

# THE SPINOR HELICITY FORMALISM IN SUGRA PHENOMENOLOGY

Bryan Larios  
J. Lorenzo Diaz Cruz

Facultad de Ciencias Físico Matemático  
BUAP

XXX Reunión Anual de la División de Partículas y  
Campos de la SMF



# Outline

- Motivation,
- Spinor Helicity Formalism (SHF),
- SHF to the rescue in SUGRA phenomenology,
- Conclusions.

# Motivation

We have been working with some process and reactions in **supersymmetric** models where the **gravitino** is very light (LSP) and then a good dark matter candidate. Recently we computed the 3-body stop decay ( $\tilde{t}_1 \rightarrow b + W + \tilde{\Psi}_\mu$ ) where we use the MATHEMATICA power to handle the huge traces that appear in the scattering amplitudes.

$$\tilde{t}_1 \rightarrow b + W + \tilde{\Psi}_\mu$$

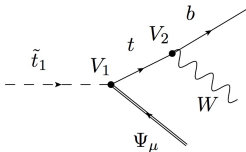


Figure 1. top diagram

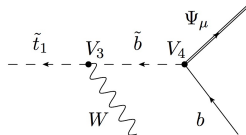


Figure 2. sbottom diagram

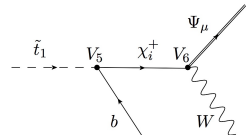


Figure 3. chargino diagram

Lorenzo Diaz-Cruz & Bryan Larios-Lopez, EPJC (2016) 76:157.





# Motivation

There are several promising search channels (in SUSY models) to find new physics at both lepton and hadron colliders but, we learned with the last process ( $\tilde{t}_1 \rightarrow b + W + \tilde{\Psi}_\mu$ ) that it is necessary to implement new calculations methods. In order to learn how to compute scattering amplitudes efficiently and then apply the new methods to some reaction with spin-3/2 particle, we revisited the monophoton plus missing energy ( $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$ ).

# The spinor helicity formalism (a pragmatic point of view)





The SHF is based in the following observation

Fields with spin-1 transform in the  $(\frac{1}{2}, \frac{1}{2})$  representation of the Lorentz group.

So we are able to express 4-moments of any particle as a bispinor:

$$p_\mu \rightarrow p_{a\dot{a}}$$

$$p_{a\dot{a}} = p_\mu \sigma_{a\dot{a}}^\mu \quad (1)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma_{a\dot{a}}^\mu = (I, \vec{\sigma}), \quad \bar{\sigma}^{\mu\dot{a}a} = (I, -\vec{\sigma})$$

We also know that:  $u_s(\vec{p})\bar{u}_s(\vec{p}) = \frac{1}{2}(1 + s\gamma_5)(-\not{p})$  with  $s = \pm$ , para  $s = -$ , tenemos:

$$u_-(\vec{p})\bar{u}_-(\vec{p}) = \frac{1}{2}(1 - \gamma_5)(-\not{p}) = \begin{pmatrix} 0 & -p_{a\dot{a}} \\ 0 & 0 \end{pmatrix}$$

where

$$u_-(\vec{p}) = \begin{pmatrix} \phi_a \\ 0 \end{pmatrix}$$

and  $\phi_a$  is a two component spinor that solves the Weyl equation. a explicit formula for this spinor is the following:

$$\phi_a = \begin{pmatrix} -\sin(\frac{\theta}{2})e^{-i\phi} \\ \cos(\frac{\theta}{2}) \end{pmatrix}$$

also  $\bar{u}_-(\vec{p}) = (0, \phi_{\dot{a}}^*)$ , and

$$p_{a\dot{a}} = -\phi_a\phi_{\dot{a}}^*$$

The key of the SHF is to considerate  $\phi_a$  as the fundamental object and express the 4-momentos of the particles in terms of  $\phi_a$ .

## Notation

If  $p$  and  $k$  are two 4-momentos and  $\phi_a, \kappa_a$  their corresponding spinors, we can define the following products of spinors:

$$[pk] = \phi^a \kappa_a = \bar{u}_+(\vec{p}) u_-(\vec{k}) = -[kp] \quad (2)$$

similarly, we also have

$$\langle pk \rangle = \phi_{\dot{a}}^* \kappa^{*\dot{a}} = \bar{u}_-(\vec{p}) u_+(\vec{k}) = -\langle kp \rangle \quad (3)$$

# Polarizations 4-vectores

- **The contraction with  $\gamma$  matrix is as follows :**

$$\begin{aligned}\not{\xi}_+(k, q) &= \frac{1}{\sqrt{2}\langle qk \rangle} \langle q | \gamma^\mu | k \rangle \gamma_\mu = \frac{2}{\sqrt{2}} (|k\rangle \langle q| + |q\rangle \langle k|) \\ &= \frac{\sqrt{2}}{\langle qk \rangle} (|k\rangle \langle q| + |q\rangle \langle k|),\end{aligned}\quad (4)$$

$$\begin{aligned}\not{\xi}_-(k, q) &= \frac{1}{\sqrt{2}[qk]} [q | \gamma^\mu | k \rangle \gamma_\mu = \frac{1}{\sqrt{2}[qk]} \langle k | \gamma^\mu | q \rangle \gamma_\mu \\ &= \frac{\sqrt{2}}{[qk]} (|k\rangle \langle q| + |q\rangle \langle k|).\end{aligned}\quad (5)$$

Before to start with our computation, I will show all the formulas that are needed to compute the scattering amplitudes:

$$[ij] = -[ji],$$

$$\langle ij \rangle = [ji]^*,$$

$$\langle ij \rangle [ji] = \langle ij \rangle \langle ij \rangle^* = |\langle ij \rangle|^2,$$

$$\langle ij \rangle [ji] = -2k_i \cdot k_j = s_{ij},$$

$$\langle i | \gamma_\mu | j \rangle = [j | \gamma_\mu | i \rangle,$$

$$\langle i | \gamma_\mu | j \rangle \langle k | \gamma^\mu | l \rangle = 2 \langle ik \rangle [lj]$$

$$\langle ab \rangle \langle cd \rangle = \langle ac \rangle \langle bd \rangle + \langle ad \rangle \langle cb \rangle,$$

$$\sum_{k=1}^n \langle ik \rangle [kj] = 0,$$

$$e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$$

Using the **SUSY QED** model constructed by Mawatari and Oexl ([arXiv:1402.3223v2](https://arxiv.org/abs/1402.3223v2)), and applying the **SHF** we will compute the scattering amplitude for the  $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$  reaction. It is important to mention that the cross sections for this reaction has been computed numerically, so it is interesting have an analytical result.

# Feynman Diagrams

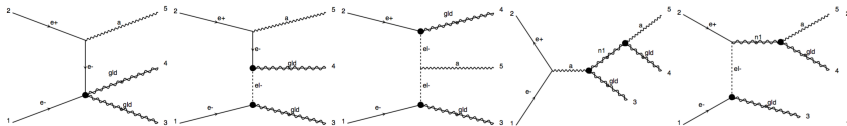


Figure: Feynman Diagrams for  $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$

$$e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$$

We have 5 Feynman diagrams each on them with 5 external (massless) particles, each particle have two helicity states ( $\pm$ ), in principle we need to compute  $2^5$  helicity amplitudes for each diagram.



# Feynman Diagrams

<p>Ampb[-1, -1, -1, -1, -1]</p> <p>Ampb[-1, -1, -1, -1, 1]</p> <p>Ampb[-1, -1, -1, 1, -1]</p> <p>Ampb[-1, -1, -1, 1, 1]</p> <p>Ampb[-1, -1, 1, -1, -1]</p> <p>Ampb[-1, -1, 1, -1, 1]</p> <p>Ampb[-1, -1, 1, 1, -1]</p> <p>Ampb[-1, -1, 1, 1, 1]</p> <p>Ampb[-1, 1, -1, -1, -1]</p> <p>Ampb[-1, 1, -1, -1, 1]</p> <p>Ampb[-1, 1, -1, 1, -1]</p> <p>Ampb[-1, 1, 1, -1, -1]</p> <p>Ampb[-1, 1, 1, -1, 1]</p> <p>Ampb[-1, 1, 1, 1, -1]</p> <p>Ampb[1, -1, -1, -1, -1]</p> <p>Ampb[1, -1, -1, -1, 1]</p> <p>Ampb[1, -1, -1, 1, -1]</p> <p>Ampb[1, -1, -1, 1, 1]</p> <p>Ampb[1, -1, 1, -1, -1]</p> <p>Ampb[1, -1, 1, -1, 1]</p> <p>Ampb[1, -1, 1, 1, -1]</p> <p>Ampb[1, -1, 1, 1, 1]</p>	<p>Ampc[-1, -1, -1, -1, -1]</p> <p>Ampc[-1, -1, -1, -1, 1]</p> <p>Ampc[-1, -1, -1, 1, -1]</p> <p>Ampc[-1, -1, -1, 1, 1]</p> <p>Ampc[-1, -1, 1, -1, -1]</p> <p>Ampc[-1, -1, 1, -1, 1]</p> <p>Ampc[-1, -1, 1, 1, -1]</p> <p>Ampc[-1, -1, 1, 1, 1]</p> <p>Ampc[-1, 1, -1, -1, -1]</p> <p>Ampc[-1, 1, -1, -1, 1]</p> <p>Ampc[-1, 1, -1, 1, -1]</p> <p>Ampc[-1, 1, 1, -1, -1]</p> <p>Ampc[-1, 1, 1, -1, 1]</p> <p>Ampc[-1, 1, 1, 1, -1]</p> <p>Ampc[1, -1, -1, -1, -1]</p> <p>Ampc[1, -1, -1, -1, 1]</p> <p>Ampc[1, -1, -1, 1, -1]</p> <p>Ampc[1, -1, -1, 1, 1]</p> <p>Ampc[1, -1, 1, -1, -1]</p> <p>Ampc[1, -1, 1, -1, 1]</p> <p>Ampc[1, -1, 1, 1, -1]</p> <p>Ampc[1, -1, 1, 1, 1]</p>	<p>Ampd[-1, -1, -1, -1, -1]</p> <p>Ampd[-1, -1, -1, -1, 1]</p> <p>Ampd[-1, -1, -1, 1, -1]</p> <p>Ampd[-1, -1, -1, 1, 1]</p> <p>Ampd[-1, -1, 1, -1, -1]</p> <p>Ampd[-1, -1, 1, -1, 1]</p> <p>Ampd[-1, -1, 1, 1, -1]</p> <p>Ampd[-1, -1, 1, 1, 1]</p> <p>Ampd[-1, 1, -1, -1, -1]</p> <p>Ampd[-1, 1, -1, -1, 1]</p> <p>Ampd[-1, 1, -1, 1, -1]</p> <p>Ampd[-1, 1, 1, -1, -1]</p> <p>Ampd[-1, 1, 1, -1, 1]</p> <p>Ampd[-1, 1, 1, 1, -1]</p> <p>Ampd[1, -1, -1, -1, -1]</p> <p>Ampd[1, -1, -1, -1, 1]</p> <p>Ampd[1, -1, -1, 1, -1]</p> <p>Ampd[1, -1, -1, 1, 1]</p> <p>Ampd[1, -1, 1, -1, -1]</p> <p>Ampd[1, -1, 1, -1, 1]</p> <p>Ampd[1, -1, 1, 1, -1]</p> <p>Ampd[1, -1, 1, 1, 1]</p>	<p>Ampa[-1, -1, -1, -1, -1]</p> <p>Ampa[-1, -1, -1, -1, 1]</p> <p>Ampa[-1, -1, -1, 1, -1]</p> <p>Ampa[-1, -1, -1, 1, 1]</p> <p>Ampa[-1, -1, 1, -1, -1]</p> <p>Ampa[-1, -1, 1, -1, 1]</p> <p>Ampa[-1, -1, 1, 1, -1]</p> <p>Ampa[-1, -1, 1, 1, 1]</p> <p>Ampa[-1, 1, -1, -1, -1]</p> <p>Ampa[-1, 1, -1, -1, 1]</p> <p>Ampa[-1, 1, -1, 1, -1]</p> <p>Ampa[-1, 1, 1, -1, -1]</p> <p>Ampa[-1, 1, 1, -1, 1]</p> <p>Ampa[-1, 1, 1, 1, -1]</p> <p>Ampa[1, -1, -1, -1, -1]</p> <p>Ampa[1, -1, -1, -1, 1]</p> <p>Ampa[1, -1, -1, 1, -1]</p> <p>Ampa[1, -1, -1, 1, 1]</p> <p>Ampa[1, -1, 1, -1, -1]</p> <p>Ampa[1, -1, 1, -1, 1]</p> <p>Ampa[1, -1, 1, 1, -1]</p> <p>Ampa[1, -1, 1, 1, 1]</p>	<p>Ampf[-1, -1, -1, -1, -1]</p> <p>Ampf[-1, -1, -1, -1, 1]</p> <p>Ampf[-1, -1, -1, 1, -1]</p> <p>Ampf[-1, -1, -1, 1, 1]</p> <p>Ampf[-1, -1, 1, -1, -1]</p> <p>Ampf[-1, -1, 1, -1, 1]</p> <p>Ampf[-1, -1, 1, 1, -1]</p> <p>Ampf[-1, -1, 1, 1, 1]</p> <p>Ampf[-1, 1, -1, -1, -1]</p> <p>Ampf[-1, 1, -1, -1, 1]</p> <p>Ampf[-1, 1, -1, 1, -1]</p> <p>Ampf[-1, 1, 1, -1, -1]</p> <p>Ampf[-1, 1, 1, -1, 1]</p> <p>Ampf[-1, 1, 1, 1, -1]</p> <p>Ampf[1, -1, -1, -1, -1]</p> <p>Ampf[1, -1, -1, -1, 1]</p> <p>Ampf[1, -1, -1, 1, -1]</p> <p>Ampf[1, -1, -1, 1, 1]</p> <p>Ampf[1, -1, 1, -1, -1]</p> <p>Ampf[1, -1, 1, -1, 1]</p> <p>Ampf[1, -1, 1, 1, -1]</p> <p>Ampf[1, -1, 1, 1, 1]</p>
--	--	--	--	--

Figure: All the helicity amplitudes

$$e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$$

One of the several and marvelous advantages of the helicity amplitudes is that it is easy to identify the symmetries as well as the null helicity amplitudes. We already know that the terms  $\langle xy \rangle$  and  $[xy]$  are zero, a small program could help us to find which helicity amplitude is zero.

$$\mathcal{A}_{\lambda_1 \lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5}^a \approx (\bar{v}_{\lambda_2}(2) \not{\epsilon}_{\lambda_3}(3) \not{u}_{\lambda_1}(1) \bar{u}_{\lambda_5}(5) v_{\lambda_4}(4)), \quad (6)$$

$$\mathcal{A}_{-+---+}^a \approx (\bar{v}_+(2) \not{\epsilon}_-(3) \not{u}_-(1) \bar{u}_+(5) v_-(4)), \quad (7)$$

$$\mathcal{A}_{-+---+}^a \approx [54] = 0. \quad (8)$$

# Counting helicity amplitudes

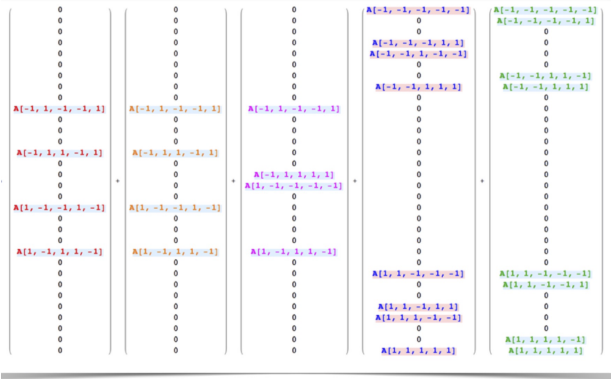


Figure: Counting helicity amplitudes

We started our problem with **160** helicity amplitudes, but just looking at the possible helicity states of the external particles we found that there are only **28** helicity amplitudes to compute. Because the charge conjugation symmetry we really need to compute half of the final helicity amplitudes. *At the end, just the **10%** of the work will be done and without the help of any machine if you desired.*

The total squared amplitud is as follows:

$$\begin{aligned}
 |\mathcal{M}|^2 &= \sum_{perm} |A_{\lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5}|^2 \\
 &= 2(|\mathcal{A}^i_{-+---}|^2 + |\mathcal{A}_{-+++}|^2 + |\mathcal{A}_{-++++}|^2 \\
 &\quad + |\mathcal{A}_{-----}|^2 + |\mathcal{A}_{-----}|^2 + |\mathcal{A}_{-----}|^2 \\
 &\quad + |\mathcal{A}_{--+-}|^2 + |\mathcal{A}_{-++-}|^2 + |\mathcal{A}^i_{-+++-}|^2) \tag{9}
 \end{aligned}$$

Each partial squared helicity amplitude is as follows:

$$|\mathcal{A}^i_{-+---+}|^2 = 2 \frac{s_{15}s_{34}}{s_{23}} \left( (B - 2Em_{\chi_0})^2 s_{23}^2 + 4C^2 s_{24}^2 \right) \quad (10)$$

$$+ 4(B - 2Em_{\chi_0})Cs_{23}s_{24}$$

$$|\mathcal{A}_{-++-+}|^2 = \frac{8C^2 s_{24}^2 s_{15}s_{34}}{s_{23}} \quad (11)$$

$$|\mathcal{A}_{-++++}|^2 = 8E^2 s_{34}^2 s_{12}s_{35} \quad (12)$$

$$|\mathcal{A}_{-----}|^2 = 2D^2 s_{34}^3 s_{12}s_{35} \quad (13)$$

$$|\mathcal{A}_{-----+}|^2 = 2D^2 s_{34}s_{12}s_{35}m_{\chi_0}^2 \quad (14)$$

$$|\mathcal{A}_{----++}|^2 = 2A^2 s_{34} s_{25} s_{15} \quad (15)$$

$$|\mathcal{A}_{--+-}|^2 = |\mathcal{A}_{----++}|^2 \quad (16)$$

$$|\mathcal{A}_{--++-}|^2 = 2D^2 s_{34}^2 s_{12} s_{54} m_{\chi_0}^2 \quad (17)$$

$$|\mathcal{A}_{--^{i}+++}|^2 = 2(s_{45} s_{25} s_{15} A^2 + D^2 s_{34}^3 s_{12} s_{35} - AD(s_{34} s_{45})) \quad (18)$$

where we have to remember that:

$$s_{ij} = -(p_i + p_j)^2$$

# Conclusions

- We compute **analytically** the total scattering amplitude for the reaction  $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$ .
- It was show that the **SHF** is a powerful method, in fact it is much more economic than the traditional approach.
- With the complete result, it is possible to compare the cross section with the numerical results.
- From this point, we can now start do physics.



Thank you very much for  
your attention.