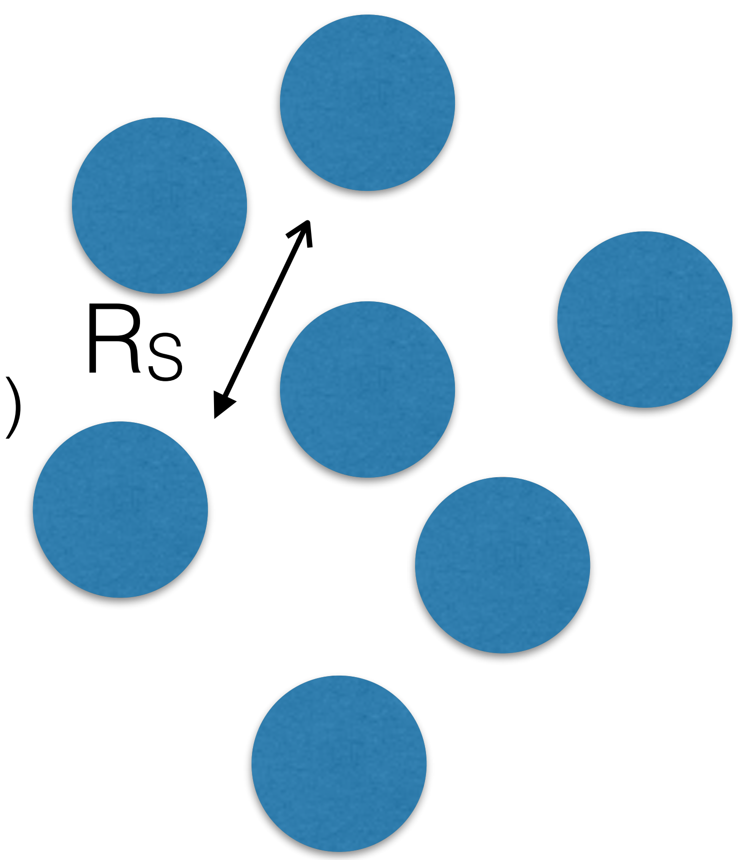
MARTIN HENTSCHINSKI (BUAP & ICN UNAM)

High gluon densities and 3 parton correlation in DIS at small x

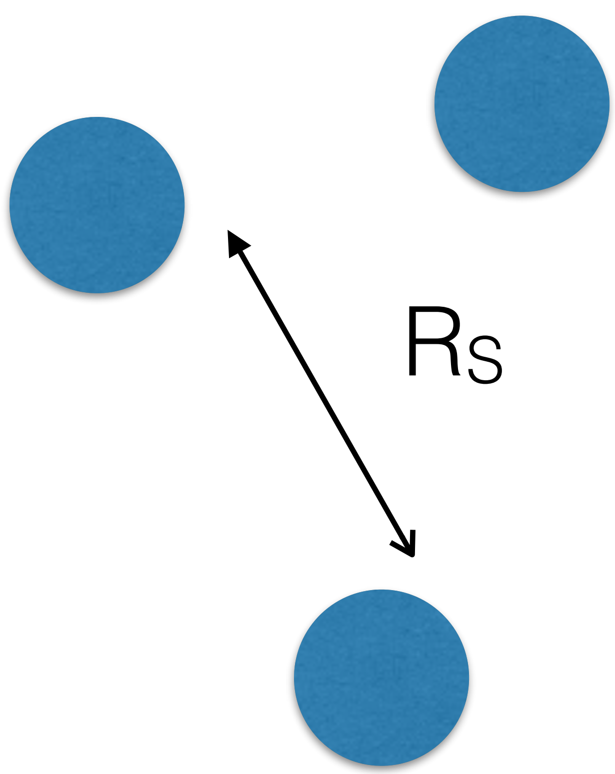
based on **Ayala, Hentschinski, Jalilian-Marian, Tejeda Yeomans arXiv:1604.08526**



high-density
$A_\mu \sim 1/g$ (non-perturbative)

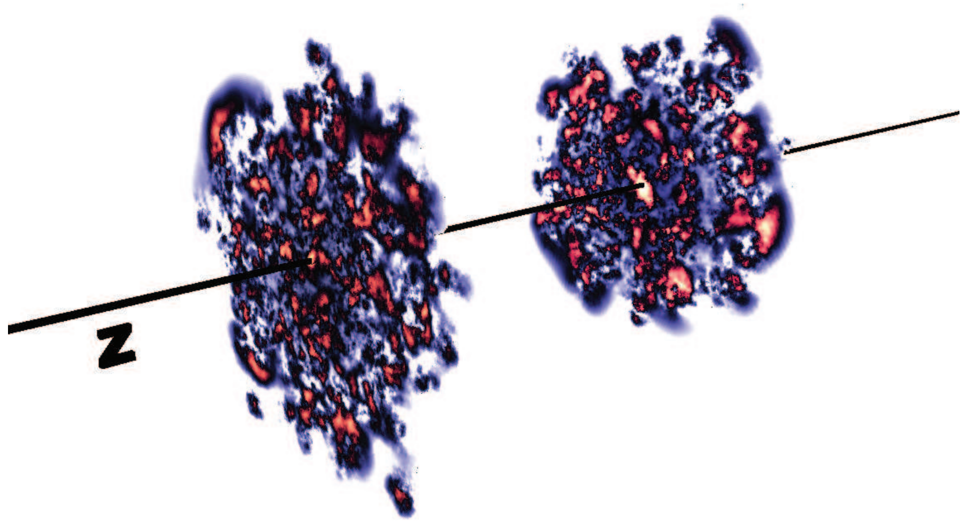
$Q_S \sim 1/R_S \gg \Lambda_{QCD}$
$\Rightarrow \alpha_S \ll 1$

weak coupling methods
can be used



low-density
$A_\mu \sim g$ (perturbative)
$Q_S \sim 1/R_S \sim \Lambda_{QCD}$

Heavy Ion Collisions + high multiplicity events (LHC, RHIC)



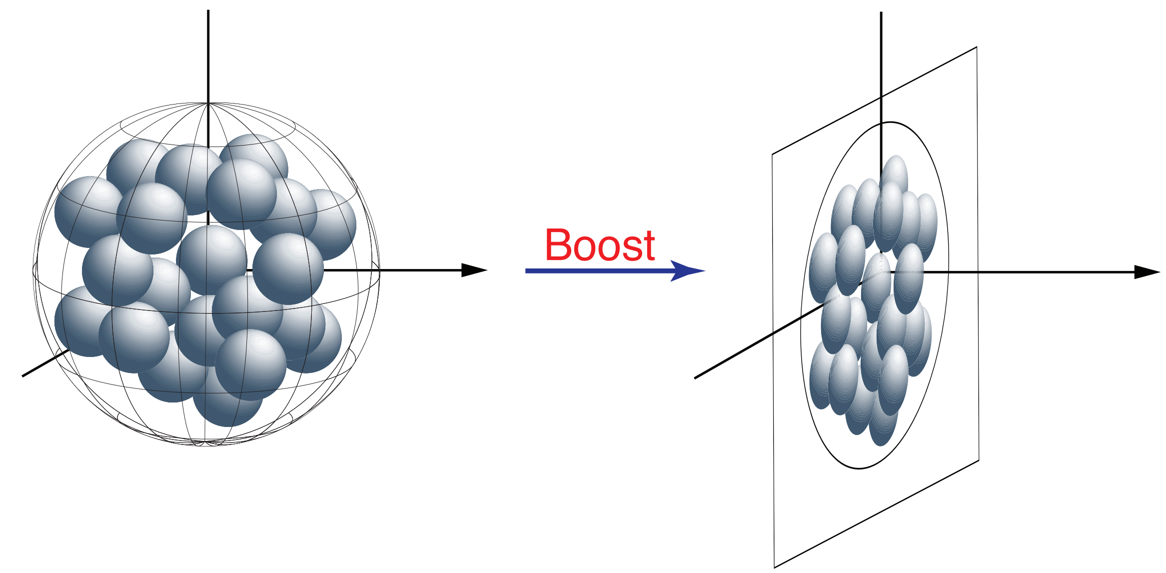
- HERA: gluon distribution grows with energy ~s^{λ}
- enhanced in nuclei (~$A^{1/3}$) & high multiplicity events

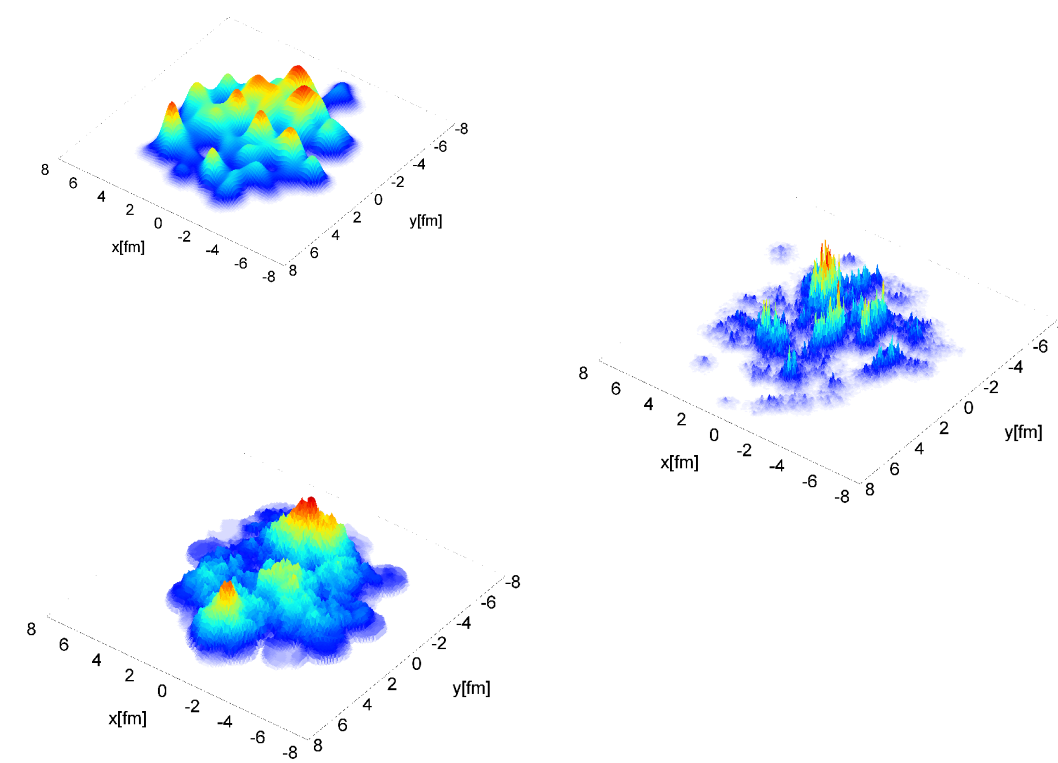
→ high gluon densities!

- Believed: heavy ion collisions at RHIC, LHC = collisions of two Color Glass Condensate

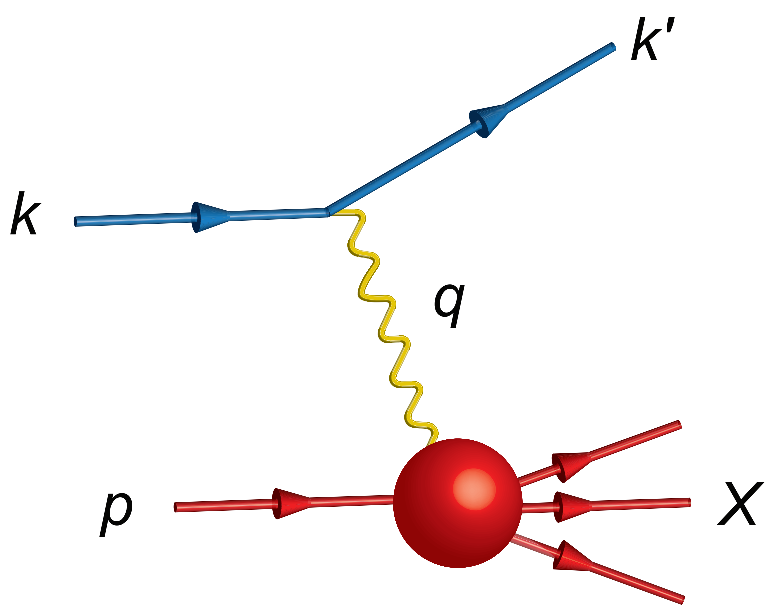
- but what are the correct initial conditions?

~ collision of two Lorentz contracted sheets of color



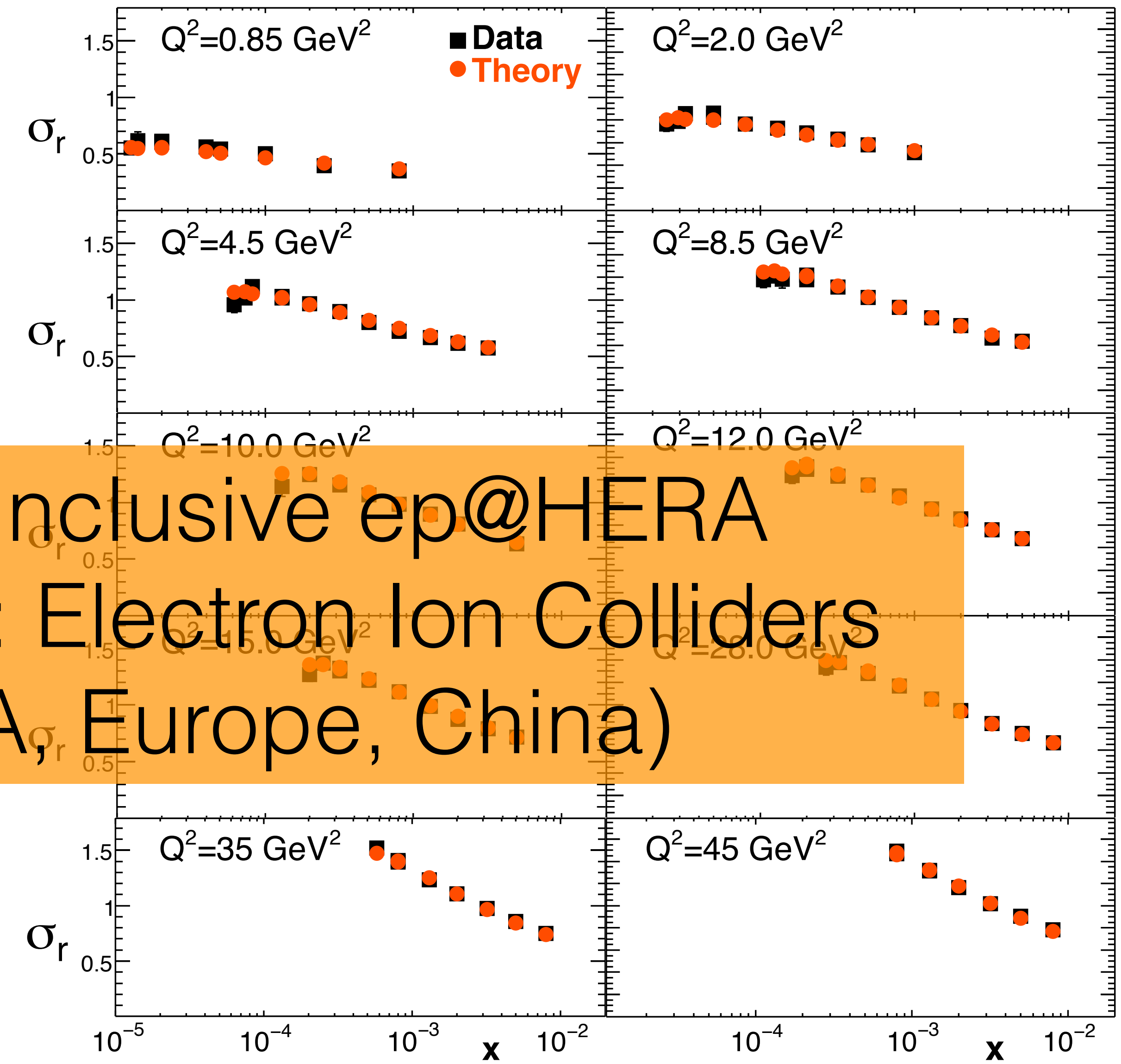


best explored in ep/eA collisions
→ control kinematics

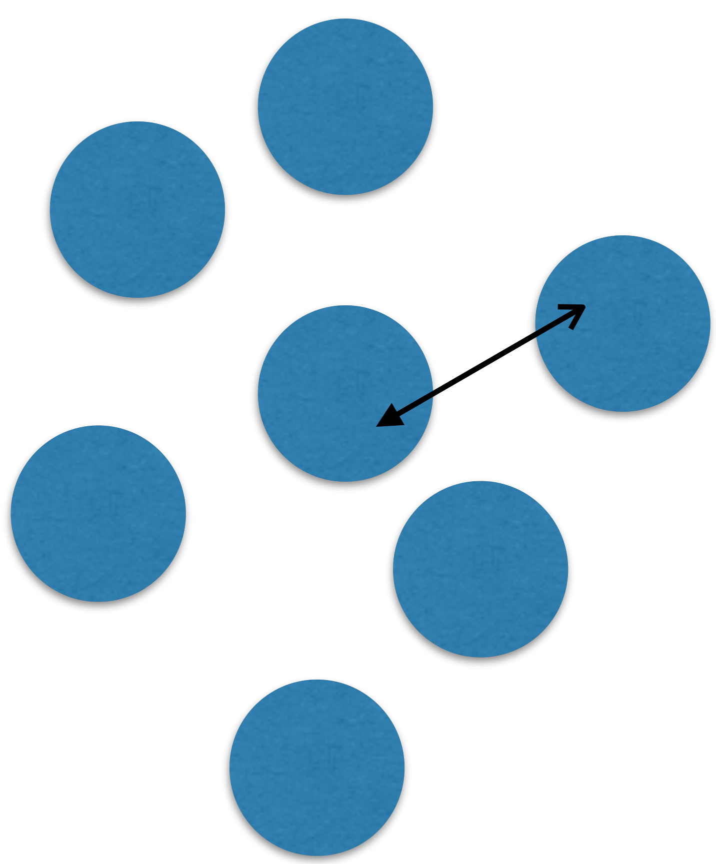


Photon virtuality $Q^2 = -q^2$

Bjorken $x = \frac{Q^2}{2p \cdot q}$



best so far: inclusive ep@HERA
future plans: Electron Ion Colliders
(USA, Europe, China)

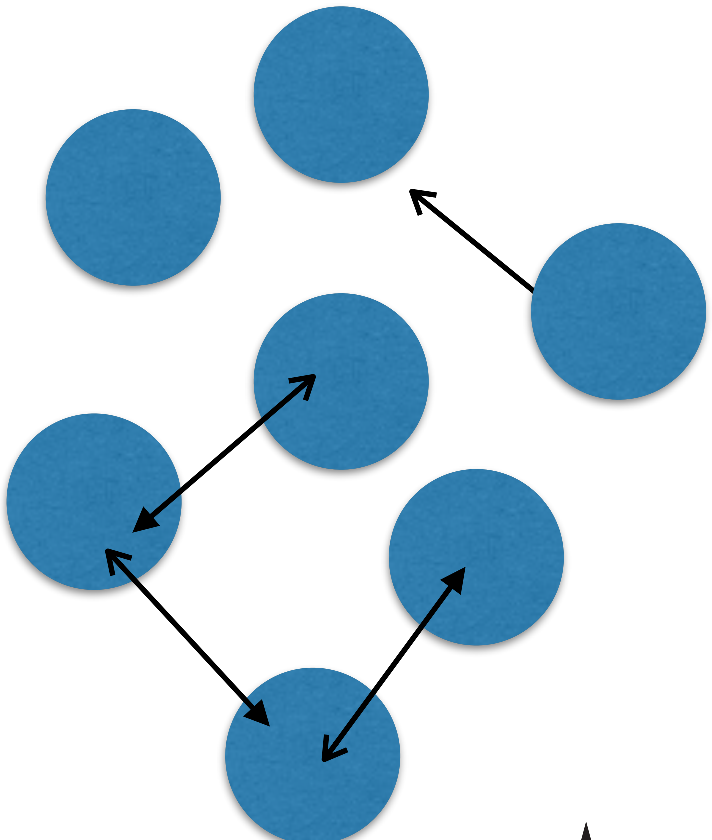


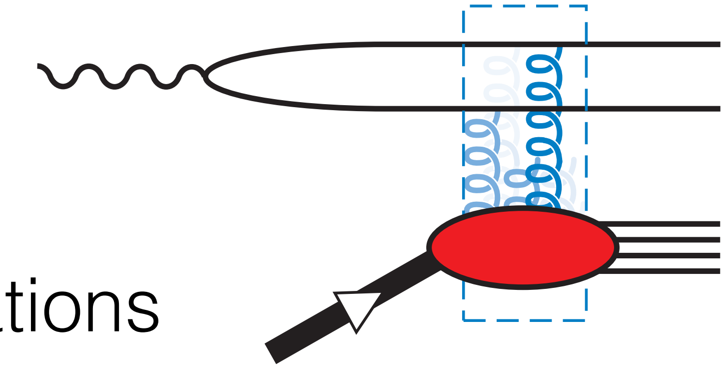
inclusive DIS: 2 point correlator only
$\Gamma(x_1,x_2) \sim \langle A(x_1)A(x_2)\rangle$

higher point correlators

$\Gamma(x_1, x_2, x_3, x_4) \sim \langle A(x_1)A(x_2)A(x_3)A(x_4)\rangle$

require exclusive final states

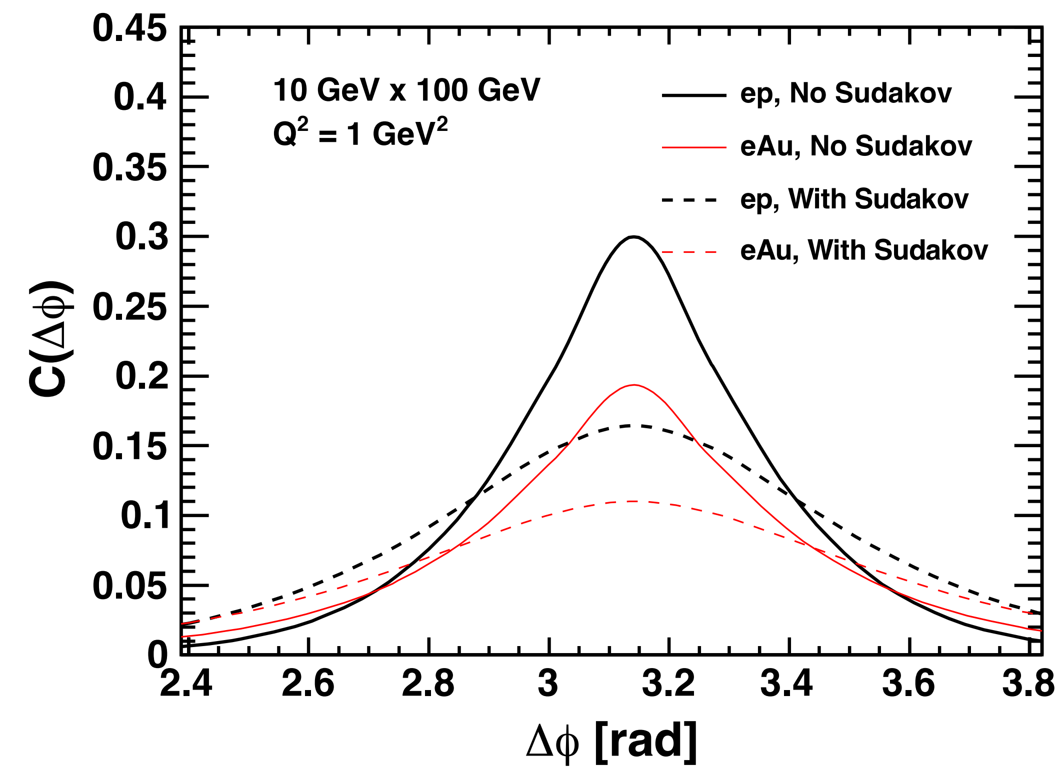




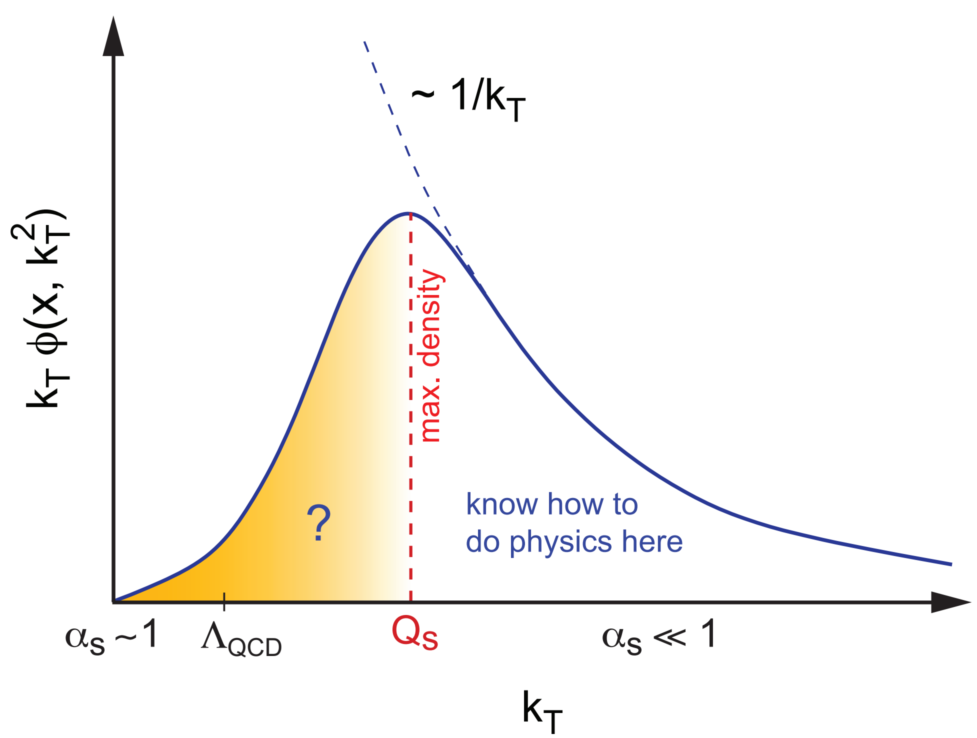
e.g. di-hadron correlations

w.r.t. their azimuthal angle

[F. Dominguez, C. Marquet, B. Xiao and F. Yuan, Phys. Rev. D 83 , 105005 (2011).

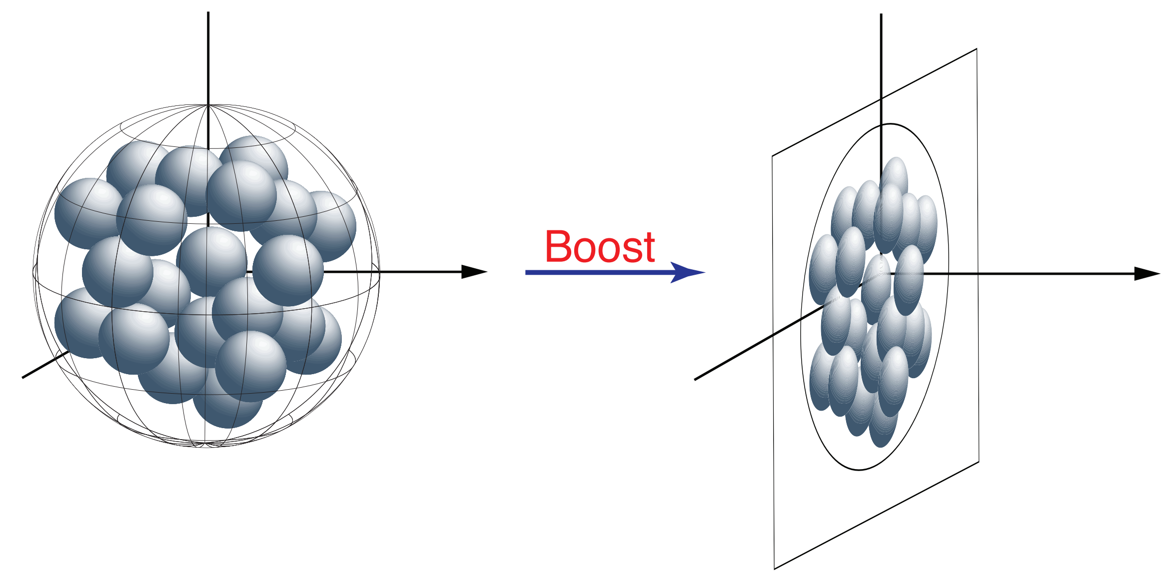


[Zheng,Aschenauer, Lee, Xiao, PRD89 (2014)7, 074037]



Azimuthal Di-hadron correlation:
Only on free parameter to explore 4-point correlator $\Gamma(x_1,x_2, x_3,x_4)$

Next step: 3 hadron/jet correlation = 2 angles
+ will see: ~ (4-point correlator)^2
(di-hadron: linear only)



<u>formal aspects:</u>

- use factorization in the high energy limit $x=Q^2/s \rightarrow 0$ (= the limit where gluon production is perturbatively enhanced!)

- use background field formalism

$$A^\mu \rightarrow A^\mu + \delta A^\mu$$

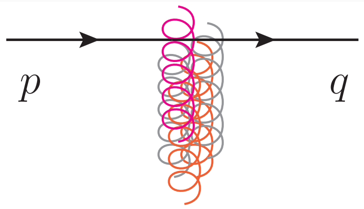
split gluon field into

strong background field $A_\mu \sim n^-_\mu \, \delta(x^-)$
(classical field with high occupation #)

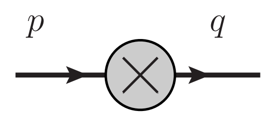
and (perturbative) quantum correction δA^μ

Theory: Propagators in background field

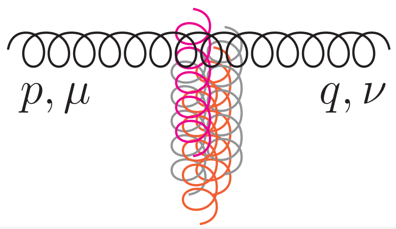
use light-cone gauge, with $k^- = n^- \cdot k$, $(n^-)^2 = 0$, $n^- \sim$ target momentum



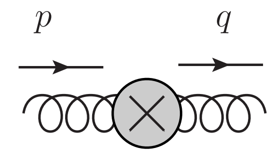
$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$



$$\tilde{S}_F^{(0)}(q)$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



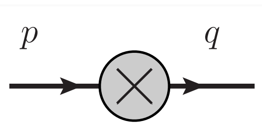
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n^-_\mu p_\nu + p_\mu n^-_\nu}{n^- \cdot p}$$

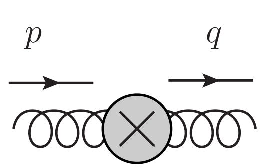
[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:



$$= 2\pi\delta(p^- - q^-)\not{n}^- \int d^{d-2}\boldsymbol{z} e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left\{\theta(p^-)[V(\boldsymbol{z}) - 1] - \theta(-p^-)[V^\dagger(\boldsymbol{z}) - 1]\right\}$$



$$= -2\pi\delta(p^- - q^-)2p^- \int d^{d-2}\boldsymbol{z} e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

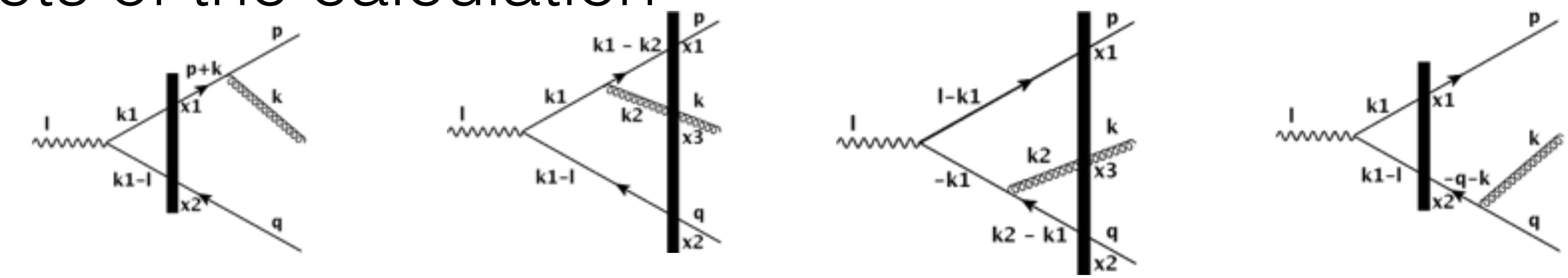
$$\cdot \left\{\theta(p^-)[U(\boldsymbol{z}) - 1] - \theta(-p^-)[U^\dagger(\boldsymbol{z}) - 1]\right\}$$

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig\int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig\int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) T^c$$

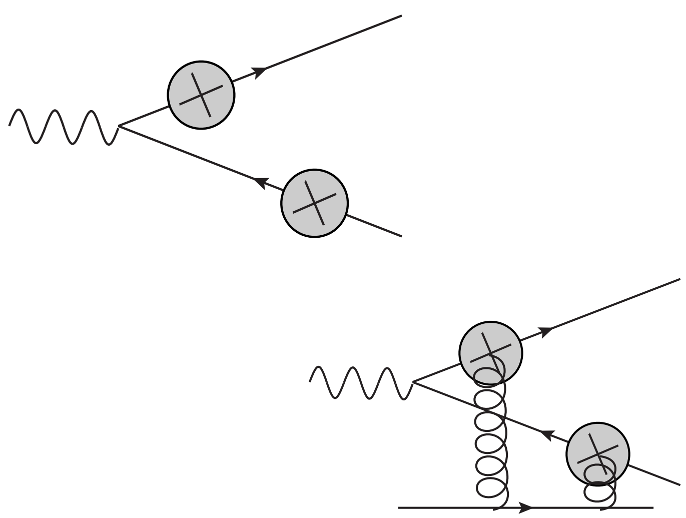
strong background field resummed into path ordered exponentials (Wilson lines)

aspects of the calculation



space time structure of background field **can reduce # of diagrams 16 →4**

loop integrals also for tree-level process



intuitive picture:

background field = t-channel gluons interacting with the target
→ naturally provide a loop which is factorized & (partially) absorbed into the projectile in the high energy limit

direct evaluation of **Dirac-Traces** using FORM, FeynCalC, FormLink possible, but gives lengthy result (100kB) far more economic: spinor helicity formalism

a popular method in modern high-energy calculation!

basic idea:express numerator in terms of spinors of mass-less momenta with definite helicity

$$|k_i^{\pm}\rangle \equiv \frac{1\pm\gamma^5}{2}u_{\pm}(k_i) \qquad \not{p} = |p^+\rangle\langle p^+| + |p^-\rangle\langle p^-| \qquad \epsilon_\mu^{\pm}(k,n) = \pm\frac{\langle n^{\mp}|\gamma^\mu|k^{\mp}\rangle}{\sqrt{2}\langle n^{\mp}|k^{\pm}\rangle}$$

important simplifications through

$$\langle i^{\pm}|j^{\pm}\rangle = 0 \qquad \langle i^{\pm}|i^{\mp}\rangle = 0.$$

allows to express full result in terms of a few helicity coefficients *e.g.*

$$\psi^L_{j,hg} = \sqrt{2}QK_0\left(QX_j\right)\cdot a^{(L)}_{j,hg}, \qquad \psi^T_{j,hg} = \frac{K_1\left(QX_j\right)}{-i|\boldsymbol{x}_{12}|e^{\mp i\phi_{\boldsymbol{x}_{12}}}}\cdot a^{\pm}_{j,hg} \qquad j=1,2$$

$$\psi^L_{3,hg} = 4\pi iQ\sqrt{2z_1z_2}K_0\left(QX_3\right)\left(a^{(L)}_{3,hg}+a^{(L)}_{4,hg}\right), \qquad \psi^T_{3,hg} = 4\pi Q\sqrt{z_1z_2}\frac{K_1\left(QX_3\right)}{X_3}\left(a^{\pm}_{3,hg}+a^{\pm}_{4,hg}\right),$$

$$a^{(L)}_{1,++} = -\frac{\left(z_1z_2\right)^{3/2}\left(z_1+z_3\right)}{z_3e^{-i\theta_p}|\boldsymbol{p}| - z_1e^{-i\theta_k}|\boldsymbol{k}|}, \qquad a^{(L)}_{1,-+} = -\frac{\sqrt{z_1}z_2^{3/2}\left(z_1+z_3\right)^2}{z_3e^{-i\theta_p}|\boldsymbol{p}| - z_1e^{-i\theta_k}|\boldsymbol{k}|}, \qquad a^{(L)}_{3,++} = \frac{z_1z_2}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}};$$

etc., see **arXiv:1604.08526** for complete expresssions

$$\frac{d\sigma^{T,L}}{d^2\boldsymbol{p}\, d^2\boldsymbol{k}\, d^2\boldsymbol{q}\, dz_1 dz_2} = \frac{\alpha_s \alpha_{em} e_f^2 N_c^2}{z_1 z_2 z_3 (2\pi)^2} \prod_{i=1}^{3} \prod_{j=1}^{3} \int \frac{d^2\boldsymbol{x}_i}{(2\pi)^2} \int \frac{d^2\boldsymbol{x}'_j}{(2\pi)^2} e^{i\boldsymbol{p}(\boldsymbol{x}_1-\boldsymbol{x}'_1)+i\boldsymbol{q}(\boldsymbol{x}_2-\boldsymbol{x}'_2)+i\boldsymbol{k}(\boldsymbol{x}_3-\boldsymbol{x}'_3)}$$

$$\Big\langle (2\pi)^4 \Big[\Big(\delta^{(2)}(\boldsymbol{x}_{13}) \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_2)$$

$$+ \Big(\delta^{(2)}(\boldsymbol{x}_{23}) \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{2;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Big) N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}'_1 | \boldsymbol{x}'_2, \boldsymbol{x}_2) \Big]$$

$$+ (2\pi)^2 \Big[\delta^{(2)}(\boldsymbol{x}_{13}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(24)}(\boldsymbol{x}_{3'}, \boldsymbol{x}_{1'} | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_1, \boldsymbol{x}_{3'}) + \{1\} \leftrightarrow \{2\}$$

$$+ \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_3 | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_{1'}) + \{1'\} \leftrightarrow \{2'\} \Big]$$

$$+ \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{3'}, \boldsymbol{x}_3 | \boldsymbol{x}_3, \boldsymbol{x}_{3'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) \Big\rangle_{A^+},$$

full (large N_c) result in terms of these wave functions + target correlators

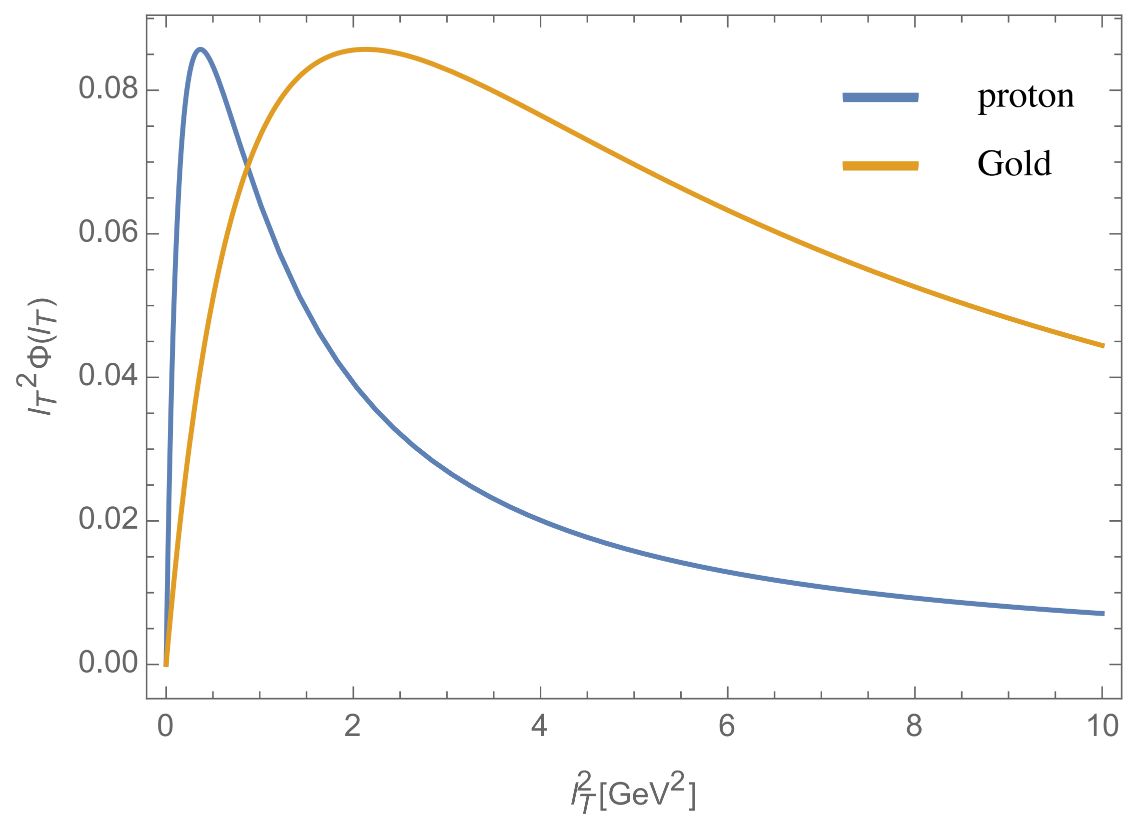
$$N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) \equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)},$$
$$N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4) \equiv$$
$$\equiv \Big[S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - 1 \Big] \Big[S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)} - 1 \Big]$$
$$N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_2 | \boldsymbol{x}_3, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6) \equiv$$
$$1 + S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} S^{(4)}_{(\boldsymbol{x}_3\boldsymbol{x}_4\boldsymbol{x}_5\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_4\boldsymbol{x}_5)},$$
$$N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4 | \boldsymbol{x}_5, \boldsymbol{x}_6, \boldsymbol{x}_7, \boldsymbol{x}_8) \equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} S^{(4)}_{(\boldsymbol{x}_5\boldsymbol{x}_6\boldsymbol{x}_7\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_4)} S^{(2)}_{(\boldsymbol{x}_5\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_2\boldsymbol{x}_3)} S^{(2)}_{(\boldsymbol{x}_6\boldsymbol{x}_7)}$$

For a first numerical study:

- Quadrupole $S^{(4)}$ expressed in terms of dipoles $S^{(2)}$ (Gaussian/dilute approximation)
- Expand result in $N^{(2)}=1-S^{(2)}$ up linear and quadratic order

$$S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} = \int d^2\boldsymbol{l}\, e^{-i\boldsymbol{l}\cdot\boldsymbol{x}_{12}}\ \Phi(\boldsymbol{l}^2)$$
$$= 2\left(\frac{Q_0|\boldsymbol{x}_{12}|}{2}\right)^{\rho-1} \frac{K_{\rho-1}(Q_0|\boldsymbol{x}_{12}|)}{\Gamma(\rho-1)},$$

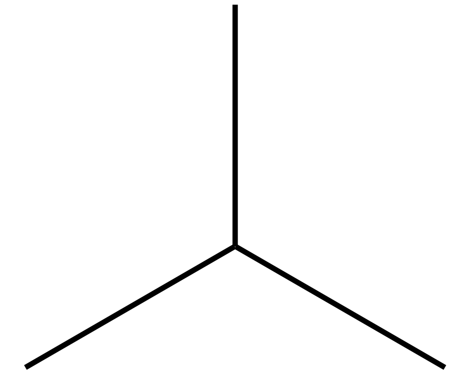
where $\Phi(\boldsymbol{l}^2) = \frac{\rho-1}{Q_0^2\pi}\left(\frac{Q_0^2}{Q_0^2+\boldsymbol{l}^2}\right)^{\rho},$



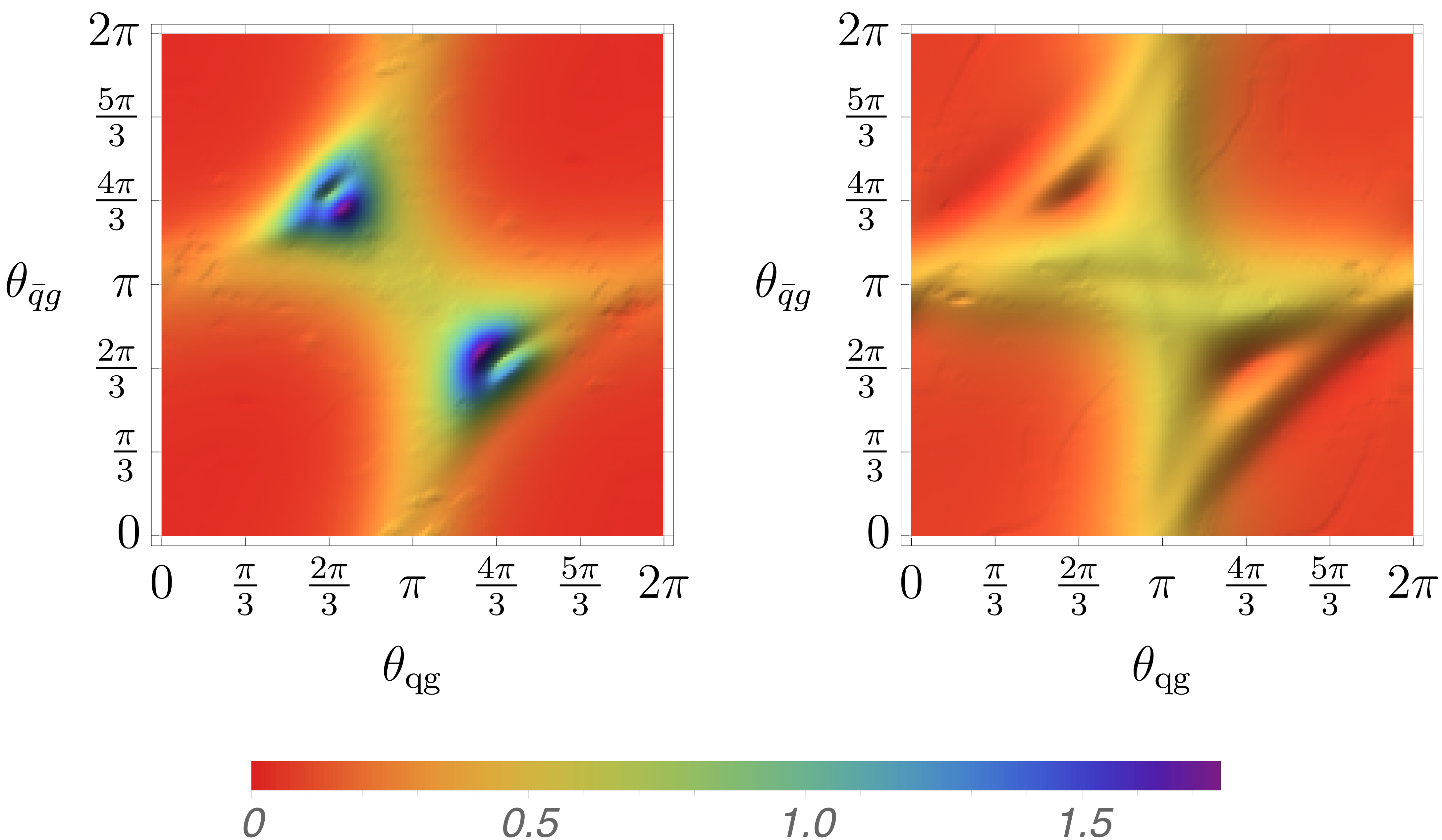
$\rho=2.3$ and $(Q_0^{proton})^2=0.48$ GeV^2
(motivated by inclusive DIS fits at x = 0.2 10^{-3})

$(Q_0^{gold})^2 = A^{1/3}\,(Q_0^{proton})^2=2.77$GeV^2

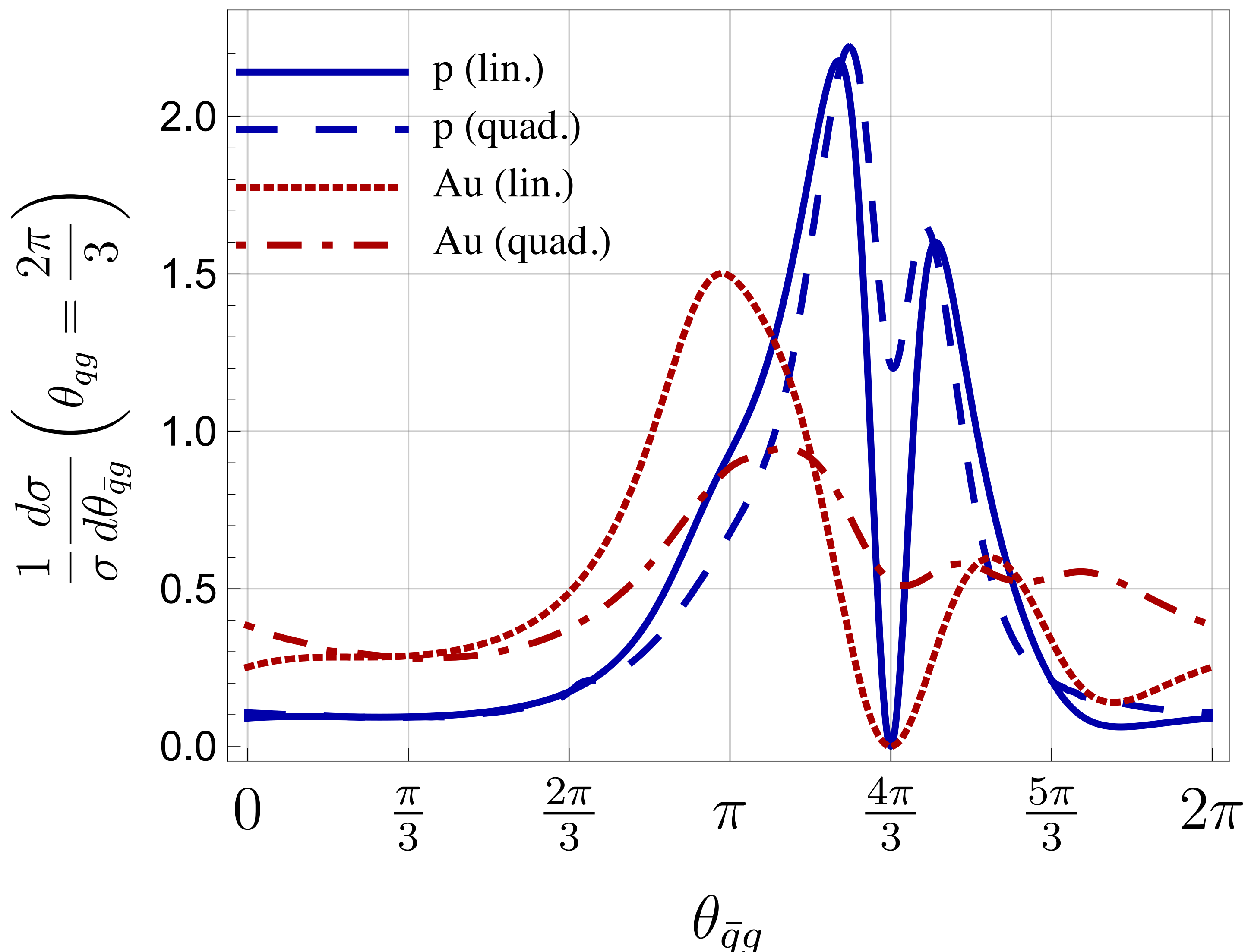
consider deviations from “Mercedes-Benz star configuration (all p_T =2 GeV)
= generalization of back-to-back configuration for 2 partons



(photon momentum fraction z_q=z_{qbar}= 0.2, Q=3 GeV)



keep 1 angle fixed at 2 π/3 p_T^2=4 GeV^2, Q^2=9 GeV^2



proton: very good convergence of expansion in $N^{(2)}=1-S^{(2)}$
(~ saturation effects weak, small saturation scale)

gold:

- linear order: significant broadening of peak at 4π/3 → presence of strong & coherent gluon field
- quadratic order: substantial correction →sensitivity to *e.g.* 4-point correlator