4\pi}\,\frac{|\alpha|}{\sqrt{\beta}} = \sqrt{\frac{\pi}{q\,C_0 T_c}}\,\frac{\mathcal{O}_3}{\xi}$$

  - We can compare the B_c calculated through the D'Hoker-Kraus solution (B_c^{DK}) with the B_c computed by our GL Approach (B_c^{GL}) by **computing the ratio** $B_c^{\mathsf{GL}}/B_c^{\mathsf{DK}}$, **evaluated at** $T = T_c$





Our Bottom-Up Experience

- **Our Minimal D=5 Model: Main Magnetic Results**

  - We have **holographically computed the Ginzburg-Landau parameter** κ and concluded that our HSC is **Type I**
  - Furthermore, the **GL parameter** κ as a function of temperature **closely resembles** the behaviour of a real world high-T_c superconductor
  - We have computed the **critical magnetic field** using the **D=5 D'Hoker-Kraus solution for the first time** in the context of HSC and saw that it is **consistent with GL computation**

A Different Model

- **Looking for Changes**
  - So, having constructed our Ginzburg-Landau effective description, it is only natural to look at **how it can be altered by considering different bulk models**. Let us then consider a **different kind of background**...

Introducing Lifshitz Scaling to Holographic Superconductivity

- **[Dector. Nucl.Phys. B898 (2015)]**

- **Enter Lifshitz Background**
[Kachru, Liu, Mulligan. Phys.Rev.D 78 (2008)], [Taylor. ITFA 48 (2008)]

  - **Motivation:** The phase transition of some condensed matter systems are governed by **Lifshitz-like Fixed Points**, which exhibit **anisotropic scaling**

$$t \to \lambda^z t \qquad\qquad x \to \lambda x$$

z = Lifshitz Dynamical Critical Exponent

  - This anisotropy breaks Lorentz invariance $\rightarrow$ Systems are **Non-Relativistic**
  - There is a **gravitational dual** to Lifshitz fixed point systems, given by the background

$$ds^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 \sum_{i=1}^{3} dx_i^2$$

$$f(u) = 1 - \frac{r_h^{z+3}}{r^{z+3}}$$

$$T = \frac{(z+3)}{4\pi} r_h^z$$

  - In the **isotropic case** $z = 1$ we recover Schwarzschild AdS BH.
  - **Our Question:** How does the **anisotropy** alters the Holographic SC phenomenology?

- **The Bulk Model**

  - We propose the same D=5 bulk-model

$$S = \int d^5 \sqrt{-g} \left(-\frac{1}{4} F^2 - |\partial \Psi - i A \Psi|^2 - m^2 |\Psi|^2 \right)$$

    under the **Fixed Lifshitz Background**.

  - We use the same Bulk-Fields Ansatz

$$\Psi(r) = \psi(r)/\sqrt{2} \qquad A = \Phi(r) dt$$

  - The scalar and gauge fields have asymptotics

$$\psi(r \to \infty) \approx \frac{\mathcal{O}_-}{r^{\Delta_-}} + \frac{\mathcal{O}_+}{r^{\Delta_+}} + \cdots \qquad \Phi(r \to \infty) \approx \mu - \frac{\rho}{r^{3-z}} + \cdots$$

$$\Delta_\pm = \frac{1}{2} \left((z+3) \pm \sqrt{(z+3)^2 + 4m^2} \right)$$

    and the **BF bound is now**

$$m^2 \geq -\frac{(z+3)^2}{4}$$

  - We will consider the integer values **z = 1, 2** to see how the SC phenomenology deviates from the **isotropic case z =1**.

- **Different Cases of Condensation**

    - To have a **fuller phenomenological picture**, we will study **two cases of condensation**
    - **Case I**: Take mass

      $$m^2 = -3z$$

      So

      $$\Psi(r \to \infty) \approx \frac{\mathcal{O}_z}{r^z} + \frac{\mathcal{O}_3}{r^3} + \cdots$$

      and set $\mathcal{O}_z = 0$, so that the SC order parameter is $\mathcal{O}_3$ of dimension 3
    - **Case II**: Take mass

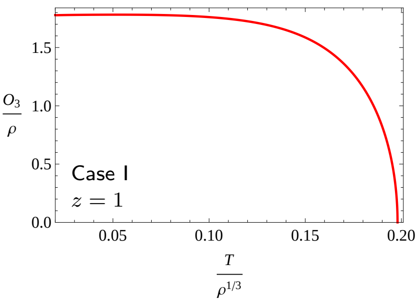
      $$m^2 = -(z+2)$$

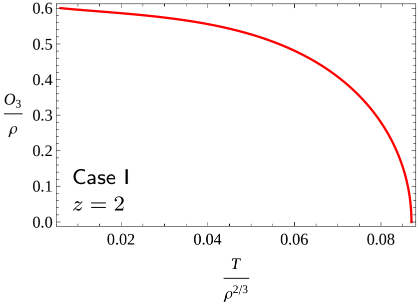
      So

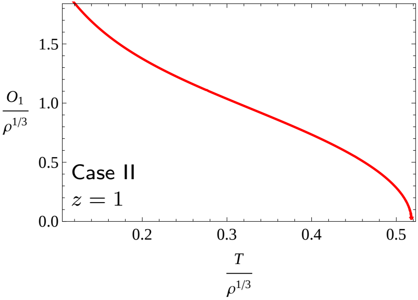
      $$\Psi(r \to \infty) \approx \frac{\mathcal{O}_1}{r} + \frac{\mathcal{O}_{z+2}}{r^{z+2}} + \cdots$$

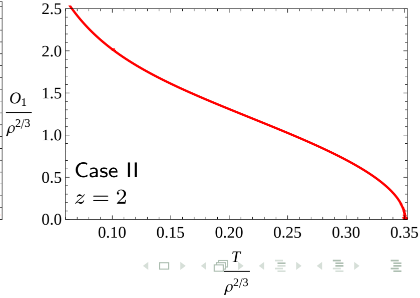
      and set $\mathcal{O}_{z+2} = 0$, so that the SC order parameter is $\mathcal{O}_1$ of dimension 1

- **Condensates and Critical Temperature**









Studying Different Condensates

- **Condensates and Critical Temperature**
  - The model predicts a near-T_c behavior as

$$\mathcal{O}_\Delta \sim (1 - T/T_c)^{1/2}$$

    as **real-world superconductors** for **all condensates and for all z**
  - **However**, we also observe that T_c changes with z:

$T_c/\rho^{z/3}$	$z = 1$	$z = 2$
Case I	0.198	0.087
Case II	0.517	0.351

  - Thus, we conclude that **anisotropy lowers the critical temperature**

Gauge Fluctuation and Penetration Length

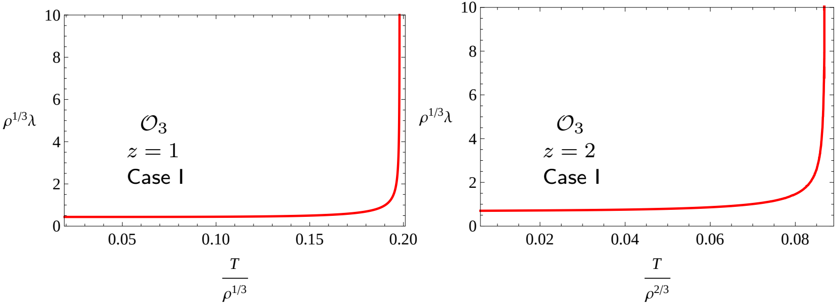
- If we add the **Gauge Fluctuation**

$$A = \Phi(r)dt + e^{-i\omega t + iky} A_x(r)$$

then we obtain holographically **SC number density** n_s

$$A_x(r \to \infty) \approx A_x^{(0)} + \frac{J_x}{r^{1+z}} + \cdots \quad \Longrightarrow \quad n_s = -\frac{J_x}{A_x^{(0)}}$$

- We can then compute the **Penetration Length** $\lambda = 1/\sqrt{4\pi n_s}$



- Near-T_c we find the behaviour

$$\lambda \sim (1 - T/T_c)^{-1/2}$$

in accordance to **real-world superconductors**, for **all condensates** and **all values of z**

Scalar Fluctuation and Coherence Length

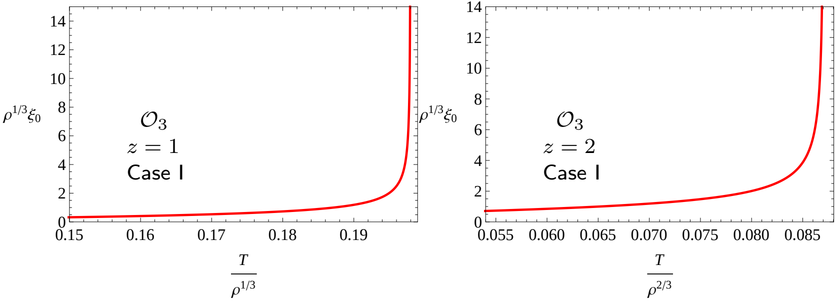
- If we add the **Scalar Fluctuation**

$$\Psi(r,y) = \left(\psi(r) + e^{iky}\eta(r)\right)/\sqrt{2}$$

then we obtain the **wave number** k for the eigenvalue equation $\mathcal{L}\{\eta\} = k^2\eta$. Then, from

$$\langle \mathcal{O}(k)\mathcal{O}(-k)\rangle \sim \frac{1}{k^2 + 1/\xi^2} \quad \Longrightarrow \quad |\xi| = 1/|k|$$

- We can then compute the **Coherence Length** ξ



- Near-T_c we find the behaviour

$$\xi \sim (1 - T/T_c)^{-1/2}$$

in accordance to **real-world superconductors**, for **all condensates** and **all values of z**

Ginzburg-Landau Approach with Lifshitz Scaling

- **Ginzburg-Landau Effective Boundary Action**
  - Following the previous exposition, we now construct an **effective GL action for the boundary theory**
  - Again, to **determine the GL order parameter** $|\Psi_{\mathsf{GL}}|$ parameter we propose

$$|\Psi_{\mathsf{GL}}|^2 = N_z \mathcal{O}_\Delta^2$$

and using the numerical equality

$$\left.\frac{\mathcal{O}_\Delta^2}{n_s}\right|_{T=T_c} = C_z$$

we obtain

$$N_z = \frac{1}{C_z}$$

Thus, the GL order parameter can again be **holographically determined**.

  - We compute the **GL coefficients** as in the previous exposition. The result is

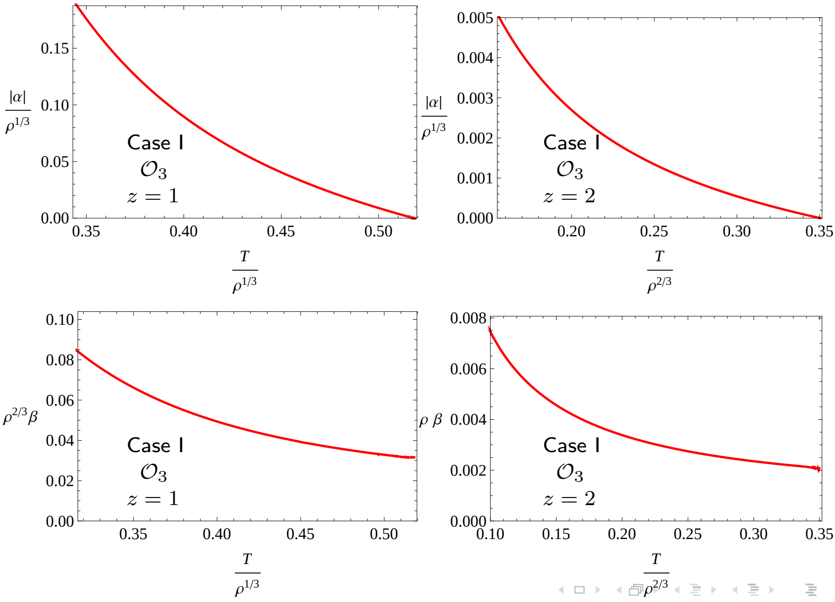
$$|\alpha| = \frac{1}{4\xi^2} \qquad \beta = \frac{1}{4N_z \xi^2 \mathcal{O}_\Delta^2}$$

  - Both coefficients retain the standard near-T_c behaviour **for all condensates and values of z**.

$$\alpha \approx \alpha_1 (1 - T/T_c) \qquad \beta \approx \beta_0 + \beta_1 (1 - T/T_c)$$

  - **However**, we also observe that **their magnitude decreases for larger values of z**

- **Ginzburg-Landau Coefficients**



Holographic Superfluid Interpretation

- **Holographic Superfluidity Point of View**
  - If we consider the case where **we keep the $U(1)$ symmetry in the boundary field theory as ungauged, that is, global** $\Longrightarrow$ We can take our system as a model for an **Holographic Superfluid**
  - Our bulk gauge field perturbation A_x has the holographic superfluid interpretation

$$A_x(r \to \infty) \approx v_x + \frac{J_x}{r^{z+1}} + \cdots$$

v_x : **Superfluid Velocity** J_x : **Supercurrent**

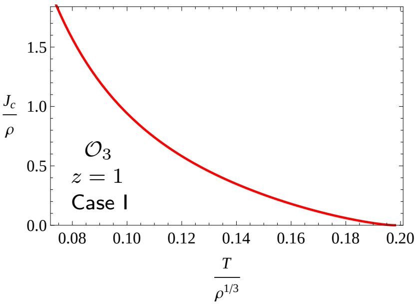
  - Particularly, we can compute the **Critical Supercurrent** J_c: The value of the supercurrent at which the superfluid system passes to the normal phase
  - **According to GL theory**, the supercurrent is given by

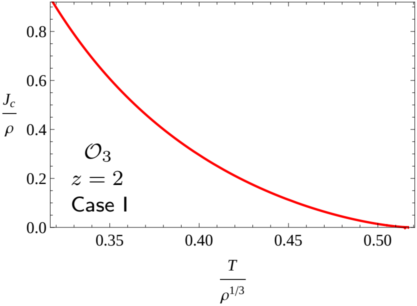
$$J_c = |\Psi_\infty|^2 \left(\frac{2}{3}\right)^{3/2} \sqrt{|\alpha|} = \frac{1}{2\,C_z} \left(\frac{2}{3}\right)^{3/2} \frac{\mathcal{O}_\Delta^2}{\xi}$$

Holographic Superfluid Interpretation

- **The Critical Supercurrents**

  - Our computation of the critical current give





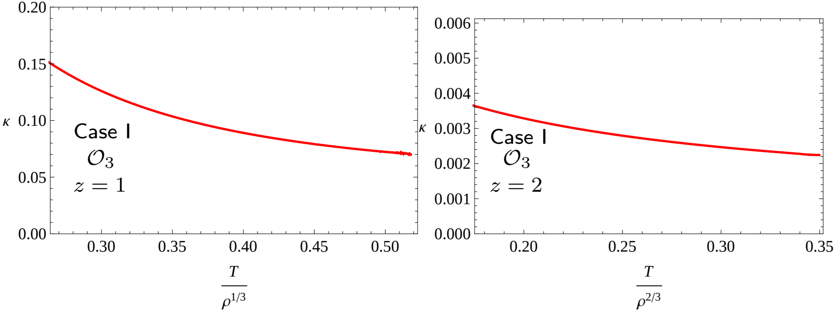
  - We find that the predicted near-T_c behaviour of J_c is

$$J_c \sim (1 - T/T_c)^{3/2}$$

which is in agreement with measured J_c in **real-world superfluids for all condensates and all values of z**
However, the magnitude is diminished by anisotropy

Ginzburg-Landau Parameter and Lifshitz Scaling

- Finally, we can compute the **GL Parameter** $\kappa = \lambda/\xi$ for different values of z



- Taking the value of κ at the critical temperature T_c we find

κ	$z = 1$	$z = 2$
Case I	0.527	0.467
Case II	0.070	0.002

- All values of κ are **lower** than $1/\sqrt{2} \sim 0.71$ for all $z \Longrightarrow$ **Our System is a Type I SC**
- κ is always lower for higher values of z

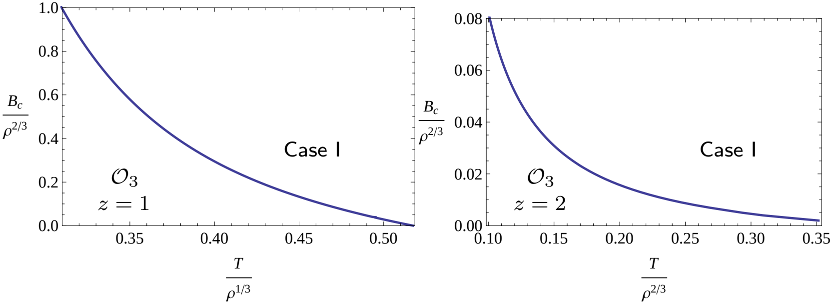
- **Vortex Lattice Solutions**
  - To study the system under the presence of a magnetic field, we follow **[Maeda, Natsuume, Okamura. Phys.Rev.D 81 (2010)]**
  - We find that the **most general** scalar field solution is **separable** as

$$\Psi^{(1)}(r,\vec{x}) = \rho(r)\exp\left(-\frac{B\,x^2}{2}\right)\vartheta_3(\nu,\tau)$$

    where ϑ_3 is the **Elliptical Theta Function** which has **pseudo-periodicity** and **periodically-located zeros** in the (x-y) plane
  - Thus, the $\Psi^{(1)}$ **solution has a lattice vortex profile in the (x-y) plane**

- **The Critical Magnetic Field**

  - Meanwhile, from the radial part of the solution $\rho(r)$ we compute the **critical magnetic field** B_c



  - Our model predicts the near-T_c behaviour **for all z and condensates** is

$$B_c \sim (1 - T/T_c)$$

in agreement with measured B_c
However, the magnitude is **diminished** for higher anisotropy

Our Bottom-Up Experience with Lifshitz Scaling

- **Minimal D=5 Model Adding Lifshitz Scaling: Main Results**
  - We have shown that the **effective GL action for the boundary theory** can be constructed **in the presence of a Lifshitz background**
  - We observe that **the critical temperature is lowered by anisotropy**
  - We observe that **near-T_c functional dependency on** T of physical quantities is **robust** and is not affected by anisotropy
  - However, the **magnitude of physical quantities (α, β, J_c, κ, B_c) is diminished by higher anisotropy**
  - The Ginzburg-Landau parameter κ is **lower than** $1/\sqrt{2}$ for **all condensates and values of z**, so the system is always **Type I**
  - We computed the **critical magnetic field** and found solutions with a vortex lattice profile

- **Final Analysis**

  - I have presented you with a good overview of the more phenomenological approach to holographic superconductivity
  - Using simple models, we constructed holographically a **Ginzburg-Landau effective action for the boundary theory**
  - In particular, we have computed the **Ginzburg-Landau parameter** κ and shown that the **system is Type I**
  - We have also computed the value of the **critical magnetic field** on different setups
  - We have seen that **holographic computations** can reproduce with quite some detail the **specific phenomenology of superconducting systems**

Muchas Gracias!

Additional Slides

Superconductivity in Just One Slide

- **We Need to Talk About SC**
  - Discovered in 1911 by K. Onnes
  - Defined by **Loss of Resistivity**+ **Perfect Diamagnetism** below a certain **Critical Temperature** T_c
  - First description given by the **London Theory** (1935).
    Very Phenomenological.
    Based on n_s (taken as constant). Gives the **London Equations.**
    However: Does not hold in strong magnetic fields.
  - Next came **Ginzburg-Landau Theory** (1950).
    Based on non-homogeneous $|\Psi_{\mathsf{GL}}(\vec{x})|$.
    Accounts for SC + magnetic phenomenology.
    However: Only valid near-T_c and is only an **effective description**.
  - Finally: **BCS Theory** (1957).
    Based on $\Delta_{\mathbf{k}} \sim \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$ and **Cooper Pairing** mediated by **Phonon Interaction**.
    Very successful microscopic theory of most superconducting materials.
  - ....And then: **High Temperature Superconductivity**.

- **General Properties**
  - Discovered first by Bednorz and Müller in 1986. (Instant Nobel Prize!)
  - A material is considered a High-Temperature Superconductor is his critical temperature is $T_c \sim 30K$ or higher. A typical High-T_c superconductor has actually $T_c \sim 90K$.
  - By a High-Temperature Superconductors, we will be referring to **cuprate** superconductors. Cuprates are ferromagnetic ceramics that after slight doping present high-T_c superconducting behaviour on cooling.
  - The cuprates are structurally composed of 2-dimensional CuO_2 layers, and superconductivity occurs in these copper-oxide layers.
  - They have an order parameter with **d-wave** symmetry

$$\Delta_{\mathbf{k}} \sim \cos k_x - \cos k_y \,.$$

Spin-Fermion Model. Advantages and Limitations

- **Advantages of the SF Model**

the Spin Fermion Model captures a lot of cuprate phenomenology

- **Superconducting instability**.
- **d-wave order parameter** $\Delta_{\mathbf{k}} \sim \cos k_x - \cos k_y$.
- **NFL liquid behaviour** from one loop corrections in the self energy.

- **Limitations of the SF Model**

However, it also has the following **Very Serious Limitations**

- It has a **strong coupling**: $\lambda \sim 2$, while for usual Superconductors $\lambda \sim 0.3$.
  This makes it hard to get information out of the theory, because of limited use of perturbative techniques
- There is a **quasi-particle picture breakdown** at the hot-spots of the Fermi Surface of the cuprates (!!!).
  This forbids us to use QFT techniques at all at some points.

- **The Superconductor Characteristic Lengths: Definitions**
  - In order to compute α and β, we first calculate holographically the **Superconductor Characteristic Lengths:** λ and ξ.
  - **Penetration Length** λ. External magnetic field have **exponential decay** inside superconductor, following

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2} \mathbf{B}$$

  - **Coherence Length** ξ. Measure of spatial **decay of small perturbations** of $|\Psi_{\text{GL}}|$ from **Minimum Value** $|\Psi_\infty|$, which is the value of the order parameter **deep-inside the SC**

$$|\Psi_{\text{GL}}| = |\Psi_\infty| + \eta(x)$$

$$\eta(x) \sim \exp(-|x|/\xi)$$

- **AdS/CFT: Minimal Elements**

Large-N **Strongly Coupled** Quantum Gauge Field Theory in **d Dimensions** **(Boundary Theory)**	$\longleftrightarrow$	**Classical** Anti-deSitter Gravitational Theory in **d+1 Dimensions** **(Bulk Theory)**

- **AdS/CFT+SC: Motivation**
- **The Dictionary**
  - **The AdS/CFT Correspondence establishes a very clear Dictionary**, between bulk and boundary physical quantities and phenomena.
  - The "**Translator**": **The Master Equation**.
    Given a bulk field ϕ with value at the boundary ϕ_0

$$e^{-\Gamma_{\mathsf{CFT}}[\phi_0]} \equiv Z_{\mathsf{String}}[\phi_0]$$

  - With

$$Z_{\mathsf{string}}[\phi_0] = \int_{\phi_0} D\phi\; e^{-S_{\mathsf{String}}} \approx e^{-S_{\mathsf{Classical\ Gravity}}}$$

$$e^{-\Gamma_{\mathsf{CFT}}[\phi_0]} = \left\langle \exp - \int \psi_0 \mathcal{O} \right\rangle_{\mathsf{CFT}}$$

- **Back to Our Choice of Bulk-Dimension.**

  - We choose **bulk-dimensions d=4+1** $\longrightarrow$ **boundary-dimension d=3+1**. **Why this choice?**
  - It is usually believed that boundary-dimension **d=2+1 HSC must be Type II**. **Why is this believed?**
  - **"The Scaling Argument":** First, consider a dual field theory on d=2+1 (SC on a plane). Then, apply a 3-dimensional magnetic field. Then, the free energy needed to expel magnetic field scales as **Volume**, while the energy the system gains from being SC scales as **Area**. Therefore, **magnetic fields are never completely expelled** $(B_{c1} = 0)$ and the system is **Type II.**
  - **However**, if both magnetic field and SC have the s**ame spatial dimension 3**, then there is a **direct thermodynamical competition** that can make the SC **Type I**. And that is why.
  - There is now evidence that a boundary dimension **d=2+1** Holographic Superconductor **can indeed be Type I**.
    [Dias, Horowitz, Iqbal, Santos. JHEP 1404 (2014)]

- **The Normal Phase**
  - The EOM of the system admit a **trivial solution for the scalar**

$$\Psi(r) = 0$$

    which corresponds to a **Null Order Parameter** in the dual QFT, i.e. **Normal Phase**
  - In this phase, for the metric and gauge field solution is exact and given by **AdS Reissner-Nordström BH**

$$g(r) = r^2 - \frac{3r_h^6 + \rho^2}{3r_h^2 r^2} + \frac{\rho^2}{3\,r^4}$$

$$\chi(r) = 0$$

$$\Phi(r) = \rho\left(\frac{1}{r_h^2} - \frac{1}{r^2}\right)$$

  - The **Hawking-Temperature** is

$$T = \frac{6r_h^6 - \rho^2}{6\pi r_0^5}$$

- **The Superconducting Instability**
  - **Summary:** The near-horizon region of the normal AdS-RN-BH at $T = 0$ has a $AdS_2 \times \mathbb{R}^3$ form. A small scalar field has an effective mass $m^2_{\text{eff}} = m^2 - 2q^2$, so the charge can drive the mass below the AdS_2 BF bound, making the near-horizon unstable and leading to hairy BH solutions. Hairy solution translates to U(1) symmetry breaking and **condensation** in the dual QFT's order parameter.
  - In the $T = 0$ limit, changing to near-horizon coordinate $\tilde{r} = r - 1$ ($r_h = 1$), the RN-BH metric is $\text{AdS}_2 \times \mathbb{R}^3$ (with a different radius)

$$ds^2 \approx -12\,\tilde{r}^2 dt^2 + \frac{1}{12\,\tilde{r}^2} d\tilde{r}^2 + d\vec{x}_3^2$$

  - The scalar field equation in this limit is

$$\psi'' + \frac{2}{\tilde{r}}\psi' + \frac{2q^2 - m^2}{12\,\tilde{r}^2}\psi = 0 \qquad m^2_{\text{EFF}} = \frac{m^2 - 2q^2}{12}$$

  - So, there is a **near-horizon instability** if the mass is below the **AdS_2 BF bound**

$$m^2 - 2q^2 < -3$$

  - Thus, in the window

$$-4 < m^2 < 2q^2 - 3$$

    one has **asymptotic AdS_{d+1} geometry** and an **instability in the near-horizon AdS_2**
  - Instability will lead to **Scalar Hair Solutions**

- **The Basic Phenomenological Elements of a ORDINARY Superconductor**

  Any very minimal SC theory must have the following properties:

  - The theory will posses the usual electromagnetic gauge invariance $\rightarrow$ **U(1) local symmetry**.
  - The **spontaneous breaking of this U(1)** symmetry leads to a **SC phase**.
  - The symmetry breaking is provoked by the condensation of a **Charged SC Order Parameter**

This elements suffice to have **Infinite Conductivity**

- **The Basic Phenomenological Elements of a HOLOGRAPHIC Superconductor**
  - **U(1) local symmetry**
    -The breaking of this U(1) in the bulk leads to a SC phase in the boundary
    -Local U(1) theory in the bulk is dual to a global U(1) theory in the boundary. We will always assume that the global U(1) theory can be promoted to local by gauging of the the dual theory
  - **Massive Charged Bulk Scalar Field** Ψ
    -This will translate to a SC **Order Parameter** in the dual theory (**Note**: s-wave)
    -**Effective holographic description** of dual multi-fermion bound state
    -The AdS$_{d+1}$ bulk theory is **stable** if scalar mass is above **BF Bound**

$$m^2 \geq -\frac{d^2}{4} \qquad \left(\; m^2 \geq -4 \quad \text{for the d=4 case} \; \right)$$

  - **U(1) Gauge Field** A_μ
    -Required by U(1) Symmetry
    -Introduces **Charge Density** in Boundary Theory
  - **Gravity**
    -**Einstein-Hilbert-Maxwell**. This will give gauge solutions with charge density
    -**Negative Cosmological Constant**: This will give **Vacuum AdS Solutions**
    -**Black Hole Solution**: This will give **Temperature** in dual QFT

- **The Ginzburg-Landau Order Parameter** $|\Psi_{\mathsf{GL}}|$
  - **GL theory predicts**

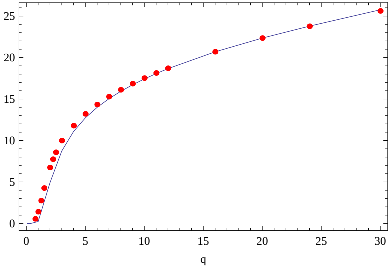
$$|\Psi_{\mathsf{GL}}| \sim (1 - T/T_c)^{1/2}$$

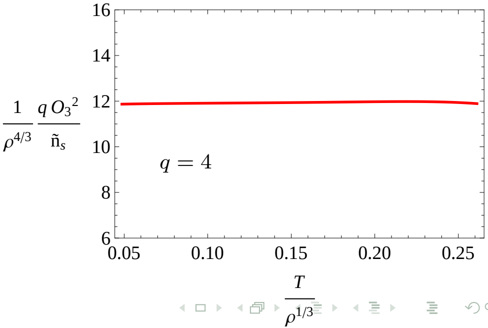
which has the same critical exponent as $\mathcal{O}_3$. We **match exponents** and simply propose

$$|\Psi_{\mathsf{GL}}|^2 = N_q \mathcal{O}_3^2$$

  - To determine N_q we use the following **Numerical Equality**

$$q \frac{\mathcal{O}_3^2}{\tilde{n}_s}\bigg|_{T=T_c} = C_0 T_c(q)$$





- Then, if we use the **Ginzburg-Landau Relation**

$$|\Psi_{\mathsf{GL}}|^2 = n_s$$

we can finally obtain

$$N_q = \frac{1}{q\, C_0 T_c(q)}$$

- Thus, the **GL order parameter is holographically identified**.

- **The Ginzburg-Landau Coefficients α and β**

Having computed the characteristic lengths, we now can compute the **GL Coefficients**

- The α coefficient is computed **directly from GL Theory**

$$|\alpha| = \frac{1}{4\xi^2}$$

- To compute β, we use the **GL Relation**

$$|\Psi_\infty|^2 = \frac{|\alpha|}{\beta}$$

where $|\Psi_\infty|$ is the value of condensate deep-inside the SC, **where external fields and gradients are negligible**.

- Since we are working with **small field perturbations, we consider ourselves in that approximation**. Then $|\Psi_{\mathsf{GL}}| \approx |\Psi_\infty|$ and

$$\beta = \frac{|\alpha|}{|\Psi_\infty|^2} = \frac{|\alpha|}{N_q \mathcal{O}_3^2} = \frac{q\, C_0 T_c}{4} \frac{1}{\xi^2 \mathcal{O}_3^2}$$

- **Constant Magnetic Field**
  - We apply a constant magnetic field following the magnetic-brane solution by **[D'Hoker, Krauss. JHEP 1003 (2010)]. First time done in HSC**.
  - **In a Nutshell:** We start with Einstein-Hilbert-Maxwell

$$S = \int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F^2\right)$$

  - We add a **Magnetic Component** to the gauge field ansatz

$$A = \Phi(r)dt + \frac{B}{2}\left(-ydx + xdy\right) \qquad F_{xy}|_{r\to\infty} = B$$

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + e^{2V(r)}\left(dx^2 + dy^2\right) + e^{2W(r)}dz^2$$

  and solve in a **perturbative manner** around $B = 0$

$$g(r) = g_0(r) + B^2 g_2(r) + \cdots \qquad \Phi(r) = \Phi_0(r) + B^2\Phi_2(r) + \cdots$$

$$V(r) = V_0(r) + B^2 V_2(r) + \cdots \qquad W(r) = W_0(r) + B^2 W_2(r) + \cdots$$

  which is a reliable expansion if $B \ll T^2$
  - The Hawking Temperature is

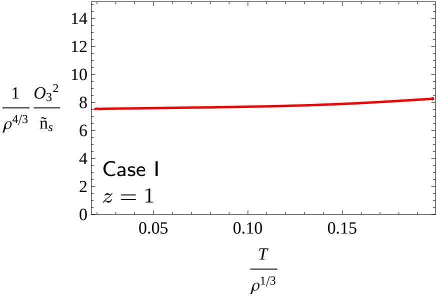
$$T = \frac{24r_h^6 - 4\rho^2 - B^2 r_h^2}{24\pi r_h^5}$$

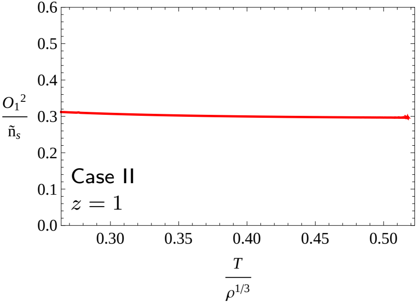
- **Confirming the Numerical Equality**

  - Having computed n_s, we can also **confirm the numerical equality**

$$\left.\frac{\mathcal{O}_{\Delta}^{2}}{n_s}\right|_{T=T_c} = C_z$$

where C_z depends only of z for each case of condensation considered





- **Magnetic Fields: A Series Expansion**

  - We follow **[Maeda, Natsuume, Okamura. Phys.Rev.D 81 (2010)]** and propose a series expansion for the bulk fields

$$\Psi(\vec{x}, r) = \epsilon^{1/2}\Psi^{(1)}(\vec{x}, r) + \epsilon^{3/2}\Psi^{(2)}(\vec{x}, r) + \cdots$$

$$A_\mu(\vec{x}, r) = A_\mu^{(0)}(\vec{x}, r) + \epsilon\, A_\mu^{(1)}(\vec{x}, r) + \cdots$$

  with

$$\epsilon \equiv \frac{B_c - B}{B_c} \qquad \epsilon \ll 1$$

  - **The Gauge Solution:** At zero-order we have solutions

$$A_t^{(0)} \equiv \Phi(r) = \mu - \frac{\rho}{r^{3-z}} \qquad A_y^{(0)} = B\, x$$

  - **The Scalar Solution:** The scalar field at first-order has a **most general**, **separable solution**

$$\Psi^{(1)}(r, \vec{x}) = \rho(r) \exp\left(-\frac{B\, x^2}{2}\right) \vartheta_3(\nu, \tau)$$

  where ϑ_3 is the **Eliptical Theta Function** which has **pseudo-periodicity** and **periodically-located zeros** in the (x-y) plane

  - Thus, the $\Psi^{(1)}$ **solution has a lattice profile in the (x-y) plane**.