

# The remnant $CP$ transformation

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- Peng Chen, Gui-Jun Ding, FGC and J. W. F. Valle,  
Generalized  $\mu - \tau$  reflection symmetry and leptonic CP violation  
Phys. Lett. B 753 (2016) 644-652  
arXiv:1512.01551
  
- Peng Chen, Gui-Jun Ding, FGC and J. W. F. Valle,  
Classifying CP transformations according to their texture zeros: theory and  
implications  
arXiv:1604.03510

# Redefinition of lepton mixing matrix

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- The invariance of the neutrino mass matrix under the action of a CP transformation

$$\nu_L \mapsto i\mathbf{X}\gamma_0\mathbf{C}\bar{\nu}_L^T \quad \Rightarrow \quad \mathbf{X}^T\mathbf{m}_\nu\mathbf{X} = \mathbf{m}_\nu^*,$$

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- The lepton mixing matrix

$$\mathbf{U} = \mathbf{\Sigma}\mathbf{O}_{3\times 3}\mathbf{Q}_\nu,$$

$\mathbf{\Sigma}$  is the Takagi factorization matrix of  $\mathbf{X}$  fulfilling  $\mathbf{X} = \mathbf{\Sigma}\mathbf{\Sigma}^T$ ,

$$\mathbf{Q}_\nu = \text{diag} \left( e^{-ik_1\pi/2}, e^{-ik_2\pi/2}, e^{-ik_3\pi/2} \right),$$

the entries of  $\mathbf{Q}_\nu$  are  $\pm 1$  and  $\pm i$  which encode the CP-parity or CP-signs of the neutrino states and it renders the light neutrino mass eigenvalues positive. 

# Redefinition of lepton mixing matrix

- The matrix  $\mathbf{O}_{3 \times 3} = \mathbf{O}_1 \mathbf{O}_2 \mathbf{O}_3$  is a generic three dimensional real orthogonal matrix, and it can be parameterized as

$$\mathbf{O}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{pmatrix}, \quad \mathbf{O}_2 = \begin{pmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{pmatrix}$$
$$\mathbf{O}_3 = \begin{pmatrix} \cos \theta_3 & \sin \theta_3 & 0 \\ -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

# The neutrino oscillation data

Parameter <sup>1</sup>	BFP $\pm 1\sigma$	$2\sigma$ range	$3\sigma$ range
$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	$7.60^{+0.19}_{-0.18}$	7.26 – 7.99	7.11 – 8.18
$\Delta m_{31}^2 [10^{-3} \text{ eV}^2]$ (NH)	$2.48^{+0.05}_{-0.07}$	2.35 – 2.59	2.30 – 2.65
$\Delta m_{13}^2 [10^{-3} \text{ eV}^2]$ (IH)	$2.38^{+0.05}_{-0.06}$	2.26 – 2.48	2.20 – 2.54
$\sin^2 \theta_{12}/10^{-1}$	$3.23 \pm 0.16$	2.92 – 3.57	2.78 – 3.75
$\sin^2 \theta_{23}/10^{-1}$ (NH)	$5.67^{+0.32}_{-1.24}$	4.14 – 6.23	3.93 – 6.43
$\sin^2 \theta_{23}/10^{-1}$ (IH)	$5.73^{+0.25}_{-0.39}$	4.35 – 6.21	4.03 – 6.40
$\sin^2 \theta_{13}/10^{-2}$ (NH)	$2.26 \pm 0.12$	2.02 – 2.50	1.90 – 2.62
$\sin^2 \theta_{13}/10^{-2}$ (IH)	$2.29 \pm 0.12$	2.05 – 2.52	1.93 – 2.65
$\delta/\pi$ (NH)	$1.41^{+0.55}_{-0.40}$	0.0 – 2.0	0.0 – 2.0
$\delta/\pi$ (IH)	$1.48 \pm 0.31$	0.00 – 0.09 & 0.86 – 2.0	0.0 – 2.0

The allowed ranges of  $|(\mathbf{U}_{\text{PMNS}})_{ij}|$  are explicitly given at the  $3\sigma$  level:

NH			IH		
$0.780 - 0.842$	$0.520 - 0.607$	$0.137 - 0.162$	$0.779 - 0.842$	$0.520 - 0.607$	$0.139 - 0.163$
$0.207 - 0.555$	$0.395 - 0.714$	$0.618 - 0.794$	$0.207 - 0.554$	$0.397 - 0.710$	$0.626 - 0.792$
$0.226 - 0.566$	$0.420 - 0.731$	$0.590 - 0.772$	$0.229 - 0.566$	$0.426 - 0.729$	$0.592 - 0.765$



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NH

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0.207 – 0.554	0.397 – 0.710	0.626 – 0.794
0.229 – 0.566	0.426 – 0.729	0.592 – 0.772

- ★ The  $|U_{\mu i}| \simeq |U_{\tau i}|$  relation. **Approximate  $\mu - \tau$  relation.**
- ★ Exact  $\mu - \tau$  relation  $|U_{\mu i}| = |U_{\tau i}|$ . **This equality holds if either of the following two sets of conditions can be satisfied.**

$$|U_{\mu i}| = |U_{\tau i}| \Leftrightarrow \begin{cases} \theta_{23} = \frac{\pi}{4}, \theta_{13} = 0; \\ \theta_{23} = \frac{\pi}{4}, \delta_{\text{CP}} = \pm \frac{\pi}{2}. \end{cases}$$

It is clear that  $\theta_{13}$  has already been ruled out, but  $\theta_{23} = \frac{\pi}{4}$  and  $\delta_{\text{CP}} = -\frac{\pi}{2}$  are both allowed at the 1 or 2 $\sigma$  level (and  $\delta_{\text{CP}} = \frac{\pi}{2}$  is also allowed at the 3 $\sigma$ ).

# The $\mu - \tau$ Flavor Symmetry

We claim that there must be a partial or approximate  $\mu - \tau$  flavor symmetry behind the observed pattern of the PMNS matrix.

The  $\mu - \tau$  symmetry gives the constraint that Lagrangian is invariant under transformation of  $\mu$  and  $\tau$  neutrinos states.

★ **The  $\mu - \tau$  permutation symmetry**

The neutrino mass term is unchanged under the transformations;

$$\nu_e \longrightarrow \nu_e, \quad \nu_\mu \longrightarrow \nu_\tau, \quad \nu_\tau \longrightarrow \nu_\mu.$$

★ **The  $\mu - \tau$  reflection symmetry**

The neutrino mass term is unchanged under the transformations<sup>2</sup>;

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---

<sup>2</sup>The superscript  $c$  denotes the charged conjugation. 

# Generalized $\mu - \tau$ reflection

This interesting CP transformation takes the following form:

$$\mathbf{X} = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} \cos \Theta & ie^{i\frac{(\beta+\gamma)}{2}} \sin \Theta \\ 0 & ie^{i\frac{(\beta+\gamma)}{2}} \sin \Theta & e^{i\gamma} \cos \Theta \end{pmatrix},$$

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where the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\Theta$  are real. The corresponding Takagi factorization matrix is given by

$$\mathbf{\Sigma} = \begin{pmatrix} e^{i\frac{\alpha}{2}} & 0 & 0 \\ 0 & e^{i\frac{\beta}{2}} \cos \frac{\Theta}{2} & ie^{i\frac{\beta}{2}} \sin \frac{\Theta}{2} \\ 0 & ie^{i\frac{\gamma}{2}} \sin \frac{\Theta}{2} & e^{i\frac{\gamma}{2}} \cos \frac{\Theta}{2} \end{pmatrix}.$$

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As a result the resulting lepton mixing angles are determined as

$$\sin^2 \theta_{13} = \sin^2 \theta_2, \quad \sin^2 \theta_{12} = \sin^2 \theta_3, \quad \sin^2 \theta_{23} = \frac{1}{2} (1 - \cos \Theta \cos 2\theta_1),$$

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while the CP violation parameters are predicted as

$$J_{\text{CP}} = \frac{1}{4} \sin \Theta \sin \theta_2 \sin 2\theta_3 \cos^2 \theta_2, \quad \sin \delta_{\text{CP}} = \frac{\sin \Theta \operatorname{sign}[\sin \theta_2 \sin 2\theta_3]}{\sqrt{1 - \cos^2 \Theta \cos^2 2\theta_1}},$$

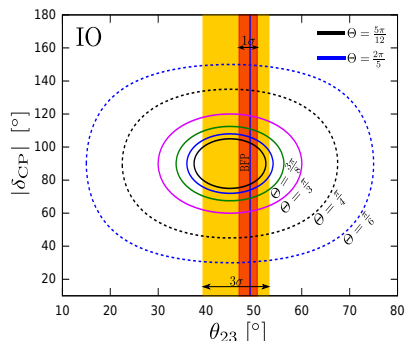
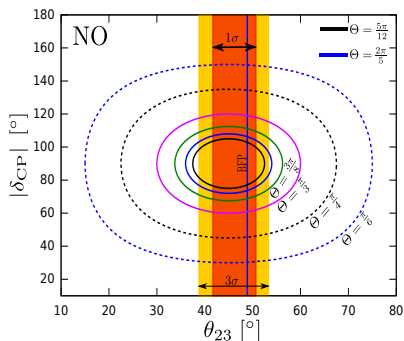
$$\tan \delta_{\text{CP}} = \tan \Theta \csc 2\theta_1, \quad \phi_{12} = \frac{k_2 - k_1}{2} \pi, \quad \phi_{13} = \frac{k_3 - k_1}{2} \pi, \quad \delta_{\text{CP}} = \frac{k_3 - k_2}{2} \pi - \phi_{23}.$$

# Generalized $\mu - \tau$ reflection

We have a correlation between  $\delta_{\text{CP}}$  and the atmospheric angle.

$$\sin^2 \delta_{\text{CP}} \sin^2 2\theta_{23} = \sin^2 \Theta.$$

Taking  $\Theta = \pm \frac{\pi}{2}$ , both  $\theta_{23}$  and  $\delta_{\text{CP}}$  are maximal, since the residual CP transformation  $\mathbf{X}$  reduces to the standard  $\mu - \tau$  reflection. When  $\theta_1 = \pm \frac{\pi}{4}$ , the atmospheric mixing angle  $\theta_{23}$  is maximal and  $\tan \delta_{\text{CP}} = \pm \tan \Theta$ .



# Neutrinoless double beta decay

The rare decay  $(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$  is the lepton number violating process “par excellence”. In the symmetric parametrization, the amplitude for the decay is proportional to the effective mass parameter

$$|m_{ee}| = \left| m_1 \cos^2 \theta_{12} \cos^2 \theta_{13} + m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} e^{-i2\phi_{12}} + m_3 \sin^2 \theta_{13} e^{-i2\phi_{13}} \right| ,$$



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Within our scheme the Majorana phases are predicted as

$$\phi_{12} = \frac{k_2 - k_1}{2} \pi \quad \text{and} \quad \phi_{13} = \frac{k_3 - k_1}{2} \pi.$$

In other words, these phase factors are predicted to lie at their CP conserving values, which correspond to the CP signs of neutrino states. This implies that the two Majorana phases  $(\phi_{12}, \phi_{13})$  can only take the following nine values

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The effective mass  $m_{ee}$  is an even function. This means that for each possible neutrino mass ordering, there are only four independent regions for the effective mass.

# Neutrinoless double beta decay

## Normal Ordering

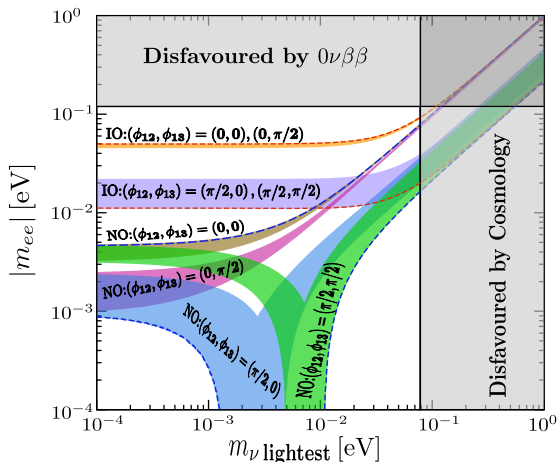
CP signs $Q_\nu$	$(\phi_{12}, \phi_{13})$	$ m_{ee} $ ( $10^{-2}$ eV)
diag(1, 1, 1)	(0, 0)	[0.32, 7.22]
diag(1, 1, $-i$ )	$(0, \frac{\pi}{2})$	$[9.50 \times 10^{-2}, 6.89]$
diag(1, $-i$ , 1)	$(\frac{\pi}{2}, 0)$	[0, 3.31]
diag(1, $-i$ , $-i$ )	$(\frac{\pi}{2}, \frac{\pi}{2})$	[0, 2.94]

## Inverted Ordering

CP signs $Q_\nu$	$(\phi_{12}, \phi_{13})$	$ m_{ee} $ ( $10^{-2}$ eV)
diag(1, 1, 1)	(0, 0)	[4.59, 8.20]
diag(1, 1, $-i$ )	$(0, \frac{\pi}{2})$	[1.10, 3.45]
diag(1, $-i$ , 1)	$(\frac{\pi}{2}, 0)$	[1.10, 3.45]
diag(1, $-i$ , $-i$ )	$(\frac{\pi}{2}, \frac{\pi}{2})$	[1.10, 3.45]

The allowed ranges for the effective mass in for the case of normal and inverted ordering. Notice that in our generalized  $\mu - \tau$  reflection scenario the Majorana phases can only be 0 and  $\pm\pi/2$ .

# Neutrinoless double beta decay



The red and blue dashed lines indicate the regions currently allowed at  $3\sigma$  by neutrino oscillation data. The most stringent upper bound  $|m_{ee}| < 0.120\text{eV}$  from EXO-200 in combination with KamLAND-ZEN. The upper limit on the mass of the lightest neutrino is derived from the latest Planck result  $\sum_i m_i < 0.230\text{eV}$  at 95% level.

# CP violation in neutrino oscillations

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$$P(\nu_\mu \rightarrow \nu_e) \simeq P_{\text{atm}} + P_{\text{sol}} \pm 2\sqrt{P_{\text{atm}}}\sqrt{P_{\text{sol}}}\cos\left(\Delta_{32} \pm \arcsin\left(\frac{\sin\Theta}{\sin 2\theta_{23}}\right)\right).$$

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- ★ The neutrino anti-neutrino asymmetry in matter is given by

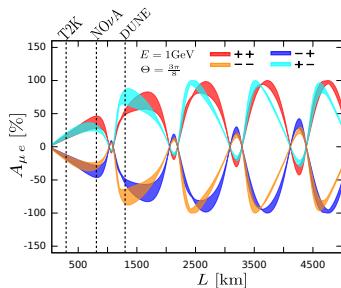
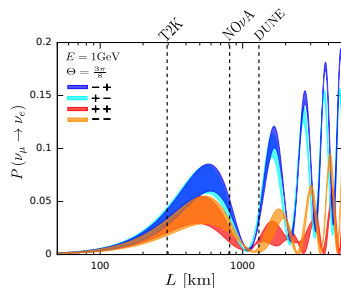
$$A_{\mu e} = \pm \frac{2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \sin \Delta_{23} \sin \Theta}{(P_{\text{atm}} + P_{\text{sol}}) \sin 2\theta_{23} \pm 2\sqrt{P_{\text{atm}}} \sqrt{P_{\text{sol}}} \sqrt{\sin^2 2\theta_{23} - \sin^2 \Theta} \cos \Delta_{23}},$$

where

$$\sqrt{P_{\text{atm}}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)} \Delta_{31}, \quad \sqrt{P_{\text{sol}}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{aL} \Delta_{21},$$

$\Delta_{kj} = \Delta m_{kj}^2 L / (4E)$  with  $\Delta m_{kj}^2 = m_k^2 - m_j^2$ ,  $L$  is the baseline,  $E$  is the energy of neutrino.  $a = G_F N_e / \sqrt{2}$ ,  $G_F$  is the Fermi constant and  $N_e$  is the density of electrons, with  $a \approx (3500\text{km})^{-1}$  for  $\rho Y_e = 3.0\text{g cm}^{-3}$ , where  $Y_e$  is the electron fraction.

# CP violation in neutrino oscillations



The mixing angle  $\theta_{23}$  is taken within the  $3\sigma$  range  $0.393 \leq \sin^2 \theta_{23} \leq 0.643$ . The remaining neutrino oscillation parameters are fixed at their best fit values:

$$\Delta m_{21}^2 = 7.60 \times 10^{-5} \text{ eV}^2,$$

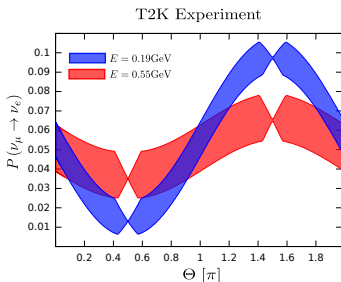
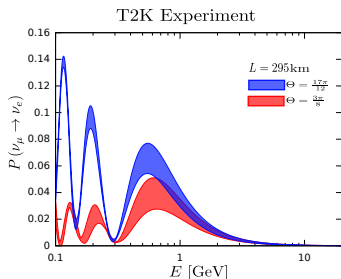
$$|\Delta m_{31}^2| = 2.48 \times 10^{-3} \text{ eV}^2,$$

$$\sin \theta_{12} = 0.323 \text{ and}$$

$\sin \theta_{13} = 0.0226$ . The  $\Theta$  parameter is fixed to the value  $3\pi/8$ . The figure corresponds to the case of normal ordering and the sign combinations.

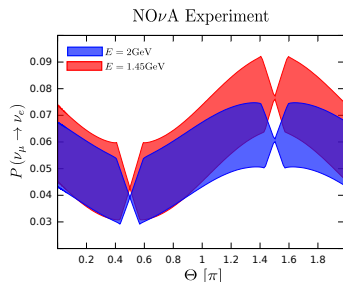
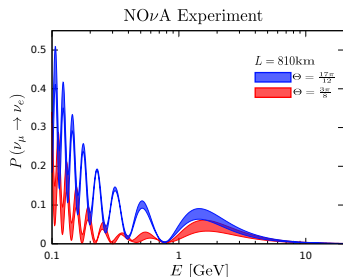


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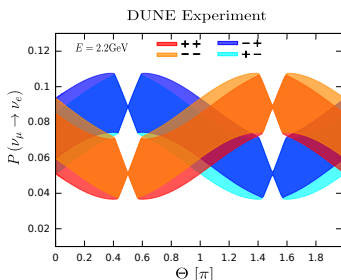
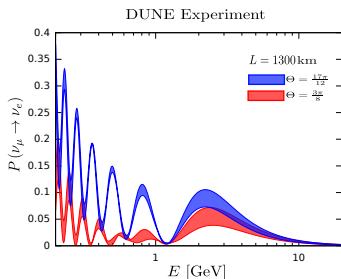
The transition probability  $P(\nu_\mu \rightarrow \nu_e)$  at a baseline of 295km which corresponds to the T2K experiment. The mixing angle  $\theta_{23}$  is taken within the  $3\sigma$  range  $0.393 \leq \sin^2 \theta_{23} \leq 0.643$ . The remaining neutrino oscillation parameters are fixed at their best fit values:  $\Delta m_{21}^2 = 7.60 \times 10^{-5} \text{ eV}^2$ ,  $|\Delta m_{31}^2| = 2.48 \times 10^{-3} \text{ eV}^2$ ,  $\sin \theta_{12} = 0.323$  and  $\sin \theta_{13} = 0.0226$ .

# CP violation in neutrino oscillations



The transition probability  $P(\nu_\mu \rightarrow \nu_e)$  at a baseline of 810km which corresponds to the NO $\nu$ A experiment. The mixing angle  $\theta_{23}$  is taken within the  $3\sigma$  range  $0.393 \leq \sin^2 \theta_{23} \leq 0.643$ . The remaining neutrino oscillation parameters are fixed at their best fit values:  $\Delta m_{21}^2 = 7.60 \times 10^{-5} \text{eV}^2$ ,  $|\Delta m_{31}^2| = 2.48 \times 10^{-3} \text{eV}^2$ ,  $\sin \theta_{12} = 0.323$  and  $\sin \theta_{13} = 0.0226$ .

# CP violation in neutrino oscillations



The transition probability  $P(\nu_\mu \rightarrow \nu_e)$  at a baseline of 1300km, which corresponds to the DUNE proposal. The mixing angle  $\theta_{23}$  is taken within the  $3\sigma$  range  $0.393 \leq \sin^2 \theta_{23} \leq 0.643$ . The remaining neutrino oscillation parameters are fixed at their best fit values:  $\Delta m_{21}^2 = 7.60 \times 10^{-5} \text{ eV}^2$ ,  $|\Delta m_{31}^2| = 2.48 \times 10^{-3} \text{ eV}^2$ ,  $\sin \theta_{12} = 0.323$  and  $\sin \theta_{13} = 0.0226$ .

# Conclusions

- We have proposed a generalized  $\mu - \tau$  reflection scenario for leptonic CP violation and derived the corresponding restrictions on lepton flavor mixing parameters.
- In contrast with flavor symmetry schemes, our generalized CP symmetry approach can constrain not only the mixing angles but also the CP violating phases in function of four parameters.
- We found that the “Majorana” phases are predicted to lie at their CP-conserving values with important implications for the neutrinoless double beta decay amplitudes.
- We have obtained a new correlation between the atmospheric mixing angle  $\theta_{23}$  and the “Dirac” CP phase  $\delta_{CP}$ .
- We have also analysed the phenomenological implications of our scheme for present as well as upcoming neutrino oscillation experiments T2K, NO $\nu$ A and DUNE.