Meson form factors through DSEs

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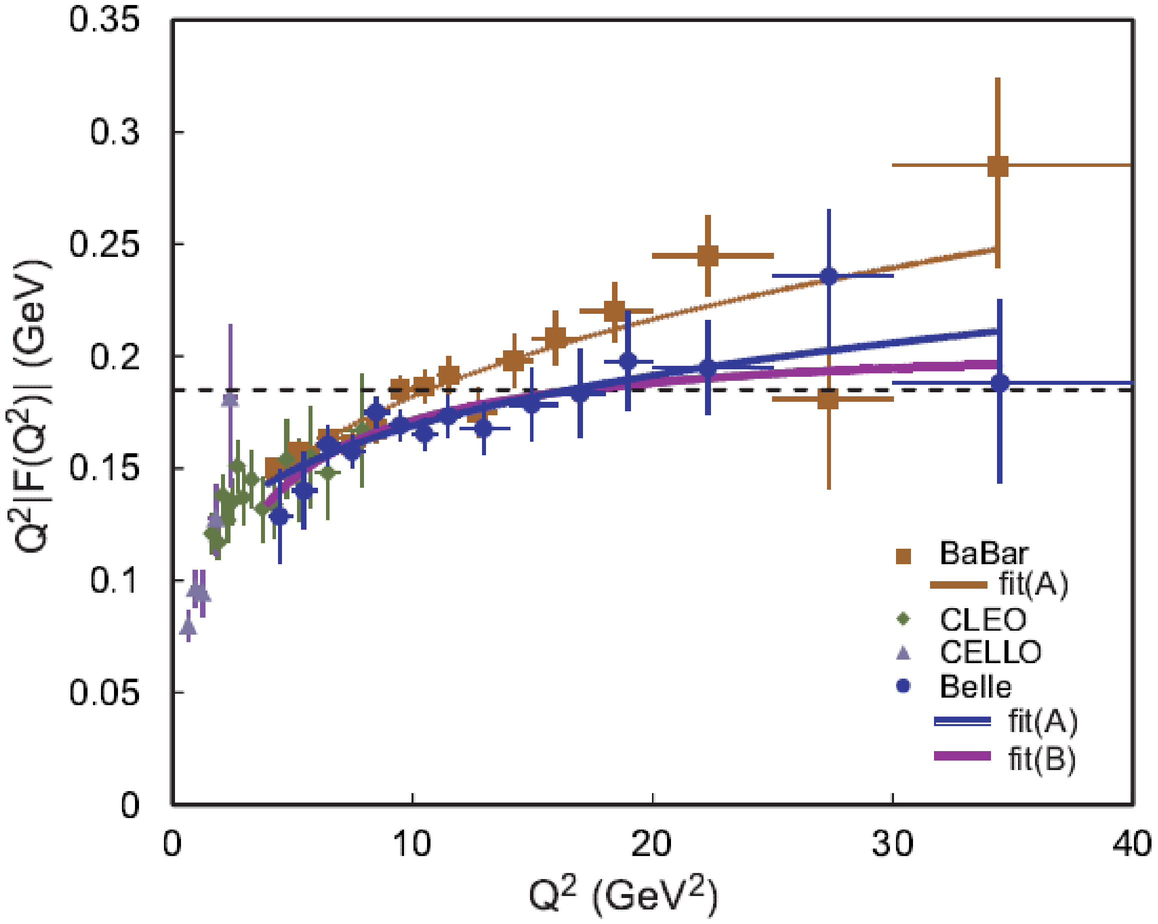
Motivation

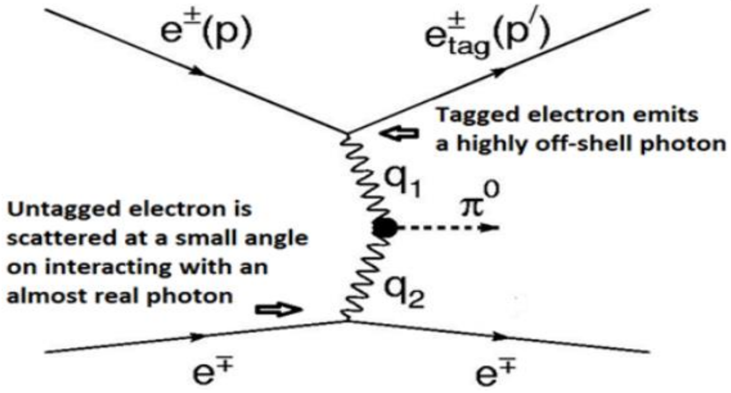
- Understanding strong interactions is still being a challenge for physicists; although scientists have developed the powerful theory of quarks and gluons, namely, Quantum Chromodynamics (QCD).
- Quarks and gluons are the fundamental degrees of freedom of such theory; however, they are not found free. Instead, they form color-singlet states known as hadrons.
- Hadron form factors are intimately related to their internal structure. But, due to the non perturbative nature of QCD, unraveling hadron form factors from first principles is an outstanding problem.
- Dyson-Schwinger equations (DSEs) are the equations of motion of QCD and they combine the IR and UV behavior of the theory at once, therefore, DSEs are an ideal platform to study quarks and hadrons.

Outline

- Pion transition form factor.
- Heavy mesons TFFs.
- Conclusions and scope.

Pion transition form factor - $\gamma\gamma^*\rightarrow\pi^0$



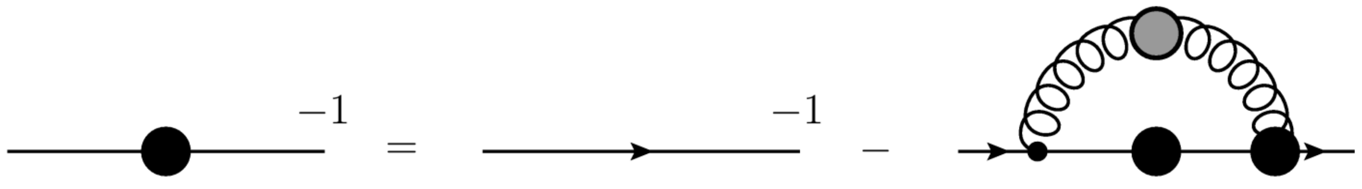


- Many experiments have been done so far; but, at large Q^2, there is no agreement between the only available data (BaBar and Belle).
- This needs to be explained, and we need to have predictions before future experiments of Belle II.

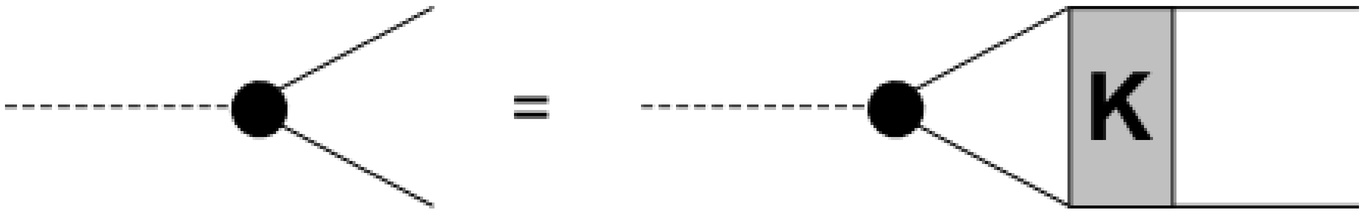
- [Dashed]: Well known asymptotic/conformal limit, $2f_{\pi}$. **G.P. Lepage, S.J. Brodsky, Phys. Rev. D22, 2157 (1980) [1].**

The DSE-BSE approach

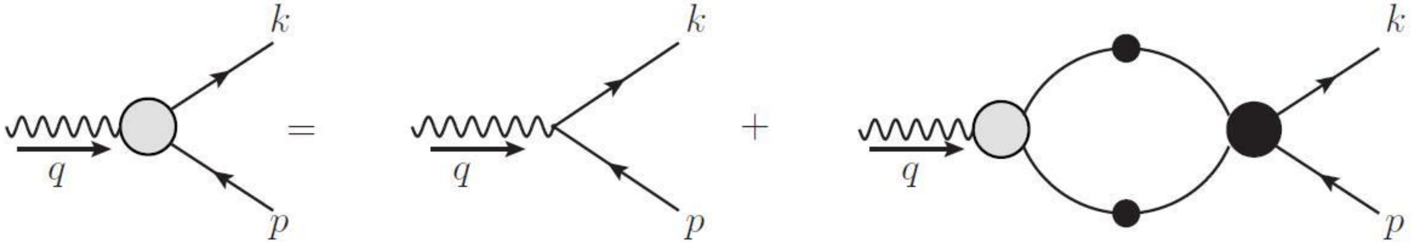
1. Quark Propagator

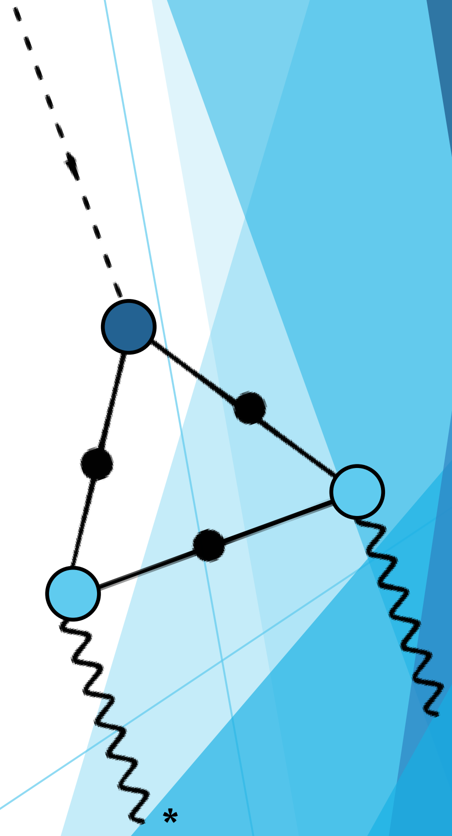


2. Bethe-Salpeter Amplitudes

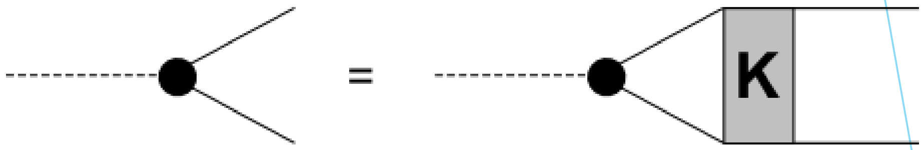


3. Quark-Photon Vertex





The DSE-BSE approach



- In the Rainbow-Ladder (RL) truncation, the renormalized DSE for the quark propagator is written as:

$$S^{-1}(p,\mu) = Z_{2F}(i\gamma\cdot p) + Z_4 m(\mu) + Z_{1F}\int_q^\Lambda G(p-q)D^0_{\mu\nu}(p-q,\mu)\frac{\lambda^a}{2}\gamma_\mu S(q,\mu)\frac{\lambda^a}{2}\gamma_\nu \, .$$

- The corresponding Bethe-Salpeter equation:

$$\Gamma^{ab}_M(p;P) = -\int_q^\Lambda \frac{G(k^2)}{k^2}D^0_{\mu\nu}(k)\frac{\lambda^c}{2}\gamma_\mu S^a(q+\eta P)\Gamma^{ab}_M(q;P_i)S^b(q-(1-\eta)P)\frac{\lambda^c}{2}\gamma_\nu \, .$$

- Where G(p-q) is an effective coupling, we model it as in **Phys. Rev. C84, 042202(R) (2011) [2]** by S.-x. Qin, L. Chang et al..

The tools: N-ccp parametrization

- The quark propagator is written as:

$$S(p,\mu) = -i\,\gamma\cdot p\,\sigma_v(p^2,\mu^2) + \sigma_s(p^2,\mu^2)\,.$$

- It can be written in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N}\left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}}\right)\,,\quad \sigma_s(q) = \sum_{k=1}^{N}\left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}}\right)\,.$$

- Constrained to the UV conditions of the free propagator form.

N. Souchlas (adv. P. Tandy), (2009). *Quark Dynamics and Constituent Masses in Heavy Quarks Systems*. **PhD. Thesis [3].**

The tools: Nakanishi representation

- We parametrize the BSA using a Nakanishi-like representation, **Phys. Rev. 130 1230-1235 (1963) [4].** Which consists in splitting the BSA into IR and UV parts and writing them as follows:

$$A(q,P) = \int_{-1}^{1} dz \int_0^{\infty} d\Lambda \left[\frac{\rho^i(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{m+n}} + \frac{\rho^u(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^n} \right] .$$

- Where the spectral density is written as:

$$\rho^{i,u}(z,\Lambda) = \rho_1(z)\delta(\Lambda - \Lambda^{i,u}) + \cdots$$

- Our form of A(q,P) is slightly different. First, we choose:

$$\rho_1(z) = \rho_\nu(z) \sim (1-z^2)^\nu .$$

The tools: Nakanishi representation

- Then we choose the following representation:

$$A^i(k,P) = c_A^i \int_{-1}^{1} dz \rho_{\nu_A^i}(z)[b_A \hat{\Delta}^4_{\Lambda_A^i}(k_z^2) + \bar{b}_A \hat{\Delta}^5_{\Lambda_A^i}(k_z^2)] \; . \quad E^u(k;P) = c_E^u \int_{-1}^{1} dz \, \rho_{\nu_E^u}(z) \hat{\Delta}^{1+\alpha}_{\Lambda_E^u}(k_z^2)$$

$$F^u(k,P) = c_F^u \int_{-1}^{1} dz \rho_{\nu_F^u}(z) k^2 \Lambda_F^u \Delta^{2+\alpha}_{\Lambda_F^u}(k_z^2) \qquad G^u(k,P) = c_G^u \int_{-1}^{1} dz \rho_{\nu_G^u}(z) \Lambda_G^u \Delta^{2+\alpha}_{\Lambda_G^u}(k_z^2)$$

- *A* stands for amplitude (E,F,G); *i*, *u* for IR and UV. H(k,P) is negligible. Λ, v, c, b are parameters fitted to the numerical data.

- With the following definitions:

$$\hat{\Delta}_\Lambda(s) = \Lambda \, \Delta_\Lambda(s) \; , \; \Delta_\Lambda(s) = (s + \Lambda^2)^{-1} \; , \; k_z^2 = k^2 + z \, k \cdot P \; .$$

Quark-Photon vertex.

- We employ unamputated vertex ansatz:

$$S\Gamma_\mu S \to \chi(k_f, k_i) = \sum_{i=1}^{3} T_{\mu i} X_i(k_f, k_i)$$

- Where the tensor structures are:

$$T_{1\mu} = \gamma_\mu$$
$$T_{2\mu} = \beta\gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{\beta}\gamma \cdot k_i \gamma_\mu \gamma \cdot k_f$$
$$T_{3\mu} = i\,\beta(\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + i\bar{\beta}(\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f)$$

$$\beta = 1 + \alpha(Q^2)$$
$$\bar{\beta} = 1 - \beta$$
$$\alpha(Q^2) = s_0 \mathrm{Exp}[-\mathcal{E}/M_E]\ .$$

- And, the dressing functions:

$$X_1(k_f, k_i) = \Delta_{k^2\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_2(k_f, k_i) = \Delta_{\sigma_V}(k_f^2, k_i^2)\,,$$
$$X_3(k_f, k_i) = \Delta_{\sigma_S}(k_f^2, k_i^2)\,.$$

$$\Delta_F(k_f, k_i) = \frac{F(k_f) - F(k_i)}{k_f - k_i}$$

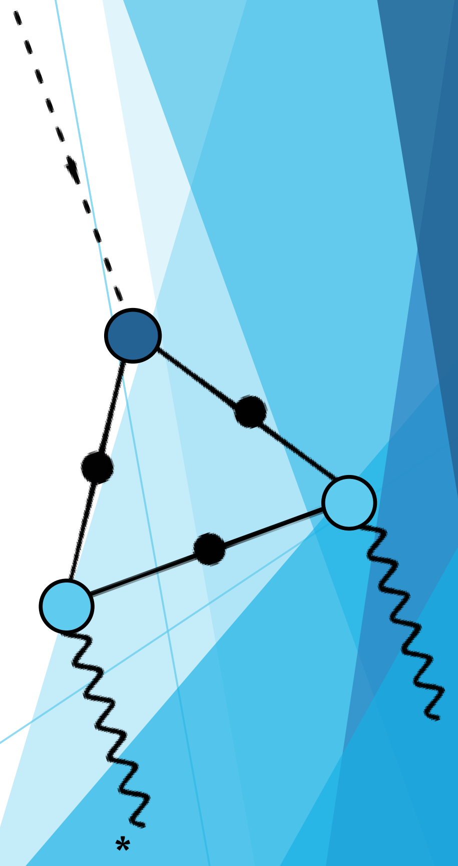
Transition form factor

- The general expression for a pseudoscalar meson transition form factor (TFF) is written as:

$$\mathcal{T}_{\mu\nu}(k_1, k_2) = T_{\mu\nu}(k_1, k_2) + T_{\nu\mu}(k_2, k_1) \ ,$$

$$T_{\mu\nu}(k_1, k_2) = \frac{\alpha_{em}}{\pi}\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G(k_1^2, k_2^2, k_1\cdot k_2)$$

$$= \mathrm{tr}\int \frac{d^4l}{(2\pi)^4}\chi_\pi(l_1, l_2)i\Gamma_\mu(l_2, l_{12})S(l_{12})i\Gamma_\nu(l_{12}, l_1) \ .$$



- The parametrizations of quark propagator and BSA allows us to solve analytically the integrations over momentum after Feynman Parametrization.

- And then, numerically integrate over Feynman Parameters.

TFF: Asymptotic limit

- The asymptotic limit is written as:

$$G(Q_1^2, Q_2^2; \mu) \to f_\pi \left\{ \frac{J_\omega(\mu)}{Q_1^2 + Q_2^2} + O\left(\frac{\alpha_s}{\pi}, \frac{1}{(Q_1^2 + Q_2^2)^2} \right) \right\} ,$$

where:

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} , \quad J_\omega(\mu) = \frac{4}{3} \int_0^1 dx \frac{\phi_\pi(x, \mu)}{1 + \omega^2 (2x - 1)} .$$

- $G(Q^2)$ has been divided by $2\pi^2$ in order to follow the convention.

- For asymptotic QCD, we have: $\phi_\pi^{asym}(x, \mu \to \infty) = 6x(1-x)$.

Therefore, we arrive at the *conformal limit* (ref. [1]): $Q^2 G(Q^2) \to 2 f_\pi$.

Pion Distribution Amplitude

- To fully understand pion, one must extract information from its PDA, the projection of the BSA onto the lightcone:

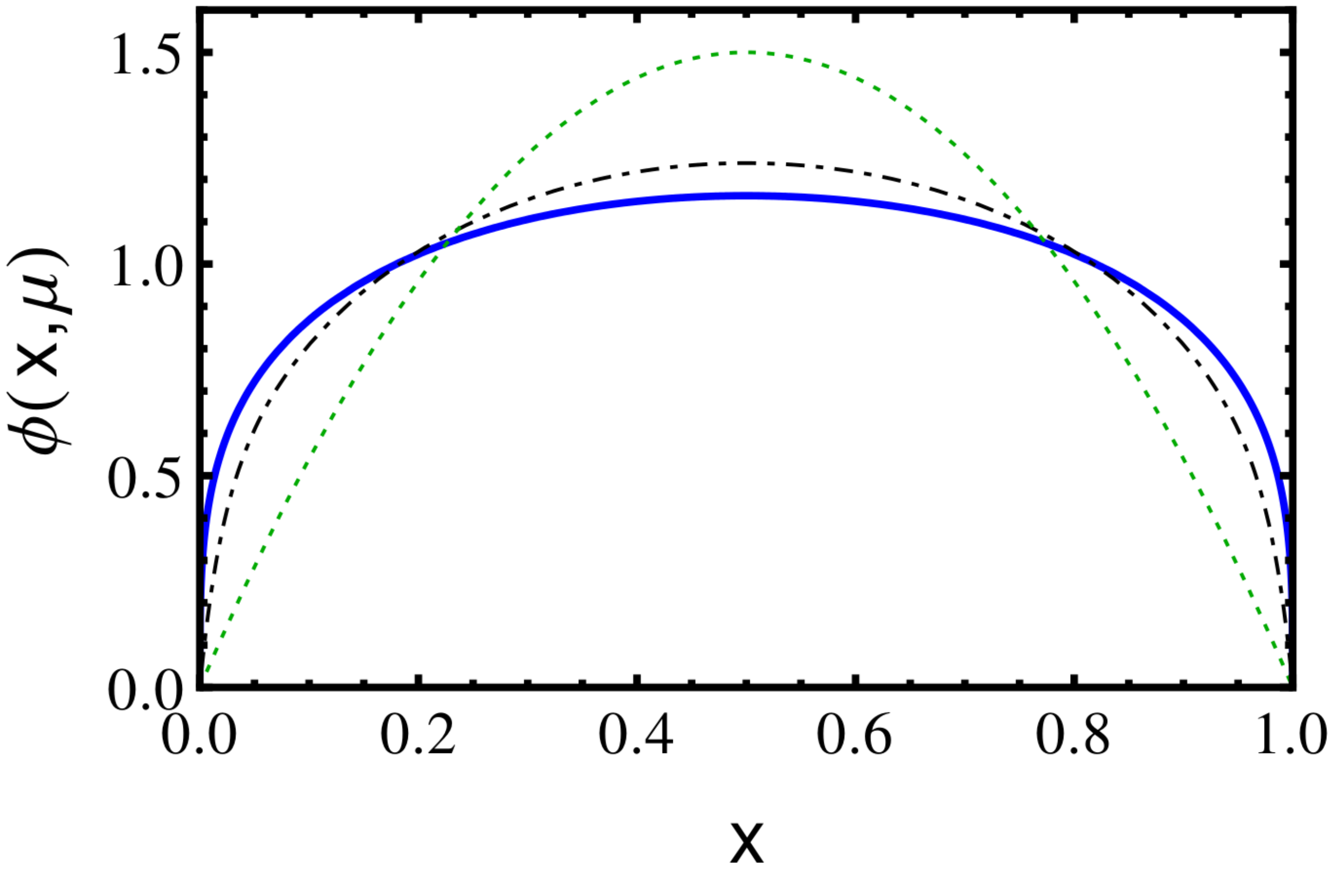
$$f_\pi \phi_\pi(x,\mu) = \mathrm{tr} \int \frac{d^4 q}{(2\pi)^4} \delta(n \cdot q^+ - xn \cdot P) \gamma_5 \gamma \cdot n \chi_\pi(q;P)$$

- It should evolve with the resolution scale $\mu^2 = Q^2$ through the ERBL evolution equations, **Phys. Rev.Lett. 11 092001 (2013) [5]**.

- Evolution enables the dressed-quark and -antiquark degrees-of-freedom, to split into less well-dressed partons via the addition of gluons and sea quarks in the manner prescribed by QCD dynamics. This can be read from the leading twist expansion (for example):

$$G(Q^2) = 4\pi^2 f_\pi \int_0^1 dx T_H(x, Q^2, \alpha(\mu); \mu) \phi_\pi(x,\mu)$$

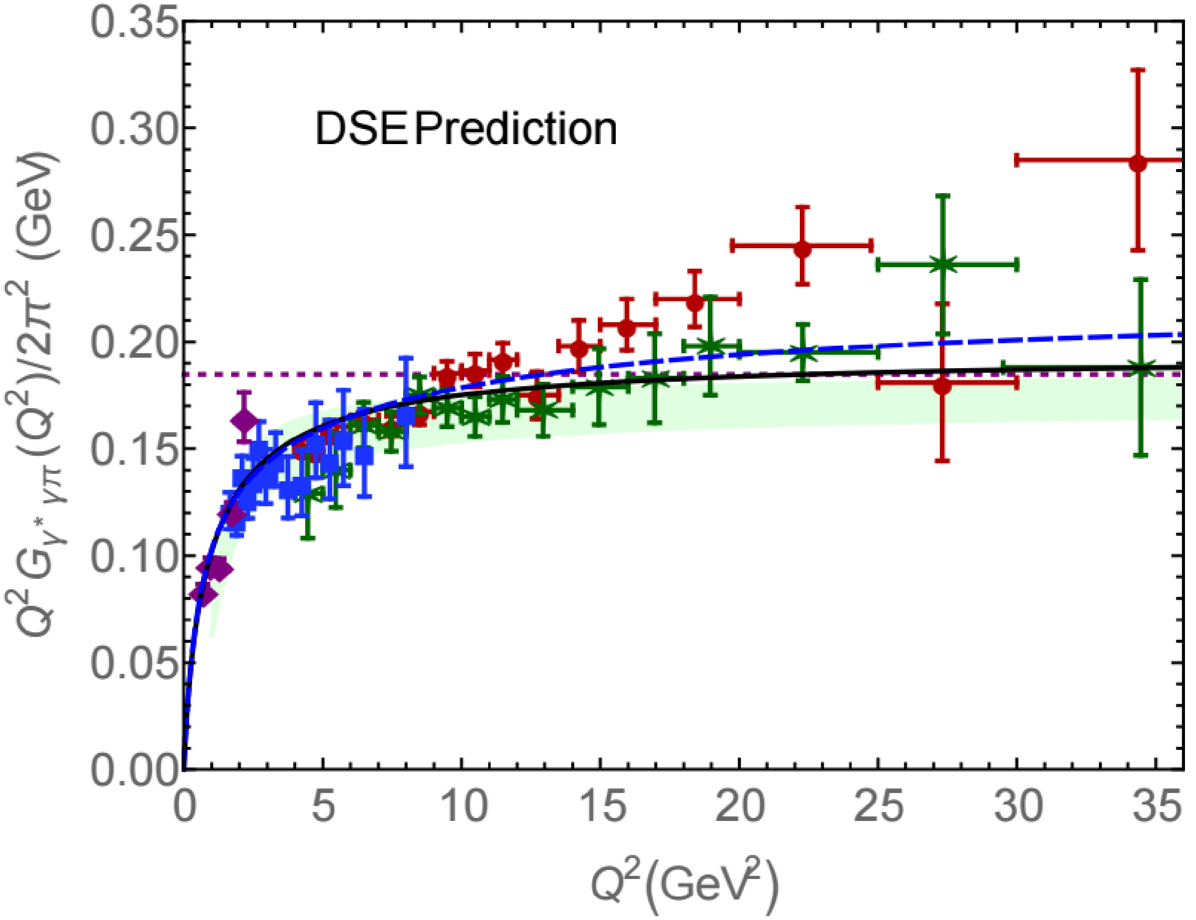
Pion Distribution Amplitude



- **PDA at different scales:** [Green, dashed] Asymptotic PDA, 6x(1-x) **[1]**. [Blue, solid] μ=2 GeV, **Phys.Rev. D93 (2016) no.7, 074017 [6]**. [Black, dot-dashed] Evolution from μ=2 GeV to μ=10 GeV.

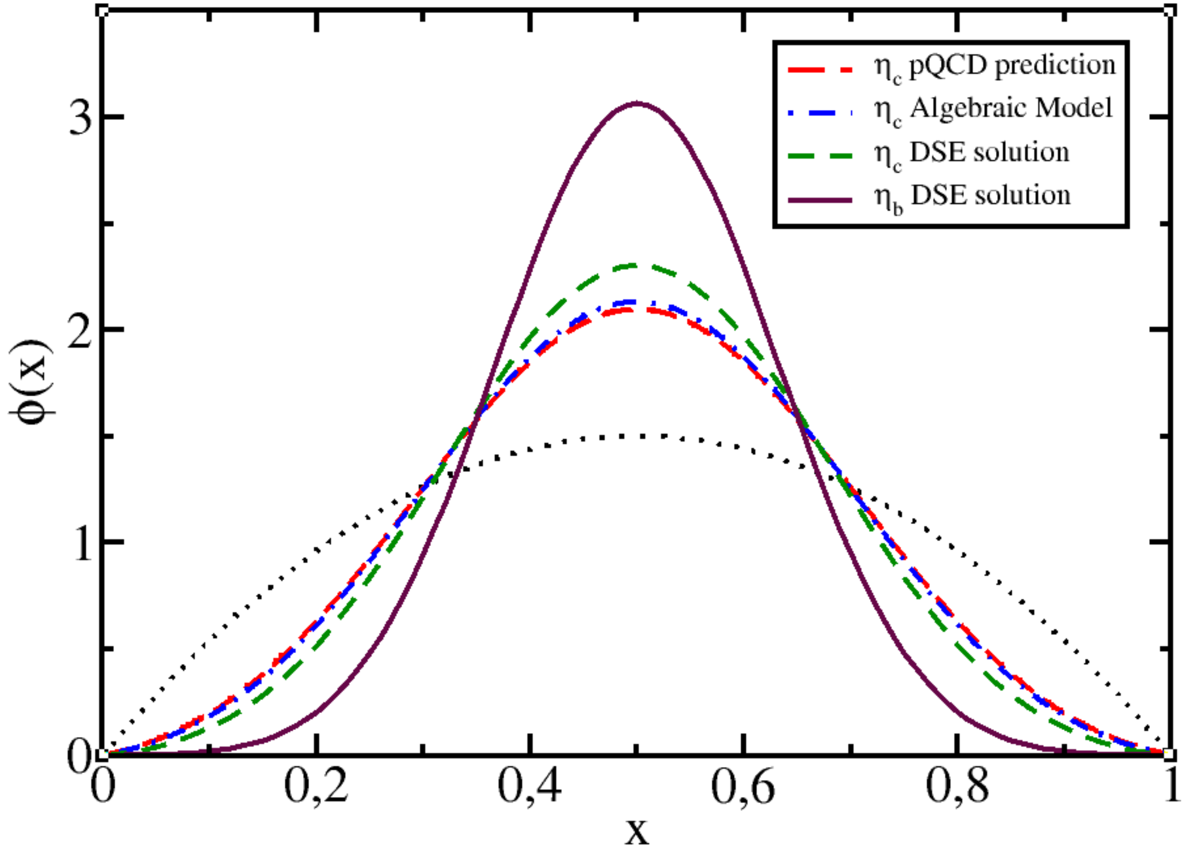
DSE Prediction

Phys.Rev. D93 (2016) no.7, 074017



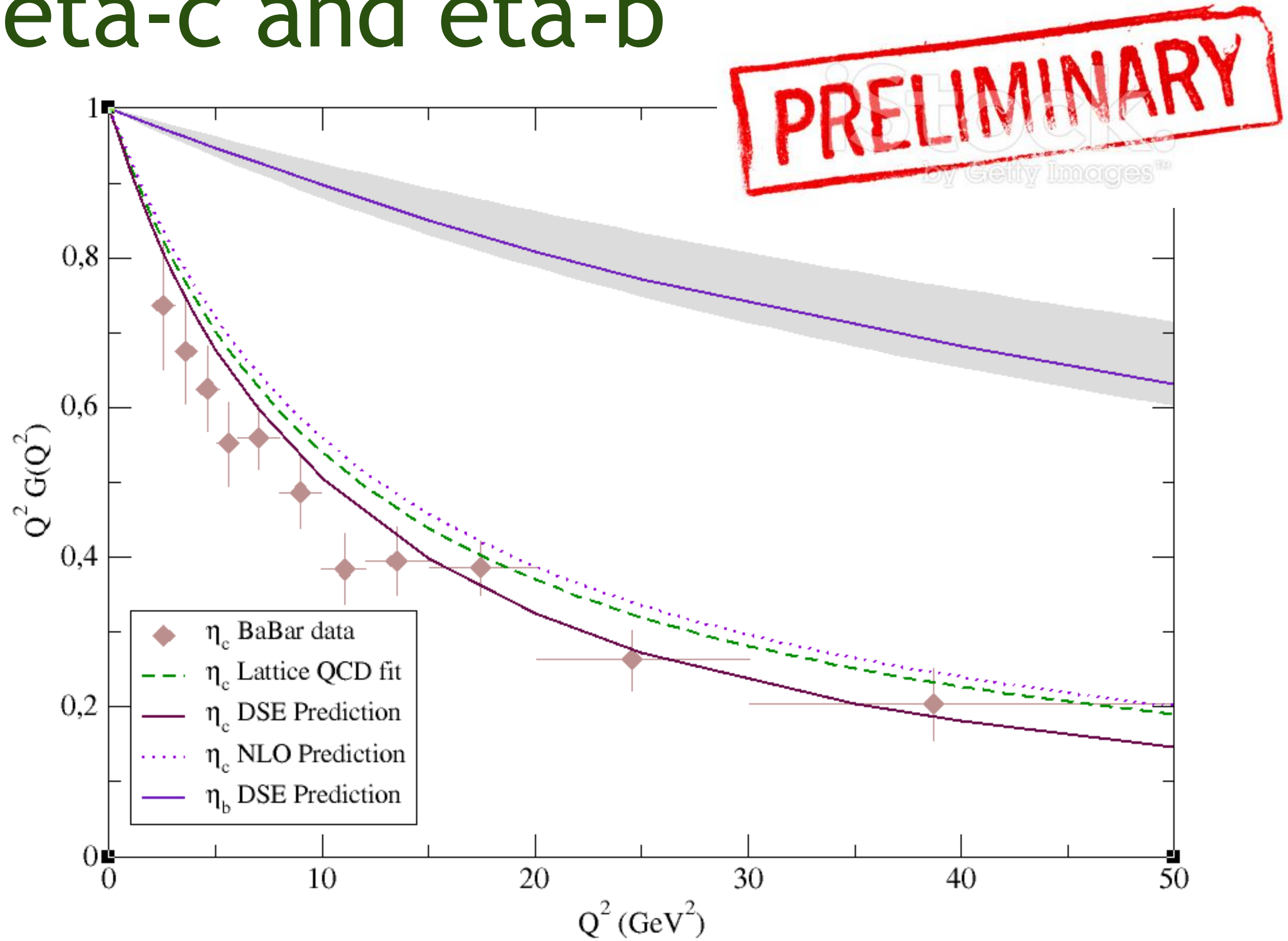
- **Transition Form Factor:** [**Black**, solid] DSE Prediction (G(0)=1/(2fpi), r=0.68 fm). [Blue, dashed] Frozen DSE Prediction (μ = 2 GeV). [Green, band] BMS model (A.P. Bakulev et al., **Phys. Rev. D84, 034015 (2011) [7]**).

PDA: eta-c and eta-b



- **Heavy mesons PDA:** As opposite to pion PDA, which is a broad concave PDA, eta-c and eta-b PDAs have a convex-concave-convex form. **DSE solutions: Phys.Lett. B753 (2016) 330-335 [8]** by M. Ding et al. **pQCD: Phys. Lett. B413, 410 (1997) [9]** by Kroll et al. **Algebraic Model: Phys. Rev. D 93, 094025 [10]**.

TFF: eta-c and eta-b



- **TFF eta-c:** [Solid, maroon] DSE result (width of 5.1 KeV) compared to BaBar data, Lattice QCD (**J.Phys.Conf.Ser. 69, 012006 (2007) [11]**) and NLO prediction (**Phys.Rev.Lett. 115 (2015) no.22, 222001 [12]**). **TFF eta-b:** [Solid, indigo] DSE result (width of 259 eV) compared to [Gray band] NNLO predictions of ref [12].

Conclusions and scope

Phys.Rev. D93 (2016) no.7, 074017

- We described a computation of the pion transition form factor, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations, obtained in the rainbow-ladder truncation. This result is relevant for BaBar and Belle experiments.
- We have unified the description and explanation of this transition with the charged pion electromagnetic form factor (**Phys.Rev.Lett. 111 (2013) no.14, 141802 [14]**) and its valence-quark distribution amplitude [6].
- The novel analysis techniques we employed made it possible to compute $G(Q^2)$, on the entire domain of space-like momenta, for the first time in a framework with a direct connection to QCD.
- Those techniques could be easily adapted to other hadrons and processes, so it was possible to extend our studies to eta-c and eta-b mesons.