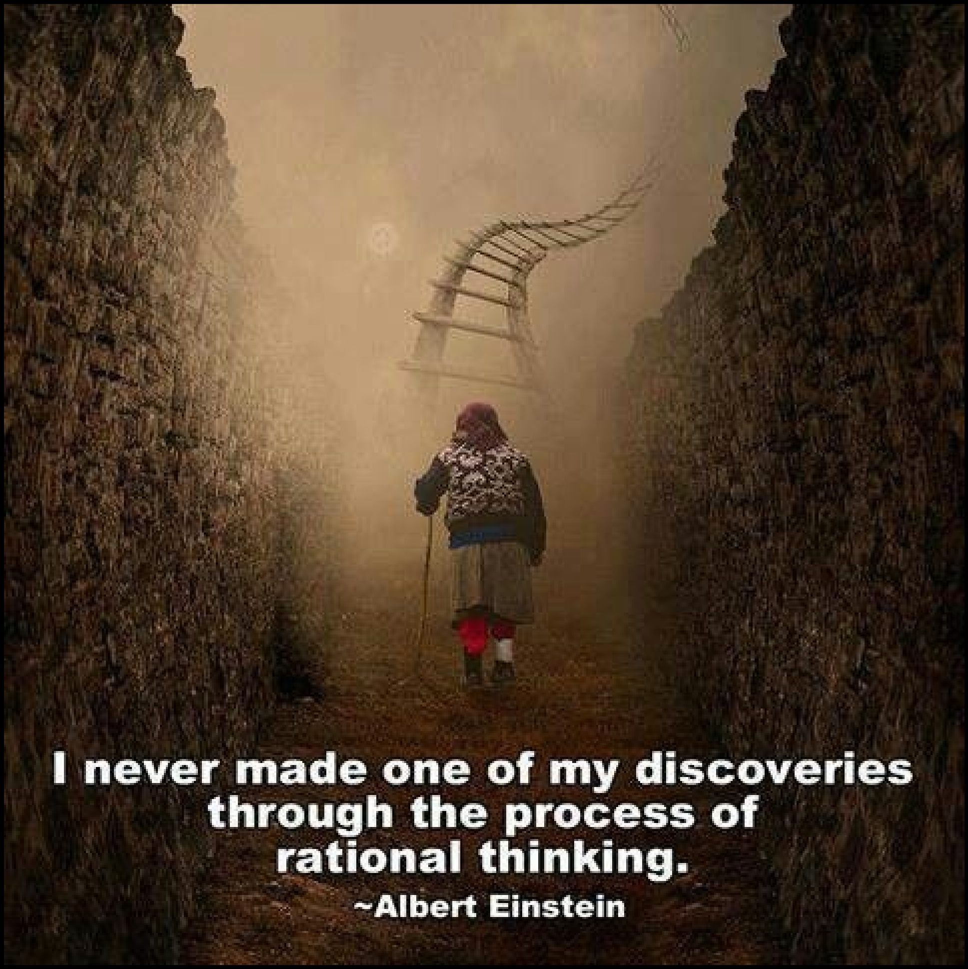
I never made one of my discoveries
through the process of
rational thinking.
~Albert Einstein

Masas de fermiones como parámetros de mezcla en el SM

U. J. Saldaña-Salazar
FCFM-BUAP

XXX Reunión Anual DPyC
Mayo 24, 2016
Puebla, Puebla

CONACyT-220498

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Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

Phys. Rev. D93 013002 (2016)
UJSS

arXiv: 1605.03860
L. Díaz-Cruz, W. G. Hollik, UJSS

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B U A P
www.fcfm.buap.mx

See Barranco's Talk

Conclusiones

¿Cuál es la escala de masas del neutrino?

¿Cuál es la jerarquía de masas?

¿Existe violación de CP en el sector leptónico?

¿Cuál es la naturaleza del neutrino: Dirac ó Majorana?

See Barranco's Talk

Conclusiones

¿Cuál es la escala de masas del neutrino?

¿Cuál es la jerarquía de masas?

¿Existe violación de CP en el sector leptónico?

~~¿Cuál es la naturaleza del neutrino: Dirac ó Majorana?~~

+ Flavor puzzle

Modelo estándar

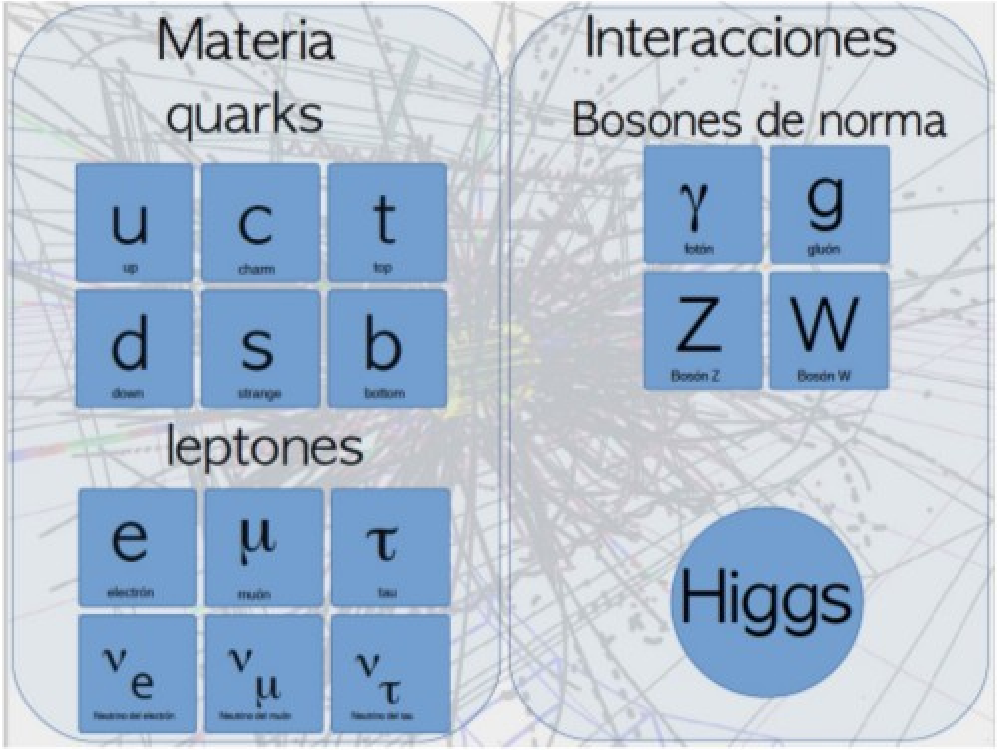
See Barranco's Talk

Nucl. Phys. B892 (2015) 364-389
W. G. Hollik & UJSS

Es un modelo de concordancia

14
~~19~~ parámetros libres:

- 6 masas de quarks
- 3 masas de leptones cargados
- ~~3 ángulos de mezcla CKM~~
- ~~1 fase CP~~
- 3 constantes de acoplamiento
- Valor de expectación del vacío
- Masa del Higgs
- ~~Θ QCD~~



arXiv: 1605.03860
L. Díaz-Cruz, W. G. Hollik, UJSS

Outline

The four mass ratios parametrization

The flavor-blind principle*

Conclusions

State-of-the-Art:

"We are struggling to find clear indications that can point us in the right direction. Some people see in this state of crisis a source of frustration. I see a source of excitement because new ideas have always thrived in moments of crisis." G. Giudice (CERN, 2016)

A grown up SM and NP

A grown up SM and NP

BSM Theoretical physicist

BSM
Theoretical
physicist

LHC
(aka
SM)

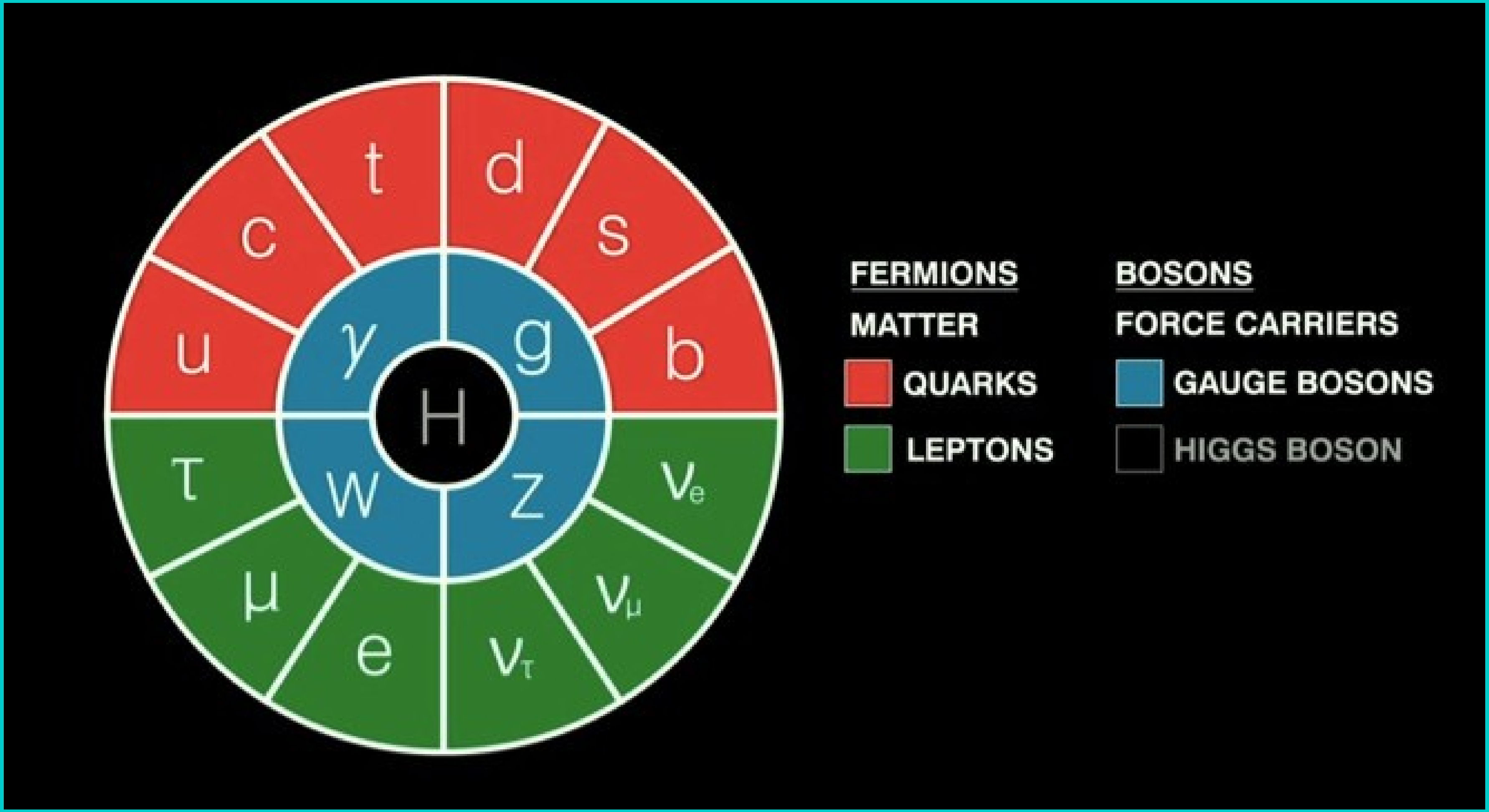
New data

Summary or logical construction

SM's free parameters → Fermion masses and mixing parameters → GST → Arbitrary Yukawa matrices → ? → Schmidt-Mirsky approximation theorem → m3 >> m2 >> m1 → systematical procedure → Construction of the mixing matrices → Discussions of ansatz → Study of complex phases → Agreement between theoretical and experimental mixing matrices (quarks and leptons) → Neutrino masses → Two opened questions (A & B) → B → Study of GST relation → Flavor-Blind principle → ?

The four mass ratios parametrization

Standard Model parameters



$g,\ g',\ \alpha_s$

$\lambda,\ v,$

$m_u,\ m_c,\ m_t,$

$m_d,\ m_s,\ m_b,$

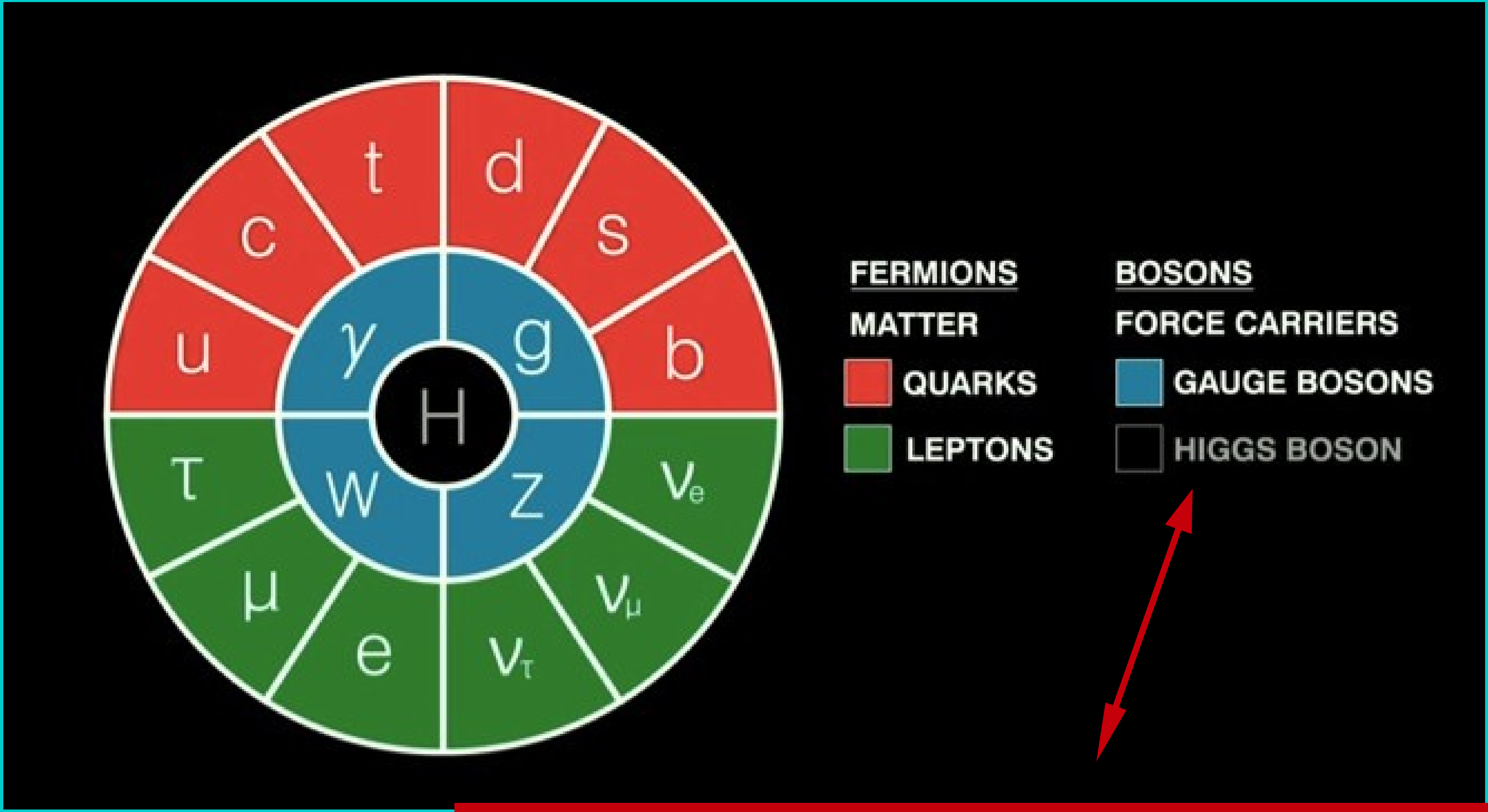
$\theta^q_{12},\ \theta^q_{13},\ \theta^q_{23}, \delta^q_{cp}$

$m_e,\ m_\mu,\ m_\tau,$

$m_{\nu 1},\ m_{\nu 2},\ m_{\nu 3},$

$\theta^\ell_{12},\ \theta^\ell_{13},\ \theta^\ell_{23}, \delta^\ell_{cp}$

Standard Model parameters



$g, \; g', \; \alpha_s$

$\lambda, \; v,$

$m_u, \; m_c, \; m_t,$
$m_d, \; m_s, \; m_b,$

$m_e, \; m_\mu, \; m_\tau,$
$m_{\nu 1}, \; m_{\nu 2}, \; m_{\nu 3},$

$\theta^q_{12}, \; \theta^q_{13}, \; \theta^q_{23}, \delta^q_{cp}$

$\theta^\ell_{12}, \; \theta^\ell_{13}, \; \theta^\ell_{23}, \delta^\ell_{cp}$

Mixing parameters

$a = u, e$ $b = d, \nu$ $\mathbf{M}_a, \quad \mathbf{M}_b$ (Weak basis)

$$\bar{\psi}_L^a \mathbf{M}_a \psi_R^a, \qquad \bar{\psi}_L^b \mathbf{M}_b \psi_R^b$$

$$\bar{U}_L \gamma_\mu D_L$$

$\mathbf{\Sigma}_a, \quad \mathbf{\Sigma}_b$ (Mass basis)

$$\bar{U}'_L \gamma_\mu \mathbf{L}_u \mathbf{L}_d^\dagger D'_L$$

$$\mathbf{V}_q = \mathbf{L}_u \mathbf{L}_d^\dagger \neq 1_{3\times 3}$$

Mixing parametrizations

$V = L_a L_b^\dagger$ $a = u, e$ $b = d, \nu$ (Mass basis)

$VV^\dagger = V^\dagger V = 1$ $\Rightarrow U(n) \to n^2$

$-(2n-1)$ (Independent field rephasings)

Num. of mixing parameters: $(n-1)^2$

$n = 3 \quad \Rightarrow \quad 4 \to (3+1)$

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PDG, Chau, Maiani, Kobayashi-Maskawa, Wolfenstein, Fritzsch-Xing, Valle-Rodejohann

Mass ratios

$$\tan^2\theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),
Mohapatra (1977), Weinberg (1977),
Fritzsch (1977), Ramond (1993), Xing (1996),
Rasin (1997), Chkareuli (1998), Mondragón (1998),
Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

$$m_1^a,\ m_2^a,\ m_3^a,\ \Big|\ m_1^b,\ m_2^b,\ m_3^b,$$

$$\Rightarrow 2(n-1) \qquad \Rightarrow n \leq 3$$

$$V = V\left(\frac{m_1^a}{m_2^a},\ \frac{m_2^a}{m_3^a},\ \frac{m_1^b}{m_2^b},\ \frac{m_2^b}{m_3^b}\right)$$

Mass ratios

$$\tan^2\theta_c \approx \frac{m_d}{m_s}$$

Gatto, Sartori, Tonin (1968),
Cabibbo (1968), Tanaka (1969),

But, is it even possible?

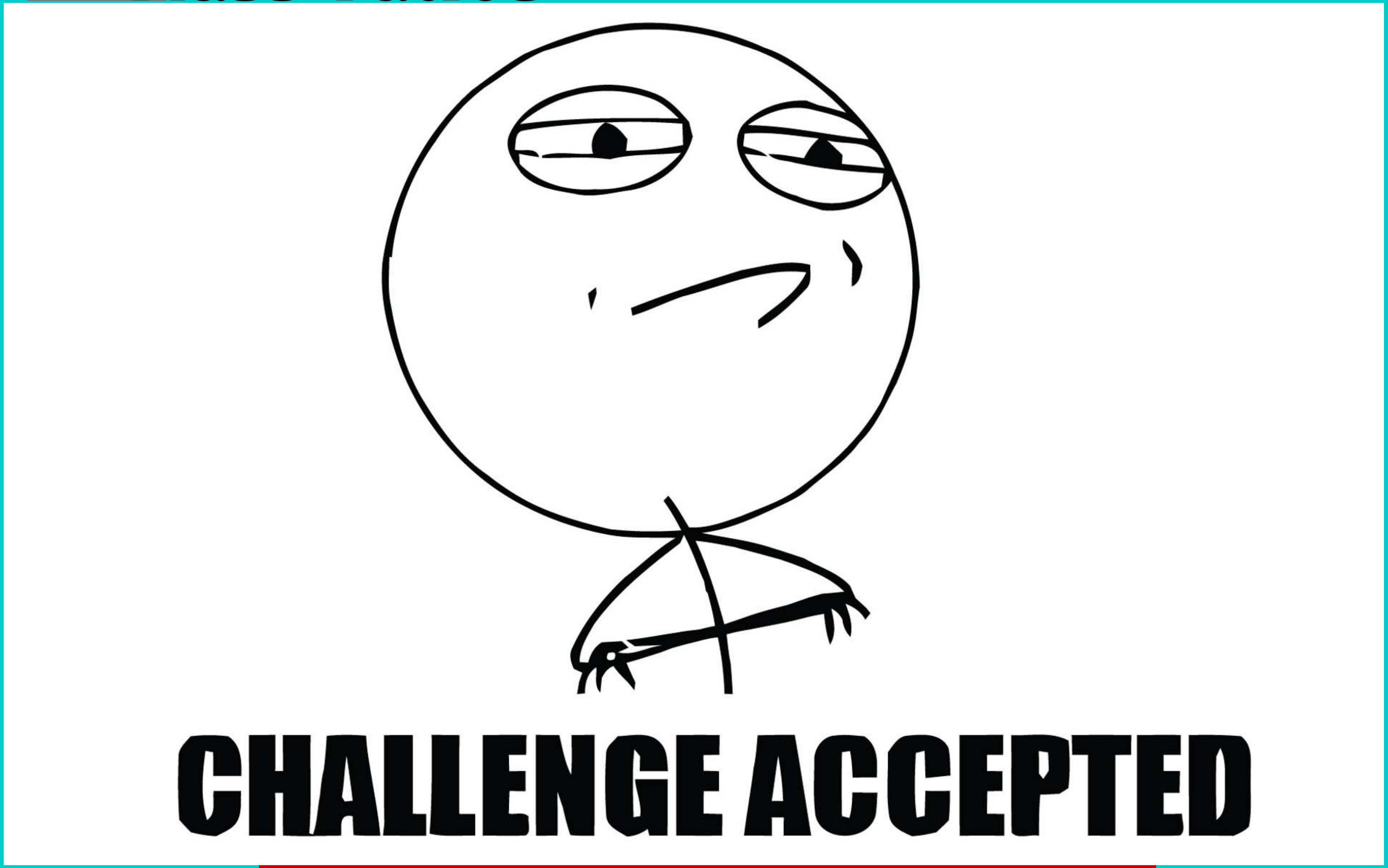
Tanimoto (1999), Fritzsch, Xing (1999), King,
Valle, Peinado, Spinrath, Antusch...

$$m_1^a,\ m_2^a,\ m_3^a,\ |\ m_1^b,\ m_2^b,\ m_3^b,$$

$$\Rightarrow 2(n-1) \qquad \Rightarrow n \leq 3$$

$$V = V\left(\frac{m_1^a}{m_2^a},\ \frac{m_2^a}{m_3^a},\ \frac{m_1^b}{m_2^b},\ \frac{m_2^b}{m_3^b}\right)$$

Mass ratios



$$V = V(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b})$$

Complex phases I

$$V = V(\frac{m_i}{m_j}, \delta_1, \delta_2, \cdots, \delta_k)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b})$$

$$\Rightarrow L_f = L_f(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f})$$

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

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$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

No exact solution.

Complex phases I

$$V = V(\frac{m_i}{m_j}, \delta_1, \delta_2, \cdots, \delta_k)$$

$$\Rightarrow \delta_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

(Masina, Savoy)

$$V = L_a L_b^\dagger = V(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b})$$

$$\Rightarrow L_f = L_f(\frac{m_1^f}{m_2^f}, \frac{m_2^f}{m_3^f})$$

An approximated solution?

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

No exact solution.

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$$+$$

$$m_1 \ll m_2 \ll m_3$$

$$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$$

$$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$$

$$\Delta m^2_{31(32)} : \Delta m^2_{21} = 1 : 10^{-2}$$

Schmidt-Mirsky approximation theorem (Schmidt, Mirsky, Eckart, Young)

$$\text{rank}[A] = n \qquad \sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$$

$$s_k = \{\sigma_k, \sigma_{k-1}, \ldots, \sigma_1\} \ll \sigma_{k+1}$$

$$||A - B||_X \geq ||A - A(s_k = 0)||_X$$

$$\text{rank}[B] = n - k \qquad ||A||_F = \sqrt{Tr(AA^\dagger)}$$

Hierarchical masses

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$m_t : m_c : m_u = 1 : 10^{-3} : 10^{-5}$

$m_b : m_s : m_d = 1 : 10^{-2} : 10^{-4}$

$m_\tau : m_\mu : m_e = 1 : 10^{-2} : 10^{-4}$

$\Delta m^2_{31(32)} : \Delta m^2_{21} = 1 : 10^{-2}$

$m_{1,2} = 0$ $+$

rank 1
rank 2

$m_1 \ll m_2 \ll m_3$

$m_1 = 0$

Schmidt-Mirsky approximation theorem (Schmidt, Mirsky, Eckart, Young)

$\mathrm{rank}[A] = n$ $\sigma_n > \sigma_{n-1} > \cdots > \sigma_2 > \sigma_1 > 0$

$s_k = \{\sigma_k, \sigma_{k-1}, \ldots, \sigma_1\} \ll \sigma_{k+1}$

$$||A - B||_X \geq ||A - A(s_k = 0)||_X$$

$\mathrm{rank}[B] = n - k$ $||A||_F = \sqrt{Tr(AA^\dagger)}$

Minimal flavor violation (MFV)

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i\gamma^\mu \partial_\mu \psi$$

$$\mathcal{L}_\psi = \sum_\psi \bar{\psi} i\gamma^\mu D_\mu \psi$$

$$\mathcal{L}_\psi = \sum_f \bar{\psi}_f (i\gamma^\mu D^f_\mu - \mathcal{M}_f)\psi_f$$

$U(48)$ → $U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu$ → $U(1)_B \times U(1)_L$

Rank 0 $U(3)^Q \times U(3)^u \times U(3)^d \times U(3)^\ell \times U(3)^e \times U(3)^\nu$ $m_1, m_2, m_3 = 0$

↓

Rank 1 $U(2)^Q \times U(2)^u \times U(2)^d \times U(2)^\ell \times U(2)^e \times U(2)^\nu$ $m_1, m_2 = 0$

↓

Rank 2 $U(1)^Q \times U(1)^u \times U(1)^d \times U(1)^\ell \times U(1)^e \times U(1)^\nu$ $m_1 = 0$

↓

Rank 3 $U(1)_B \times U(1)_L$

(Barbieri, Hall, Romanino, Dvali, Straub, Blankenburg, Grinstein)

Electroweak basis*

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\uparrow$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\uparrow$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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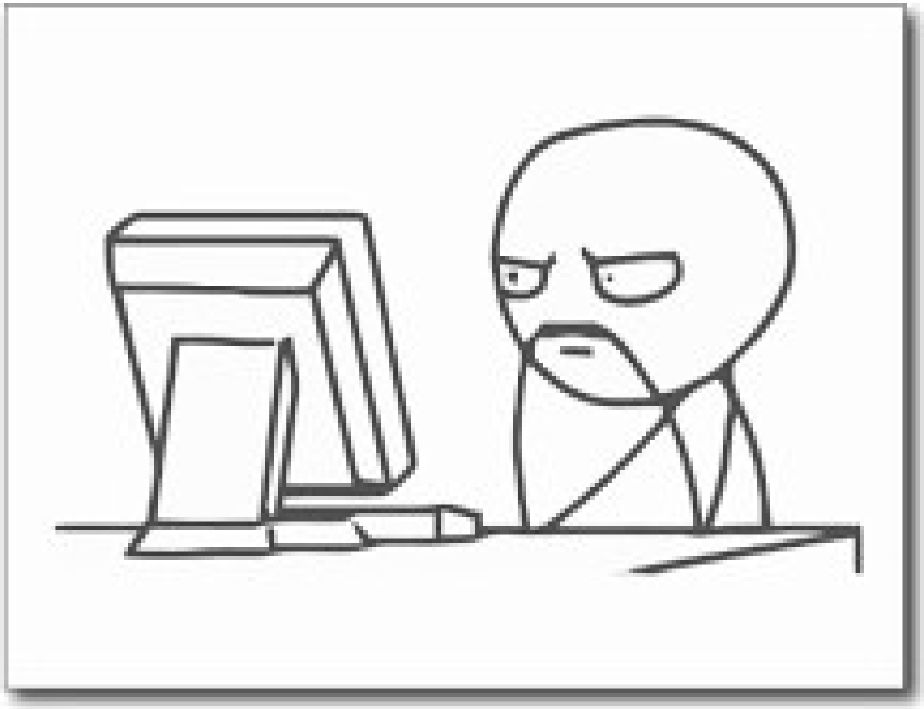
$\uparrow$

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$



Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

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↑

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↑

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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↑

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

↑

$$L_f M_{f,r=1} M_{f,r=1}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

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$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

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$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f M_{f,r=2} M_{f,r=2}^\dagger L_f^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{f,r=2} M_{f,r=2}^\dagger \approx m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \theta_{23}^2 & \theta_{23} \\ 0 & \theta_{23} & 1+\theta_{23}^2 \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

$$M_{f,r=1} M_{f,r=1}^\dagger = m_3^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Electroweak basis* + mixing

$$L_f M_f M_f^\dagger L_f^\dagger = \Sigma_f^2$$

$a = u, e$

$b = d, \nu$

$M_a,\ M_b \quad \Rightarrow \quad M_f \neq \Sigma_f$

$$L_f M_f M_f^\dagger L_f^\dagger = m_3^2 \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

?

$$\tan^2\theta_{23} = \frac{m_2}{m_3}$$

$$M_{f,r=2} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & m_{23} \\ 0 & m_{23} & m_{33} \end{pmatrix}$$

$$L_f(0, \frac{m_2}{m_3}) = L_{23}(\frac{m_2}{m_3})$$

$$|M_{f,r=1}| = m_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$L_f = L_f(0,0) = 1_{3\times 3}$$

Complex phases II

$$V = L_a L_b^\dagger \rightarrow V_{23} = L_{23}^a L_{23}^b{}^\dagger$$

$$V_{ij} = \sqrt{\frac{\hat{m}_{ij}^a + \hat{m}_{ij}^b - 2\hat{m}_{ij}^a \hat{m}_{ij}^b \cos(\delta_{ij}^a - \delta_{ij}^b)}{(1+\hat{m}_{ij}^a)(1+\hat{m}_{ij}^b)}}$$

- Minimal mixing

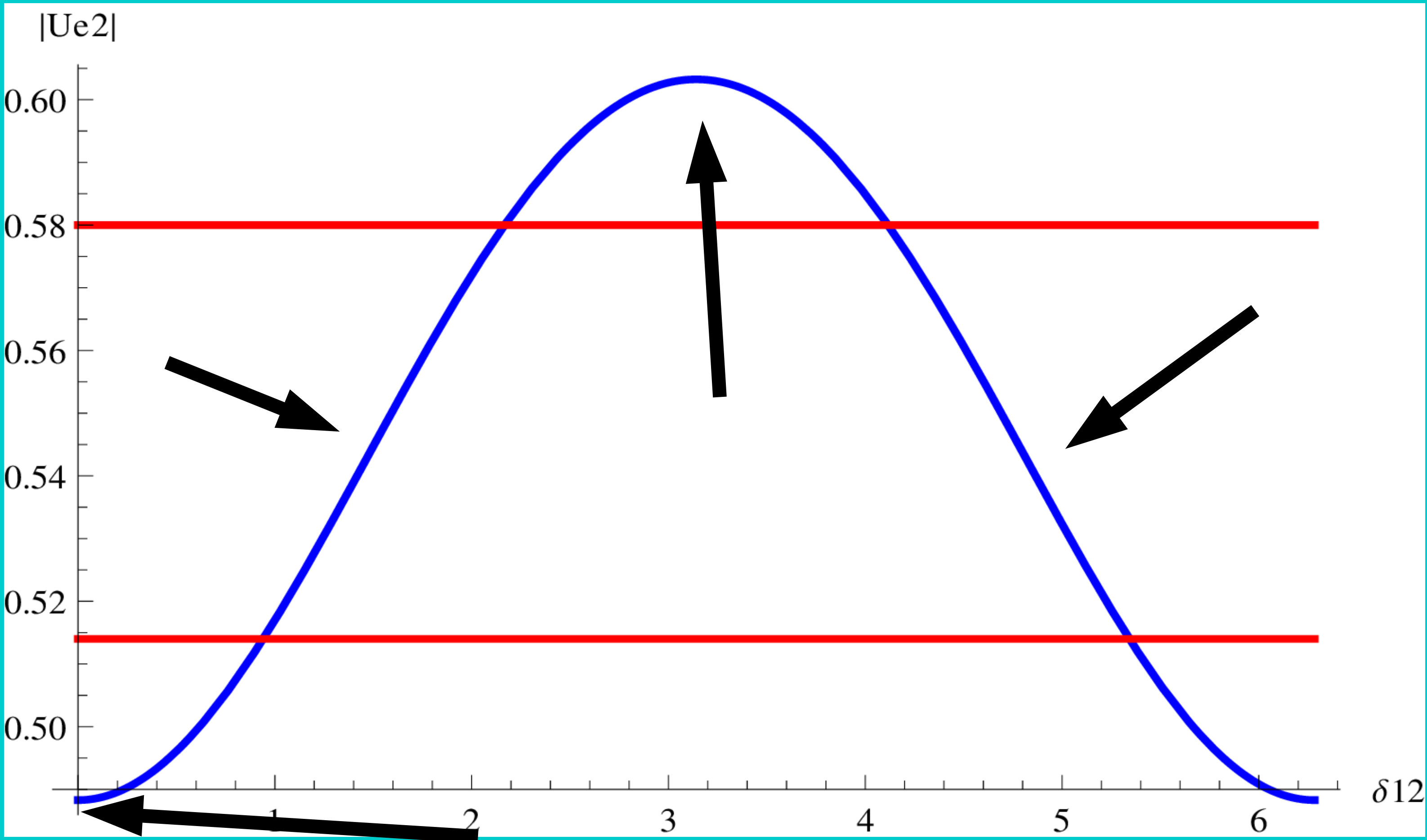
$$\Delta\delta_{ij} = 0$$

- Maximal mixing

$$\Delta\delta_{ij} = \pi$$

- CP Violation

$$\Delta\delta_{ij} = (3)\frac{\pi}{2}$$



Ansatz

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^f = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{23}^{(1)}(\frac{m_1}{m_3}) L_{23}^{(0)}(\frac{m_2}{m_3})$$

$$L_f = L_{12}(\theta_{12}^f, \pi/2) L_{13}(\theta_{13}^f, 0) L_{23}(\theta_{23}^f, 0)$$

CKM (other authors)

$$\mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 & 0 \\ f_7(m_1) & 0 & m_3 \end{pmatrix}$$

(Fritzsch, Xing, Chkareuli, Froggatt, Nielsen, Rasin, Hall)

$$V_{CKM}^{\text{th}} = \begin{pmatrix} 0.974 & 0.225 & 0.032 \\ 0.225 & 0.971 & 0.076 \\ 0.033 & 0.076 & 0.997 \end{pmatrix}$$

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

CKM (ours)

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$$\Rightarrow \mathcal{M}_f = \begin{pmatrix} f_1(m_1) & f_2(m_1) & f_3(m_1) \\ f_4(m_1) & m_2 + f_5(m_1) & f_6(m_1) \\ f_7(m_1) & f_8(m_1) & m_3 + f_9(m_1) \end{pmatrix}$$

$$L_{23}^f = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{23}^{(1)}(\frac{m_1}{m_3}) L_{23}^{(0)}(\frac{m_2}{m_3})$$

$$|V_{\rm CKM}^{\rm th}| = \begin{pmatrix} 0.974^{+0.004}_{-0.003} & 0.225^{+0.016}_{-0.011} & 0.0031^{+0.0018}_{-0.0015} \\ 0.225^{+0.016}_{-0.011} & 0.974^{+0.004}_{-0.003} & 0.039^{+0.005}_{-0.004} \\ 0.0087^{+0.0010}_{-0.0008} & 0.038^{+0.004}_{-0.004} & 0.9992^{+0.0002}_{-0.0001} \end{pmatrix}$$

$$J_q^{\rm th} = (2.6^{+1.3}_{-1.0}) \times 10^{-5}$$

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

PDG 2014

$$J_q = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$$

CKM (ours)

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$\Rightarrow \mathcal{M}_f = \ldots m_1) \ldots m_1) \ldots f_9(m_1)$

$|V^{\rm th}_{\rm CKM}| = \ldots {}^{+0.0018}_{-0.0015} \ldots {}^{+0.005}_{-0.004} \ldots {}^{+0.0002}_{-0.0001}$

$|V_{\rm CKM}| = \begin{pmatrix} 0.974\ldots & \ldots & \ldots 0.00355 \pm 0.00015 \\ 0.225\ldots & \ldots & \ldots 0.0414 \pm 0.0012 \\ 0.00\ldots & \ldots & \ldots 0.99914 \pm 0.00005 \end{pmatrix}$

PDG 2014

$J_q = (3.06^{\ldots}_{-0.20}) \times 10$

PMNS (Neutrino masses)

$$m_{\nu 2} = \sqrt{\Delta m_{21}^2/(1-\hat{m}_{\nu 12}^2)},$$
$$m_{\nu 1} = \sqrt{m_{\nu 2}^2 - \Delta m_{21}^2},$$
$$m_{\nu 3} = \sqrt{\Delta m_{31}^2 - \Delta m_{21}^2 + m_{\nu 2}^2}.$$

$$|V_{12}^{f=q,\ell}| \approx \sqrt{\frac{\hat{m}_{12}^a + \hat{m}_{12}^b}{(1+\hat{m}_{12}^a)(1+\hat{m}_{12}^b)}}$$

$$\frac{m_e}{m_\mu},\ |U_{e2}|,\ \Delta m_{21}^2,\ \Delta m_{31}^2$$

$$\text{NH:}\quad \Delta m_{31}^2 = +2.457 \pm 0.002 \times 10^{-3}\,\text{eV}^2,$$
$$\text{IH:}\quad \Delta m_{32}^2 = -2.448 \pm 0.047 \times 10^{-3}\,\text{eV}^2,$$
$$\Delta m_{21}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\text{eV}^2,$$
NuFit14

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\text{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\text{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\text{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

PMNS (our predictions)

$$|U_{PMNS}| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

$$J_\ell = -0.033 \pm 0.010$$

NuFit14

$$|U^{\rm th}_{\rm PMNS}| = \begin{pmatrix} 0.83^{+0.04}_{-0.05} & 0.54^{+0.06}_{-0.09} & 0.14 \pm 0.03 \\ 0.38^{+0.04}_{-0.06} & 0.57^{+0.03}_{-0.04} & 0.73 \pm 0.02 \\ 0.41^{+0.04}_{-0.06} & 0.61^{+0.03}_{-0.04} & 0.67 \pm 0.02 \end{pmatrix},$$

$$J_\ell = -0.031^{+0.006}_{-0.007}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\text{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\text{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\text{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

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PMNS (without fine tuning)

$$\sin^2\theta^\ell_{12} = 0.323 \pm 0.016, \qquad \sin^2\theta^\ell_{23} = 0.567^{+0.032}_{-0.128}, \qquad \sin^2\theta^\ell_{13} = 0.0234 \pm 0.0020,$$

Forero et al

$$\frac{\delta_{\rm CP}}{\pi} = 1.34^{+0.64}_{-0.38}$$

$$\sin^2\theta^\ell_{13} = 0.020 \pm 0.001$$

Daya Bay 2016

$$\sin^2\theta^{\ell,\rm th}_{12} = 0.30^{+0.07}_{-0.09}, \qquad \sin^2\theta^{\ell,\rm th}_{23} = 0.54 \pm 0.03, \qquad \sin^2\theta^{\ell,\rm th}_{13} = 0.020^{+0.009}_{-0.007},$$

$$\frac{\delta^{\rm th}_{\rm CP}}{\pi} = 1.36^{+0.05}_{-0.16}$$

$$m_{\nu 1} = (0.0041 \pm 0.0015)\,\text{eV},$$
$$m_{\nu 2} = (0.0096 \pm 0.0005)\,\text{eV},$$
$$m_{\nu 3} = (0.050 \pm 0.001)\,\text{eV}.$$

$$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$$

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PMNS (without fine tuning)

$\sin^2\theta^\ell_{12} = 0.323$ … $0.0234 \pm 0.0020,$

Forero et al

$\frac{\delta_{\rm CP}}{\pi}$ … 001

2016

$\sin^2\theta^{\ell,\rm th}_{12} = 0.3$ … $= 0.020^{+0.009}_{-0.007},$

$m_{\nu 3} > m_{\nu 2} > m_{\nu 1}$

Nucl. Phys. B892 (2015) 364-389
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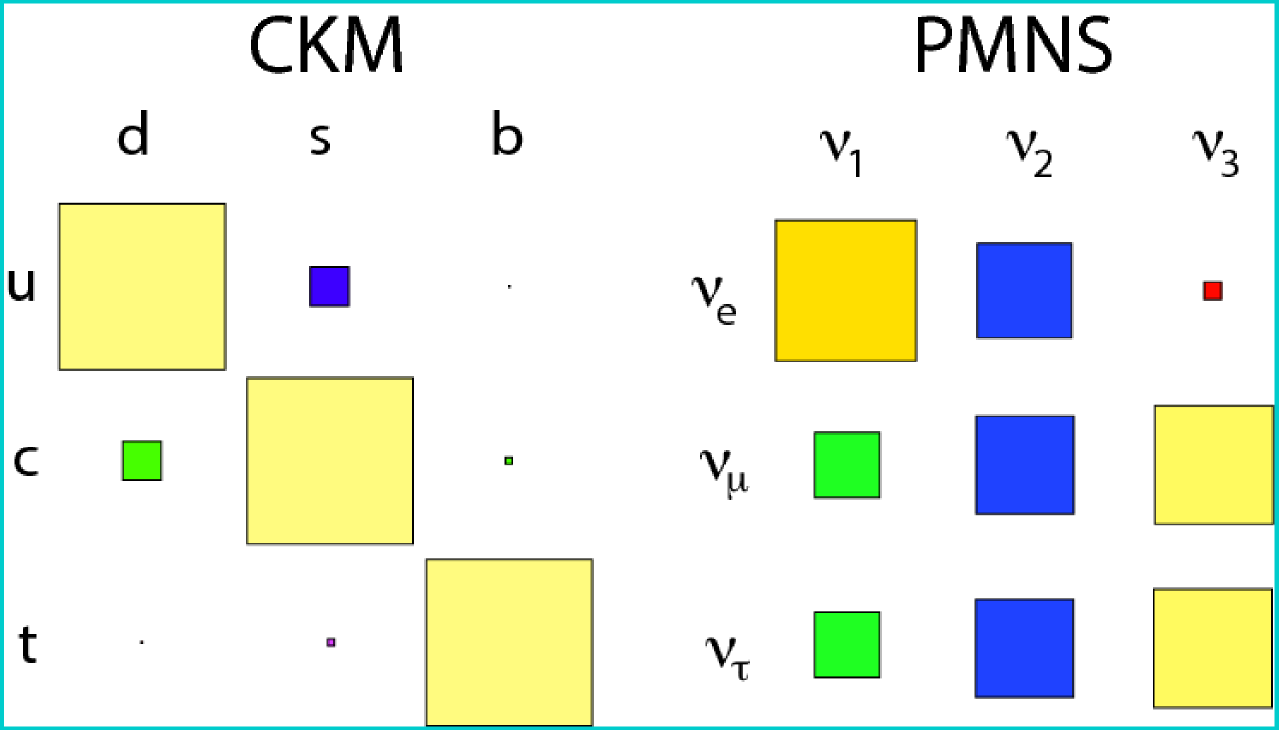
Some insight into the *flavor puzzle*



- Strong hierarchical masses
- Minimal mixing in the 1-3 and 2-3 sectors
- CP Violation in the 1-2 sector

- Weak hierarchy in neutrino masses
- Minimal mixing in the 1-3 sector
- Maximal mixing in the 2-3 sector
- CP Violation in the 1-2 sector

See Barranco's Talk

Conclusiones

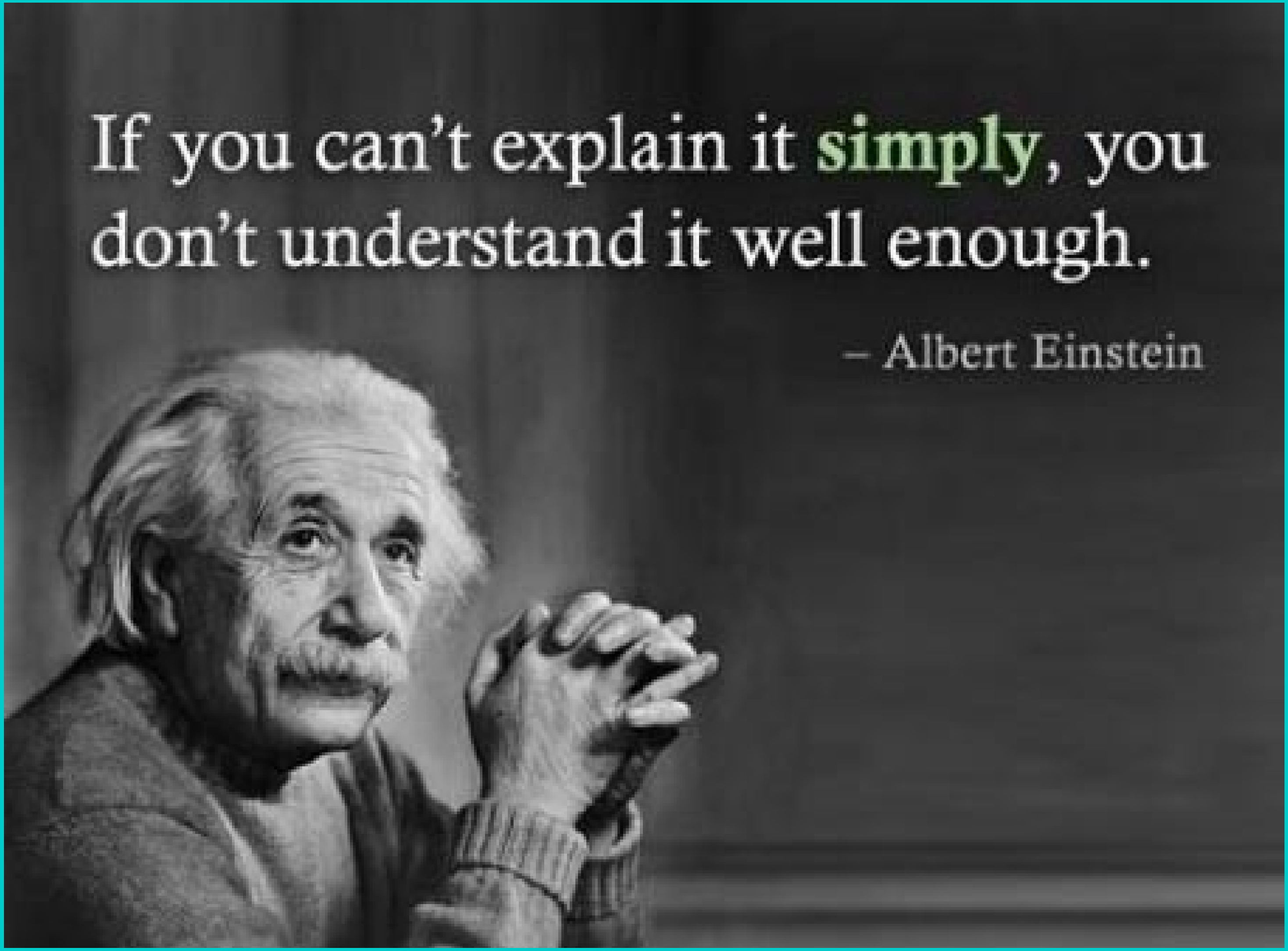
¿Cuál es la escala de masas del neutrino?

¿Cuál es la jerarquía de masas?

¿Existe violación de CP en el sector leptónico?

~~¿Cuál es la naturaleza del neutrino: Dirac ó Majorana?~~

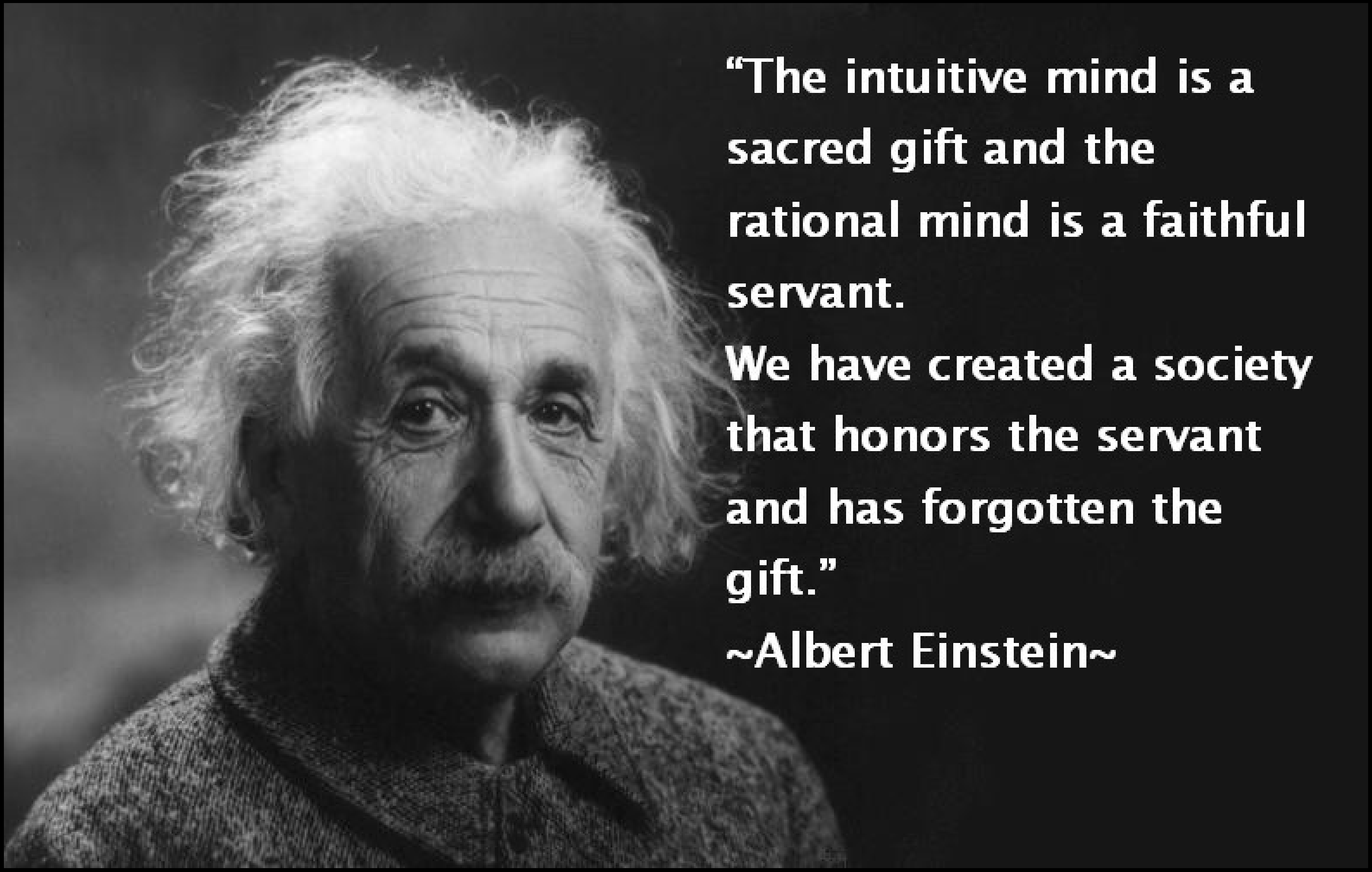
+ Flavor puzzle


If you can't explain it simply, you don't understand it well enough.
– Albert Einstein

Conclusions

- The hierarchy in the masses provides the *simplest* way to study fermion mixing
- We have built a new mixing parametrization using four mass ratios
- The flavor puzzle is understood as a direct consequence of the fermion masses
- For the parametrization it was necessary to use the Schmidt-Mirsky approximation theorem
- Application of this theorem was equivalent to ask Minimal Flavor Violation
- We found an excellent agreement in the quark mixing sector (CKM)
- Application to the lepton sector provided the absolute value of neutrino masses (which gave an excellent agreement to the PMNS matrix) and pointed to which 2-3 octant

Thanks for your attention!



The flavor-blind principle

Two inquiries

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$m_3 \gg m_2 \gg m_1$$

- Hierarchical contributions*

  (work in preparation with J. Hoff, W. G. Hollik, L. Flores, UJSS)
- Ordered Yukawas

What principle or symmetry could lay behind such sequential Yukawas?

*(Froggatt-Nielsen, Arkani Hamed, Ibarra-Solaguren, Altmannshofer, Knapen-Robinson)

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$ $|m_{22}| \neq |m_{22}|(\delta_{ij})$

$$|\mathbf{m}| = \begin{pmatrix} 0 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

Phys. Rev. D93 013002 (2016), UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$ $|m_{22}| \neq |m_{22}|(\delta_{ij})$

$$|\mathbf{m}| = \begin{pmatrix} 0 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$M = L^\dagger \Sigma R$

$\theta_L = \theta_R$

$\delta_L = \delta_R + \pi$

Phys. Rev. D93 013002 (2016), UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$m_2 \gg m_1$

$$|\mathbf{m}| = \begin{pmatrix} |m_{12}|^2 & |m_{12}| \\ |m_{12}| & |m_{22}| \end{pmatrix}$$

$$\tan^2\theta_{ij} = \frac{m_i}{m_j}$$

$|m_{12}| \neq |m_{12}|(\delta_{ij})$

$|m_{22}| \neq |m_{22}|(\delta_{ij})$

Antisymmetric

$\delta = 0,\ \pi$

$$M = \begin{pmatrix} 0 & \sqrt{m_1 m_2} e^{-i\delta} \\ -\sqrt{m_1 m_2} e^{i\delta} & m_2 - m_1 \end{pmatrix}$$

Symmetric

$\delta = \frac{\pi}{2},\ \frac{3\pi}{2}$

Phys. Rev. D93 013002 (2016), UJSS

The flavor-blind principle:

"Yukawa couplings shall be either flavor blind or decomposed into several sets obeying distinct permutation symmetries."

$$S_{nL} \otimes S_{nR} \rightarrow S_{(n-1)L} \otimes S_{(n-1)R} \rightarrow \cdots \rightarrow S_{2L} \otimes S_{2R} \rightarrow S_{2A}$$

$$\mathcal{Y}^{1\leftrightarrow 2\leftrightarrow \cdots\ (n-1)\leftrightarrow n} + \mathcal{Y}^{1\leftrightarrow 2\leftrightarrow \cdots (n-2)\leftrightarrow (n-1)} + \cdots + \mathcal{Y}^{1\leftrightarrow 2} + \mathcal{Y}^{2A}$$

Phys. Rev. D93 013002 (2016), UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann $$\mathcal{Y} = y \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} \beta & \alpha \\ -\alpha & -\beta \end{pmatrix}$$

$\psi_1, \ \psi_2$

$S_{2L} \times S_{2R}$

S_{2A}

Phys. Rev. D93 013002 (2016), UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann

$$\mathcal{Y} = 2y \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \alpha + \beta \\ \beta - \alpha & 0 \end{pmatrix}$$

$$\psi_1^m, \ \psi_2^m$$

$$S_{2L} \times S_{2R}$$

$$O_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$S_{2A}$$

$$m_2 \neq 0, \quad m_1 = 0$$

$$m_1 \neq 0$$

Phys. Rev. D93 013002 (2016), UJSS

Two family case

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

Lehmann $$\mathcal{Y} = x_1 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & x_2 e^{-i\delta} \\ x_2 e^{i\delta} & 0 \end{pmatrix}$$

$$x_1 = m_2 - m_1$$

$$x_2 = \sqrt{m_1 m_2}$$

$$\delta = 0, \ \pi$$

Symmetric

$$\tan^2 \theta_{ij} = \frac{m_i}{m_j}$$

$$\delta = \frac{\pi}{2}, \ \frac{3\pi}{2}$$

Antisymmetric

Phys. Rev. D93 013002 (2016), UJSS

Three family case

Harari 78, Kaus, Lavoura, Fritzsch, Tanimoto, Meshkov, Babu, Mohapatra, Mondragón, Rodríguez Jauregui, González Canales, Barranco

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$\mathcal{Y} = y \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \longrightarrow \begin{array}{c} S_{3L} \times S_{3R} \\ m_3 \neq 0,\ m_2, m_1 = 0 \end{array}$$

$$+ \begin{pmatrix} \beta & \beta & \alpha_1 \\ \beta & \beta & \alpha_1 \\ \alpha_2 & \alpha_2 & \gamma \end{pmatrix} \longrightarrow \begin{array}{c} S_{2L} \times S_{2R} \\ m_3, m_2 \neq 0,\ m_1 = 0 \end{array}$$

$$+ \begin{pmatrix} \tau & i\mu & \nu_1 \\ -i\mu & -\tau & -\nu 1 \\ \nu_2 & -\nu_2 & 0 \end{pmatrix} \longrightarrow \begin{array}{c} S_{2A} \\ m_3, m_2, m_1 \neq 0 \end{array}$$

Phys. Rev. D93 013002 (2016), UJSS

Three family case

Harari 78, Kaus, Lavoura, Fritzsch, Tanimoto, Meshkov, Babu, Mohapatra, Mondragón, Rodríguez Jauregui, González Canales, Barranco

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} + \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}$$

$$\mathcal{Y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} \longrightarrow U(2)^3, \quad m_3 \neq 0,\ m_2, m_1 = 0$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} \longrightarrow U(1)^3, \quad m_3, m_2 \neq 0,\ m_1 = 0$$

$$+ \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix} \longrightarrow U(1)_{B(L)}, \quad m_3, m_2, m_1 \neq 0$$

Phys. Rev. D93 013002 (2016), UJSS

Conclusions

Conclusions

- The hierarchy in the masses provides the *simplest* way to study fermion mixing
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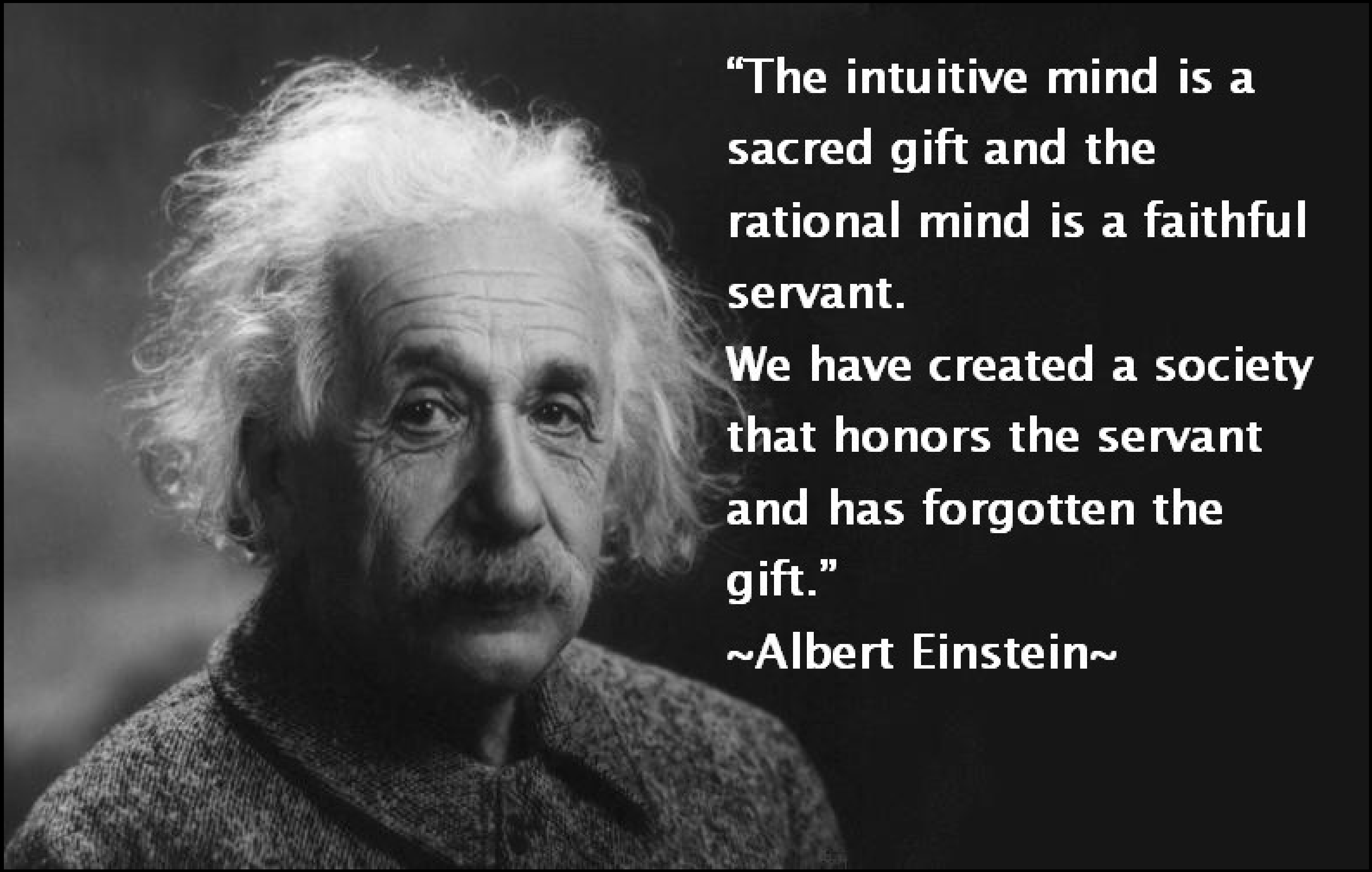
- For the parametrization it was necessary to use the Schmidt-Mirsky approximation theorem
- Application of this theorem was equivalent to ask Minimal Flavor Violation
- We found an excellent agreement in the quark mixing sector (CKM)
- Application to the lepton sector provided the absolute value of neutrino masses (which gave an excellent agreement to the PMNS matrix) and pointed to which 2-3 octant
- The study of the flavor-blind principle provided a way to understand origin of the sequential Yukawa terms noticed in the study of the four mass ratios parametrization

Thanks for your attention!

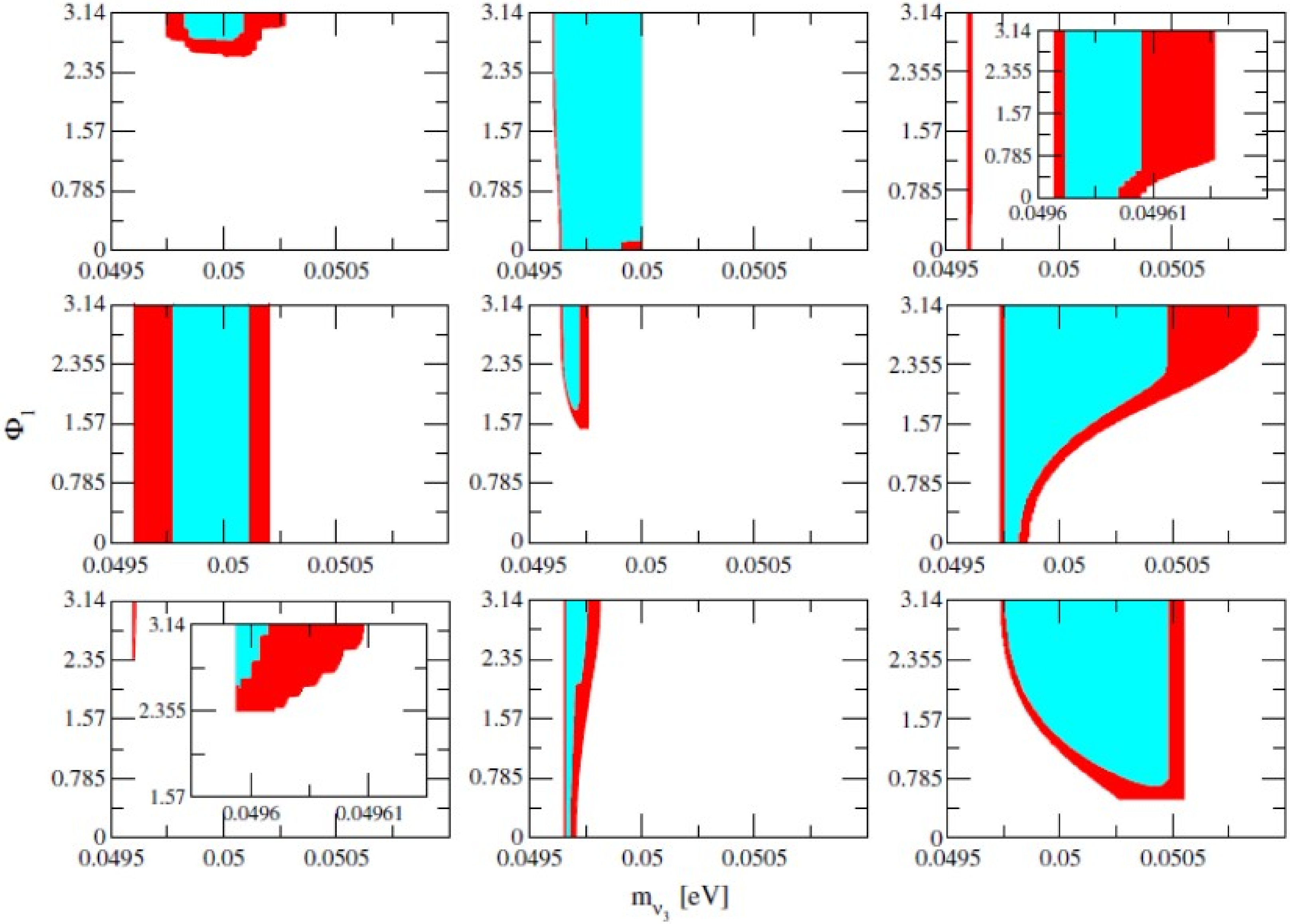


Back up slides

$$\chi^2_{\text{leptons}}(a_l, a_\nu, \Phi_1, m_{\nu 3}) = \sum_{i=1}^{9}\left(\frac{U^{\text{th}}_{\text{PMNS}}(a_l, a_\nu, \Phi_1, m_{\nu 3}) - |U_{\text{PMNS}}|}{\delta U_{\text{PMNS}}}\right)^2.$$

$$U^{\text{th}}_{\text{PMNS}} = O_l^T P^{l-\nu} O_\nu,$$

See Barranco's Talk



J. Barranco, D. Delepine, L. Lopez-Lozano **Phys.Rev. D86 (2012) 053012**

	δ_{12}	$\delta_{13}^{(0)}$	$\delta_{13}^{(1)}$	$\delta_{13}^{(2)}$	$\delta_{23}^{(0)}$	$\delta_{23}^{(1)}$	$\delta_{23}^{(2)}$
CKM	$\frac{\pi}{2}$	0	π	π	0	π	π
PMNS	$\frac{\pi}{2}$	0	π	π	π	π	0

$$L_{ij}(\theta_{ij}, \delta_{ij}) = \begin{pmatrix} c\theta_{ij} & s\theta_{ij} e^{-i\delta_{ij}} \\ -s\theta_{ij} e^{i\delta_{ij}} & s\theta_{ij} \end{pmatrix}$$

$$L_{23}^{f} = L_{23}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{23}^{(1)}(\frac{m_1}{m_3}) L_{23}^{(0)}(\frac{m_2}{m_3})$$

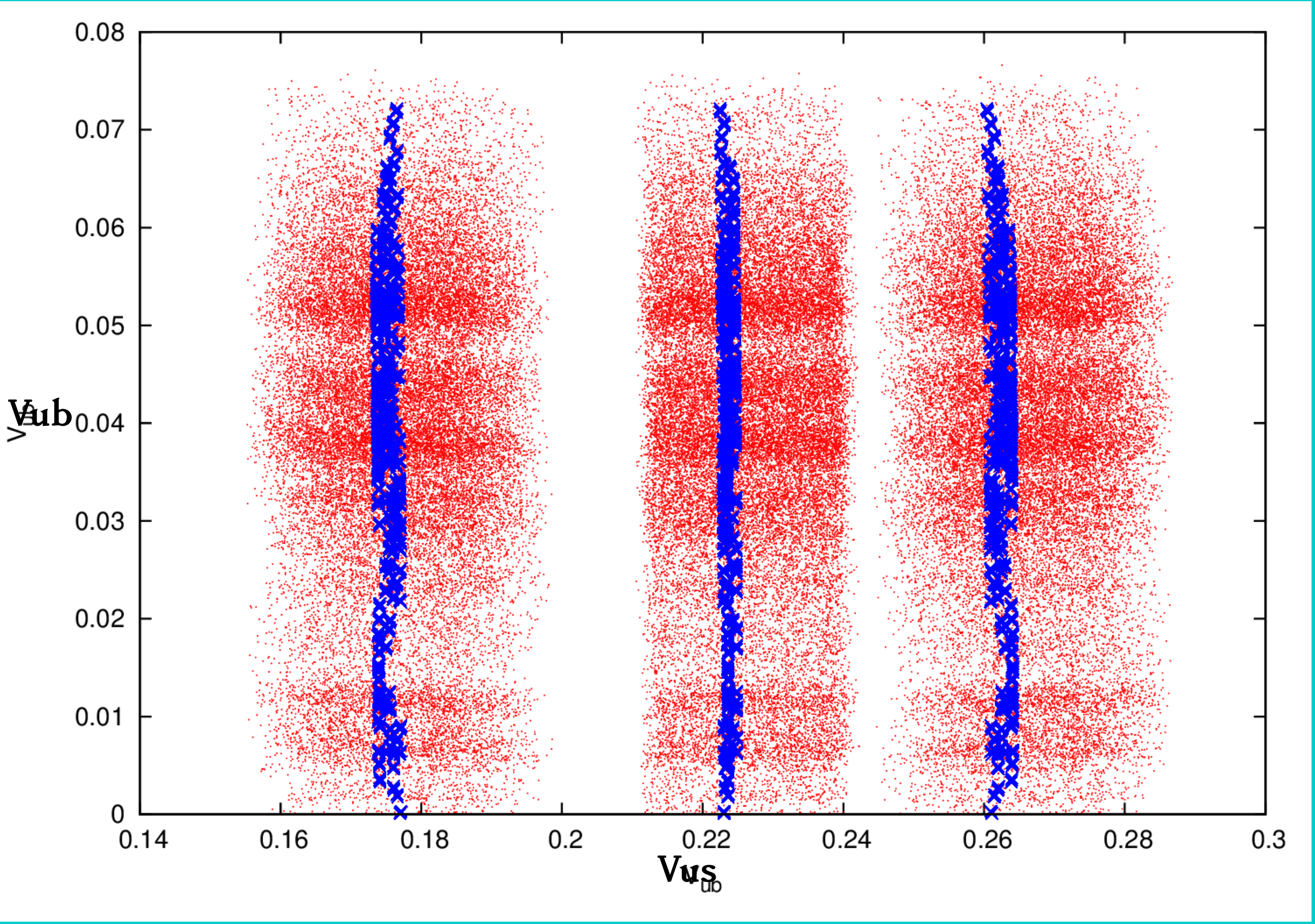
$$L_{13}^{f} = L_{13}^{(2)}(\frac{m_1 m_2}{m_3^2}) L_{13}^{(1)}(\frac{m_2^2}{m_3^2}) L_{13}^{(0)}(\frac{m_1}{m_3})$$

$$L_{12}^{f} = L_{12}^{(0)}(\frac{m_1}{m_2})$$

$$x_f^r \equiv \frac{\sqrt{(r-1)m_{f,2}^2 + m_{f,3}^2}}{\parallel \mathcal{M}_f \parallel_{\mathbf{F}}} = \sqrt{\frac{(r-1)m_{f,2}^2 + m_{f,3}^2}{m_{f,1}^2 + m_{f,2}^2 + m_{f,3}^2}},$$

x_f^r	u	d	e	ν
$r=1$	0.999993	0.999816	0.998274	0.978894
$r=2$	0.999999	0.999999	0.999999	0.996773

$$\parallel \mathcal{M}_f \parallel_{\mathbf{F}} = \sqrt{\sum_{i=1,2,3} m_{f,i}^2}.$$



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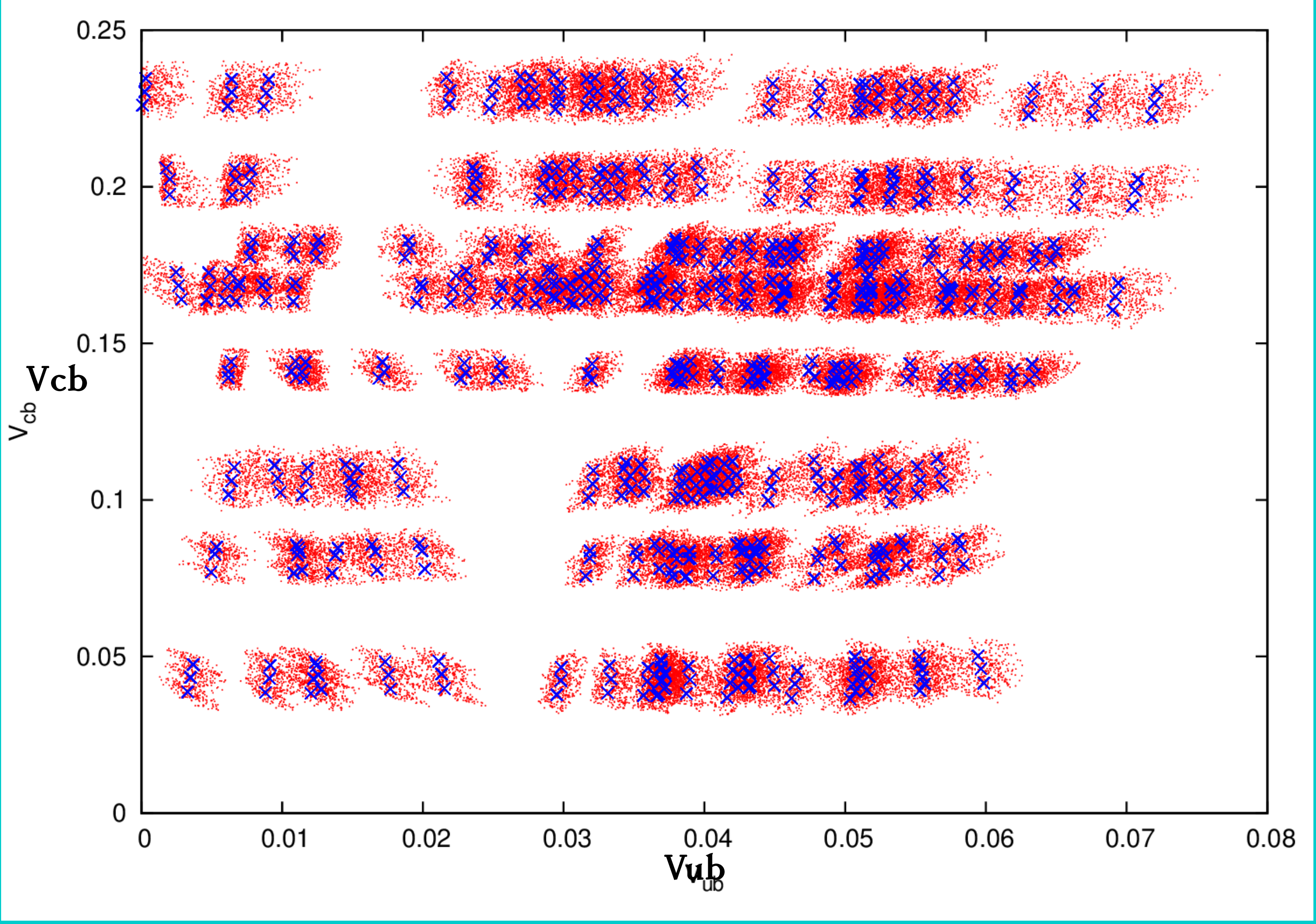

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Vcb
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