Non-Extensive Statistical Approach for Hadronization and its Application

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Approach: Eur. Phys. J. A49 (2013) 110, Physica A 392 (2013) 3132

Application: J.Phys.CS 612 (2015) 012048 arXiv:1405.3963, 1501.02352, 1501.05959

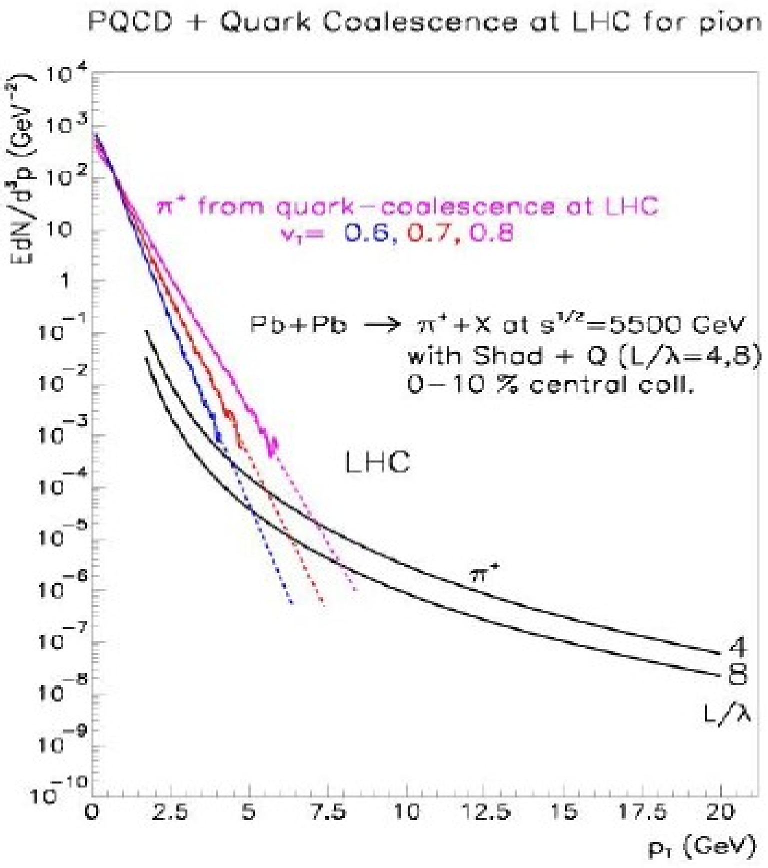
Seminar at ICN UNAM, Ciudad Mexico, Mexico, 15th January 2016

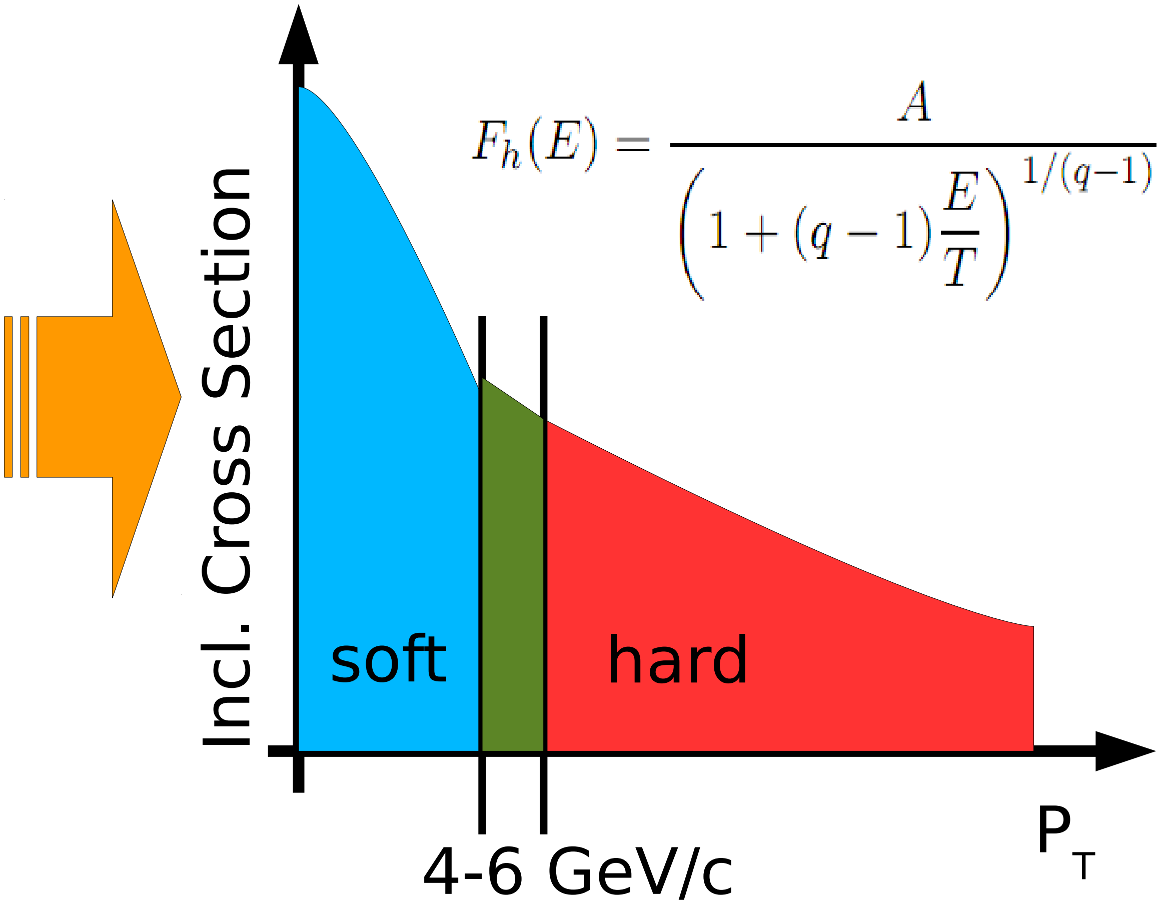
O U T L I N E

- Motivation...
  - by a student exercise
- Non-extensive statistical approarch
  - Fits of experimental spectra from e^+e^-, pp
  - Non-extensive statistical approach
- Can Tsallis – Pareto fit spectra of HIC?
  - The soft+hard model and its applications
  - Spectra fit and extraction of q and T
  - Asimuthal anisotropy from the model

MOTIVATION

- Simplest and best fit to hadron spectra at low-p_T & high-p_T





$$F_h(E) = \frac{A}{\left(1+(q-1)\dfrac{E}{T}\right)^{1/(q-1)}}$$

P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

The student exercise...

- Why use Tsallis–Pareto distribution?
    - Is it true Boltzmann-Gibbs fits better at low momenta?
    - Is it true Power-law distribution is better at high momenta?
    - Is it true Tsallis – Pareto fits the whole mumentum range?
    - Can we apply this for any system: ee, pp, pA, AA?

- Let's see first a 'known' case:
    - PYTHIA6.4: π, K and p production in proton-proton @ 14 TeV
    - Fits of Boltzmann-Gibbs, Power law, and Tsallis–Pareto distributions
    - Low momenta: [1.2 GeV/c : 2.0 GeV/c] or [1.2GeV/c : 5.0 GeV/c]
    - High momenta: [5.0 GeV/c : 15.0 GeV/c]
    - Full range: [1.2 GeV/c : 15.0 GeV/c]

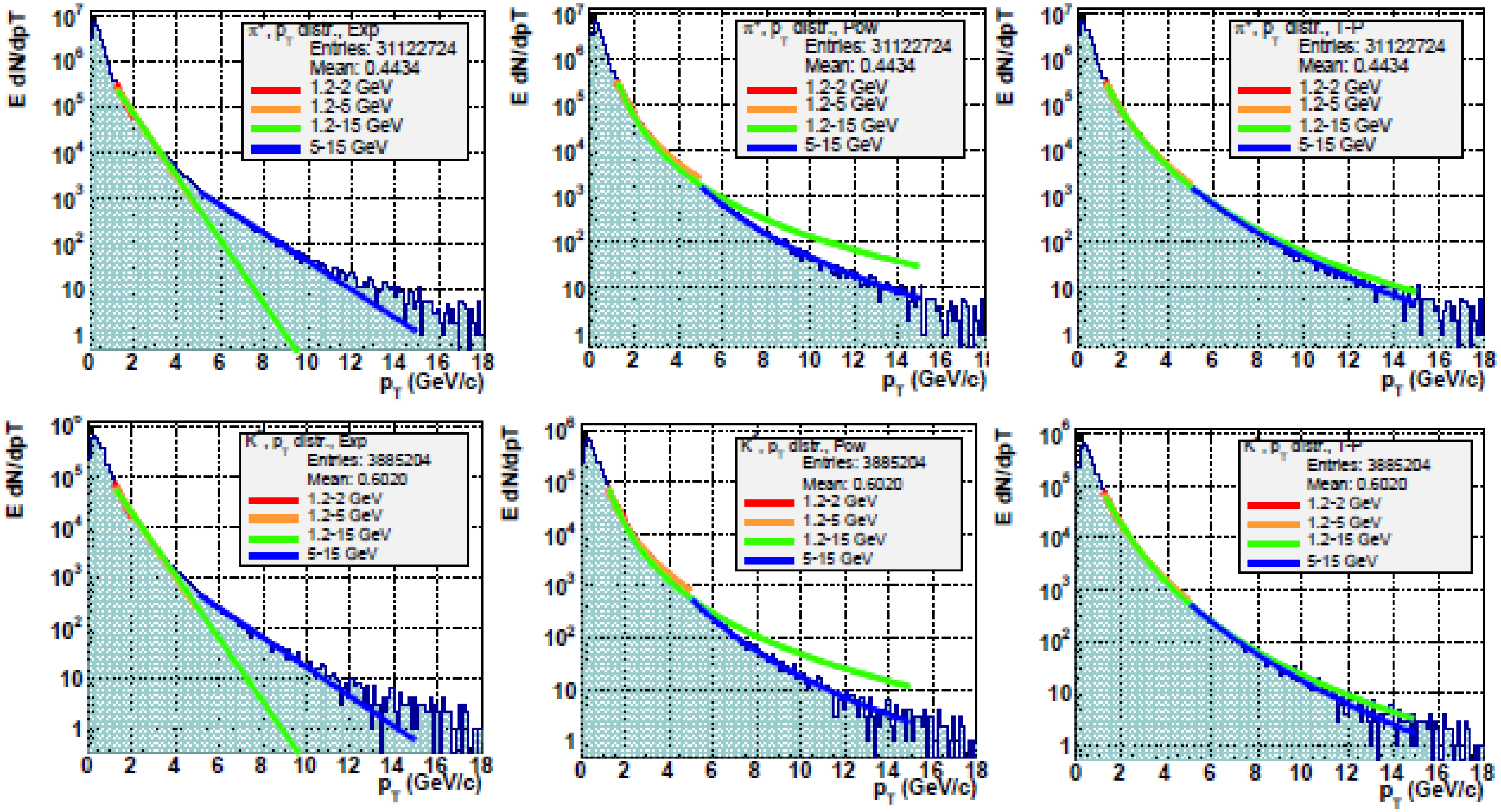
What can we learn form a simply exercise?

The student exercise...

Boltzmann–Gibbs **Power Law** **Tsallis–Pareto**

Pions

Kaons



The fitted momentum regions:

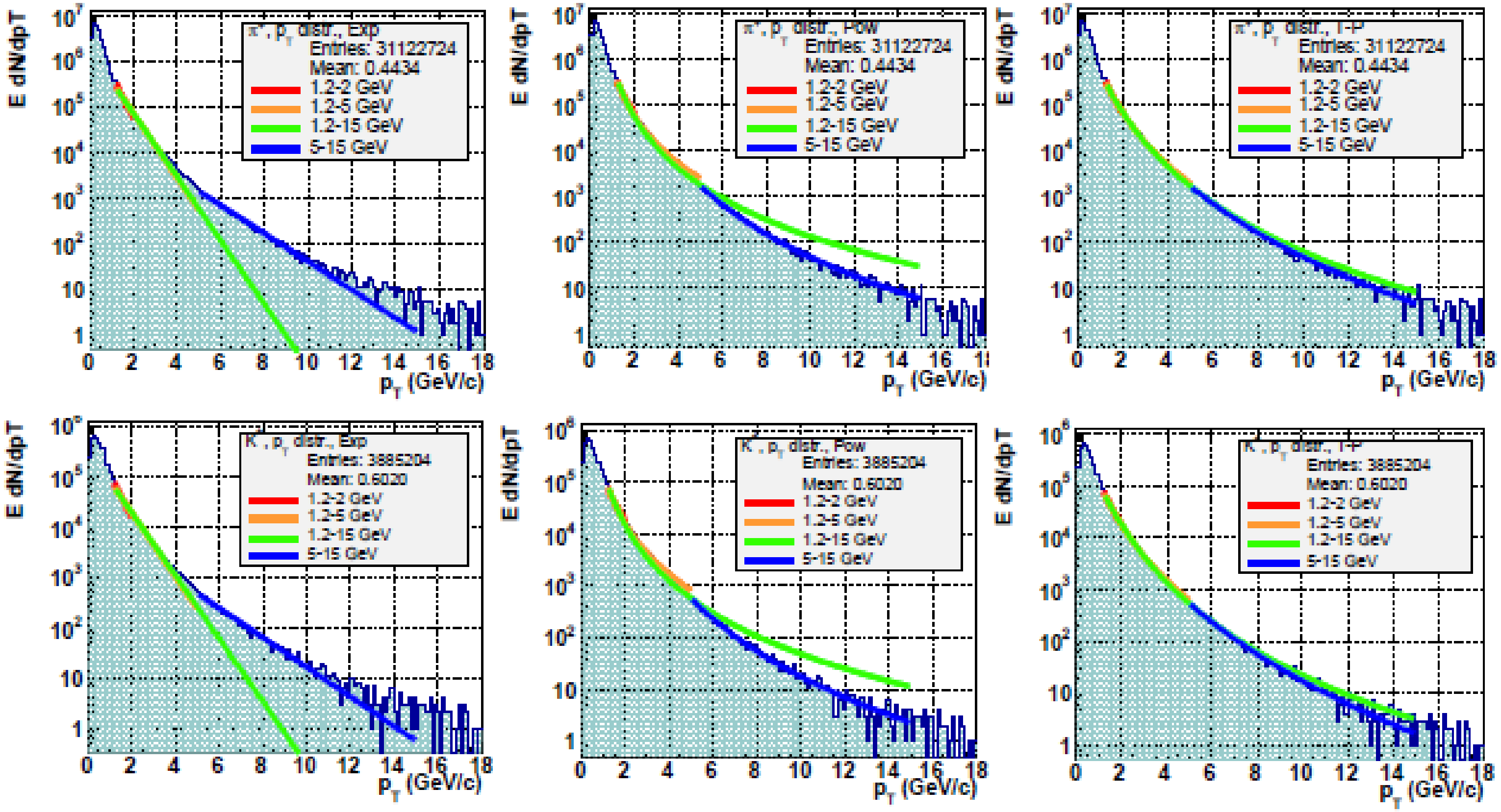
🟥 1.2-2 GeV
🟧 1.2-5 GeV
🟩 1.2-15 GeV
🟦 5-15 GeV

The student exercise...

Boltzmann–Gibbs **Power Law** **Tsallis–Pareto**

Pions

Kaons



χ2 values:

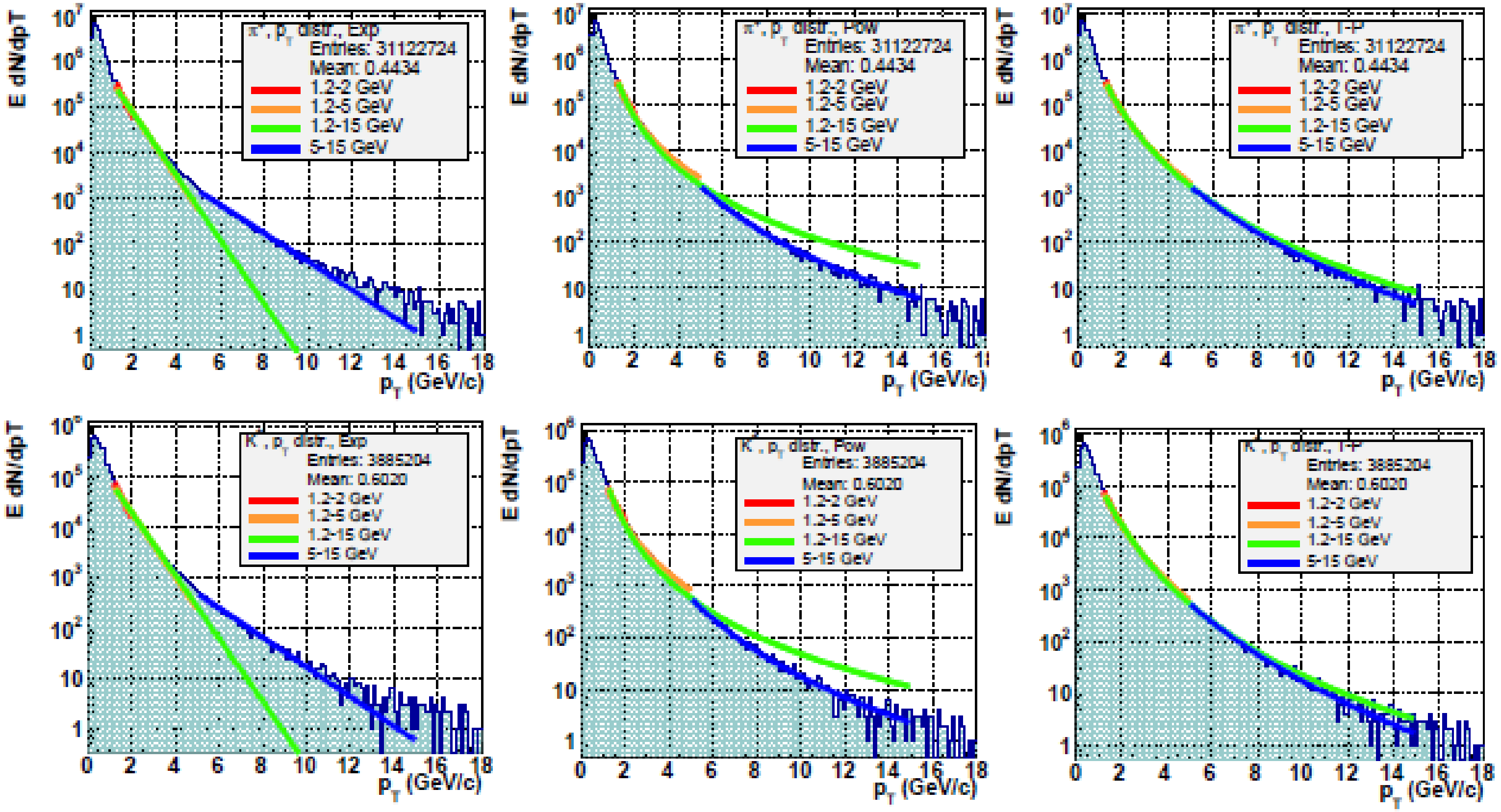
	[1,2:2] GeV/c	[1,2:5] GeV/c	[1,2:15] GeV/c	[5:15] GeV/c
Exp	112,37/29,81/27,34	623,89/130,48/109,26	254,12/61,71/48,13	3,01/1,44/1,45
Pow	1,71/0,98/0,47	161,27/55,68/56,08	214,12/76,92/77,26	1,37/1,144/0,91
TP	0,45/1,19/0,56	12,21/5,55/11,06	10,39/4,37/7,77	1,14/0,97/0,91

The student exercise...

Boltzmann–Gibbs **Power Law** **Tsallis–Pareto**

Pions

Kaons



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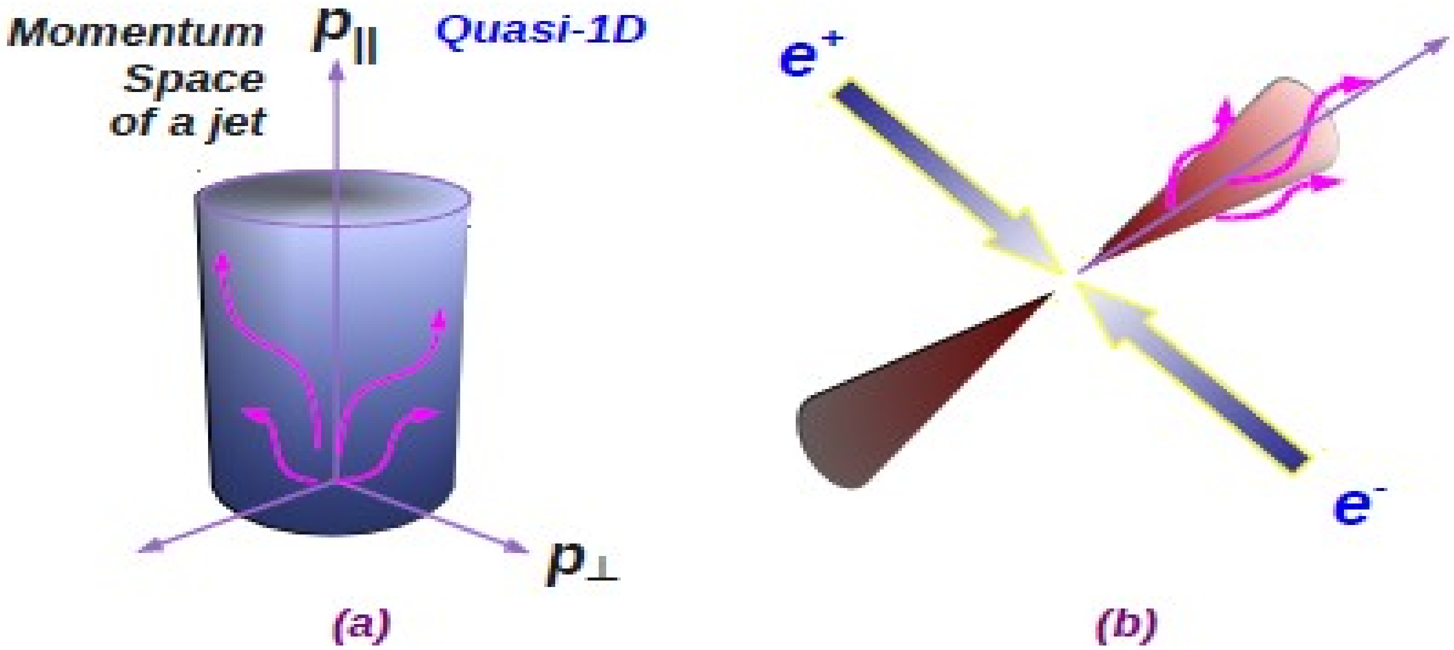
The student exercise...

- Why fit Tsallis–Pareto distribution?
  - Yes, it is true Boltzmann-Gibbs fits better at low momenta.
  - Yes, it is true Power-law distribution is better at high momenta.
  - Yes, it is true Tsallis – Pareto fits the whole mumentum range.
  - Can we apply this for any system: ee, pp, pA, AA?

- But carefully
  - BODY vs. TAIL (dependence on the momentom regions)
  - Need to find the proper variable E_{jet}, p_T, m_T, m_T^*
  - Need for
    - High-p_T PID hadron data
    - High statistic data
    - Spectra in several multiplicity bins
    - Dream: all of these on track-by-track basis

Application of the non-extensive statistical approach on small systems using experimental data.

The 'Thermodynamics of Jets'

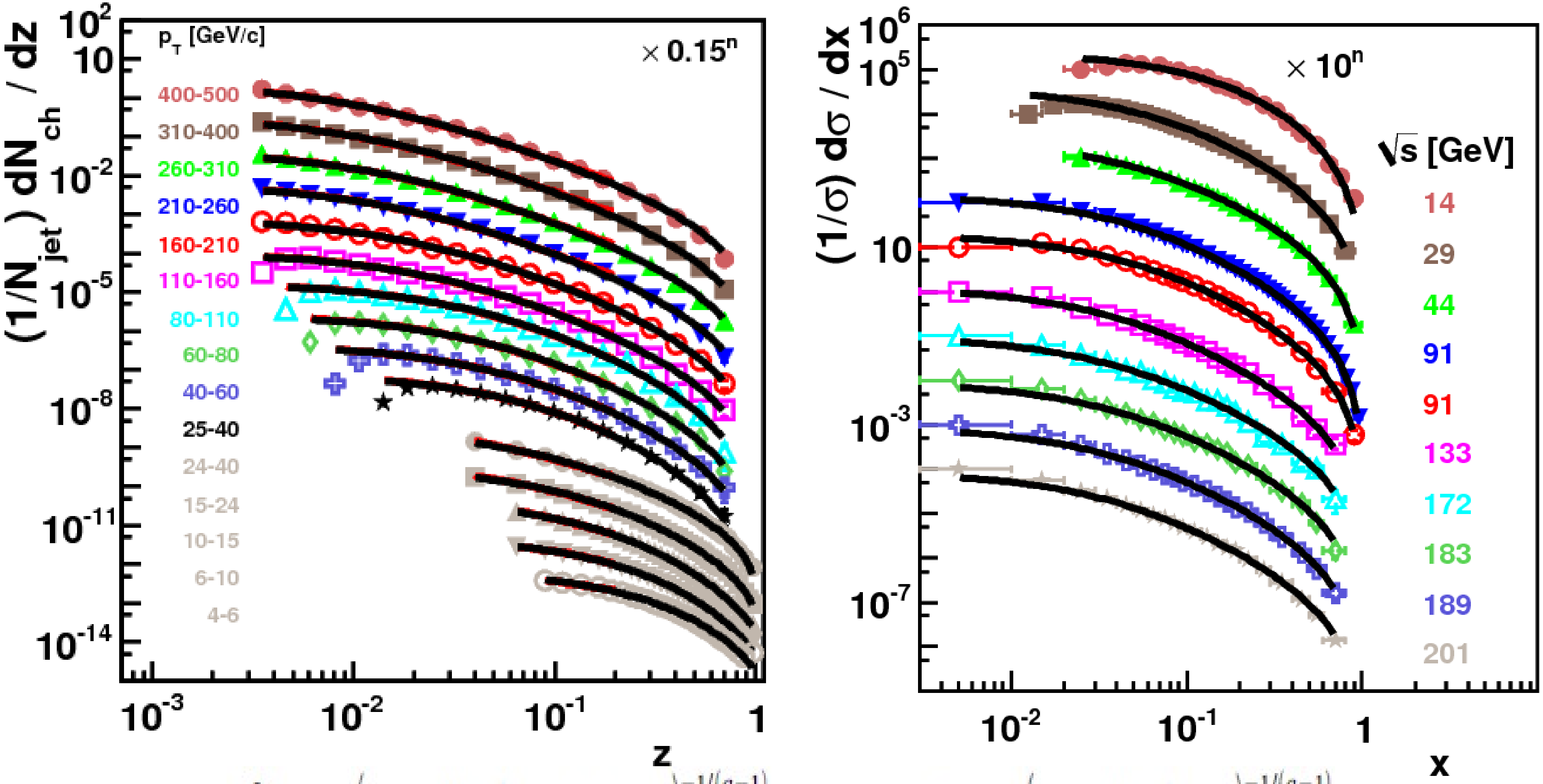


(a) (b)

K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

- Microcanonical Jet-Fragmentation in pp at LHC energies:
  Phys. Lett. B701 (2011) 111
- Generalized Tsallis distribution in e^+e^- collisons
  Phys. Lett. B718 (2012) 125

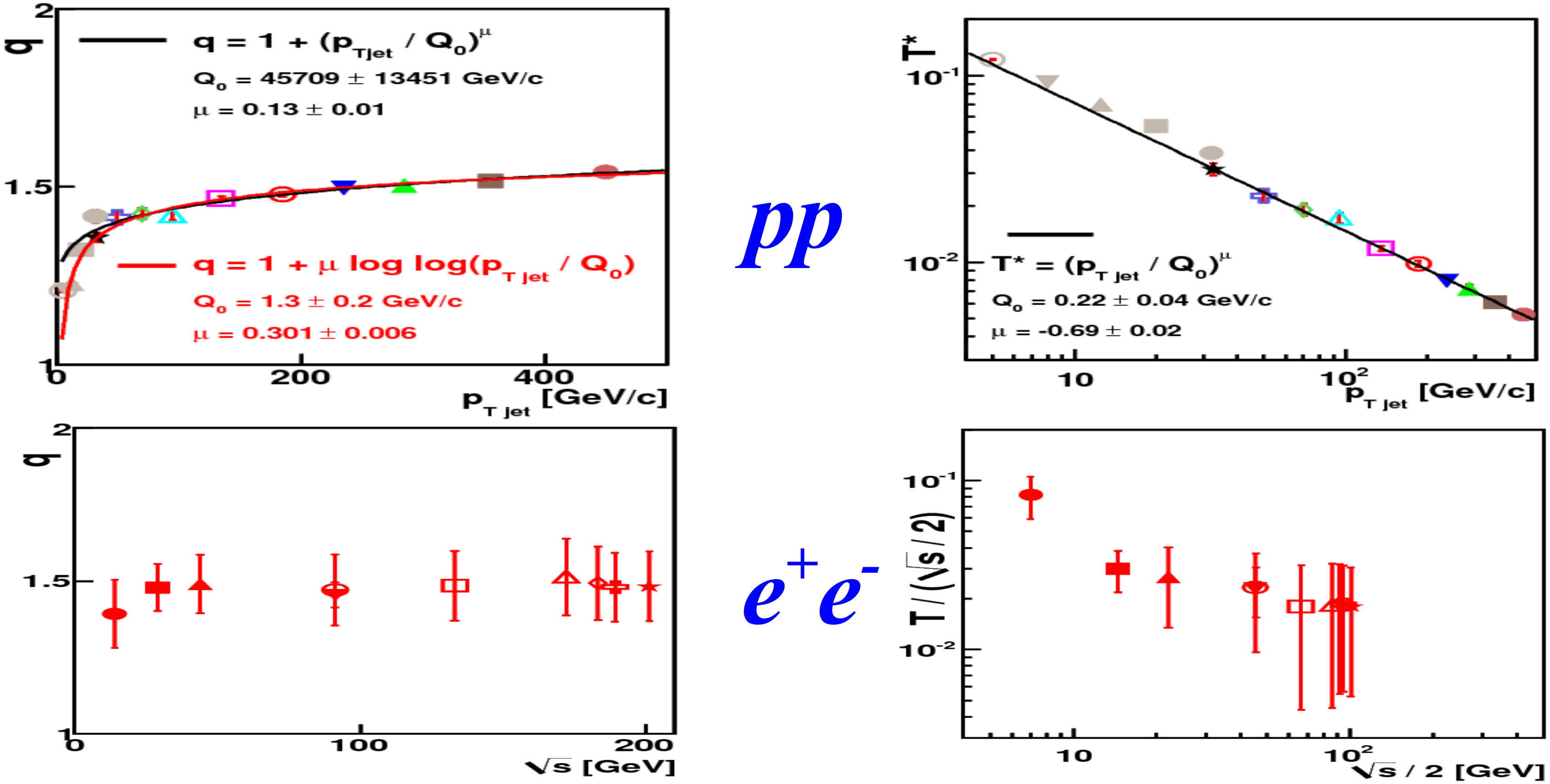
Fits for jet spectra in pp (left) and e^+e^- (right)



$$\frac{d\sigma}{dx} \propto \left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{-1/(q-1)} \longrightarrow \left(1+\frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$

Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

The evolution of q and T parameters



K Ürmössy, GGB, TS Biró,
PLB 710 (2011) 111, PLB 718 (2012) 125.

- Energy dependence (hard)
  - Parameters *q* seem to saturate at high energies *q>1.1*
  - Parameter *T* is decreading with increasing energy

What is the physical meaning of these '*q*' and '*T*' parameters?

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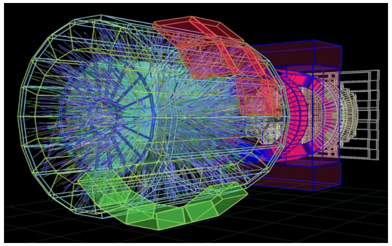
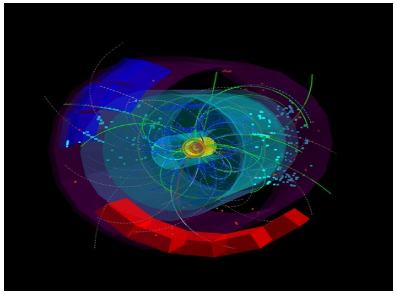
The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$\begin{aligned} S_{12} &= S_1 + S_2 \\ E_{12} &= E_2 + E_2 \end{aligned} \Rightarrow S_B = -\sum_i p_i \ln p_i$$

The non-extensive statistical approach





- Extensive Boltzmann – Gibbs statistics

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➡

$$S_B = -\sum_i p_i \ln p_i$$

- Non-extensivity → generalized entropy

$$\begin{aligned} \hat{L}_{12}(S_{12}) &= \hat{L}_1(S_1) + \hat{L}_2(S_2), \\ L_{12}(E_{12}) &= L_1(E_1) + L_2(E_2) \end{aligned}$$

➡

$$S_T = \frac{1}{1-q} \sum_i (p_i^q - p_i)$$

- Tsallis entropy

$$S_{12} = S_1 + S_2 + (q-1) S_1 S_2$$

➡

$$\hat{L}(S) = \frac{1}{q-1} \ln\left(1 + (q-1)S\right)$$

from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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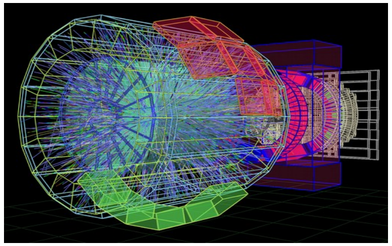
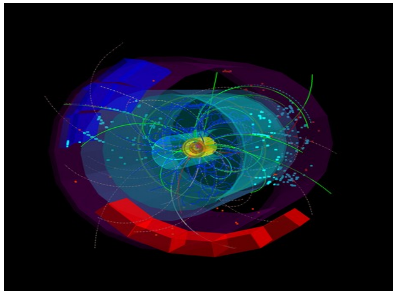
$$\frac{1}{T} = \langle S'(E)\rangle$$





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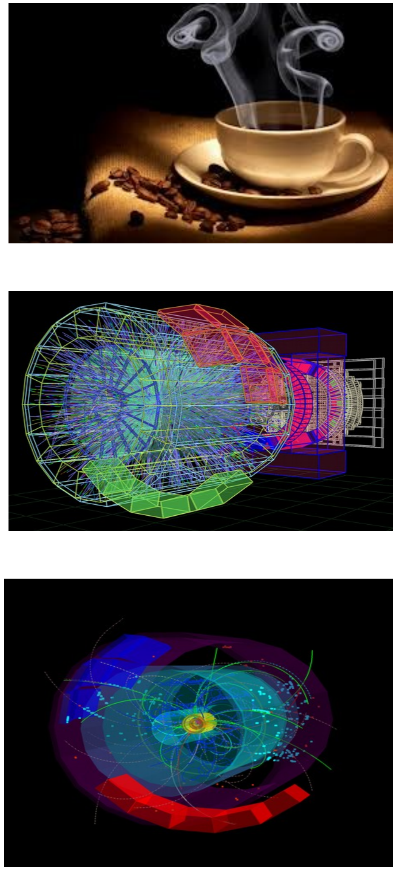
$$\frac{1}{T} = \langle S'(E)\rangle$$

$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}.$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = -\frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$



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The non-extensive statistical approach

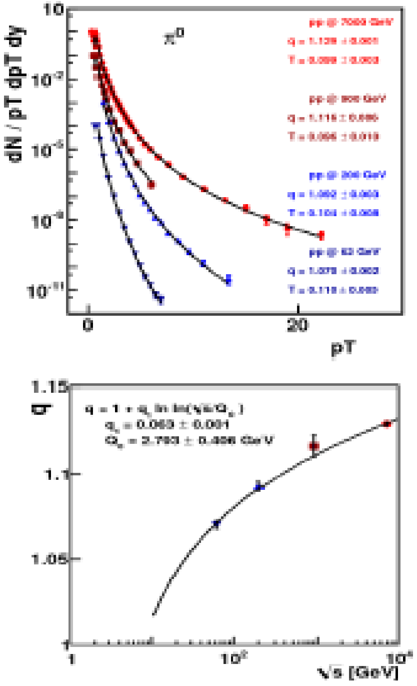
Hadron spectra in *pp* collisions can be described by the *Tsallis distribution*:

$$\frac{dN}{d^3\mathbf{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

***π* spectra in *pp* collisions depends *similarly on √s* and on the multiplicity**

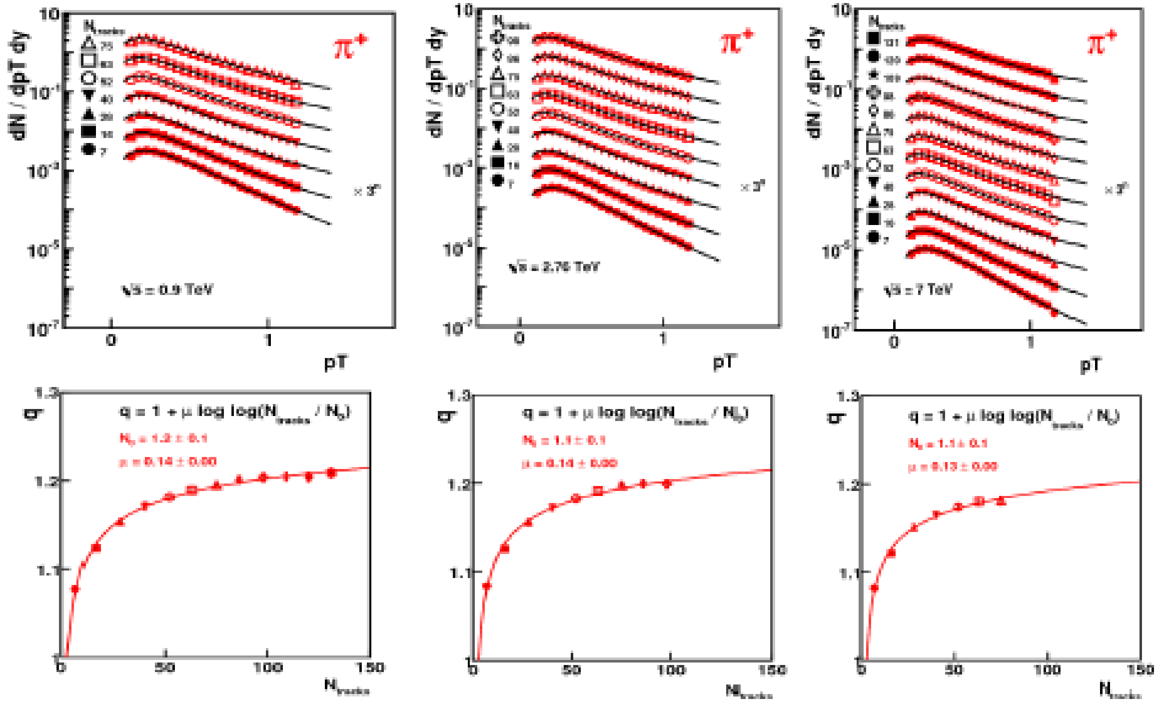
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

√s = fix



N = fix

arXiv:1405.3963, 1501.02352, 1501.05959



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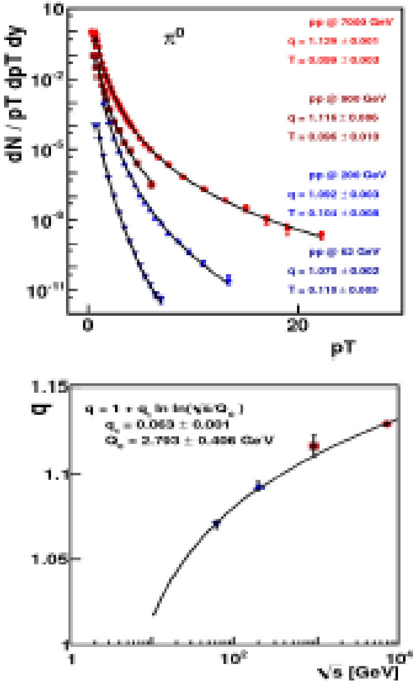
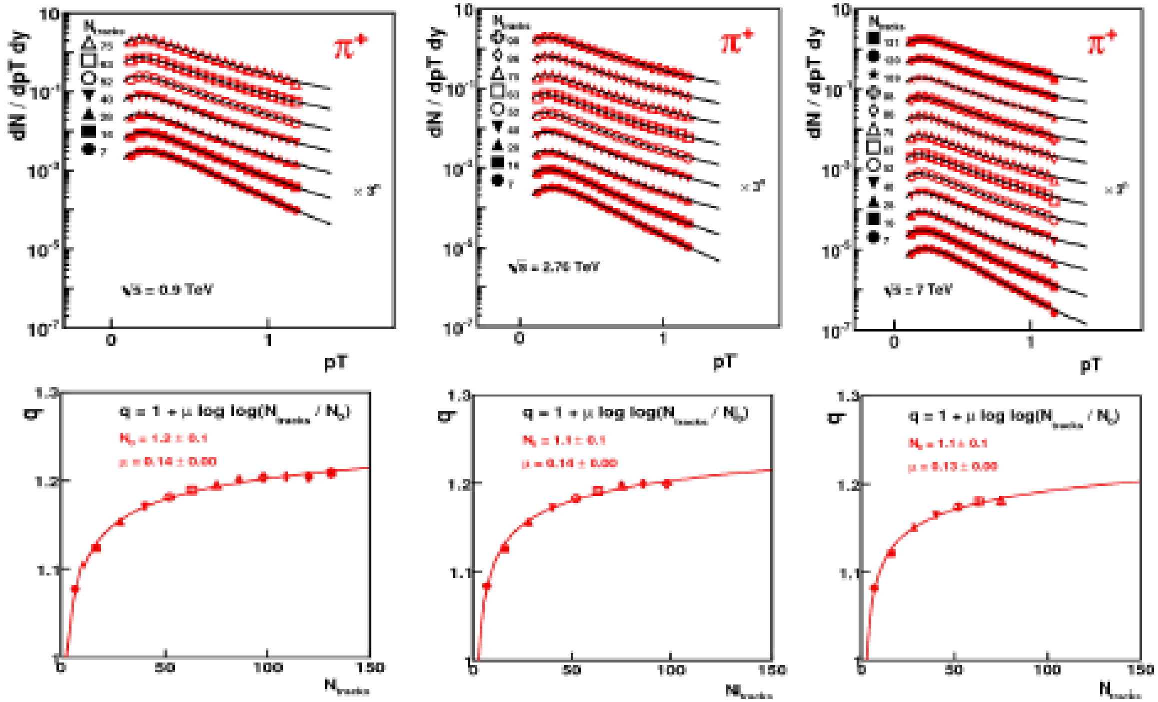
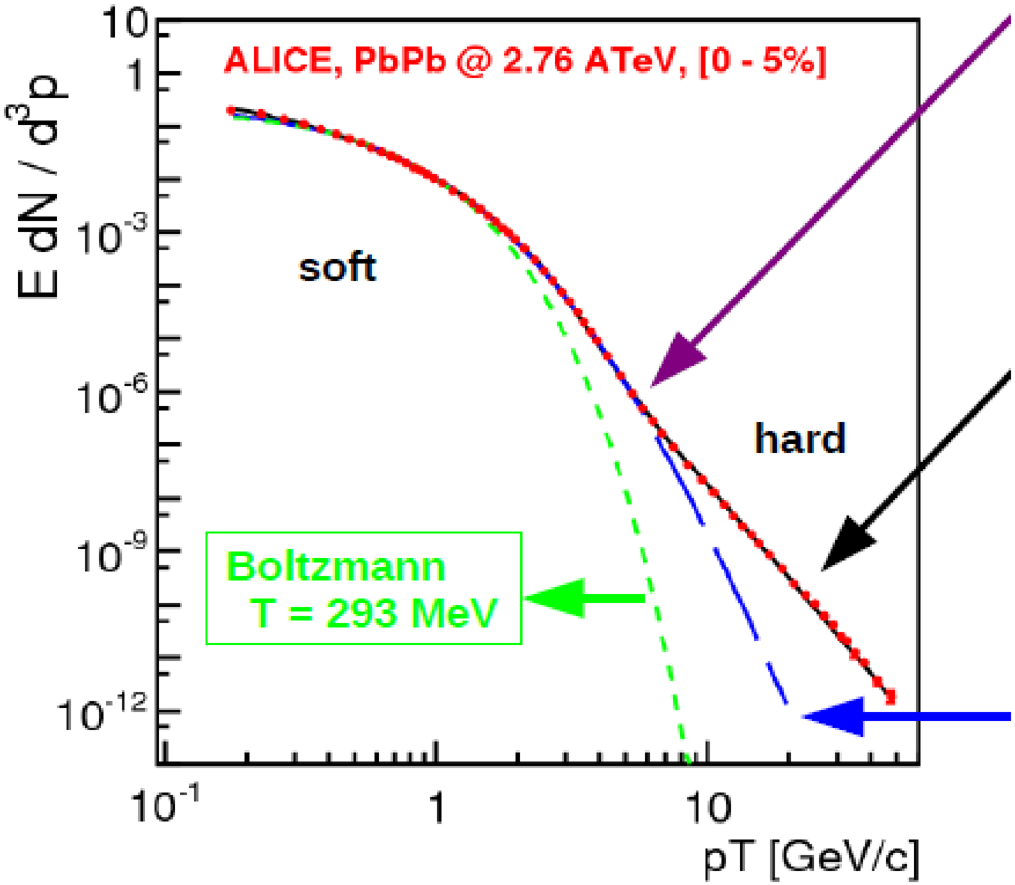




What if, we would apply this for
a bigger system (AA)
where
Boltzmann–Gibbs
use to work?

Test with real data in PbPb

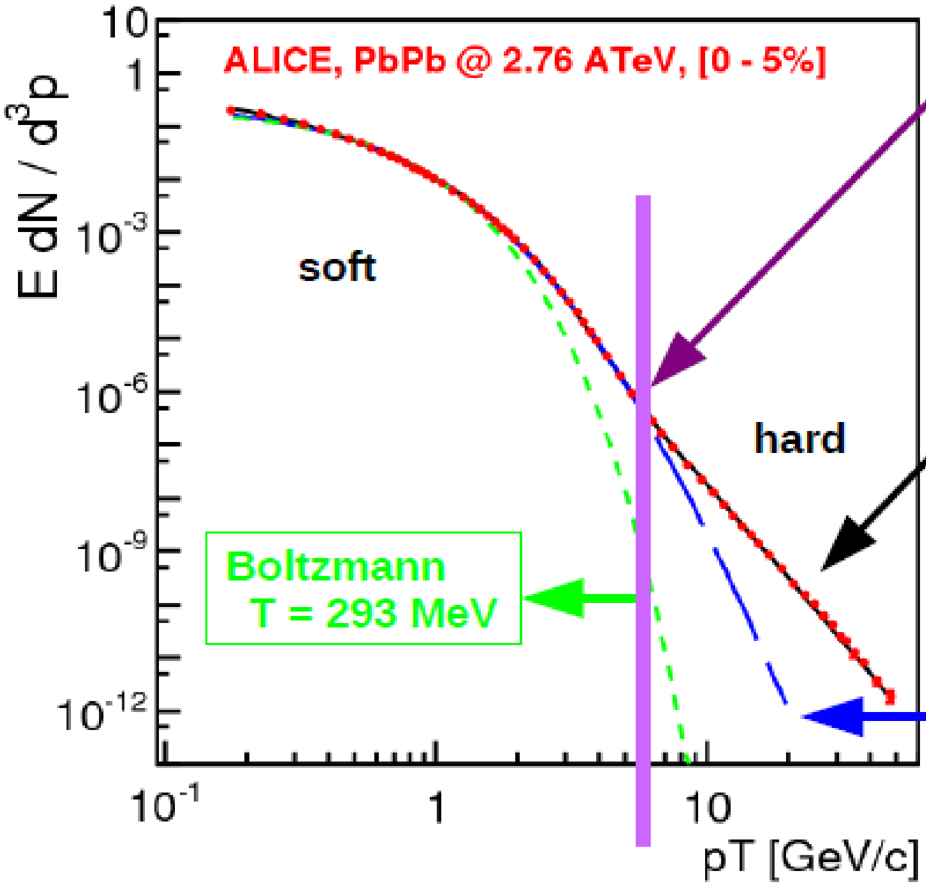


The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

Test with real data in PbPb



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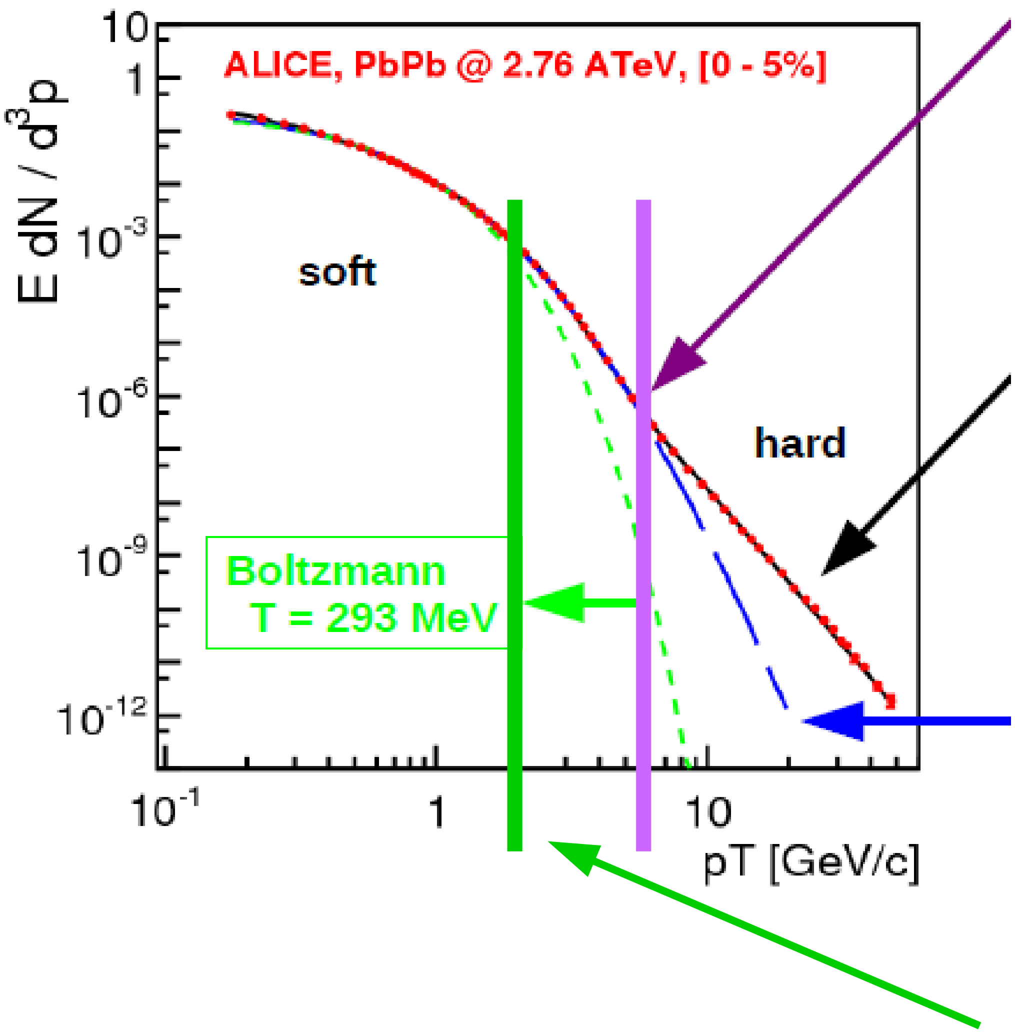

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Handling soft/hard regime with a new approach, using not only the temperature, T

The soft + hard model

- Simplest approximation: soft ('bulk') + hard ('jet') contribution

$$p^0 \frac{dN}{d^3\mathbf{p}} = p^0 \frac{dN}{d^3\mathbf{p}}^{\text{hard}} + p^0 \frac{dN}{d^3\mathbf{p}}^{\text{soft}}$$

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- Identified hadron spectra is given by double Tsallis–Pareto:

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft} \qquad f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i}[\gamma_i (m_T - v_i p_T) - m]\right]^{-1/(q_i-1)}$$

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in where parameters are given by

- Lorentz factor $\gamma_i = 1/\sqrt{1-v_i^2}$
- Transverse mass $m_T = \sqrt{p_T^2 + m^2}$
- Doppler temperature $T_i^{Dopp} = T_i \sqrt{\frac{1+v_i}{1-v_i}}$

- Finally we assume N_{part} scaling for the parameters

$$\begin{aligned} q_i &= q_{2,i} + \mu_i \ln(N_{part}/2) \\ T_i^{Dopp} &= T_{1,i} + \tau_i \ln(N_{part}) \, . \end{aligned}$$

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Fit of pp and PbPb (centra/peripheral) data

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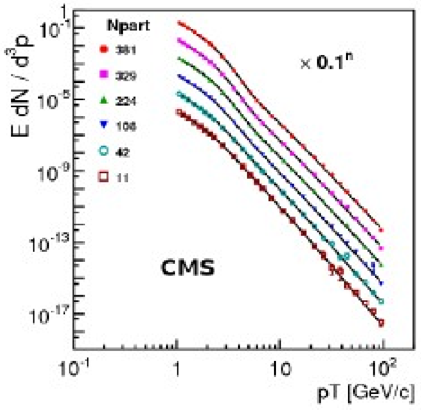
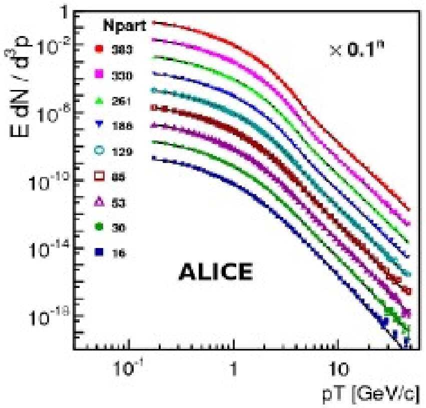
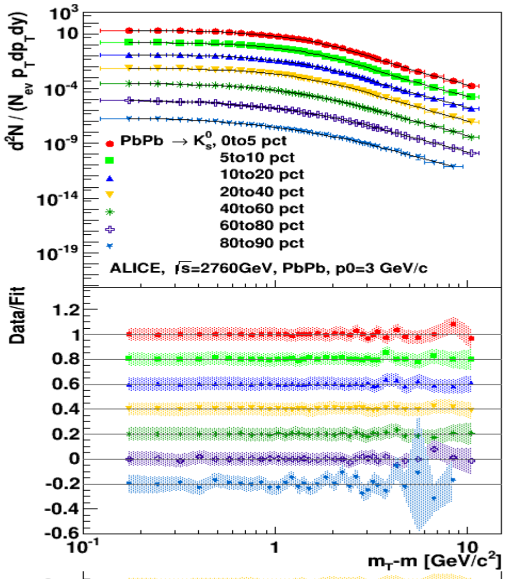
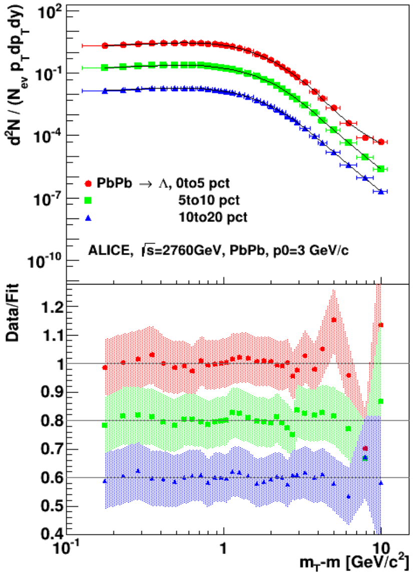
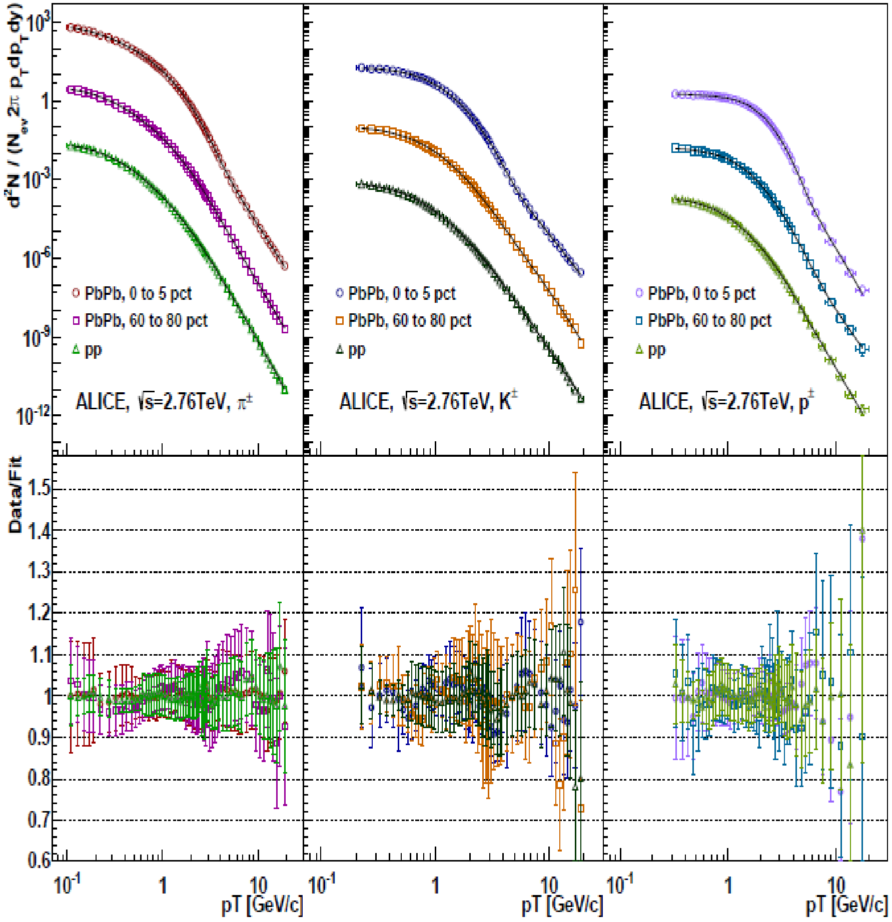








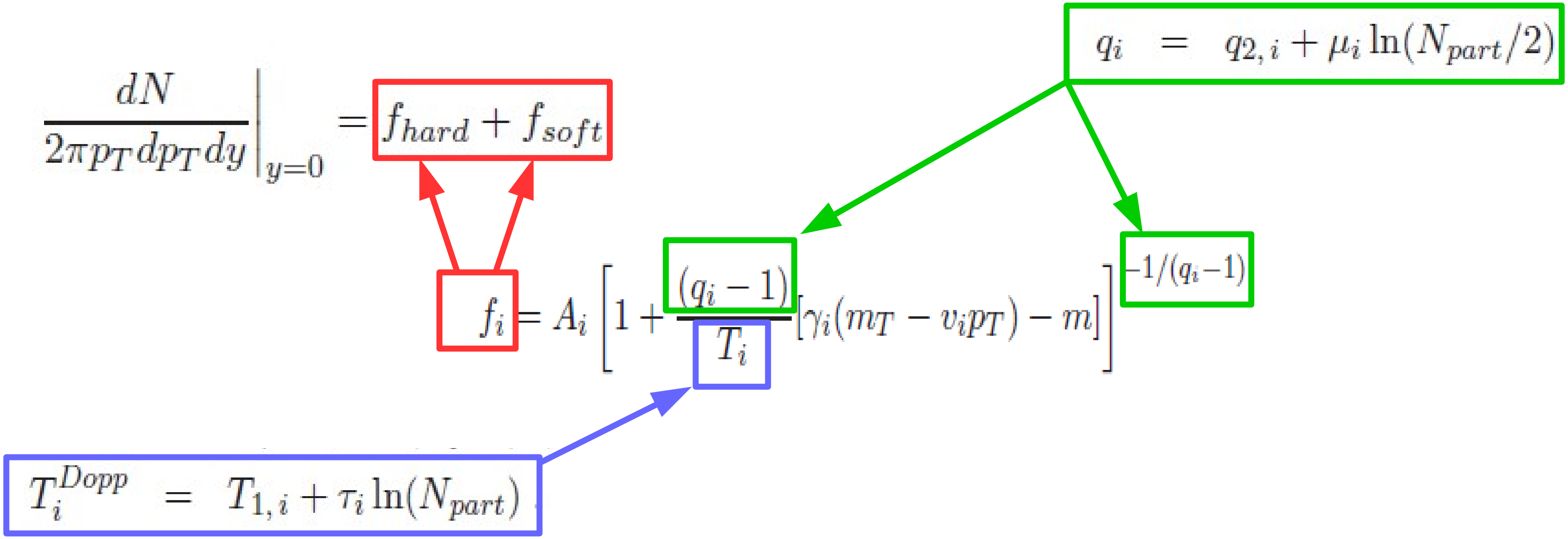


Parameters of the soft+hard model

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \boxed{f_{hard} + f_{soft}}$$

$$\boxed{f_i} = A_i \left[1 + \frac{(q_i - 1)}{T_i}[\gamma_i(m_T - v_i p_T) - m]\right]^{-1/(q_i-1)}$$

Parameters of the soft+hard model



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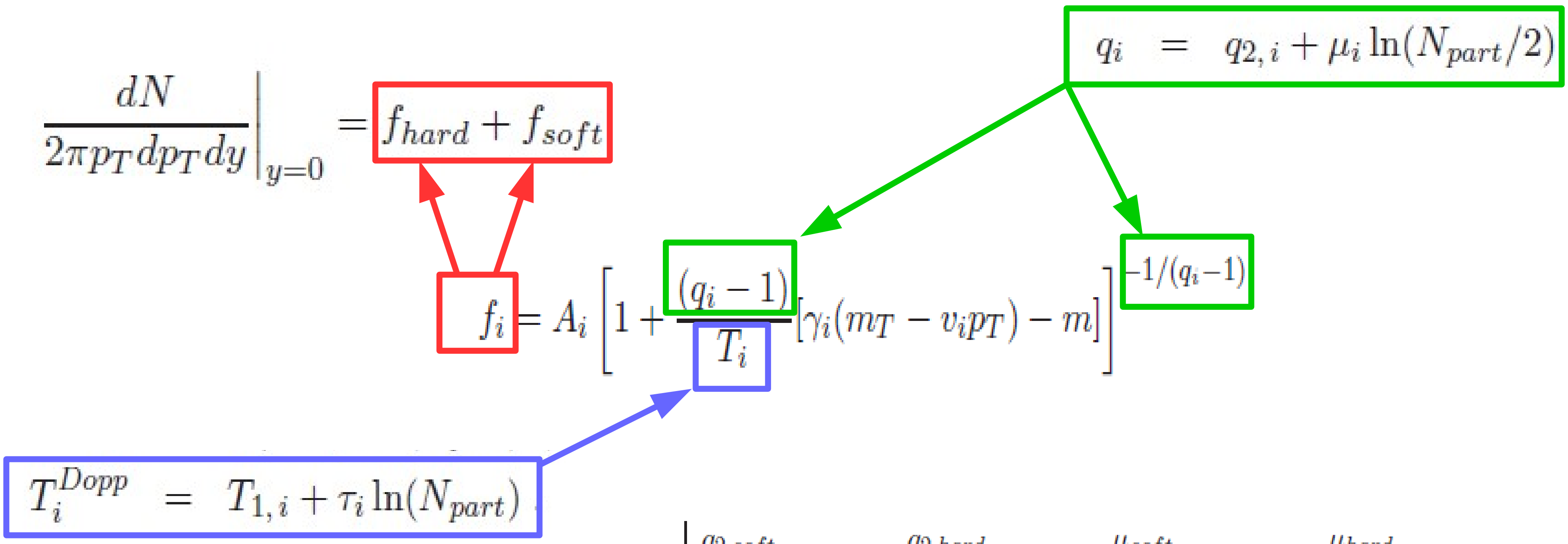
$$\boxed{q_i = q_{2,i} + \mu_i \ln(N_{part}/2)}$$

$$\boxed{f_i} = A_i \left[1 + \frac{\boxed{(q_i - 1)}}{\boxed{T_i}}[\gamma_i(m_T - v_i p_T) - m]\right]^{\boxed{-1/(q_i-1)}}$$

$$\boxed{T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})}$$

Parameters of the soft+hard model



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$$\boxed{T_i^{Dopp} = T_{1,i} + \tau_i \ln(N_{part})}$$

	$q_{2,soft}$	$q_{2,hard}$	μ_{soft}	μ_{hard}
CMS	1.058 ± 0.025	1.136 ± 0.001	-0.008 ± 0.005	0.005 ± 0.0003
ALICE	1.074 ± 0.018	1.131 ± 0.002	-0.009 ± 0.004	0.006 ± 0.0006
PHENIX	1.073 ± 0.016	1.100 ± 0.002	-0.005 ± 0.004	0.000 ± 0.0006

	T_1^{soft} [MeV]	T_1^{hard} [MeV]	τ_{soft} [MeV]	τ_{hard} [MeV]
CMS	310 ± 20	126 ± 5	9.9 ± 3.7	5.3 ± 0.8
ALICE	266 ± 16	194 ± 2	11.5 ± 2.9	-12.5 ± 0.5
PHENIX	165 ± 26	192 ± 20	9.3 ± 5.5	18.7 ± 4.6

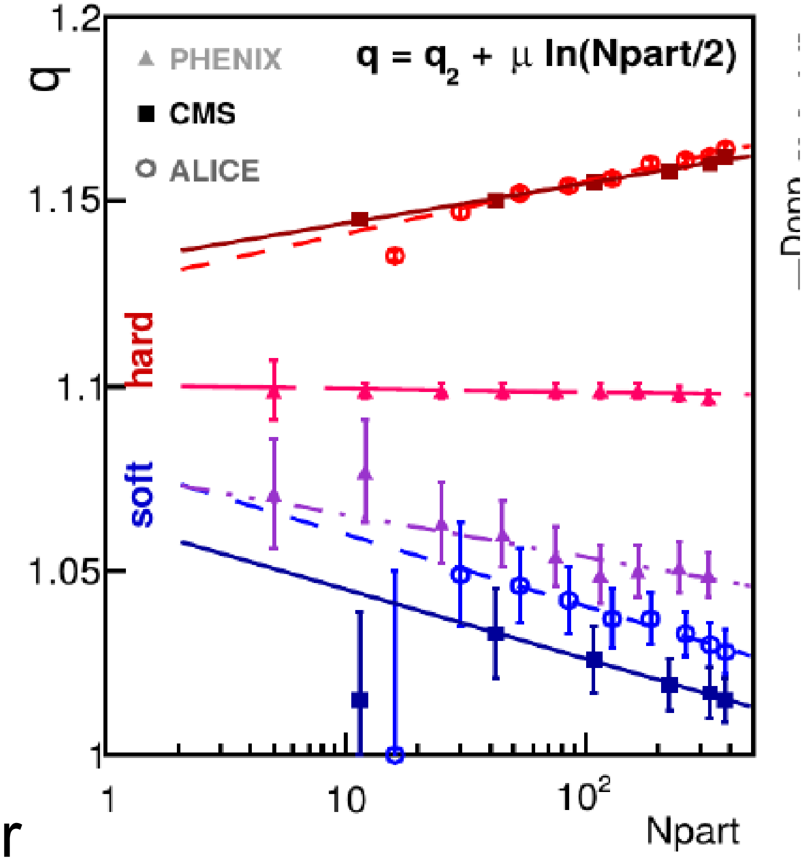
The N_{part} scaling of the *q* & *T* parameters

- Scaling of the $q_i \;=\; q_{2,i} + \mu_i \ln(N_{part}/2)$
    - Soft component, *q*→*1*
        - LHC: dereasing
        - RHIC: decreasing

      Higher N_{part} result BG statistics

    - Hard component, *q >1.1*
        - LHC: slight increasing
        - RHIC: constant

      Without the soft part result clearer non-extensive behaviour, like e^+e^-



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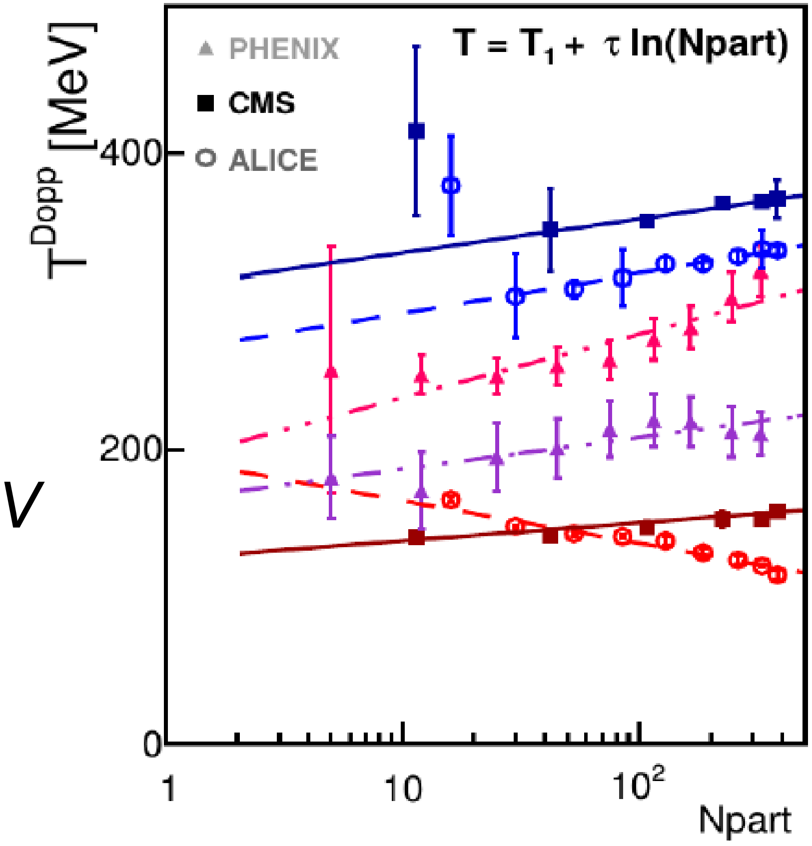
The N_{part} scaling of the *q* & *T* parameters

- Scaling of the $T_i^{Dopp} \;=\; T_{1,i} + \tau_i \ln(N_{part})$
  - Soft component, *T~200-400 MeV*
    - LHC: constant/increasing
    - RHIC: slightly increasing

  higher N_{part} results bit higher T

  - Hard component, *T ~100-300 MeV*
    - LHC: decreasing
    - RHIC: increasing

  N_{part} scalimg seems sensitive...



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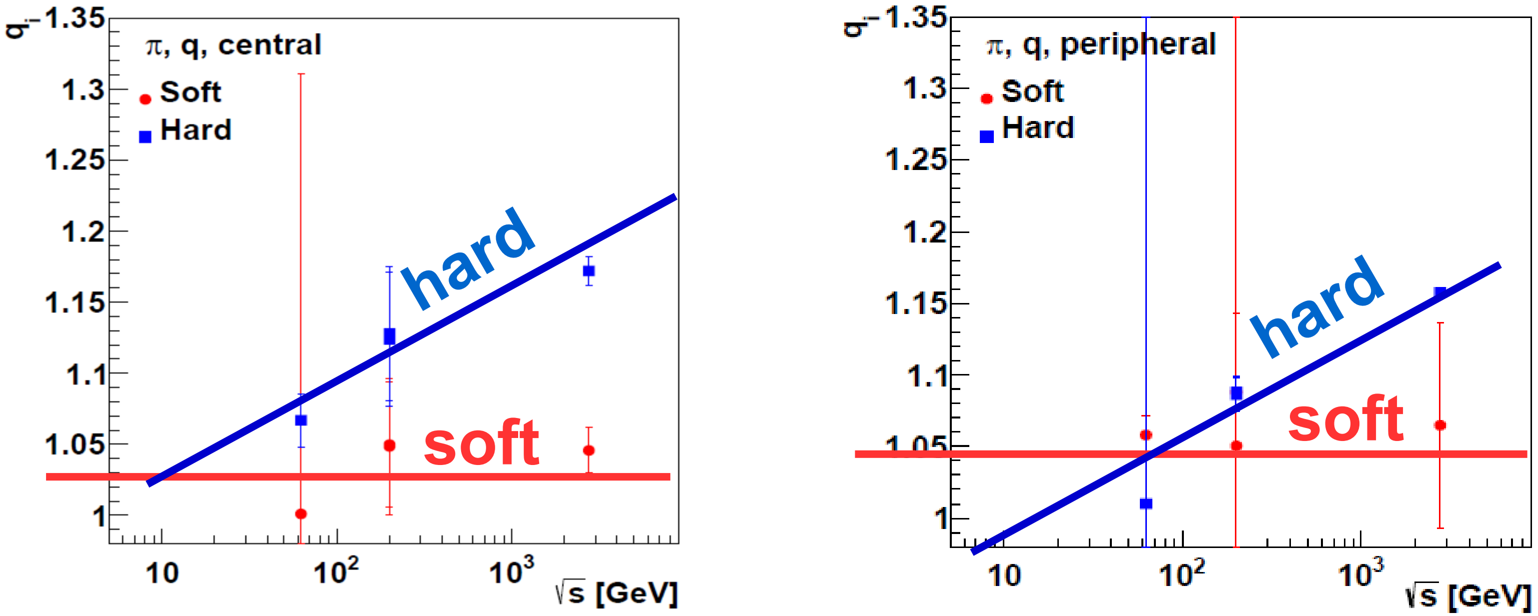
The c.m. energy dependence of *q* & *T*

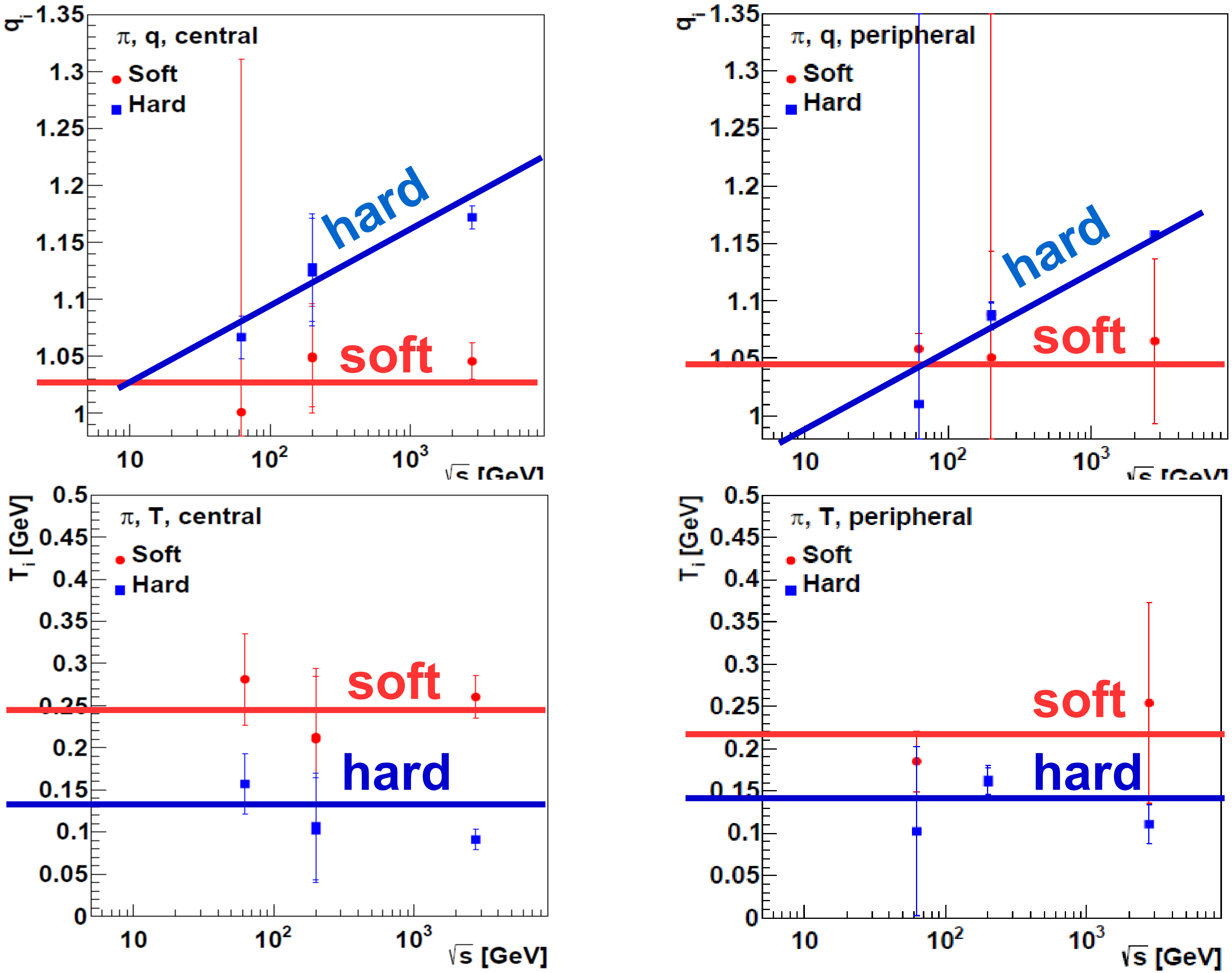
***q* measures non-extensivity**



The c.m. energy dependence of *q* & *T*

***q* measures non-extensivity**

***T* measures average *E* per multiplicity**

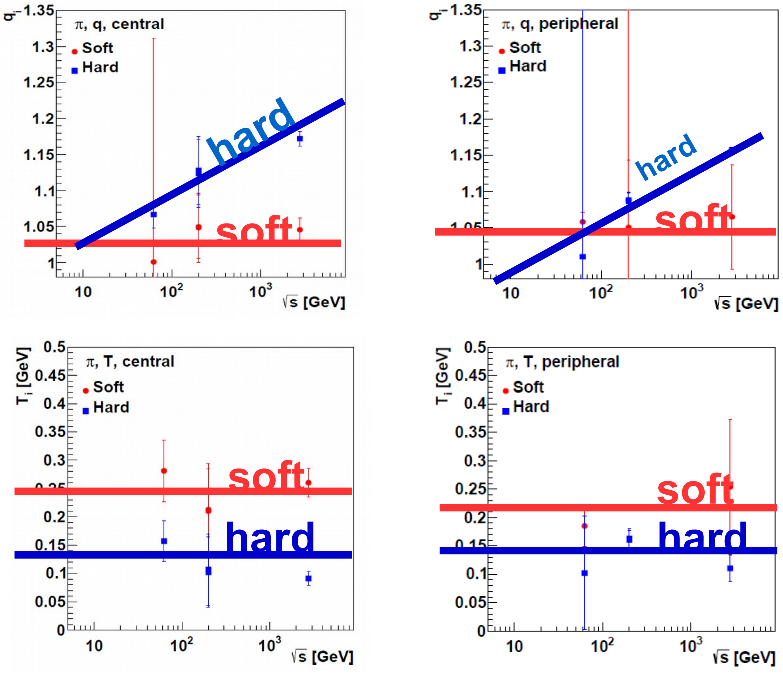


The c.m. energy dependence of *q* & *T*

- Energy dependence
  - Parameter q
    - HARD: clearly increasing
    - SOFT: no relevant change
  - Parameter *T*
    - HARD: central decreasing
peripheral const?

      $T_{centr} = T_{periph}$
    - SOFT: similar trend

      $\mathrm{T}_{\mathrm{centr}}$ ~100 MeV higher



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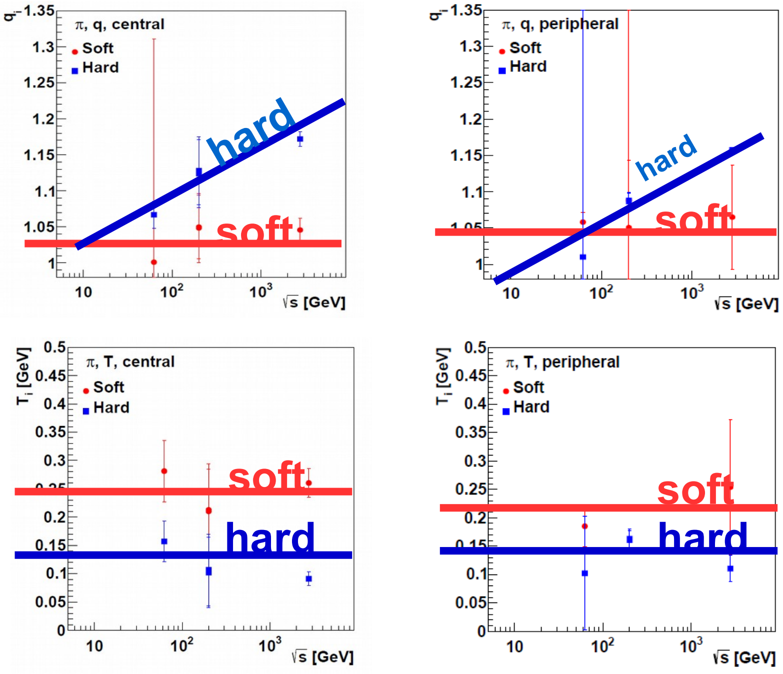

- Energy dependence
  - Parameters q & T present different values for centr./periph.
  - Above RHIC soft is BG-like and hard is more TP-like.

Can we connect this to asimuthal anisotropy?

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$p^0 \frac{dN}{d^3p}\bigg|_{y=0} = \int\limits_{-\infty}^{+\infty} d\zeta \int\limits_{0}^{2\pi} d\alpha\, f[u_\mu p^\mu] \longrightarrow \frac{dN}{2\pi p_T dp_T dy}\bigg|_{y=0} = \int\limits_{0}^{2\pi} \frac{d\varphi}{2\pi}\, p^0 \frac{dN}{d^3p}\bigg|_{y=0}$$

where we used parameters and assumptions

- Hadron momenum: $p^\mu = (m_T \cosh y, m_T \sinh y, p_T \cos\varphi, p_T \sin\varphi)$
- Cylindric symmetry: $u^\mu = (\gamma\cosh\zeta, \gamma\sinh\zeta, \gamma v\cos\alpha, \gamma v\sin\alpha)$

  where $\zeta = \frac{1}{2}\ln[(t+z)/(t-z)]$ and $\gamma = 1/\sqrt{1-v^2}$,
- Co-moving energy: $u_\mu p^\mu\big|_{y=0} = \gamma[m_T\cosh\zeta - v p_T\cos(\varphi-\alpha)]$
- Transverse flow: $v(\alpha) = v_0 + \sum\limits_{m=1}^{\infty} \delta v_m \cos(m\alpha) \equiv v_0 + \delta v(\alpha)$
- Taylor expansion: $f[u_\mu p^\mu]\big|_{y=0} = \sum\limits_{m=0}^{\infty} \frac{[\delta v(\alpha)]^m}{m!} \frac{\partial^m}{\partial v_0^m} f[u_\mu p^\mu]\Big|_{y=0}^{v(\alpha)=v_0}$

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi}\frac{d\varphi}{2\pi}\, p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty}\frac{a_m}{m!}\frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O\left(\delta v^2\right)$$

where $E(v_0) = \gamma_0(m_T - v_0 p_T)$ and $a_m = \int_0^{2\pi} d\alpha\, [\delta v(\alpha)]^m$.

- Azimuthal anisotropy:

$$v_n = \frac{\int_0^{2\pi} d\varphi \cos(n\varphi)\; p^0 \left.\frac{dN}{d^3p}\right|_{y=0}}{\int_0^{2\pi} d\varphi\; p^0 \left.\frac{dN}{d^3p}\right|_{y=0}} \approx \frac{\delta v_n \gamma_0^3}{2}\frac{(v_0 m_T - p_T) f'[E(v_0)]}{f[E(v_0)]} + O\left(\delta v^2\right)$$

  - Boltzmann–Gibbs: → $v_n^{\mathrm{BG}} \approx \frac{\delta v_n \beta \gamma_0^3}{2}(p_T - v_0 m_T) + O\left(\delta v^2\right)$

    $f \sim \exp[-\beta E(v_0)]$.

  - Tsallis–Pareto: → $v_n^{\mathrm{TS}} \approx \frac{\delta v_n \beta \gamma_0^3}{2}\frac{p_T - v_0 m_T}{1+(q-1)\beta\gamma_0(m_T - v_0 p_T)} + O\left(\delta v^2\right)$

    $f \sim [1+(q-1)\beta E(v_0)]^{-1/(q-1)}$

Connecting spectra and v_2

- Spectra originating from hadronic sources

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = \int_0^{2\pi} \frac{d\varphi}{2\pi}\, p^0 \left.\frac{dN}{d^3p}\right|_{y=0} = \sum_{m=0}^{\infty} \frac{a_m}{m!}\frac{\partial^m}{\partial v_0^m} f[E(v_0)] \approx f[E(v_0)] + O\left(\delta v^2\right)$$

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- Using the soft+hard model:

$$v_2 = \frac{w_{hard} f_{hard} + w_{soft} f_{soft}}{f_{hard} + f_{soft}}$$ with the coefficient $$w_i = \frac{\delta v_i \gamma_i^3}{2T_i}\frac{p_T - v_i m_T}{1 + \frac{q_i - 1}{T_i}\left[\gamma_i(m_T - v_i p_T) - m\right]}$$

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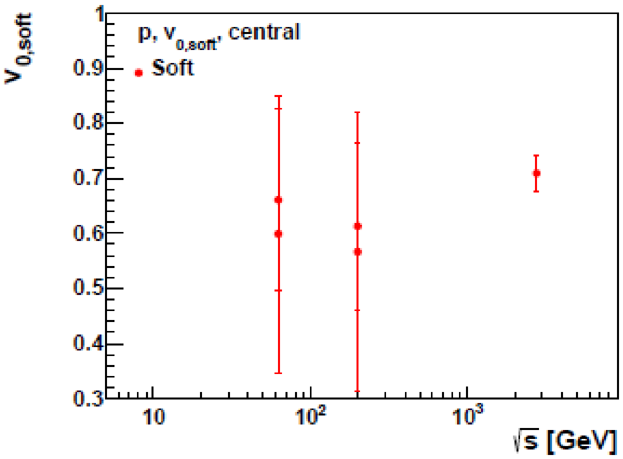
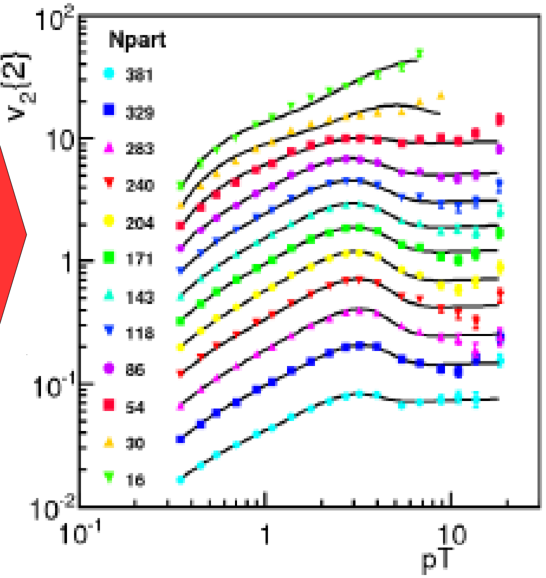
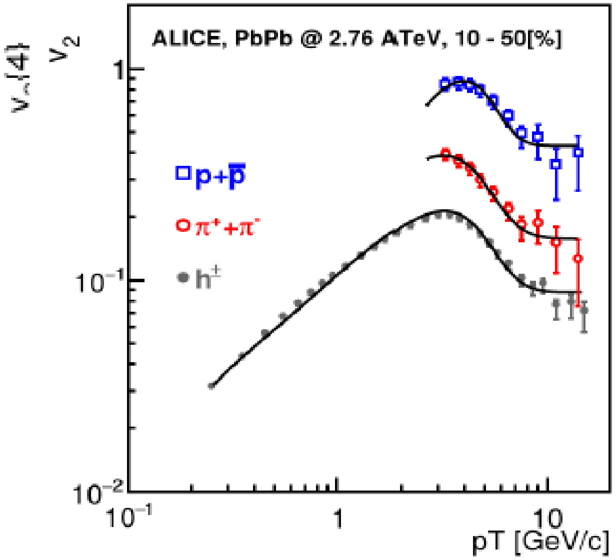
- Assuming v_0 only for the soft component v_2 can be obtained







arXiv:1405.3963, 1501.02352, 1501.05959

J.Phys.CS 612 (2015) 012048

SUMMARY

- Non-extensive statistical approach in e^+e^- & pp
  - Obtained Tsallis/Rényi entropies from the first principles.
  - Providing phyiscal meaning of $q=1-1/C + \Delta T^2/T^2$
  - *Boltzmann Gibbs limit* $C \rightarrow \infty$ & $\Delta T^2/T^2 \rightarrow 0$ $(q \rightarrow 1)$,
  - *Tsallis – Pareto fits on spectra in* e^+e^-, *pp*
  - *Not working for larger system, like pA, AA and no flow.*

- Application of 'soft+hard' model in AA
  - Tsallis – Pareto + Exp does not working.
  - Double Tsallis – Pareto measures non-extensitivity
  - SOFT: $q \rightarrow 1$, suggest Boltzmann Gibbs (QGP)
  - HARD: $q > 1.1$, Tsallis – Pareto like
  - Asimuthal anisotropy can be obtained too.

ADVERTISEMENT

11th international workshop on High-pT Physics in the RHIC & LHC Era

BNL, USA in April 12-15, 2016.

Topics

- Nuclear modifications of the parton distribution functions.
- High-pT jet production in pp, pA and AA.
- High-pT parton propagation in matter.
- Nuclear modifications of the fragmentation functions.
- Correlations of jets and leading particles.
- Direct photons, heavy flavor, quarkonia.
- Multiparticle effects (net-charge, net-proton, p-p ridge).

High pT Physics in the RHIC-LHC Era

RIKEN BNL Research Center Workshop
April 12-15, 2016 at Brookhaven National Laboratory

High pT Physics in the RHIC-LHC Era

General Workshop Registration (Deadline: March 1, 2016)
Additional BNL Guest Registration (Deadline: March 1, 2016) ⓘ

Begin Workshop Registration

Motivation

This will be the 11th Workshop in the series which began at the ECT* in Trento, Italy in September 2006 as a "Workshop on Jet Physics in Heavy Ion Collisions at the LHC", continued in Jyvaskyla, Finland in February 2007, Tokaj, Hungary in March 2008, Prague, Czech Republic in 2009, etc. This will be the first meeting in the United States.

The purpose of this workshop is to offer an opportunity for both experimentalists and theorists to get together and discuss the latest experimental results from RHIC and LHC and

Workshop Dates
April 12-15, 2016

Workshop Venue
Brookhaven National Laboratory
Upton, NY 11973 USA

Workshop Location
Physics Department (Bldg. 510)
Large Seminar Room

Map and Directions
To Event | To BNL

Workshop Coordinator
Pam Esposito
Bus: 631-344-3097
Fax: 631-344-4047

Information

For more details see https://www.bnl.gov/pt2016/

Please reply to: HighPT-workshop@cern.ch by January 20.

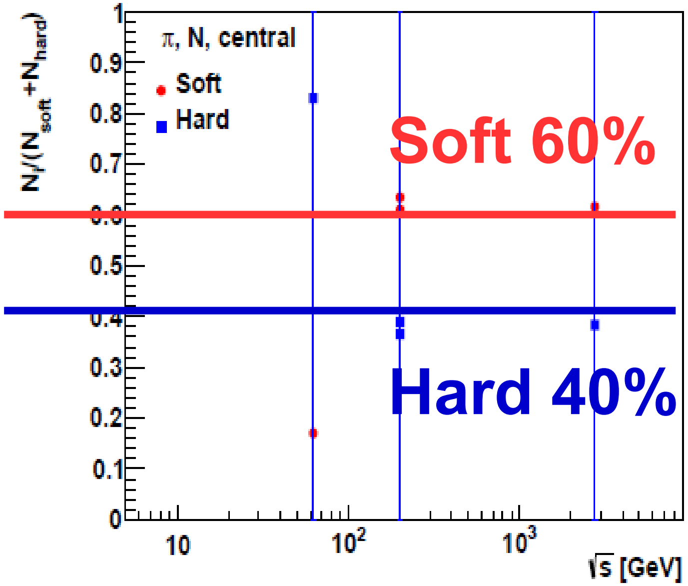
Workshop organizers

Yasuyuki Akiba, Gergely Gábor Barnaföldi, Megan Connors, Gabor David, Andreas Morsch, Takao Sakaguchi, Jan Rak, Michael J. Tannenbaum (local organizer)

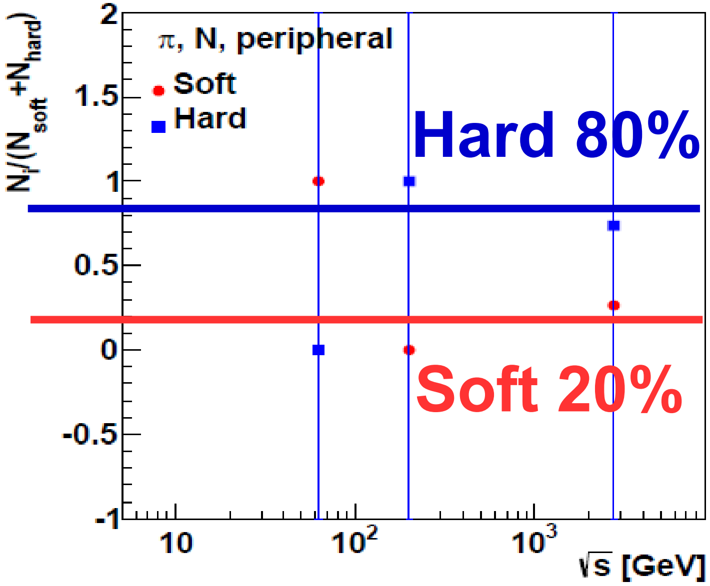
BACKUP

The c.m. Energy Dependence of N_{soft} & N_{hard}

- Energy dependence N_i/N_{tot}
  - Central
    - LHC: HARD 40% + SOFT 60%
    - RHIC: HARD 80% + SOFT 20%



  - Peripheral
    - LHC: HARD 80% + SOFT 20%
    - RHIC: HARD 10% + SOFT 90%



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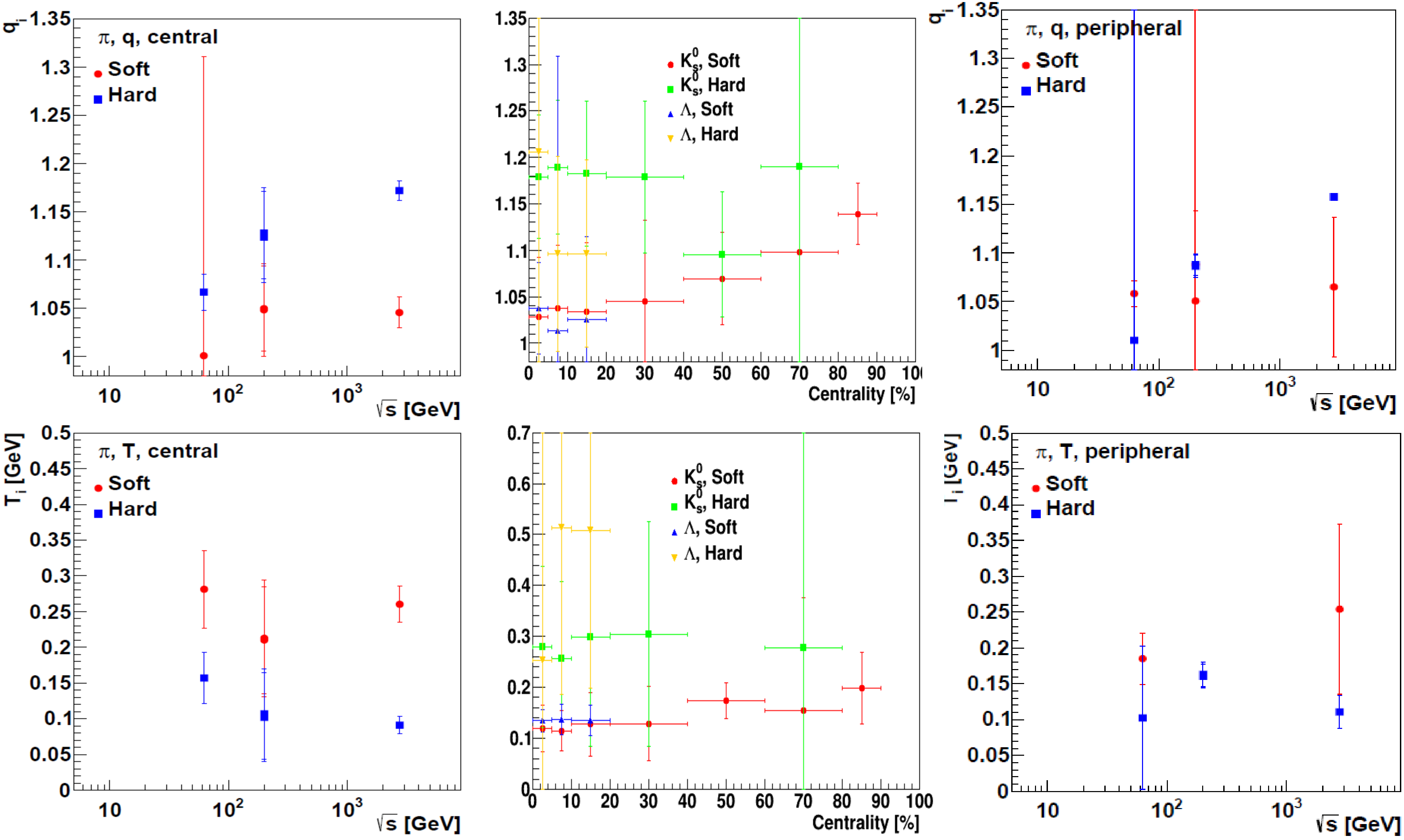


Related publications..

1. arXiv:1409.5975: Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle

2. arXiv:1405.3963 Disentangling Soft and Hard Hadron Yields in PbPb Collisions at $\sqrt{s_{NN}}$ = 2.76 ATeV

3. arXiv:1405.3813 New Entropy Formula with Fluctuating Reservoir, Physica A (in Print) 2014

4. arXiv:Statistical Power-Law Spectra due to Reservoir Fluctuations

5. arXiv:1209.5963 Nonadditive thermostatistics and thermodynamics, Journal of Physics, Conf. Ser. V394, 012002 (2012)

6. arXiv:1208.2533 Thermodynamic Derivation of the Tsallis and Rényi Entropy Formulas and the Temperature of Quark-Gluon Plasma, EPJ A 49: 110 (2013)

7. arXiv:1204.1508 Microcanonical Jet-fragmentation in proton-proton collisions at LHC Energy, Phys. Lett. B, 28942 (2012)

8. arXiv:1101.3522 Pion Production Via Resonance Decay in a Non-extensive Quark-Gluon Medium with Non-additive Energy Composition Rule

9. arXiv:1101.3023 Generalised Tsallis Statistics in Electron-Positron Collisions, Phys.Lett.B701:111-116,2011

10. arXiv:0802.0381 Pion and Kaon Spectra from Distributed Mass Quark Matter, J.Phys.G35:044012,2008

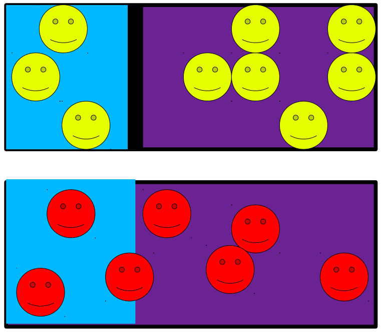
General derivation as inproved canonical

The story is about...

- Two body thermodynamics:

  1 subsystem *(E_1)* +one reservoir *($E-E_1$)*

- Finite system, finite energy → microcanonical description
  - microcanonical $\sum_j \epsilon_j = E$
  - canonical $\sum_j <\epsilon_j> = E$



- Maximize a monotonic function of the Boltzmann-Gibbs entropy, *L(S)* (0^{th} law of thermodynamics)

- Taylor expansion of the *L(S)* = max, principle beyond $-\beta E$

Description of a system & reservoir

- For generalized entropy function $L(S_{12}) = L(S_1) + L(S_2)$
- In order to exist β of the system $L\left(S(E_1)\right) + L\left(S(E - E_1)\right) = \max$

TS Biró P. Ván: Phys Rev. E84 19902 (2011)

- Thermal contact between system (E_1) & reservoir $(E\text{-}E_1)$, requires to eliminate E_1 :

$$\begin{aligned} \beta_1 &= L'\left(S(E_1)\right) \cdot S'(E_1) \\ &= L'\left(S(E - E_1)\right) \cdot S'(E - E_1) \end{aligned}$$

- This is usually handled in canonical limit, but now, we keep higher orders in the Taylor-expansion in E_1/E

$$\beta_1 = L'\left(S(E)\right) \cdot S'(E) \boxed{- \left[S'(E)^2 L''(S(E)) + S''(E) L'(S(E))\right] E_1 + \ldots}$$

Description of a system & reservoir

- Assuming $\beta_1=\beta$, the Lagrange multiplicator become familiar for us:

$$\beta = L'\left(S(E)\right)\cdot S'(E) = L'(S)\cdot\frac{1}{T}$$

- To satisfy this, need simply to solve

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}$$

- *Universal Thermostat Independence (UTI)* Principle: l.h.s. must be as an S-independent constant for solving $L(S)$,

$$\frac{L''(S)}{L'(S)} = a$$

- Based on $L(S) \rightarrow S$ for small S, coming from 3rd law of the thermodynamics $L'(0)=1$ and $L(0)=0$

$$L(S) = \frac{e^{aS}-1}{a}$$

- EoS derivatives do have physical meaning:

$$\boxed{\begin{aligned} S'(E) &= 1/T \\ S''(E) &= -1/CT^2 \end{aligned}}$$

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- Simly the heat capacity of the reservoir:

$$\boxed{a = 1/C}$$

From two system to many...

- Analogue to Gibbs ensamble generalize

$$S = -\sum_i P_i \ln P_i \quad \rightarrow \quad L(S) = \sum_i P_i L(-\ln P_i)$$

-
- The *L*-additive form of a generally non-additive entropy, given by:

$$L(S(E_1)) - \beta E_1 = \frac{1}{a}\left(\mathrm{e}^{aS(E_1)} - 1\right) - \beta E_1 = \max.$$

- Introducing $a = 1/C(E) \quad \rightarrow \quad L(S(E_1)) = L(-\ln P_1) = \frac{1}{a}\left(P_1^{-a} - 1\right)$
- we need to maximize: $\frac{1}{a}\sum_i \left(P_i^{1-a} - P_i\right) - \beta \sum_i P_i E_i - \alpha \sum_i P_i = \max.$

which, results Tsallis:

$$S_{\mathrm{Tsallis}} := L(S) = \frac{1}{q-1}\sum_i \left(P_i - P_i^q\right)$$

and its inverse Rényi:

$$S_{\mathrm{Rényi}} := S = \frac{1}{1-q}\ln \sum_i P_i^q$$

The temperature slope

- Taking P_i weights of system, E_i , results cut power law:

$$P_i = \left(Z^{1-q} + (1-q)\frac{\beta}{q} E_i \right)^{\frac{1}{q-1}} = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}\, e^{S/C}}{C-1} \frac{E_i}{T} \right)^{-C}$$

- Partition sum is related to Tsallis entropy, $L(S_1)$ and E_1

$$\ln_q Z := C\left(Z^{1/C} - 1\right) = L\left(S_1\right) - \frac{1}{1-1/C}\beta E_1$$

- In $C \rightarrow \infty$ limit, the inverse log slope of the energy distribution:

$$T_{\text{slope}}(E_i) = \left(-\frac{d}{dE_i} \ln P_i \right)^{-1} = T_0 + E_i/C, \quad \text{with} \quad T_0 = T e^{-S/C} Z^{1/C} (1-1/C)$$