Radiative Corrections of O(α) to $B^- \to V^0 \ell^- \bar{\nu}_\ell$ decays

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Charged current and the CKM matrix

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}} \left(J_W^{\mu} W_\mu^- + J_W^{\mu\dagger} W_\mu^+ \right),$$

$$\begin{aligned} J_W^{\mu\dagger} &= \sum_{m=1}^{F} \left[\bar{\nu}_m^0 \gamma^\mu (1-\gamma^5) e_m^0 + \bar{u}_m^0 \gamma^\mu (1-\gamma^5) d_m^0 \right] \\ &= (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau) \gamma^\mu (1-\gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t}) \gamma^\mu (1-\gamma^5) \boldsymbol{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \end{aligned}$$

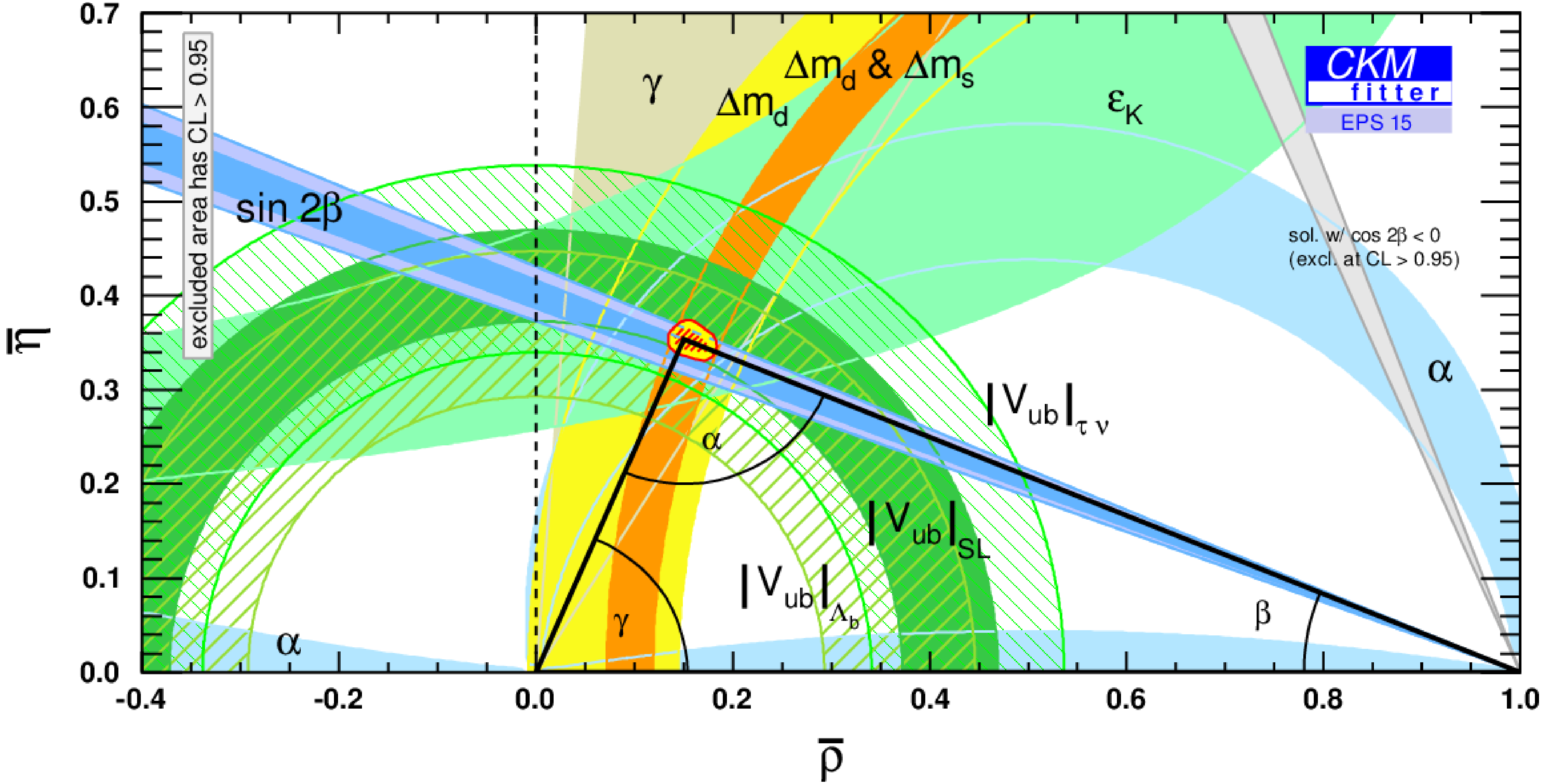
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\sum_j V_{ij} V_{jk}^* = \delta_{ik};$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

A precise determination of the |Vub | quark mixing at a few percent level, is crucial for future tests of the Standard Model (SM) picture of CP violation as it fixes one of the sides of the db unitarity triangle.



(CKM fitter 2015)

Several leptonic and semileptonic exclusive decays induced by the b → u l ν transition can be used to extract |Vub| in an independent way.

The cleanest channel owing to the better control on theoretical and experimental inputs is $B \to \pi \ell \nu_\ell$

$$|V_{ub}|_{\rm excl} = (3.28 \pm 0.29) \times 10^{-3}$$

When compared to the most precise determination from inclusive b → u l ν transitions $|V_{ub}| = (4.49 \pm 0.16 \, {}^{+0.16}_{-0.18}) \times 10^{-3}$

one is led to discrepant result :

$$\Delta|V_{ub}| = |V_{ub}|_{\rm incl} - |V_{ub}|_{\rm excl} = (1.13 \pm 0.36) \times 10^{-3}$$ **(3.1 σ)**

In the isospin symmetry limit, the following relation holds:

$$\Gamma^I(B^0 \to \rho^- \ell^+ \nu_\ell) = 2\Gamma^I(B^+ \to \rho^0 \ell^+ \nu_\ell)$$
$$= 2\Gamma^I(B^+ \to \omega \ell^+ \nu_\ell) \ ,$$

Departures from the isospin symmetry relations are expected at the few percent level owing to effects of electromagnetism and u − d quark mass difference.

Channel	PDG 2014 [2]
$B^0 \to \rho^- \ell^+ \nu_\ell$	1.61± 0.21
$B^+ \to \rho^0 \ell^+ \nu_\ell$	0.87± 0.14
$B^+ \to \omega \ell^+ \nu_\ell$	0.73± 0.06

Improved measurements of the different charged decay channels of $B \to V \ l^- \nu$ branching ratios need RC and isospin corrections in order to test consistency.

$$d\Gamma = d\Gamma_{\mathrm{LD}}(k < \Lambda) + d\Gamma_{\mathrm{SD}}(k > \Lambda)$$

Virtual + real photons in effective
theory (hadrons): scalar QED, ChPT, SCET,…

Virtual corrections (quarks, leptons, W,…): EW theory

Matching at scale Λ

Corrections to decay rate, Dalitz Plot, spectrum.
Photon inclusive (integrate over all k);
Discriminate photons (partial integration over k)

Long distance radiative corrections to $B^- \to V^0 \ell^- \bar{\nu}_\ell$

Tree level amplitude:

$$\mathcal{M}^0 = \frac{G_F}{\sqrt{2}} V_{qb} c_V W_\nu(P_V, P) L^\nu ,$$

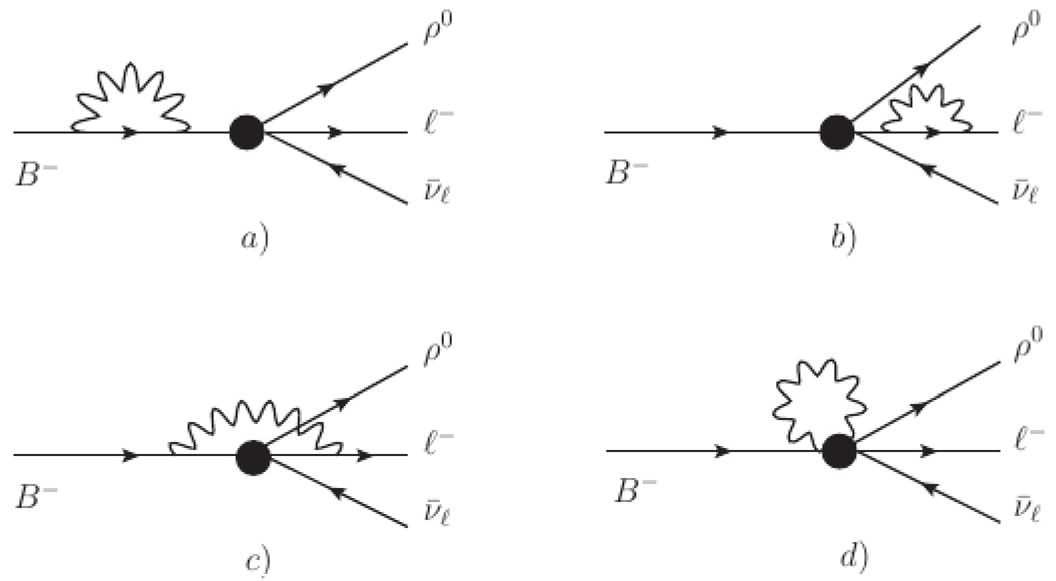


FIG. 1. QED virtual corrections to $B^- \to \rho^0 \ell^- \bar{\nu}_\ell$. Figs. $a)$ and $b)$ correspond to the self-energies of charged particles and $c)$, $d)$ to vertex contributions.

$$\frac{d^2\Gamma_v^1}{dE\,dE_V} = \frac{\alpha}{4\pi}\frac{d\Gamma^0}{dE\,dE_V}\left\{-6 + 2\ln\left(\frac{M^2}{\lambda^2}\right) + 2\ln\left(\frac{m^2}{\lambda^2}\right)\right.$$
$$+2\left(\frac{3}{2} + \frac{1-\beta^2}{\beta^2}\right) B_0[m^2, 0, m^2]$$
$$+2\left(2 - \frac{M}{E\beta^2}\right) B_0^M[M^2, 0, M^2]$$
$$-2\left(2 + \frac{1-\beta^2}{\beta^2} - \frac{M}{E\beta^2}\right) B_0^{lM}[u, m^2, M^2]$$
$$\left.-4ME\ F_2(E) - \frac{2}{\beta}\left[\ln\left(\frac{1+\beta}{1-\beta}\right)\ln\left(\frac{u}{\lambda^2}\right)\right]\right\}$$
$$-\frac{\alpha}{4\pi}\frac{d\Gamma_{NF}^1}{dE\,dE_V}, \qquad (9)$$

S.L.T. and G. López-Castro
arXiv:1510.08020

UV-div in B[...] (regulated by Λ) and IR-div in λ.

Λ dependence partially cancels when the universal short distance (SD) correction is added (Marciano and Sirlin 1993)

$$\delta_{SD}^1 = \frac{2\alpha}{\pi}\ln\left(\frac{m_Z}{\Lambda}\right)$$

To address the IR divergence we consider the real photon emission:

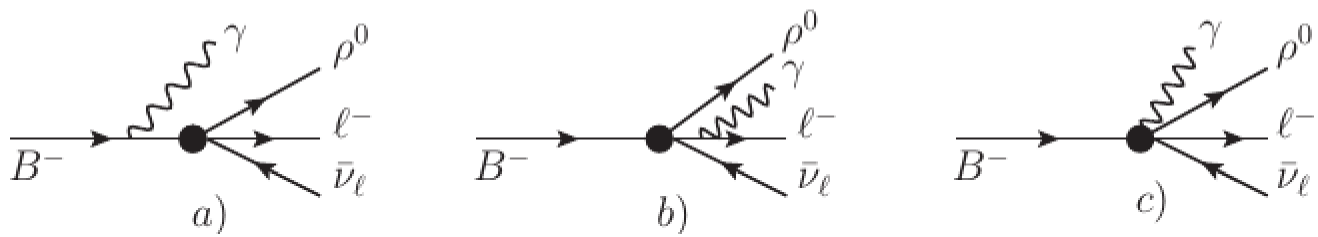


FIG. 2. Feynman diagrams for real photon emission.

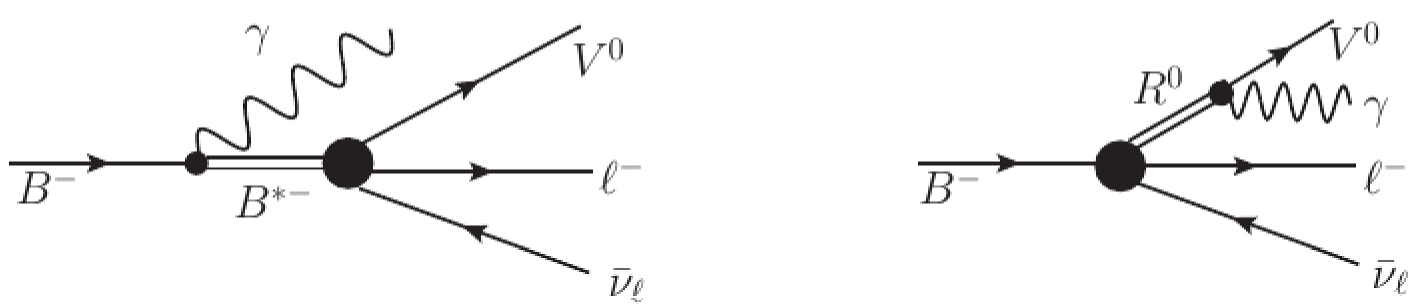


FIG. 3. Model-dependent real photon corrections; R^0 denotes a pseudoscalar resonance.

Model-independent (MI):

$$\mathcal{M}^{\mathrm{Low}} = e\frac{G_F}{\sqrt{2}}V_{qb}c_V\left\{ W_\mu(P_V,P) \right.$$

$$\times\, \bar{u}_\ell\left[-\frac{P\cdot\varepsilon}{P\cdot k}+\frac{2p\cdot\varepsilon+\not\varepsilon\not k}{2p\cdot k}\right]\gamma^\mu(1-\gamma_5)v_{\nu_\ell}$$

$$\left. +L^\mu D^\lambda W_{\mu\lambda} - 2P_V\cdot D\; L^\mu\frac{\partial W_\mu(P_V,P)}{\partial q^2}\right\}, \quad (10)$$

Model-dependent (MD):

$$\mathcal{M}^{MD} = e\frac{G_F}{\sqrt{2}}V_{qb}c_V\epsilon^\mu\;(V^{MD}_{\mu\nu}-A^{MD}_{\mu\nu})L^\nu\;.$$

$$V^{MD}_{\mu\nu}-A^{MD}_{\mu\nu} = \frac{\langle V^0|(V_\nu-A_\nu)|B^{*-}(P_1)\rangle\langle B^{*-}(P_1)|J_\mu|B^-\rangle}{P_1^2-m^2_{B^*}+i\epsilon} + \frac{\langle V^0|J_\mu|R^0(P_2)\rangle\langle R^0(P_2)|(V_\nu-A_\nu)|B^-\rangle}{P_2^2-m_R^2+i\epsilon} \quad (16)$$

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E\mathrm{d}E_\rho} = \frac{\mathrm{d}^2\Gamma^0}{\mathrm{d}E\mathrm{d}E_\rho}\left[1+\tilde{\delta}^1_{LD}(E,E_\rho,\Lambda)\right] + \frac{\mathrm{d}^2\Gamma^{MI}_{IV-III}(E,E_\rho)}{\mathrm{d}E\mathrm{d}E_\rho} \frac{\mathrm{d}^2\Gamma^{MI-MD}(E,E_\rho)}{\mathrm{d}E\mathrm{d}E_\rho} + \frac{\mathrm{d}^2\Gamma^{MD}(E,E_\rho)}{\mathrm{d}E\mathrm{d}E_\rho}$$

Similar in the ω mode

$\bar{B}^- \to \rho^0 \ell^- \bar{\nu}_\ell$

Λ	δ^1_{SD}	$\delta^1_{LD}(\mu)$	$\delta^1_T(\mu)$	$\delta^1_{LD}(e)$	$\delta^1_T(e)$
$m_\rho/2$	0.0254	-0.0085	0.0168	-0.0083	0.0171
m_ρ	0.0221	-0.0060	0.0162	-0.0058	0.0163
2 GeV	0.0177	-0.0025	0.0152	-0.0025	0.0152

$$\delta^1_T(\ell) = \begin{cases} (1.62 \pm 0.10 \pm 0.04)\% \ , & \text{for } \ell = \mu \\ (1.63 \pm 0.11 \pm 0.04)\% \ , & \text{for } \ell = e \end{cases}$$

$B^- \to D^{*0} \ell^- \bar{\nu}_\ell$

Λ	δ^1_{SD}	$\delta^1_{LD}(\mu)$	$\delta^1_T(\mu)$	$\delta^1_{LD}(e)$	$\delta^1_T(e)$
$m_{D^{*0}}/2$	0.0209	-0.0051	0.0159	-0.0048	0.0161
$m_{D^{*0}}$	0.0177	-0.0024	0.0153	-0.0024	0.0153
$2m_{D^{*0}}$	0.0145	0.0003	0.0148	0.0001	0.0146

$$\delta^1_T(\ell) = \begin{cases} (1.53 \pm 0.06 \pm 0.04)\%, & \text{for } \ell = \mu \\ (1.53 \pm 0.08 \pm 0.04)\%, & \text{for } \ell = e \end{cases}$$

Corrections to the Dalitz Plot:

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E\mathrm{d}E_\rho} = \frac{\mathrm{d}^2\Gamma^0}{\mathrm{d}E\mathrm{d}E_\rho}\left[1+\Delta^1_{LD}(E,E_\rho)\right]$$

$B^- \to \rho^0 e^- \bar{\nu}_e$

$E_\rho \setminus E$	200	450	700	950	1200	1450	1700	1950	2200	2450
2650	0.1001	0.0501	0.0251	0.0071	-0.0084	-0.0230	-0.0390	-0.0560	-0.0800	-0.1200
2450		0.0422	0.0232	0.0072	-0.0073	-0.0219	-0.0369	-0.0549	-0.0790	-0.1300
2250			0.0183	0.0040	-0.0094	-0.0228	-0.0378	-0.0559	-0.0799	-0.1300
2050			0.0143	0.0010	-0.0117	-0.0247	-0.0397	-0.0568	-0.0809	-0.1300
1850				-0.0019	-0.0136	-0.0267	-0.0407	-0.0578	-0.0819	-0.1299
1650					-0.0157	-0.0287	-0.0427	-0.0588	-0.0839	-0.1399
1450						-0.0308	-0.0438	-0.0609	-0.0849	-0.1399
1250							-0.0459	-0.0630	-0.0880	-0.1499
1050								-0.0652	-0.0901	-0.1699
850									-0.0964	

$B^- \to \rho^0 \mu^- \bar{\nu}_\mu$

2650	-0.0200	0.0035	0.0007	-0.0032	-0.0073	-0.0120	-0.0170	-0.0220	-0.0300	-0.0450
2450		0.0078	0.0044	0.0002	-0.0041	-0.0089	-0.0139	-0.0199	-0.0290	-0.0460
2250			0.0041	0.0000	-0.0043	-0.0090	-0.0138	-0.0209	-0.0289	-0.0460
2050			0.0035	-0.0005	-0.0047	-0.0093	-0.0147	-0.0208	-0.0299	-0.0470
1850				-0.0010	-0.0051	-0.0097	-0.0147	-0.0208	-0.0299	-0.0479
1650					-0.0056	-0.0097	-0.0147	-0.0218	-0.0299	-0.0489
1450						-0.0108	-0.0158	-0.0219	-0.0309	-0.0509
1250							-0.0159	-0.0230	-0.0320	-0.0539
1050								-0.0232	-0.0331	-0.0619
850									-0.0344	

Conclusions

- Vub and Vcb determination at the few percent level requires the consideration of full RC of O(α)
- We have calculated the LD RC to $B^- \to V^0 \ell^- \bar{\nu}_\ell$ decays.
- In addition to Low's soft photon amplitude, we have included some MD contributions that originate in the exchange o meson resonances.
- There exist large cancellations between LD corrections to the decay rates coming from the three- and four-body regions of the Dalitz plot.
- The total RC are dominated by SD corrections.
- Our results can be useful for future/improved measurements of the different charged decay channels of $B \to V\ l^-\ \nu$ branching ratios in order to test consistency.

Backup

Corrections to the Dalitz Plot:

$B^- \to D^{*0} e^- \bar{\nu}_e$

$E_{D^{*0}} \setminus E$	200	400	600	800	1000	1200	1400	1600	1800	2000
3000	0.0931	0.0501	0.0271	0.0111	-0.0034	-0.0170	-0.0300	-0.0450	-0.0620	-0.0870
2900	0.0772	0.0484	0.0294	0.0144	0.0003	-0.0127	-0.0268	-0.0418	-0.0599	-0.0859
2800		0.0435	0.0266	0.0126	-0.0005	-0.0135	-0.0266	-0.0417	-0.0598	-0.0859
2700		0.0384	0.0236	0.0107	-0.0016	-0.0144	-0.0275	-0.0416	-0.0607	-0.0869
2600			0.0216	0.0093	-0.0027	-0.0143	-0.0274	-0.0426	-0.0607	-0.0889
2500				0.0079	-0.0038	-0.0153	-0.0284	-0.0436	-0.0617	-0.0919
2400				0.0065	-0.0048	-0.0163	-0.0294	-0.0446	-0.0638	-0.0989
2300					-0.0058	-0.0174	-0.0305	-0.0456	-0.0658	-0.1200
2200						-0.0185	-0.0316	-0.0477	-0.0709	
2100							-0.0337	-0.0518	-0.0870	

$B^- \to D^{*0} \mu^- \bar{\nu}_\mu$

$E_{D^{*0}} \setminus E$	200	400	600	800	1000	1200	1400	1600	1800	2000
3000	0.0032	0.0042	0.0014	-0.0019	-0.0052	-0.0087	-0.0120	-0.0170	-0.0220	-0.0300
2900	0.0140	0.0113	0.0080	0.0042	0.0004	-0.0036	-0.0081	-0.0128	-0.0189	-0.0279
2800		0.0124	0.0085	0.0047	0.0009	-0.0032	-0.0076	-0.0127	-0.0188	-0.0289
2700		0.0123	0.0084	0.0048	0.0009	-0.0031	-0.0076	-0.0126	-0.0187	-0.0289
2600			0.0082	0.0046	0.0009	-0.0032	-0.0077	-0.0126	-0.0197	-0.0289
2500				0.0045	0.0007	-0.0032	-0.0077	-0.0126	-0.0197	-0.0299
2400				0.0042	0.0006	-0.0034	-0.0079	-0.0136	-0.0198	-0.0319
2300					0.0003	-0.0037	-0.0082	-0.0136	-0.0208	-0.0380
2200						-0.0040	-0.0086	-0.0137	-0.0229	
2100							-0.0093	-0.0158	-0.0280	

The form factor parametrization:

$$W_\nu(P_V,P) = \frac{2V^{B\to V}}{M+m_V}\epsilon_{\nu\alpha\beta\gamma}\varphi^\alpha P^\beta P_V^\gamma$$
$$-i(M+m_V)A_1^{B\to V}\varphi_\nu + i\frac{A_2^{B\to V}}{M+m_V}q\cdot\varphi(P+P_V)_\nu$$
$$-i\frac{2m_V A^{B\to V}}{q^2}q\cdot\varphi q_\nu \ . \qquad (5)$$

Additional terms in the Low's amplitude:

$$W_{\mu\lambda} = -\frac{2}{M+m_V}V\epsilon_{\mu\lambda\nu\alpha}\varphi_V^\nu P_V^\alpha + i\frac{q\cdot\varphi}{M+m_V}A_2\delta_{\mu\lambda}$$
$$+i\frac{A_2}{M+m_V}(P+P_V)_\mu\varphi_\lambda - 2i\frac{m_V A}{q^2}q_\mu\varphi_\lambda$$
$$-2i\frac{m_V A}{q^2}q\cdot\varphi\delta_{\mu\lambda} \ . \qquad (11)$$

$$D^\lambda = (\varepsilon\cdot P/k\cdot P)k^\lambda - \varepsilon^\lambda$$

Dalitz plot for the 3 and 4 body decay:

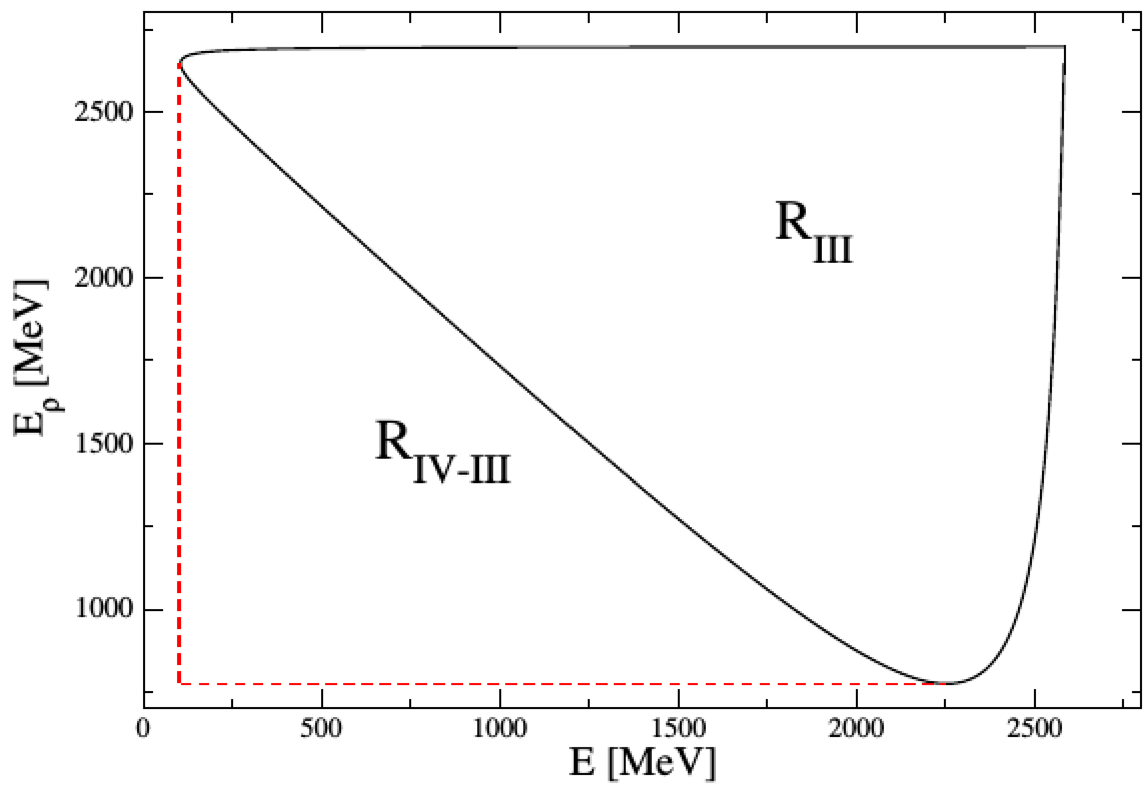


FIG. 4. Phase-space region accessible to $B^- \to \rho^0 \mu^- \nu_\ell(\gamma)$ decays. The region R_{III} is allowed to three- and four-body (radiative) decays, while R_{IV-III} is accesible only in the radiative mode.