Flavored Axion Monodromy Inflation[1]

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Overview

- Problems in Big Bang Cosmology
- Slow-Roll Inflation
- Axion Monodromy Inflation
- Flavor Symmetries
- Final Remarks

Problems in Big Bang Cosmology

Despite the success of the big bang cosmology (BBC), some puzzling mysteries remain unexplained.

The horizon problem: In the original BBC, several regions of the CMB where causally disconnected at the time of recombination. However, the CMB shows a high degree of homogeneity among all its regions.

The flatness problem: Measurements indicate that the current universe is nearly flat. Predictions indicate that its curvature should have deviated to an open or a closed universe over time, unless it was nearly exactly flat from the beginning to an enormous degree of precision.

Slow-Roll Inflation

Considering a flat universe and a homogeneus scalar field ϕ, the expressions for the energy density and pressure are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

Note that when $\dot{\phi}^2 \ll V(\phi)$, the potential V can contribute dominantly to the energy density and the pressure, the resulting equation of state is

$$p_\phi = -\rho_\phi$$

Accelerated expansion!

A scalar with *slow roll* dynamics may accelerate the expansion of the universe.

Slow-Roll Inflation: Parameters

Let's define *slow-roll parameters* and set conditions over their values. The main slow-roll parameters are:

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} + \epsilon. \tag{1}$$

- ϵ<1 sets the limit for inflation to happen
- η ensures that the second derivative $\ddot{\phi}$ remains negligible as required by slow rolling.

The slow roll parameters in terms of $V(\phi)$ are

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv \frac{M_P^2}{8\pi}\frac{V''}{V}, \tag{2}$$

were the primes indicate the number of derivatives with respect of ϕ.

Slow-Roll Inflation: Parameters

We can write the CMB properties as functions of the slow-roll parameters

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon,$$

$$n_r = 16\epsilon\eta - 24\epsilon^2 - 2\gamma, \qquad \Delta_R^2 = \frac{V}{24\pi^2\epsilon},$$

where $\gamma \equiv [M_P^2/(64\pi)]V'V'''/V^2$ is a higher order slow-roll parameter.

To measure the amount of inflation between some time t_i and t_{end}, we define the number of *e-folds*, the exponential growth of the scale factor, as

$$N \equiv \ln\left(\frac{a(t_{\text{end}})}{a(t_i)}\right) = \frac{2\sqrt{\pi}}{M_P}\int_{\phi_{\text{end}}}^{\phi_i} \frac{d\phi}{\sqrt{\epsilon}}. \tag{3}$$

Axion Monodromy Inflation

Consider two complex scalar fields Φ and χ that transform under two discrete symmetry groups $\mathbf{Z}_p^{\Phi} \times \mathbf{Z}_r^{\chi}$ as

$$\Phi \to \exp(2\pi i/p)\Phi \qquad \text{and} \qquad \chi \to \exp(2\pi i/r)\chi$$

where p and r are integers. For $p \geq 5$ and $r \geq 5$, the renormalizable terms in the potential are

$$V(\Phi, \chi) = -m_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4 - m_\chi^2|\chi|^2 + \frac{\lambda_\chi}{2}|\chi|^4 + \lambda_p|\Phi|^2|\chi|^2,$$

Is easy to see that the following reparameterization leaves a flat potential for θ and ρ

$$\Phi = \frac{\phi_0 + f_\theta}{\sqrt{2}}\exp(i\theta/f_\theta) \qquad \text{and } \chi = \frac{\chi_0 + f_\rho}{\sqrt{2}}\exp(i\rho/f_\rho).$$

Axion Monodromy Inflation

The discrete symmetry $\mathbf{Z}_p^{\Phi} \times \mathbf{Z}_r^{\chi}$ serves four purposes:

1. it assures that there are goldstone bosons that have no potential generated by renormalizable couplings,
2. it will serve as a flavor symmetry to create a hierarchy of standard model fermion Yukawa couplings,
3. it will lead to the correct pattern of couplings in a new gauge sector that provides for the desired form of the inflaton potential,
4. it will keep quantum gravitational corrections to the potential highly suppressed.

Axion Monodromy Inflation

Extend the SM gauge group to include SU(N_1)×SU(N_2), together with the new fermions with charges

$$A_L, A_R \sim (\mathbf{N_1}, \mathbf{1}) \quad \text{and} \quad B_L^{(i)}, B_R^{(i)}, C_L, C_R \sim (\mathbf{1}, \mathbf{N_2}).$$

The Yukawa type interactions between the new fermions and the scalar Φ and χ are

$$\mathcal{L} \supset h_1 \bar{A}_R A_L \chi + \sum_{i=1}^{n} h_2^{(i)} \bar{B}_R^{(i)} B_L^{(i)} \chi + h_3 \bar{C}_R C_L \Phi^* + \text{ h.c. } . \tag{4}$$

The anomalous global U(1) symmetries lead to the interactions

$$\frac{g_1^2}{32\pi^2}\left(\frac{\rho}{f_\rho}\right) F_1 \widetilde{F}_1 + \frac{g_2^2}{32\pi^2}\left(\frac{n\rho}{f_\rho} - \frac{\theta}{f_\theta}\right) F_2 \widetilde{F}_2. \tag{5}$$

Axion Monodromy Inflation

The resulting potential is, after a field redefinition of the form $\rho = c\,\tilde{\rho} + s\,\tilde{\theta}$ and $\theta = c\,\tilde{\theta} - s\,\tilde{\rho}$

$$V(\tilde{\rho}, \tilde{\theta}) = \Lambda_1^4 \left[1 + \cos\left(\frac{c\tilde{\rho} + s\tilde{\theta}}{f_1} \right) \right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\tilde{\rho}}{f} \right) \right], \tag{6}$$

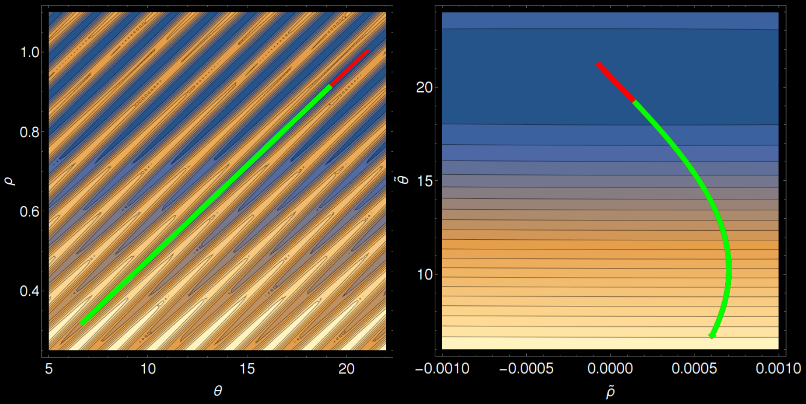
where $f_1 = f_\rho = f_\theta = nf_2$ and $f = f_1 f_2 / \sqrt{f_1^2 + f_2^2}$. The potential presents trenches whose position is given by

$$\sin\left(\frac{\tilde{\rho}}{f} \right) - s\,c\,\frac{\Lambda_1^4}{\Lambda_2^4} \sin\left(\frac{c\tilde{\rho} + s\tilde{\theta}}{f_1} \right) = 0. \tag{7}$$

Using the equations of motion for ρ and θ we can find the trajectory followed by the inflaton

$$\ddot{\rho} + 3H\dot{\rho} + \frac{\partial V}{\partial \rho} = 0, \qquad \ddot{\theta} + 3H\dot{\theta} + \frac{\partial V}{\partial \theta} = 0.$$

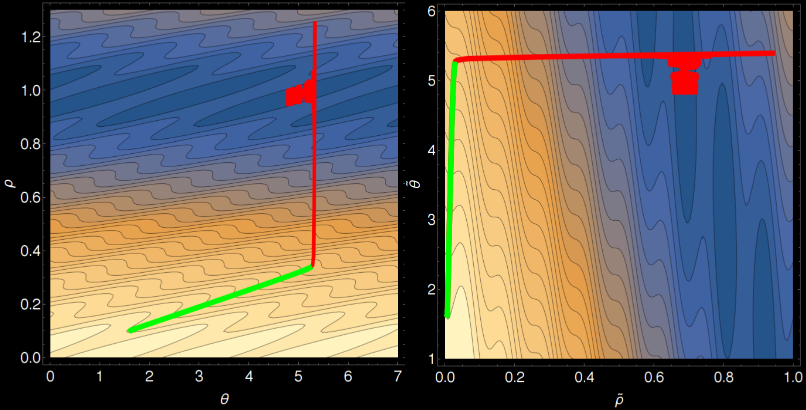
Termination without Waterfalling



$$f_1 = 0.22\sqrt{2}, \qquad f_2 = f_1/21, \qquad \Lambda_1 = \Lambda_2 = 0.006.$$

$$n_s = 0.96, \qquad r = 0.060, \qquad n_r = -0.00046, \qquad \Delta_R^2 = 2.2 \times 10^{-9}.$$

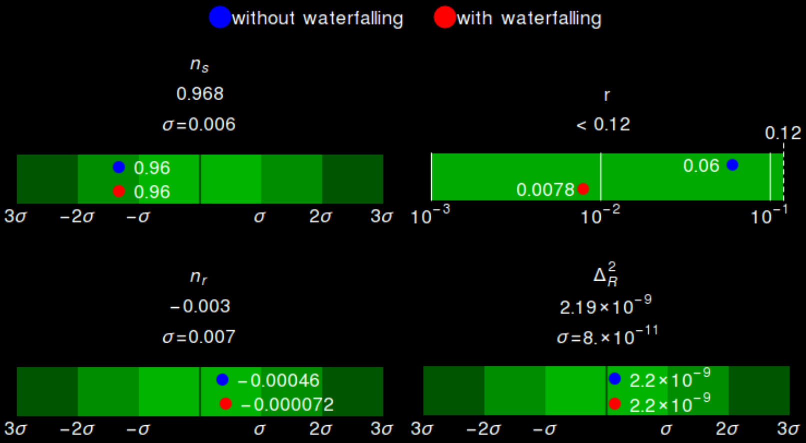
Termination with Waterfalling



$$f_1 = 0.22\sqrt{2}, \quad f_2 = f_1/17, \quad \Lambda_1 = 3.38 \times 10^{-3}, \quad \Lambda_2 = 1.61 \times 10^{-3}.$$

$$n_s = 0.96, \quad r = 0.0078, \quad n_r = -7.2 \times 10^{-5}, \quad \Delta_R^2 = 2.2 \times 10^{-9}.$$

Comparing with Observations



Data taken from "Planck 2015 Results. XX. Constraints on inflation."

Flavor Symmetries

- There are 27 free parameters in the standard model related to flavor physics: Yukawa couplings
- There is a notable hierarchy between the masses of the charged fermions ($\lambda \approx 0.22$ is the Cabibbo angle)

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1, \qquad m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1,$$

$$m_e : m_\mu : m_\tau \approx \lambda^5 : \lambda^2 : 1$$

- Small mixings for quarks. Neutrino oscillation imply large mixing for leptons

We approach these issues through discrete flavor symmetries.

Flavor Symmetries

The flavon fields Φ and χ will form higher-dimension operators with the Yukawa couplings of the standard model. Given our choice

$$\langle\Phi\rangle/M_* = \langle\chi\rangle/M_* = \lambda$$

the size of these entries will be determined by powers of the Cabibbo angle λ. As an example we consider the special case $\mathbf{Z}_{21}^{\Phi}$. The couplings to the flavon will have the form

$$\frac{1}{M_*^k}\overline{Q}_{jL} H \Phi^k u_{jR} + \text{ h.c.},$$

where k is an integer that depends on the charge assignments. The corresponding Yukawa coupling will be of order λ^k.

Flavor Symmetries

The charge assignments for the standard model fermions under $\mathbf{Z}_{21}^{\Phi}$ and the are given below

Q_{1L}	Q_{2L}	Q_{3L}	u_R^c	c_R^c	t_R^c	d_R^c	s_R^c	b_R^c
6	5	3	2	-1	-3	-1	-2	-2
L_{1L}	L_{2L}	L_{3L}	e_R^c	μ_R^c	τ_R^c	ν_{1R}^c	ν_{2R}^c	ν_{3R}^c
0	0	0	5	3	1	-3	-3	-3

Flavor Symmetries

Using the previous table we obtain the following Yukawa matrices by considering the lowest order operators.

$$Y_u = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \qquad Y_d = \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^2 & \lambda & \lambda \end{pmatrix},$$

$$Y_e = \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda \\ \lambda^5 & \lambda^3 & \lambda \\ \lambda^5 & \lambda^3 & \lambda \end{pmatrix},$$

These achieve the desired masses ratios

$$m_u/m_t \sim \lambda^8, \qquad m_c/m_t \sim \lambda^4, \qquad m_d/m_b \sim \lambda^4, \qquad m_s/m_b \sim \lambda^2.$$

Final Remarks

With this project we achieved a number of things:

1. Inflation presents a solution to two notable problems of big bang cosmology: the horizon and the flatness problems.
2. Our setup suggests that the complex scalars involved in inflation may be the flavor symmetry breaking fields that give structure to the Yukawa matrices.
3. We showed that the current data on the CMB can be accommodated by a multi-field axion-monodromy inflation.
4. By considering this approach, the vevs of the complex scalars remained sub-Planckian.
5. The suggested approach is not unique. The vast amount of possibilities surely allows more compelling or clever constructions.