(SDE-BSE perspective on) Nonperturbative QCD and Hadron Physics

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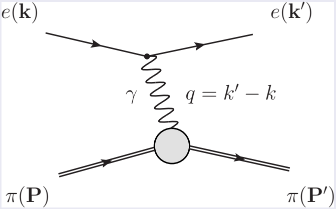
Electromagnetic Structure of Hadrons

Pion-photon vertex $\langle \pi(P')|J_\mu|\pi(P)\rangle$; EM current conservation

- $F_\pi(Q^2)$ parametrises the distribution of charge inside the hadron.

$$\mathcal{M} = (-ie)\overline{u}(k')\gamma_\mu u(k)\frac{-i}{q^2}\langle \pi^+(P')|J_\mu|\pi^+(P)\rangle$$



$$\langle \pi^+(P')|J_\mu|\pi^+(P)\rangle = (P' + P)_\mu F_\pi(Q^2)$$

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2; \quad \frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left|F_\pi(Q^2)\right|^2$$

pQCD analysis of form factors

Brodsky-Lepage, 1979; Radyushkin, 1977

- pQCD yields predictions for pion elastic and transition form factors at asymptotically high energies. But how high?
- QCD factorisation property:

$$F_\pi(Q^2) = \int\int dx\, dy\, \phi_\pi(x,Q)\, T_B(x,y,Q)\phi_\pi(y,Q)$$

- The quark-gluon subprocesses are encoded in the hard-scattering amplitude $T_B(x,y,Q)$ and can be computed order-by-order in pQCD.
- The nonperturbative effects are absorbed into a universal pion distribution amplitude (PDA) $\phi_\pi(x,Q)$.

pQCD analysis of form factors

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- pQCD yields predictions for pion elastic and transition form factors at asymptotically high energies. But how high?
- pQCD: Large Q^2. But how large?

$$T_B(x,y,Q) = 16\pi C_F \frac{\alpha_s(Q^2)}{Q^2}\frac{1}{xy}$$
$$\phi_\pi(x) = \phi_\pi^{asy}(x) = 6x(1-x)$$

- pQCD prediction for $F_\pi(Q^2)$

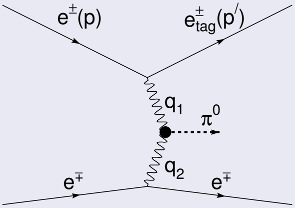
$$Q^2 F_\pi(Q^2) = 16\pi\alpha_s(Q^2) f_\pi^2 \left|\frac{1}{3}\int_0^1 \mathrm{d}x \frac{1}{x}\phi_\pi(x)\right|^2$$
$$= 16\pi\alpha_s(Q^2) f_\pi^2 .$$

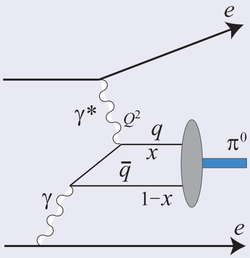
- The normalization of $F_\pi(Q^2)$ is controlled by $\left|\frac{1}{3}\int_0^1 \mathrm{d}x \frac{1}{x}\phi_\pi(x)\right|^2$.
- More about $\phi_\pi(x)$ later.

Electromagnetic structure of hadrons

$\gamma\gamma^* \to \pi^0$ transition form factor

- The measured $\gamma\gamma^* \to \pi^0$ TFF $G_{\pi\gamma\gamma^*}$ parametrizes the inner part of the diagram: $\mathcal{M} \propto e^2 \epsilon_{\mu\nu\alpha\beta}\epsilon_1^\alpha \epsilon_2^\beta Q_1^\alpha Q_2^\beta \, G_{\pi\gamma\gamma^*}(Q_1^2, Q_2^2)$.





- $G_{\pi\gamma\gamma^*}$ can be computed in various approaches—See Khepani's talk for a nonperturbative computation.
- $\gamma\gamma^* \to \pi^0$ has been measured at at BABAR [PRD 80, 052002 (2009)] and BELLE [PRD 86, 092007 (2012)].

pQCD analysis of form factors; Brodsky-Lepage, 1979

$\gamma\gamma^* \to \pi^0$ Transition Form Factor

- QCD factorisation:

$$G_{\pi\gamma\gamma^*}(Q^2) = 4\pi^2 f_\pi \int_0^1 \mathrm{d}x\, T_H(x, Q^2, \alpha_s)\phi_\pi(x, Q)$$

- The quark-gluon subprocesses are encoded in the hard-scattering amplitude $T_H(x, y, Q)$ and can be computed order-by-order in pQCD.
- The nonperturbative effects are absorbed into the universal PDA $\phi_\pi(x, Q)$.
- pQCD:

$$T_H = \frac{e_u^2 - e_d^2}{xQ^2}.$$

$$\phi_\pi(x) = \phi_\pi^{asy}(x) = 6x(1-x).$$

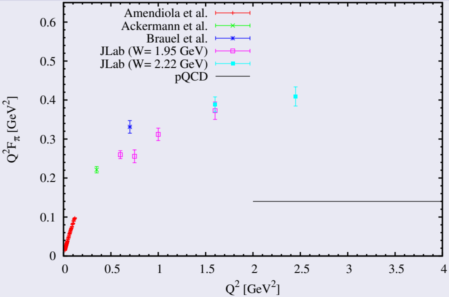
- pQCD prediction for $G_{\pi\gamma\gamma^*}(Q^2)$:

$$Q^2 G_{\pi\gamma\gamma^*}(Q^2) \to 4\pi^2 f_\pi.$$

- More about $\phi_\pi(x)$ later.

Comparision to experimental data

Brodsky-Lepage, 1979; Radyushkin, 1977



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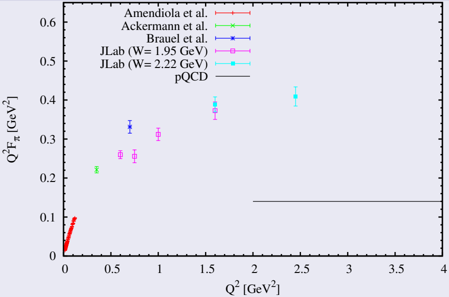

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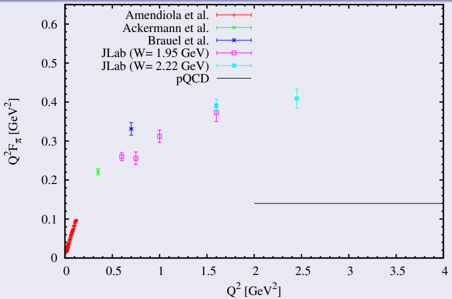

- There is a weak suggestion that $Q^2 F_\pi(Q^2) = $ constant.
- Nonperturbative effects dominate; e.g. for $Q^2 = 2.45\text{GeV}^2$ $Q^2 F_\pi(Q^2) = 0.41\,\text{GeV}^2$; compare to $Q^2 F_\pi(Q^2) = 0.15$ using pQCD for $Q^2 = 4\text{GeV}^2$.

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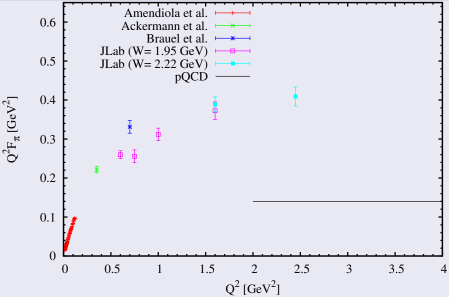

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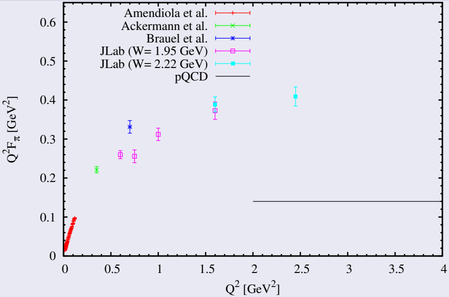

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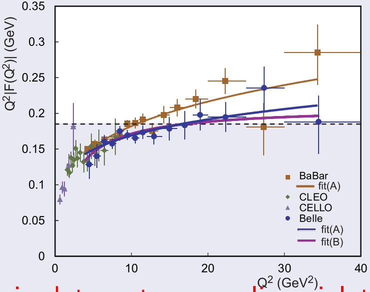
- Has $\phi_\pi(x)$ reached its asymptotic value ϕ_π^{asy} at present energies?
- We are awaiting for JLab 12 GeV upgrade experimental data.

Comparision to experimental data

$\gamma\gamma^* \to \pi^0$ transition form factor

- The $\gamma\gamma^* \to \pi^0$ TFF has attracted a lot of attention since the publication of the BABAR and BELLE data



- There is controversy in data—strong scaling violation from BABAR.
- BELLE II (2018) may help to resolve these controversies.
- Computation of this quantity is of course very important—See Khepani's talk

Pion Distribution Amplitude

$\pi \to q\bar{q}$ transition $\phi_\pi(x)$

- The PDA $\phi_\pi(x, Q^2)$ plays an important role in the theoretical description of many QCD processes (e.g. $\gamma^*\pi^+ \to \pi^+$, $\gamma\gamma^* \to \pi^0$)

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- Its x-dependence cannot be computed from pQCD but its evolution with Q^2 can (ERBL evolution eqn, similar to DGLAP eqn for PDFs)
- Later we will see its formal definition and computation in the SDE-BSE approach.

Quantum Chromodynamics—a reminder

- QCD is the fundamental theory of quarks (spin 1/2 fermions), gluons (spin 1 gauge bosons), and their interactions
- It is a consistent QFT with a simple and elegant Lagrangian, based entirely on the invariance under the local non-Abelian $SU(3)$ colour gauge group and renormalisability
- QCD is a powerful tool in the description of large momentum transfer experiments due to its property of asymptotic freedom eg QCD backgrounds at the LHC
- Over the years QCD has become the accepted theory of the strong interactions at the fundamental level.
- What about exclusive reactions?
- What about nonperturbative objects like the PDA?

Strong QCD

- The Lagrangian of QCD written on the blackboard does not by itself explain the data of strong interacting matter. Furthermore, it is not clear how the plethora of the observed bound state objects, the hadrons, and their properties arise from the fundamental quarks and gluons of QCD.
- Emergent Phenomena
  - Confinement (means that) quarks and gluons cannot be removed from hadrons and studied in isolation.
  - Dynamical Chiral Symmetry Breaking (is) responsible for the existence of light pions and the generation of quark masses via interactions.
- Neither DCBS nor Confinement can be accounted for in pQCD, and are therefore genuine effects of strong QCD .
- Both can be studied in the functional approach to hadron physics, ie SDE-BSE, in particular the SDE for the quark propagator.

QCD generating functional

quarks, gluons, ghosts and all that

$$\mathcal{Z}[J,\xi,\xi^*,\overline{\eta},\eta] = \int \mathcal{D}A\mathcal{D}\overline{\eta}\mathcal{D}\eta\mathcal{D}\xi\mathcal{D}\xi^* \exp\left\{\mathrm{i}\int \mathrm{d}^4x \left(\mathcal{L}_{\text{eff}} + \text{Sources}\right)\right\},$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{G}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{F}},$$

$$\text{Sources} = A^a_\mu J^{a\mu} + \chi^{a*}\xi^a + \xi^{a*}\chi^a + \overline{\psi}\eta + \overline{\eta}\psi,$$

where

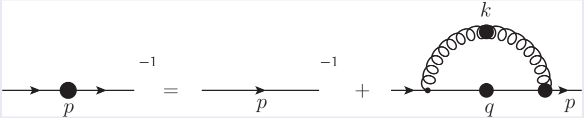
$$\mathcal{L}_{\text{G}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a, \qquad \mathcal{L}_{\text{GF}} = -\frac{1}{\xi}\left(\partial^\mu A^a_\mu\right)^2,$$

$$\mathcal{L}_{\text{FP}} = \left(\partial^\mu\chi^a\right)^* D^{ab}_\mu\chi^b, \qquad \mathcal{L}_{\text{F}} = \sum_{k=1}^{N_f}\overline{\psi}_k\left(\mathrm{i}\gamma_\mu D^\mu - m_k\right)\psi_k$$

Schwinger-Dyson Equations—npQCD formulation

$\int \mathcal{D}\phi \frac{\delta}{\delta\phi} = 0$

- The SDEs are the eqns for the Green functions of the theory.
- The SDEs provide a nonperturbative formulation of QCD in the continuum.
- The QCD quark propagator (a 2-point function) SDE eqn (valid for any value of the coupling α_S).

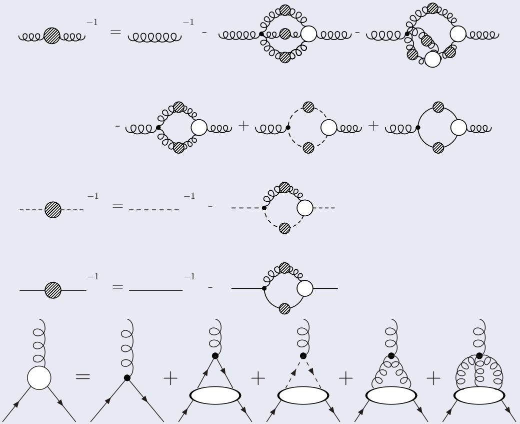


$$S_F{}^{-1}(p) = Z_2 \left[S_F^{\text{bare}}(p) \right]^{-1} - \Sigma(p)$$

$$\Sigma(p) = Z_{1F} \mathrm{i} g^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} D^{\mu\nu}(k) t^a \gamma_\mu S_F(q) \Gamma_\nu^a(p,q;k)$$

SDEs form an infinite set of coupled equations

- The SDEs form an infinite tower of coupled nonlinear integral eqns.

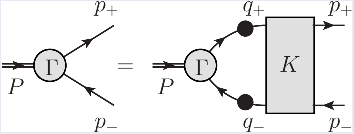


- There is a need to introduce a truncation scheme.

Hadron physics—real-world QCD

Meson Bound States: The Bethe-Salpeter Equation

- Quarks and gluons are not free so we must study bound state's properties to test our understanding of npQCD—and explain/understand hadron physics.
- Mesons are bound states of a nonperturbative quark-antiquark pair.



$$[\Gamma^H(p;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4}[K(p,q;P)]_{tu;rs}[S^a(q_+)\Gamma^H(q;P)S^b(q_-)]_{sr}$$

- The quark propagators are solns to the quark SDE.
- How do we determine $K(p,q;P)$?
- Need to determine the quark SDE kernel that appears in Σ first.

Hadron physics—Axial-Vector Ward-Takahashi identity

- How do we determine K? Look at the chiral symmetry breaking ppties of QCD.
- The AxWTI relates $K(p, k; P)$ in the BSE to $\Sigma(p)$ in the quark SDE:

$$[\Sigma(p_+)\gamma_5 + \gamma_5\Sigma(p_-)]_{tu} = \int \frac{d^4q}{(2\pi)^4} K^{rs}_{tu}(p, q; P)[\gamma_5 S(q_-) + S(q_+)\gamma_5]_{sr}$$

- A truncation in Σ implies a truncation in K.
- The chiral symmetry breaking pattern of QCD guarantees a massless pion in the chiral limit when DCSB occurs (Goldstone theorem!), among other relations (GMOR).
- When chiral symmetry is explicitly broken, it ensures a light pion.

The Quark propagator—Perturbative solution

- General form of the quark propagator:

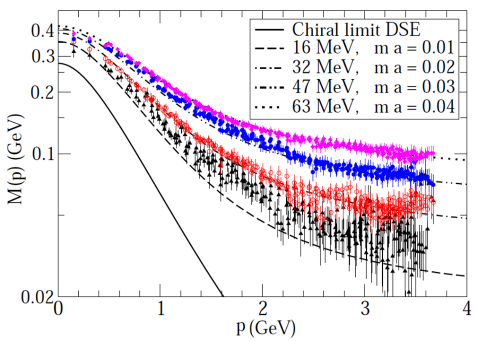
$$S^{-1}(p) = \mathrm{i}\not{p}A(p^2,\mu^2) + B(p^2,\mu^2) = Z^{-1}(p^2,\mu^2)\left[\mathrm{i}\not{p} + M(p^2)\right],$$
$$M(p^2) = \frac{B(p^2)}{A(p^2)};\; Z(p^2) = A^{-1}(p^2)$$

- pQCD prediction (i.e. perturbative truncation of the quark SDE)

$$M(p^2) = m\left[1 - \frac{\alpha}{\pi}\ln\left(\frac{p^2}{m^2}\right) + \cdots\right];\; \lim_{m\to 0} M(p^2) = 0$$

- It is always true that at any order in perturbation theory there is no dynamical mass generation, i.e. $M(p^2) = 0$.

The Quark propagator—Nonperturbative solutions



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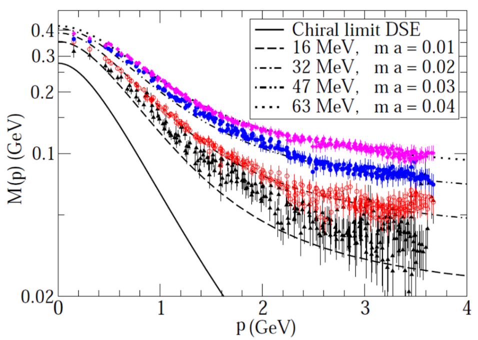

- DCSB IS realized in the nonperturbative solutions.

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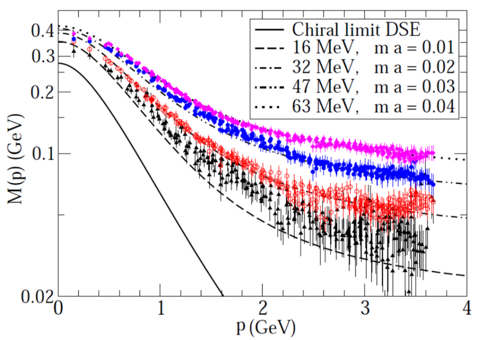

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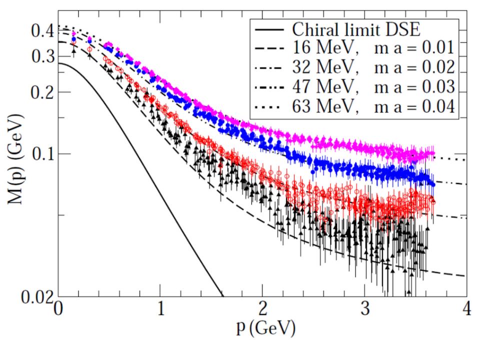

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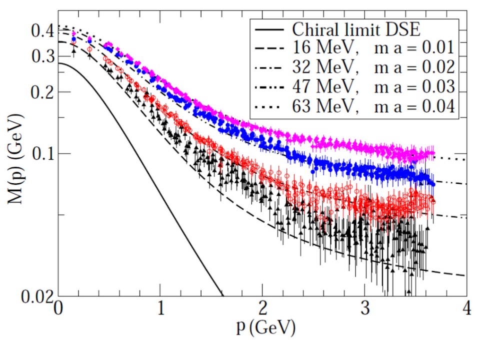

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- The $M(p^2)$ connects the current-quark mass (UV) to a constituent-like quark mass (IR)—in agreement with pQCD
- DCSB is the most important mass generating mechanism for visible matter in the Universe (98% of the proton's mass).

Hadron physics

Static properties: mass spectrum, decay constants, etc

- A meson is characterized by its Dirac structure.
- In the pseudoscalar chanel, $J^P = 0^-$, the lowest mass solution is the π (excited states can also be studied!)

$$\Gamma_\pi(k;P) = \gamma_5[\mathrm{i}E_\pi(k;P) + \not{P}F_\pi(k;P) \\ \not{k}(k\cdot P)G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)$$

- By solving the BSE we obtain the meson mass m_π and its BSA $\Gamma_\pi(k;P)$
- The pseudoscalar leptonic decay constant $(\pi^+ \to \mu^+ + \nu_\mu)$

$$f_\pi P_\mu = Z_2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{Tr}\left[\gamma_5\gamma_\mu S^a(q_+)\Gamma_\pi(q;P)S^b(q_-)\right],$$

Hadron physics

The Pion Distribution Amplitude $\phi_\pi(x)$ at the fixed scale of 2 GeV

- Recall $\phi_\pi(x)$ that appears in $F_\pi(Q^2)$?

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- The leading twist PDA:

$$f_\pi \phi_\pi(x) = Z_2 N_c \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - xn\cdot P) \mathrm{Tr}\left[\gamma_5 \not{n} S(k) \Gamma_\pi(k;P) S(k-P)\right]$$

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- Calculate its moments $\langle x^m \rangle = \int_0^1 \mathrm{d}x x^m \phi_\pi(x)$:

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- We can compute arbitrarily many of them [Lei Chang et al PRL 110, 132001 (2013)].

$\phi_\pi(x)$ reconstruction at 2 GeV

- $\phi_\pi(x)$ can be parametrised as

$$\phi_\pi(x,\mu) = 6x(1-x)[1 + \sum_{n=2,4,\ldots} a_n(\mu) C_n^{3/2}(2x-1)]$$

- It is more efficient to fit the moments with order α Gegenbauer pols

$$\phi_\pi(x,\mu) = N(\alpha)\,[x(1-x)]^{\alpha-1/2}\,[1 + \sum_{n=2,4,\ldots} a_n^\alpha(\mu) C_n^\alpha(2x-1)]$$

- For our (RL) model $\alpha_{\rm RL} = 0.79$, $a_2^{\rm RL} = 0.0029$—This is sufficient to get a very accurate fit.
- We then project onto order $\alpha = 3/2$ Gegenbauers: $a_2^{3/2} = 0.23, \ldots,$ $a_{14}^{3/2} = 0.022$—This underscores the merit of reconstruction via Gegenbauer-α pols (many $a_n^{3/2}$ are needed!).

Comparison with Lattice QCD

The Pion Distribution Amplitude $\phi_\pi(x, 2\text{ GeV})$

- Lattice QCD can calculate only one nontrivial moment:

$$\begin{aligned}\langle(2x-1)^2\rangle &= 0.27 \pm 0.04 \ \text{(Braun et al, 2006)} \\ \langle(2x-1)^2\rangle &= 0.24 \pm 0.01 \ \text{(Braun et al, 2015)}\end{aligned}$$

- Our RL and DB results give

$$\langle(2x-1)^2\rangle^{\text{RL}} = 0.28, \quad \langle(2x-1)^2\rangle^{\text{DB}} = 0.25$$

- While the asymptotic PDA $[6x(1-x)]$ gives

$$\langle(2x-1)^2\rangle^{\text{Asy}} = 0.2$$

Hadron physics

$\phi_\pi(x, Q^2)$ scale evolution

- The Q^2 scale evolution can be computed from pQCD (similar to DGLAP equation for PDFs).
- At leading order, the expansion coefficients evolve according to

$$a_n^{3/2}(\mu) = a_n^{3/2}(\mu_1)\left[\frac{\alpha_s(\mu_1)}{\alpha_s(\mu)}\right]^{\gamma_n^{(0)}/\beta_0}, \quad \alpha_s(Q^2) = (4\pi/\beta_0)\ln^{-1}\left(Q^2/\Lambda_{\rm QCD}^2\right),$$

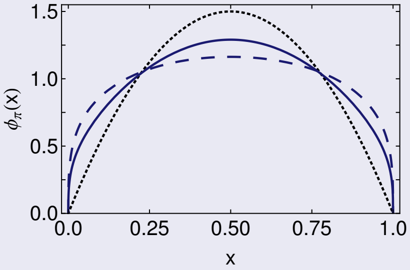
$$\beta_0 = 11 - (2/3)N_f, \quad \gamma_n^{(0)} = \frac{4}{3}\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right]$$

- Recall $a_2^{3/2}(2\,\mathrm{GeV}) = 0.23, \ldots, a_{14}^{3/2}(2\,\mathrm{GeV}) = 0.022$.
- It is necessary to evolve to $\mu = 100\,\mathrm{GeV}$ before $a_2^{3/2}(2\,\mathrm{GeV})$ falls to 50 % of its value!($N_f = 4$, and $\Lambda_{\rm QCD} = 0.234\,\mathrm{GeV}$)
- The asymptotic limit $\phi_\pi^{asy}(x) = 6x(1-x)$ is a poor approximation to the PDA at currently accessible energies.

Hadron physics

The PDA $\phi_\pi(x)$–Lei Chang et al PRL 110, 132001 (2013)

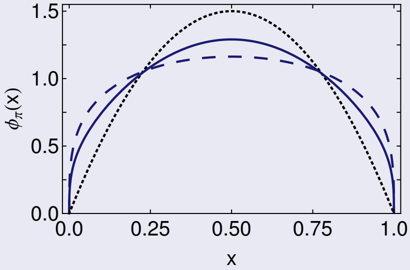
- RL result (dashed curve); DB result (solid curve); Asymptotic PDA (dotted curve)



- Need to evolve to $\mu = 100\,\text{GeV}$ before $a_2^{3/2}(2\,\text{GeV})$ reduces $50\,\%$.
- The asymptotic domain (ϕ^{asy}) lies at very very large momenta.
- The dilation is an expression of dynamical chiral symmetry breaking on the light front.

Back to the perturbative analysis of $F_\pi(Q^2)$

$\phi_\pi(x)$ and the normalization of $F_\pi(Q^2)$



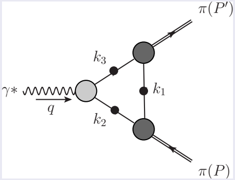
- In evaluating $Q^2 F_\pi$, ϕ_π has not yet reached its asymptotic value:

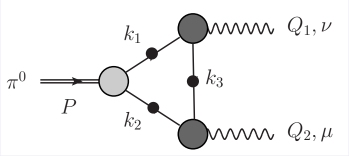
$$\left| \frac{1}{3} \int_0^1 \mathrm{d}x \frac{1}{x} \phi_\pi^{\text{asy}}(x) \right|^2 = 1, \qquad \left| \frac{1}{3} \int_0^1 \mathrm{d}x \frac{1}{x} \phi_\pi(x, 2\,\text{GeV}) \right|^2 = 3.2$$

- The pQCD result has to be multiplied by 3.2 (at the scale of 2 GeV) and the asymptotic analysis of various models have to be compared to this new result—the normalization of F_π runs with Q^2.

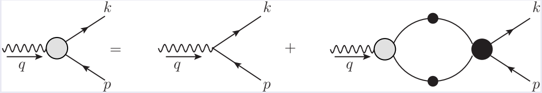
EM interactions of hadrons: the nonperturbative quark-photon vertex

- Once we have $S_{u/d}(k)$, m_π, $\Gamma_\pi(k;P)$ we can calculate anything that involves these objects, e.g. $F_\pi(Q^2)$, $G_{\pi\gamma\gamma^*}(Q^2)$:





- Quark electromagnetic interaction: quark-photon vertex SDE



- The quark-photon vertex $\Gamma_\mu(k,p;Q)$ is constrained by the WTI of QED.

$F_\pi(Q^2)$: RL Nonperturbative computation

Impulse approximation for F_π

- Once we have $S_{u/d}(k)$, m_π, $\Gamma_\pi(k;P)$ we can calculate anything that involves these objects.

$$\Lambda^\pi_\mu(P,P';q) \equiv \langle \pi(P')|J_\mu|\pi(P)\rangle = (P+P')_\mu F_\pi(Q^2),$$

- Flavor decompostion

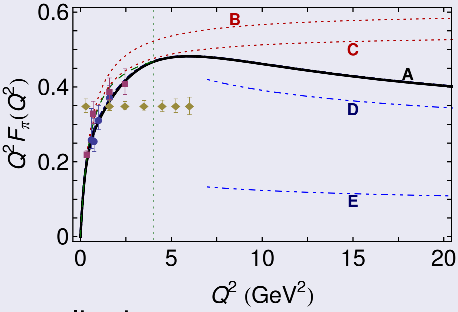
$$\Lambda^\pi_\mu(P,P';q) = \hat{Q}^u \Lambda^{\pi,u}_\mu(P,P';q) + \hat{Q}^{\overline{d}} \Lambda^{\pi,\overline{d}}_\mu(P,P';q)$$

$$\Lambda^{\pi,\overline{d}}_\mu(P,P';q) = N_c \int d\ell \, \mathrm{Tr}\Big[S^u(k_1)\Gamma_\pi(k_1,k_2;P)S^d(k_2) \\ i\Gamma^d_\mu(k_2,k_3;q)S^d(k_3)\overline{\Gamma}_\pi(k_3,k_1;-P')\Big]$$

The pion form factor: RL Nonperturbative computation

Full calculation of F_π [Lei Chang et al PRL 111, 141802 (2013)]

- A: New computation of F_π (any Q^2).
- B: Monopole parametrisation; C: Old NBF computation (up to 4 GeV).



- E: Perturbative normalization.
- D: NonPerturbative normalization.

Summary and outlook

- The SDE-BSEs are well suited to the study of hadrons as composites of dressed quarks and gluons:
  - Static properties (light and heavy mesons): masses, decay constants.
  - Electromagnetic and Transition Form Factors (light and heavy mesons).
  - Parton distribution functions (PDFs).
  - Parton distribution amplitudes (PDAs).
  - Generalized parton distrubution functions (GPDs), etc.
  - QCD at finite temperature and chemical potential (QCD phase diagram).
- Due to their infinite-set nature a truncation must be introduced.
- There is an interconnection between truncation schemes and symmetries.
- Phenomenology and Lattice QCD provide valuable information to design a robust truncation scheme.