The University of Manchester

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Coulomb gluons and colour evolution

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in collaboration with
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JHEP 1512 (2015) 091
& arXiv:1602.00623 (accepted for publication)

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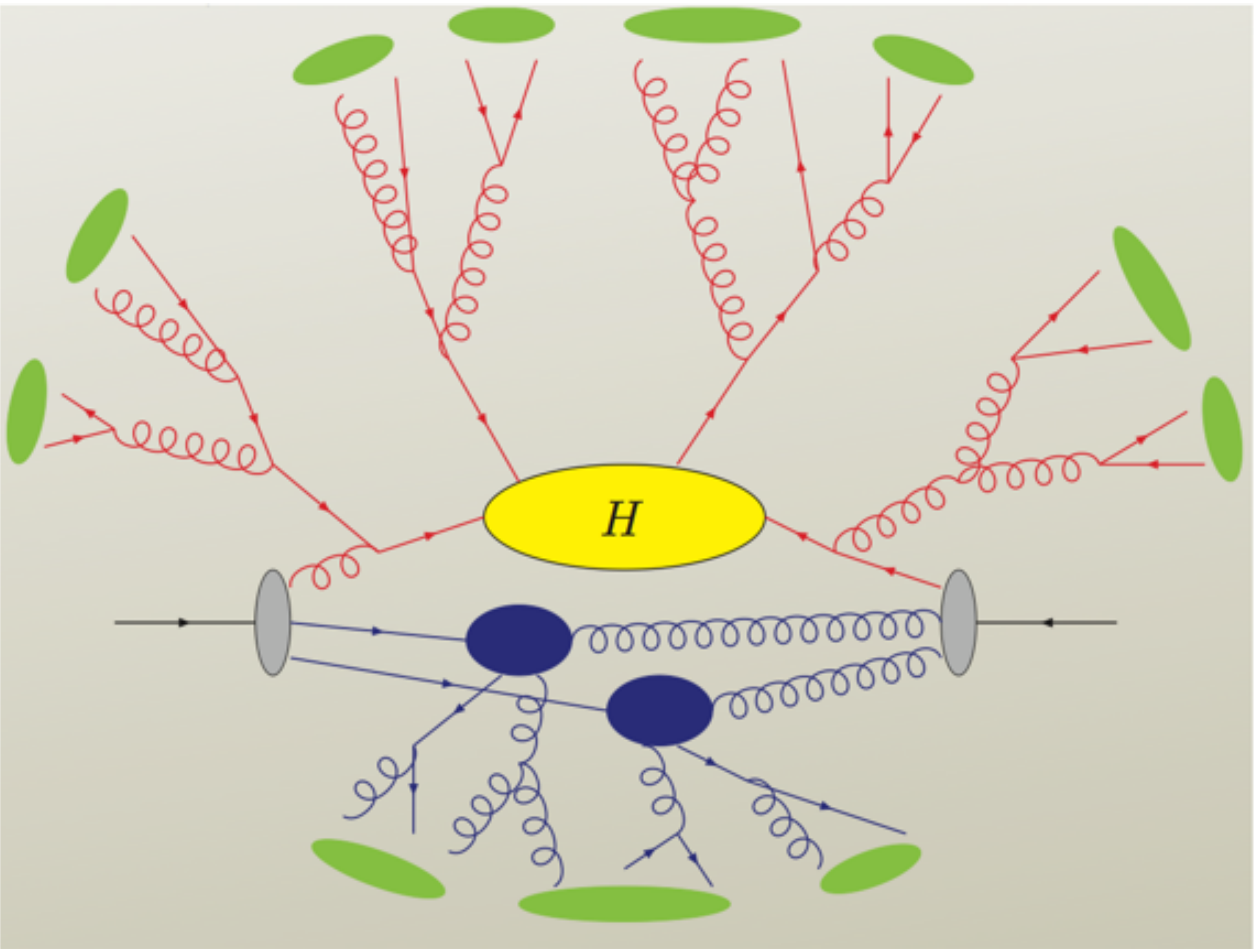
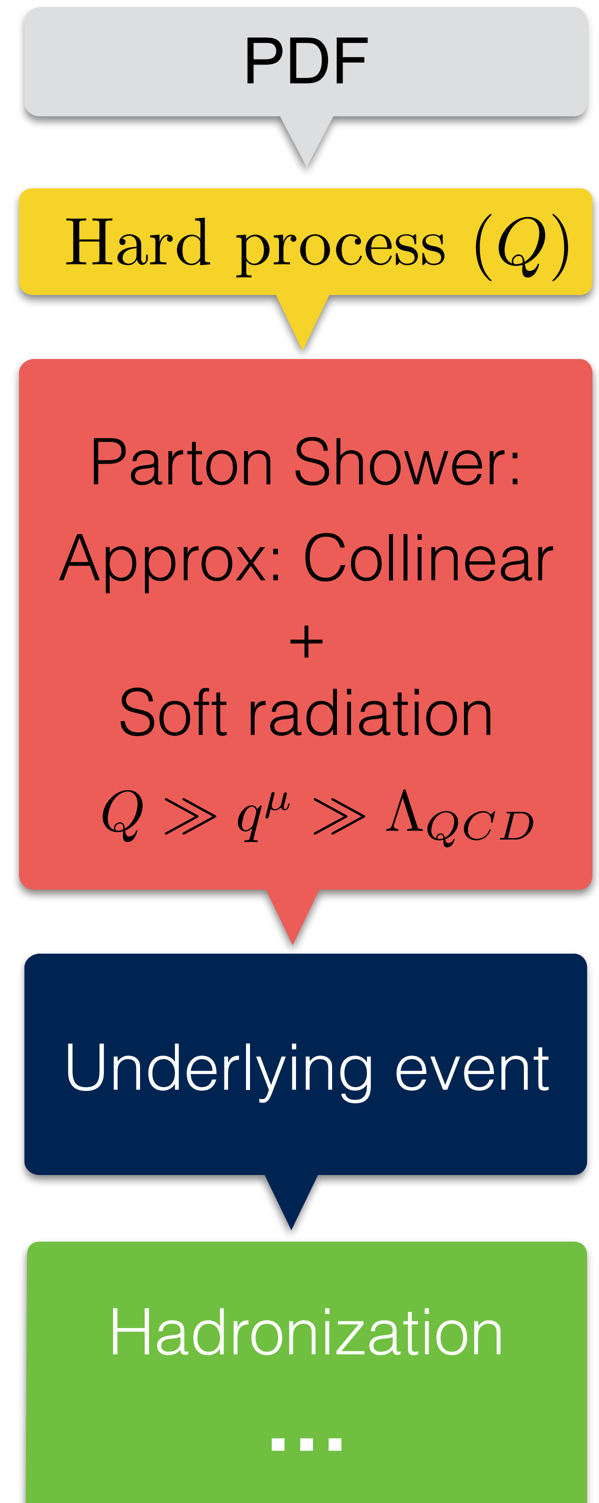
In this talk: Progress towards including the colour interference of soft gluons in partons showers.

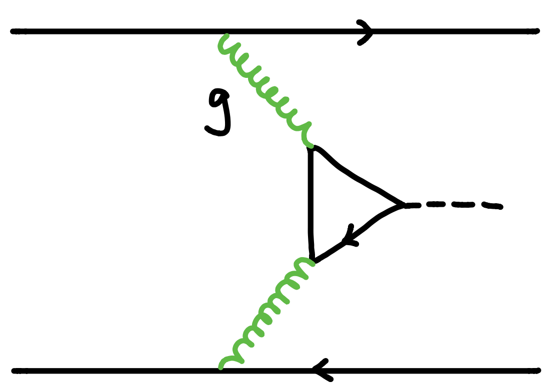


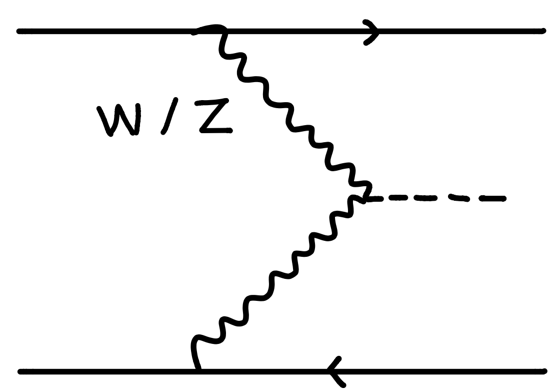
Hadron-Hadron collision, Soper (CTEQ School)



Motivation

- Why? Increase precision of theoretical predictions for the LHC
- Is this necessary? Yes, for particular non-inclusive observables.
- Are those relevant to search for new physics? Yes, these can tell us about the (absence of) colour of the production mechanism (couplings).





Outline

- Coulomb gluons, collinear factorisation & colour interference.

- Concrete effect: super-leading-logs.

- Including colour interference in partons showers. (Also see JHEP 07, 119 (2015), arXiv:1312.2448 & 1412.3967)

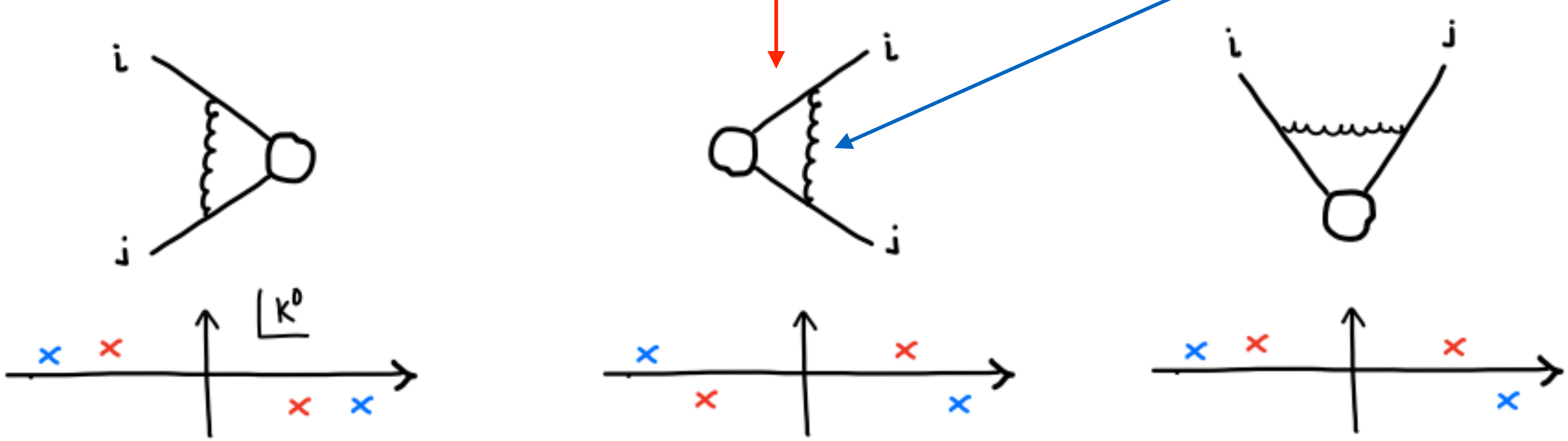
One-loop in the soft approximation

$$k^\mu \ll Q_{ij}$$

Colour matrices acting on $|2\rangle$

Hard subprocess is a vector colour + spin

$$i g_s^2 \mathbf{T}_i \cdot \mathbf{T}_j \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{-p_i \cdot p_j}{[p_j \cdot k \pm i0][-p_i \cdot k \pm i0][k^2 + i0]} |2\rangle$$



$i: \ -i0$

$j: \ -i0$

$i: \ +i0$

$j: \ +i0$

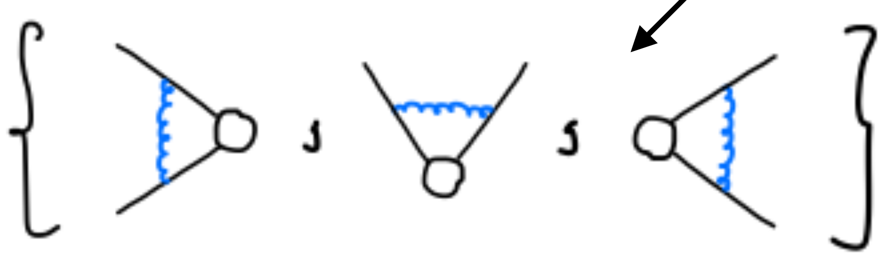
$i: \ -i0$

$j: \ +i0$

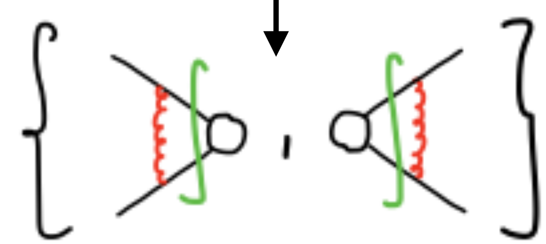
Introduction: one-loop soft gluon correction

After contour integration:

$$g_s^2 \mu^{2\epsilon} \mathbf{T}_i \cdot \mathbf{T}_j p_i \cdot p_j \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[\frac{(2\pi)\delta(k^2)\theta(k^0)}{[p_j \cdot k][p_i \cdot k]} + i\tilde{\delta}_{ij} \frac{(2\pi)^2 \delta(p_i \cdot k)\delta(-p_j \cdot k)}{2[k^2]} \right] |2\rangle$$



(On-shell gluon: Purely real)

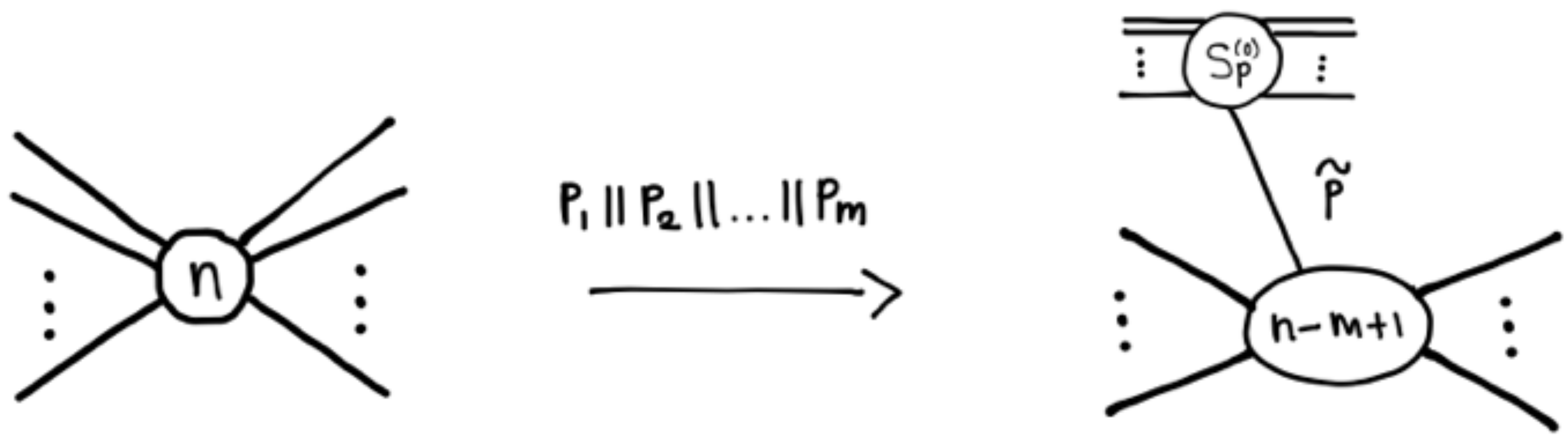


(Coulomb gluon: Purely imaginary)

$$\tilde{\delta}_{ij} = \begin{cases} 1 & \text{if } i, j \text{ in }, \\ 1 & \text{if } i, j \text{ out }, \\ 0 & \text{otherwise.} \end{cases}$$

Tree-level collinear factorisation

For a general on-shell scattering:



$$|n\rangle \simeq \mathrm{Sp}^{(0)}\,|n-m+1\rangle$$

($\tilde{P}$: SINGLE PARTON (1+…+m))

Colour + Spin operator.
Depends only on collinear partons

$$\mathrm{Sp}^{(0)}(\{C\},\tilde{P})$$

Depends only on non-collinear partons

$$|n-m+1(\{NC,\tilde{P}\})\rangle$$

Generalised factorisation beyond tree level

Catani, De Florian & Rodrigo JHEP 1207 (2012) 026

This collinear factorisation generalises to all orders

$$|n\rangle \simeq Sp\,|n-m+1\rangle$$

$$Sp = Sp^{(0)} + Sp^{(1)} + Sp^{(2)} + \ldots$$

but

$$Sp^{(\ell)} = Sp^{(\ell)}\left(\tilde{P}, \{C\}, \{NC\}\right)$$

Violation of strict (process-independent) factorisation!

Generalised factorisation: one loop

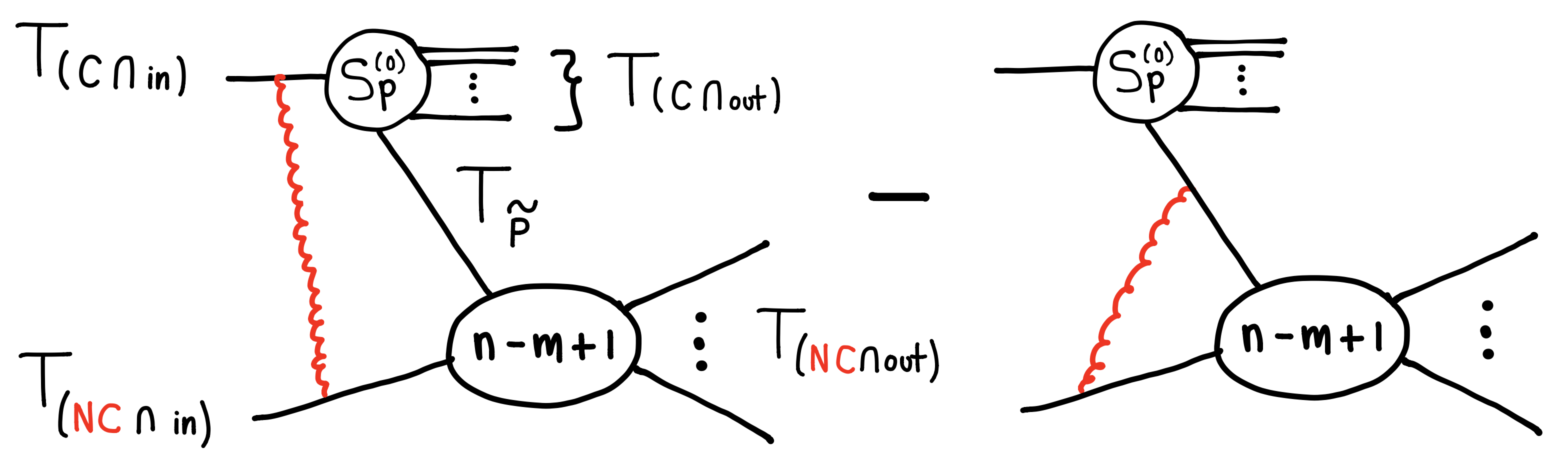
The problem first seed at this order

$$S_P^{(1)} = S_{P_f}^{(1)}(\tilde{P}, \{C\}) + \tilde{\Delta}(\tilde{P}, \{C\}, \{NC\})\, S_P^{(0)}$$

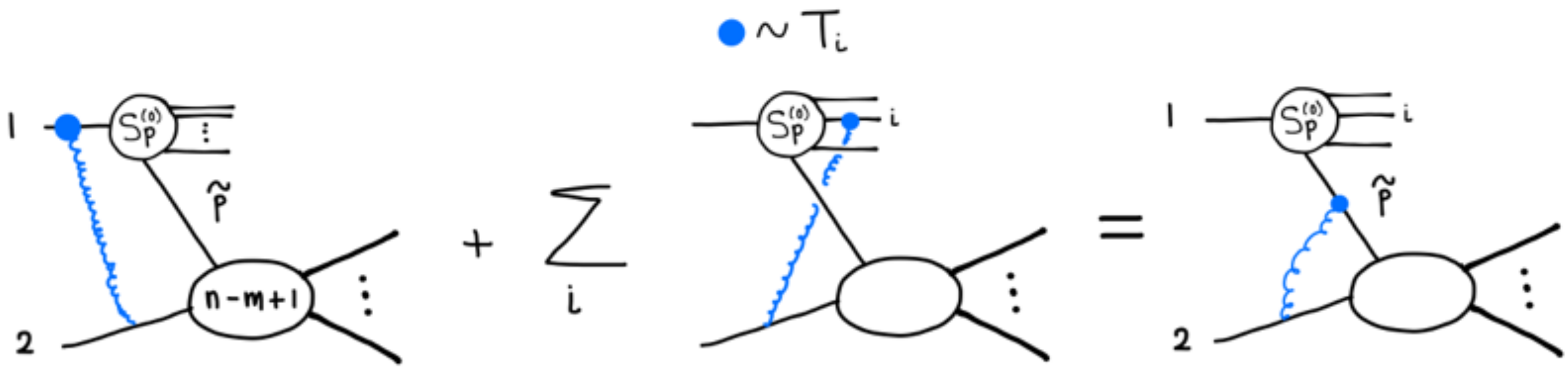
$$\Updownarrow$$

$$T_{(C\cap in)},\ T_{(C\cap out)},\ T_{(NC\cap in)},\ T_{(NC\cap out)} \neq 0$$

$$\tilde{\Delta}\, S_P^{(0)}\, |n-m+1\rangle =$$

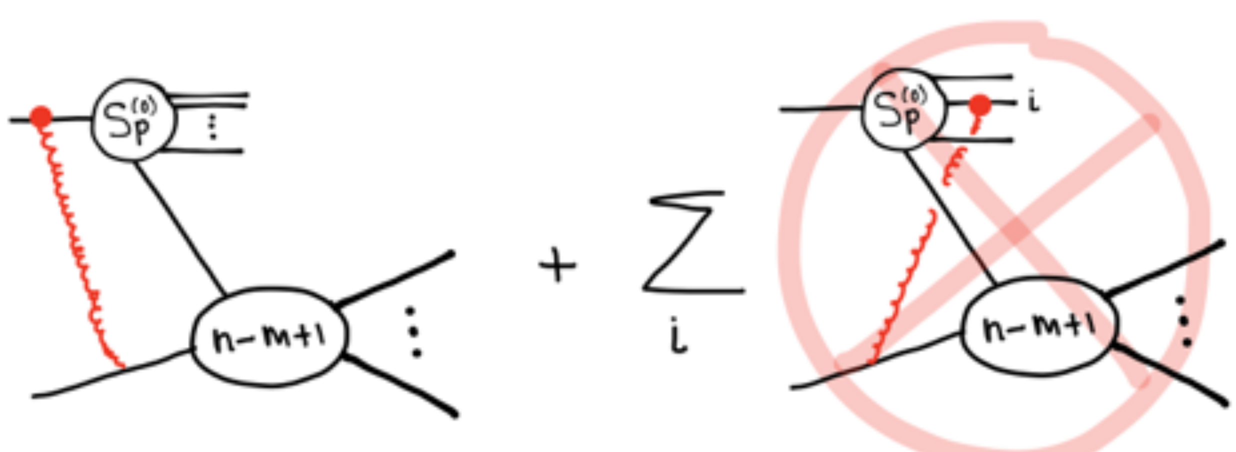


Breakdown of color coherence



$$T_2 \cdot \left(T_1 + \sum_i T_i\right) Sp^{(0)} |n\text{-}m\text{+}1\rangle = Sp^{(0)} \left[T_2 \cdot T_{\tilde{p}} |n\text{-}m\text{+}1\rangle\right]$$

Effectively, correction to

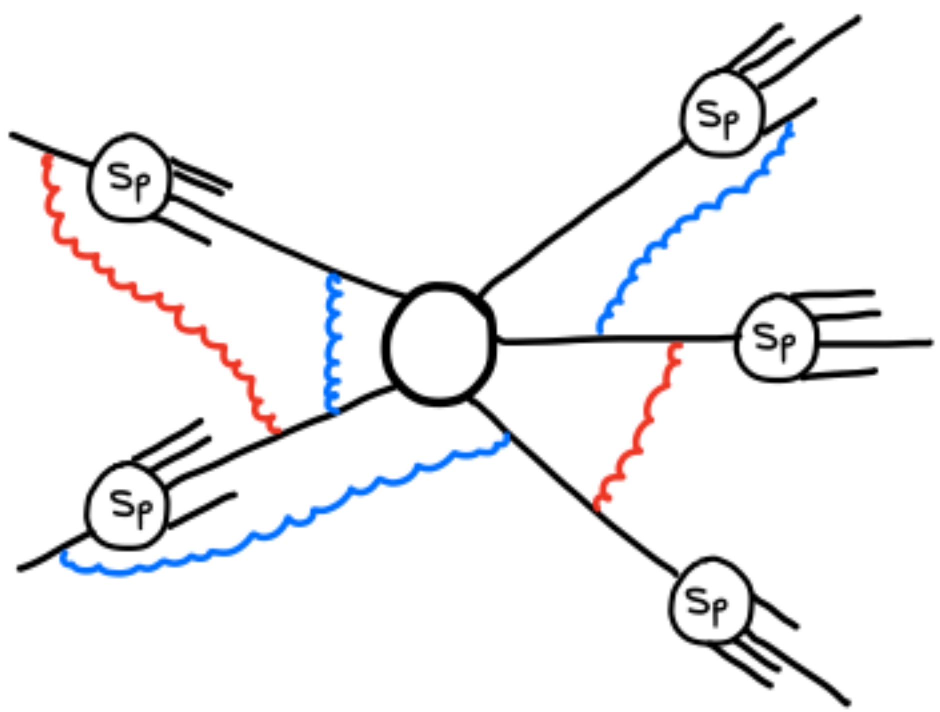


$$= T_1 \cdot T_2 \, Sp^{(0)} |n\text{-}m\text{+}1\rangle$$

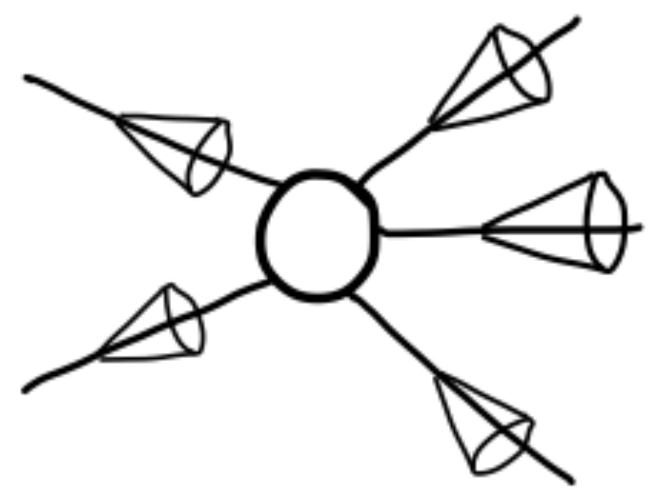
Unavoidable colour correlation

Coulomb gluons and (the lack of) coherence

Conclusion: coherence allows us to “unhook” on-shell gluons and recover process independent factorisation. But it fails for Coulomb gluons.



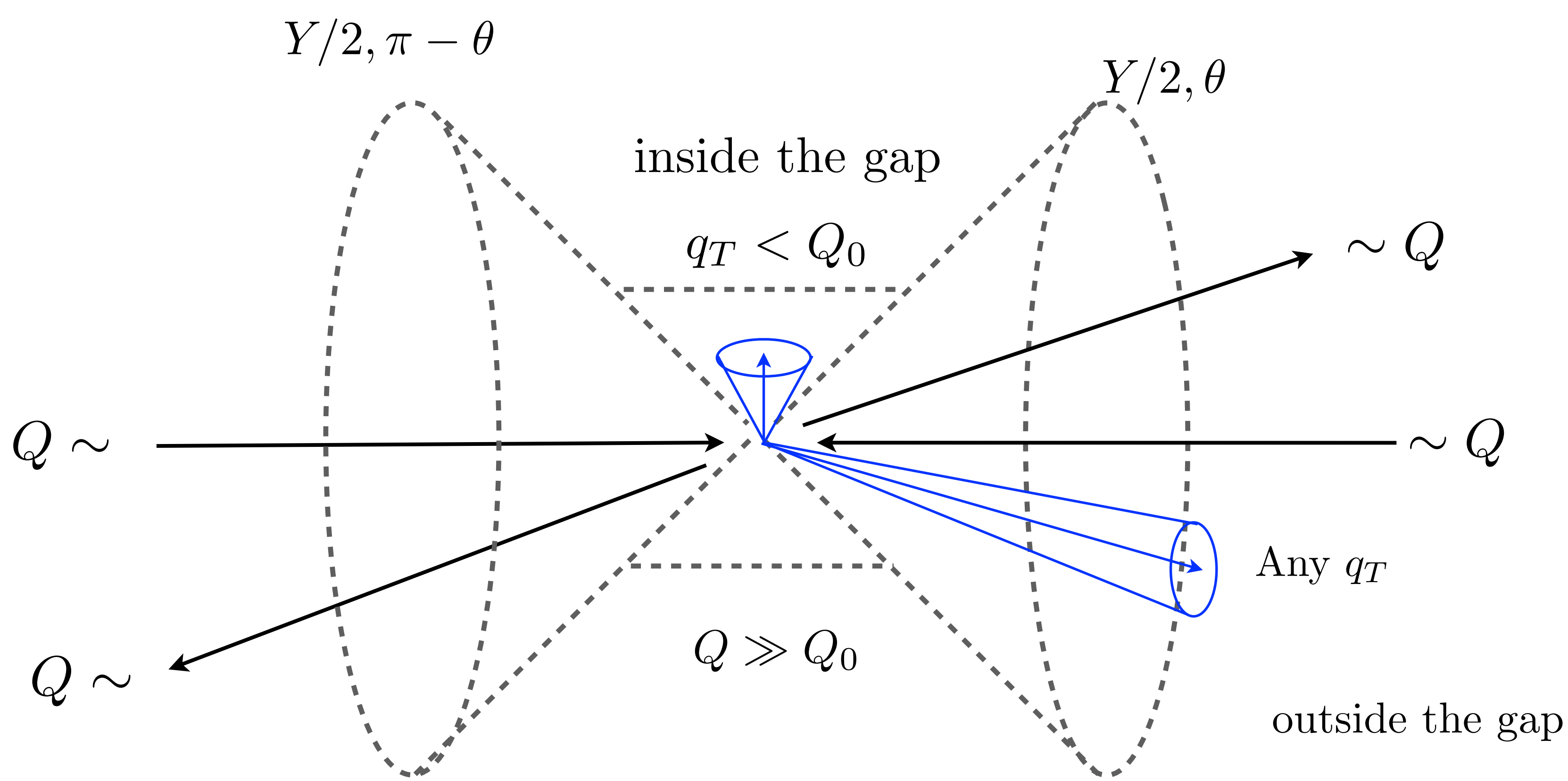
instead of



Can we make sense of these nested structure?

Concrete case: gaps-between-jets

(Forshaw, Kyrieleis & Seymour hep /0604094 ; /0808.1269)



Soft corrections

$$\sigma_m = \int |\mathcal{M}(q_1, \ldots, q_m)|^2 \, \mathrm{dPS}$$

$$Q_0 \ll q_i \ll Q$$

(On-shell gluons)

$$\sim \alpha_s^n \ln^n \left(\frac{Q^2}{Q_0^2} \right)$$

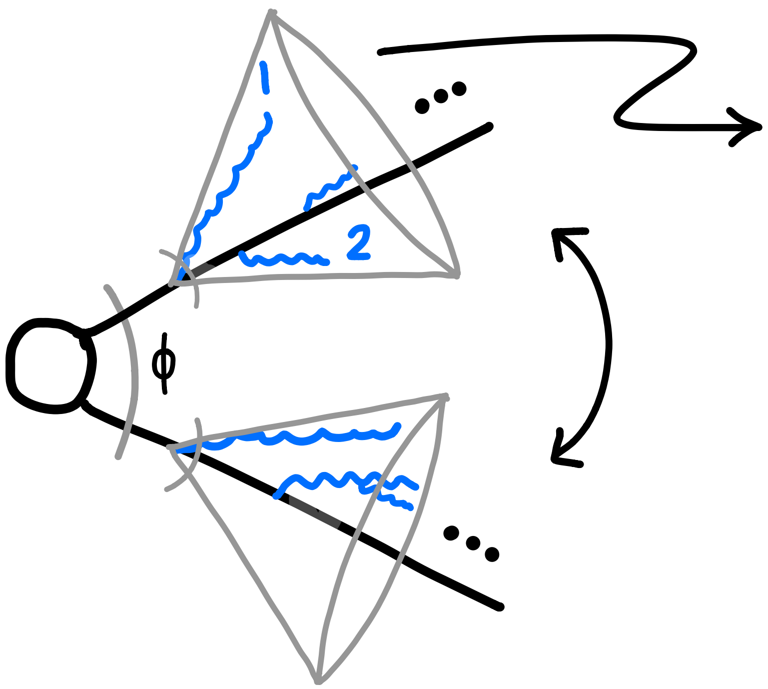
Super-leading logs
(On-shell + Coulomb gluons)

$$\sim \alpha_s^3 \ln^4 \left(\frac{Q^2}{Q_0^2} \right), \alpha_s^4 \ln^5 \left(\frac{Q^2}{Q_0^2} \right)$$

Origin: lack of coherence (strict factorisation).

Parton showers

(Produce events from approximate x-sections)



$$Q > q_1^0 > q_2^0 > \cdots$$

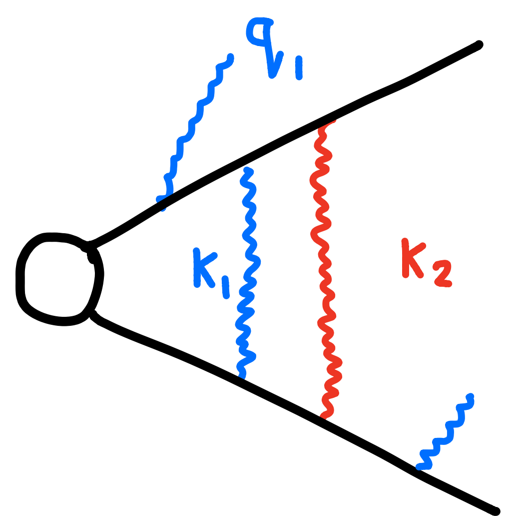
$$\phi > \phi_1 > \phi_2 > \cdots$$

Typically:

- X-section approximated by "ordering" real radiation
- Soft radiation included but no colour interference.
- Virtual radiation included indirectly via unitarity.

Colour interference:

- Ansatz (hep /0604094): Order soft radiation, real & virtual, according to its "hardness".
- Is the specific ordering variable relevant?



$$\theta(q_1) > \theta(k_1) > \theta(k_2) > \cdots$$

$$f(k_1)\,?$$

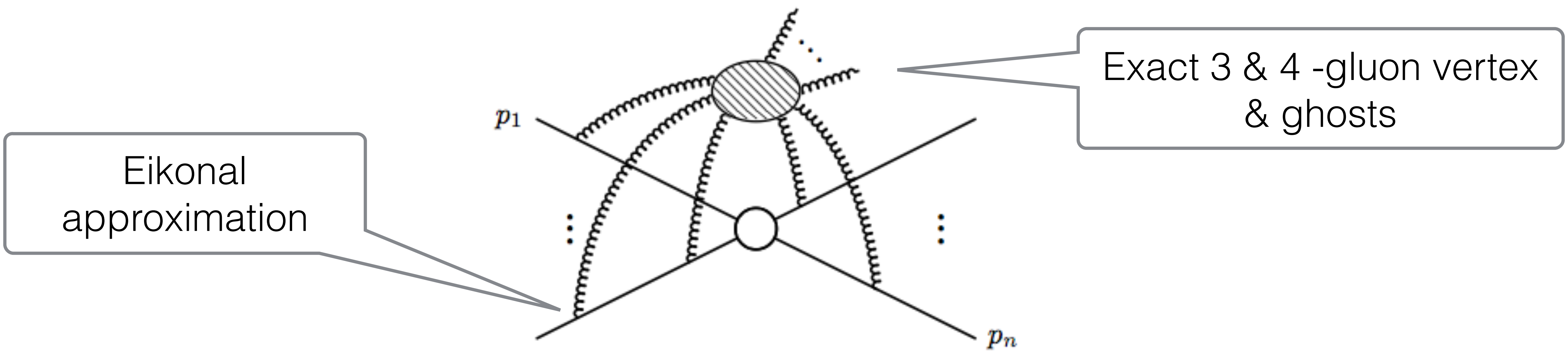
The role of the ordering variable is crucial!

(Banfi, Salam, Zanderighi JHEP06(2010)038)

The coefficients of super-log varies for different ordering variables:

- Angular ordering: zero.
- Energy ordering: infinite.
- Transverse momentum ordering: finite.
- Virtuality ordering: 1/2.

Our strategy to solve this problem: Brute force!



Complete (1-loop) diagrammatic calculation assuming that all gluons are soft, but not relatively softness (RAM, Forshaw, Seymour: PhD thesis, JHEP 1512 (2015) 091 & arXiv:1602.00623)

Coulomb gluons and colour evolution

Our fixed order calculations suggest that the one-loop amplitude of a general hard scattering with N soft-gluon emissions (ordered in softness $q_i \lambda \sim q_{i+1}$) is

$$\begin{aligned} \left| n_N^{(1)} \right\rangle =& \sum_{m=0}^{N} \sum_{i=2}^{p} \sum_{j=1}^{i-1} \mathbf{J}^{(0)}(q_N) \cdots \mathbf{J}^{(0)}(q_{m+1})\, \mathbf{I}_{ij}(q_{m+1}^{(ij)}, q_m^{(ij)}) \mathbf{J}^{(0)}(q_m) \cdots \mathbf{J}^{(0)}(q_1)\, |n_0^{(0)}\rangle \\ &+ \sum_{m=1}^{N} \sum_{j=1}^{n+m-1} \sum_{k=1}^{n+m-1} \mathbf{J}^{(0)}(q_N) \cdots \mathbf{J}^{(0)}(q_{m+1})\, \mathbf{I}_{n+m,j}(q_{m+1}^{(ij)}, q_m^{(jk)})\, \mathbf{d}_{jk}(q_m) \mathbf{J}^{(0)}(q_{m-1}) \cdots \mathbf{J}^{(0)}(q_1) |n_0^{(0)}\rangle, \end{aligned}$$

where the virtual insertion operator:

$$\mathbf{I}_{ij}(a, c) = \mathbf{I}_{ij}(a, b) + \mathbf{I}_{ij}(b, c)$$

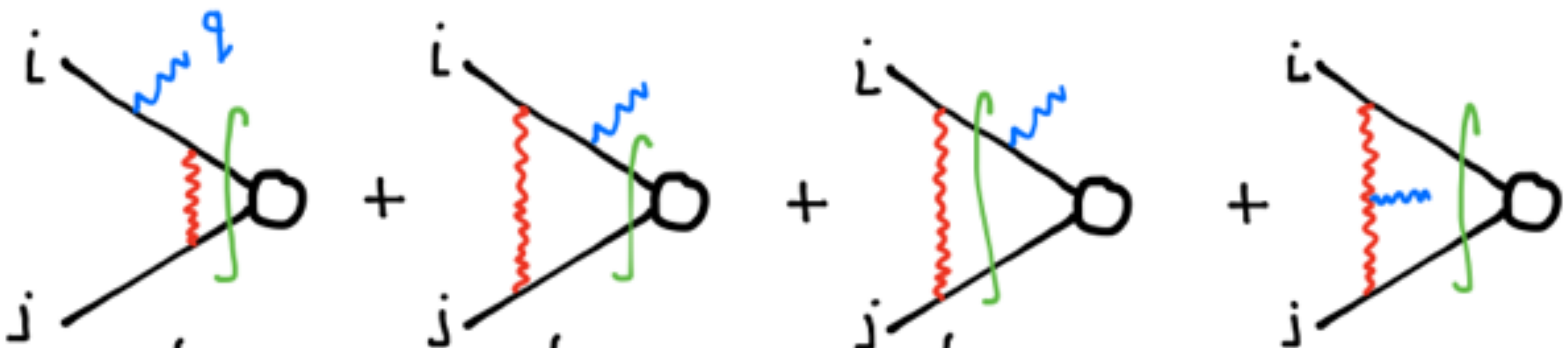
describes the non-emission evolution of partons i and j from a to c.

$$q^\mu = \alpha p_i^\mu + \beta p_j^\mu + (q_T^{(ij)})^\mu$$

Key point: The Ordering variable is dipole kT

- Gauge invariant.
- Correct IR poles
- Interpretation: ordered evolution!
- …

Sketch in the simplest case



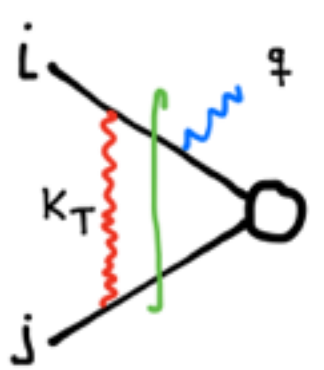
$$i\pi \frac{p_i\cdot\epsilon}{p_i\cdot q}\int_0^{Q^2}\frac{dk_T^2}{k_T^2}\left\{ T_i(T_i\cdot T_j) - T_i\cdot T_j\, T_i + T_i\cdot T_j\, T_i \right\}|2\rangle$$

Decomposition in colour and spin

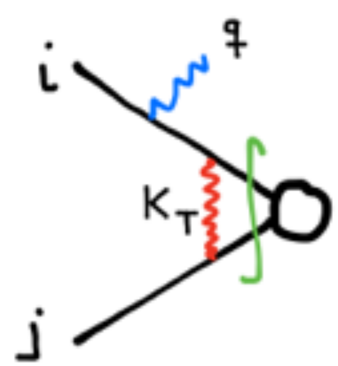
$$-i\pi \frac{p_i\cdot\epsilon}{p_i\cdot q}\underbrace{\int \frac{dk_T^2}{k_T^2}\frac{q_T^2}{k_T^2+q_T^2}}_{\simeq \int_0^{q_T^2}\frac{dk_T^2}{k_T^2}} \left[T_i\cdot T_j\, T_i - T_i\, T_i\cdot T_j\right]|2\rangle$$

$$+(i\leftrightarrow j)$$

$$\simeq \int_0^{q_T^2}\frac{dk_T^2}{k_T^2}$$



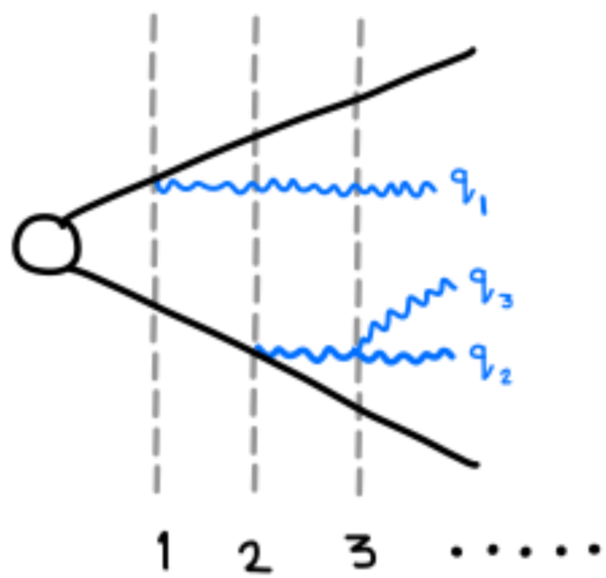
$$+\int_{q_T^2}^{Q^2}\frac{dk_T^2}{k_T^2}$$



i.e. non-trivial test of k_T ordering!

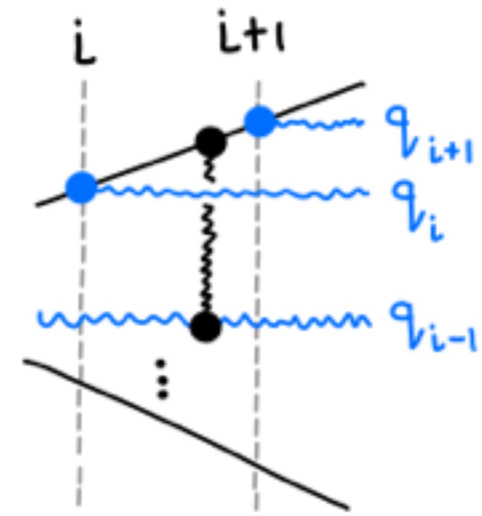
Diagrammatics of dipole: kT ordering

1.- Add N-emissions on external legs

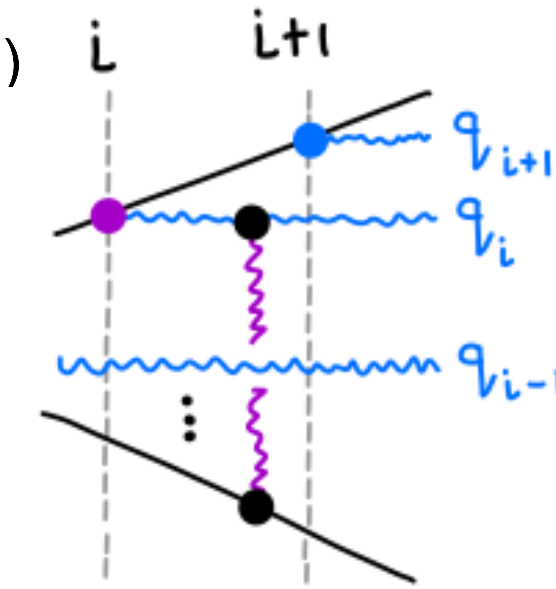


2.- Add virtual exchanges

Case a)

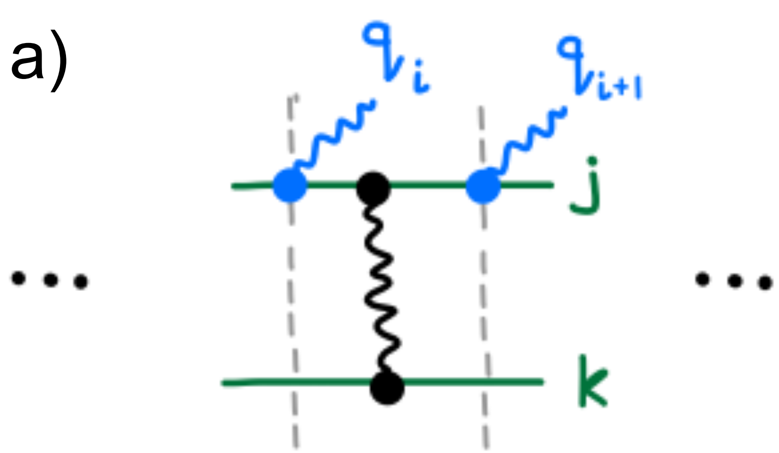


Case b)

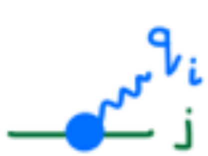


and apply effective rules:

a)



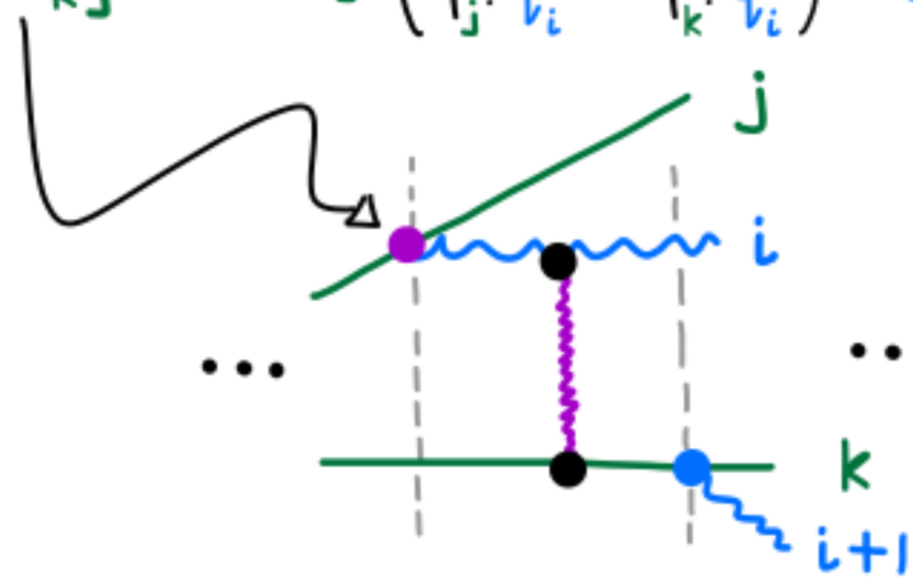
 $= I_{jk}\left(q_{i+1}^{(jk)}, q_i^{(jk)}\right)$



$= T_j \dfrac{p_j \cdot \varepsilon_i}{p_j \cdot q_i}$

b)

$$\bullet = d_{kj}(i) = T_j \left(\frac{p_j}{p_j \cdot q_i} - \overbrace{\frac{p_k}{p_k \cdot q_i}}^{\text{Gauge invariance}} \right) \cdot \varepsilon_i$$



 $= I_{q_i k}\left(q_{i+1}^{(ik)}, q_i^{(jk)}\right)$

Non-emission evolution operator

On-shell

Coulomb

$$\mathbf{I}_{ij}(a,b) = \frac{\alpha_s}{2\pi}\mathbf{T}_i\cdot\mathbf{T}_j c_\Gamma \int \mathrm{d}(k^{(ij)})^2 (k^{(ij)})^{-2\epsilon}\left[\int_{-\ln\sqrt{2}p_j^-/k^{ij}}^{\ln\sqrt{2}p_i^+/k^{ij}} \mathrm{d}y \frac{p_i\cdot p_j}{2[p_j\cdot k][p_i\cdot k]} - \frac{i\pi\delta_{ij}}{(k^{(ij)})^2}\right]$$

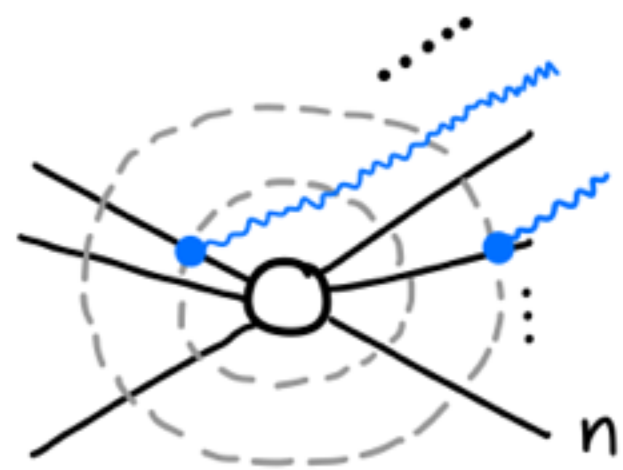
$$\times\,\theta(a < k^{(ij)} < b)$$

This is the same one-loop operator that appears at one-loop but kT ordered!

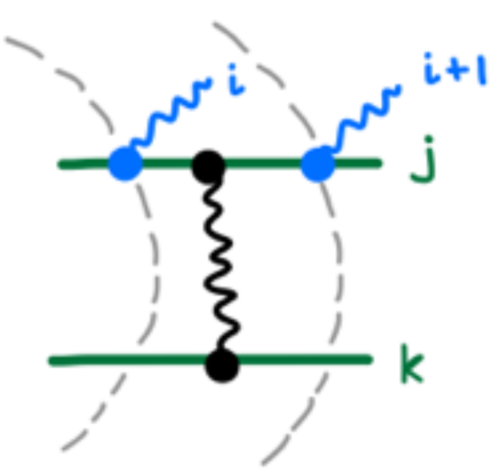
$$k^\mu = \alpha p_i^\mu + \beta p_j^\mu + (k^{(ij)})^\mu$$

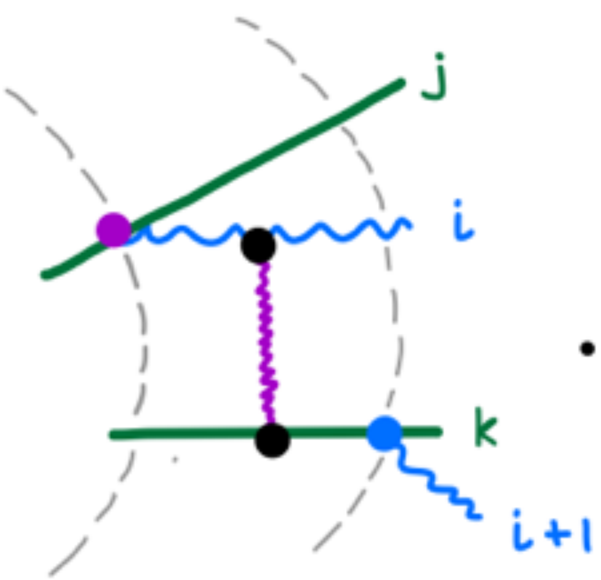
Diagrammatics of dipole kT evolution

For a general scattering $|n\rangle$ we need spheres



The effective rules are the same:

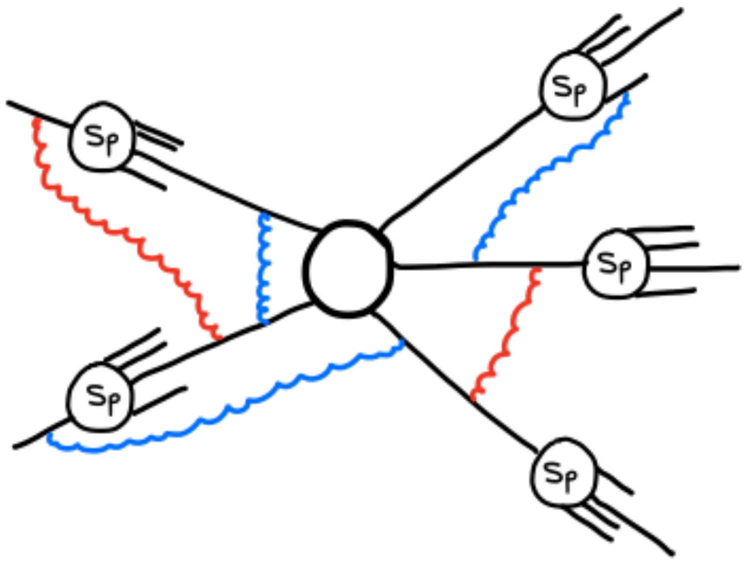




Summary / Conclusion

- Coulomb gluons introduce colour-interference & play an essential role in the evolution of hard processes:
  – super-leading logs
  – violations of coherence

- Can be incorporated at *amplitude level* as an evolution in dipole transverse momentum, making sense of



- Future: Monte Carlo Parton Shower for general observables.