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Comments on the Polymer field quantization.

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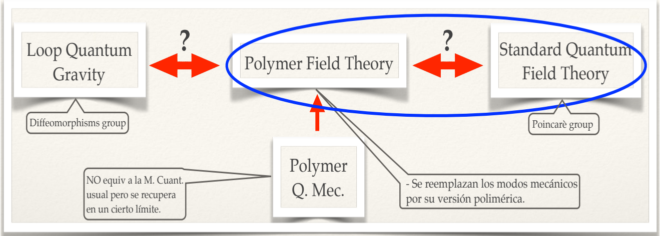
Map of ideas, Motivation and Outline



Steps...

- Algebraic construction of Polymer Field Theory.
- Derivation of the Hilbert space features.
- Symmetry Group of PFT.
- Comparison with the Standard Quantum Field Theory

OUTLINE

- Polymer Quantum Mechanics (PQM): a summary.
  — Idea.
  — Weyl algebra.
  — Kinematic analysis.
  — Algebraic analysis.
- Polymer Field Theory (PFT): algebraic analysis.
  — Symmetry group.
  — Poly. Hilbert space.
  — Comparison with the Fock representation.
- Conclusions.

Polymer Quantum Mechanics

Idea

- Polymer QM is a model that implements some of the techniques used in LQG.
- Its peculiarity resides in choosing a Hilbert space different from that used in the usual quantization.
- As a result there is no momentum operator and therefore a hamiltonian-type operator is proposed in order to get a dynamical description.
- Such a description requires insert, by hand, a length scale parameter which in the limit of small values conduces to the usual amplitudes of the operators, i.e., the standard Schröedinger representation.

- The Weyl algebra of the mechanical system is formed by the elements of the form $W(\mu,\lambda) \sim e^{\frac{i}{\hbar}(\lambda\widehat{x}-\mu\widehat{p})}$ with multiplication

$$W(\mu_1,\lambda_1)\cdot W(\mu_2,\lambda_2) = e^{-\frac{i}{2\hbar}\Omega((\mu_1,\lambda_1),(\mu_2,\lambda_2))}W(\mu_1+\mu_2,\lambda_1+\lambda_2)$$

- The standard polarizations are

$$W(\mu,\lambda)\Psi(x) = e^{i\frac{\mu\lambda}{2\hbar}}e^{i\frac{\lambda x}{\hbar}}\Psi(x+\mu), \quad \Psi \in L^2(\mathbb{R},dx)$$

$$W(\mu,\lambda)\Psi(p) = e^{-i\frac{\mu\lambda}{2\hbar}}e^{-i\frac{\mu}{\hbar}p}\Psi(p+\lambda), \quad \Psi \in L^2(\mathbb{R},dp)$$

- The Weyl generator can be decomposed as the product of the generators of two Abelian algebras $W(\mu,\lambda) = e^{\frac{i}{2}\lambda\mu}U(\lambda)V(\mu)$

$$U(\lambda)\Psi(x) = e^{i\frac{\lambda x}{\hbar}}\Psi(x), \quad V(\mu)\Psi(x) = \Psi(x+\mu)$$

$$U(\lambda)\Psi(p) = \Psi(p+\lambda), \quad V(\mu)\Psi(p) = e^{-i\frac{\mu p}{\hbar}}\Psi(p)$$

x-Rep. (polymer)

$$\tilde{\Psi} \in L^2(\mathbb{R}_d, d\mu_c)$$

$$\tilde{\Psi}(x) = \sum_j \tilde{\Psi}_{x_j} \delta_{x,x_j}$$

$$\langle \tilde{\Psi} | \tilde{\Phi} \rangle = \sum_n \tilde{\Psi}^*_{x_n} \tilde{\Phi}_{x_n}$$

$$U(\lambda)\tilde{\Psi}(x) = e^{i\lambda x}\tilde{\Psi}(x),$$

$$V(\mu)\tilde{\Psi}(x) = \tilde{\Psi}(x+\mu)$$

p-Rep. (polymer)

$$\tilde{\Psi} \in L^2(\overline{\mathbb{R}}, d\mu_{Bohr})$$

$$\tilde{\Psi}(p) = \sum_j \tilde{\Psi}_{x_j} e^{ipx_j/\hbar}$$

$$\langle \tilde{\Psi} | \tilde{\Phi} \rangle = \lim_{L\to\infty} \frac{1}{2L} \int_{-L}^{L} \tilde{\Psi}^*(p)\tilde{\Phi}(p)dp$$

$$U(\lambda)\tilde{\Psi}(p) = \tilde{\Psi}(p+\lambda)$$

$$V(\mu)\tilde{\Psi}(p) = e^{-i\frac{\mu p}{\hbar}}\tilde{\Psi}(p)$$

There is no momentum-operator $\widehat{p}$ on any of these singular representations.

x-Rep. (polymer)

$$\langle \delta_{x,x_i} | V(\mu)\delta_{x,x_i} \rangle = \delta_{x_i,x_i-\mu}$$

$$\widehat{x}\,\tilde{\Psi}(x) = x\,\tilde{\Psi}(x)$$

p-Rep. (polymer)

$$\langle e^{ipx_i/\hbar} | V(\mu) e^{ipx_i/\hbar} \rangle = \delta_{x_i,x_i-\mu}$$

$$\widehat{x}\,\tilde{\Psi}(p) = \frac{\hbar}{i}\frac{d}{dp}\tilde{\Psi}(p)$$

The representation of generator V is not weakly continuous, therefore, accordingly to von Neumann's theorem it is not equivalent(unitarily) to the Schrödinger representation.

Complex structure and algebraic state

$$J(x,p) = (\frac{d^2}{\hbar^2}p, -\frac{1}{d^2}x)$$

$$\omega_d(\widehat{W}(\mu,\lambda)) = e^{-\frac{1}{2}\Omega((\mu,\lambda);J(\mu,\lambda))} = e^{-\frac{1}{2}\left(\frac{1}{d^2}\mu^2+\frac{d^2}{\hbar^2}\lambda^2\right)}.$$

Singular polarizations

$$\omega_x(\widehat{W}(\mu,\lambda)) := \lim_{d\to 0}\omega_d(\widehat{W}(\mu,\lambda)) = \delta_{\mu,0}, \quad \text{x-polarization}$$
$$\omega_p(\widehat{W}(\mu,\lambda)) := \lim_{1/d\to 0}\omega_d(\widehat{W}(\mu,\lambda)) = \delta_{\lambda,0}. \quad \text{p-polarization}$$

Standard Quantization

Fourier decomposition and Poincarè invariant state

$$J^{(P)}(\varphi, \pi) = (-(-\Delta + m^2)^{-1/2}\pi, (-\Delta + m^2)^{1/2}\varphi).$$

$$\Omega((\varphi_1, \pi_1), (\varphi_2, \pi_2)) := \int_{\mathbb{R}^3} d^3x \, (\pi_1\varphi_2 - \varphi_1\pi_2),$$

$$\omega_{J^{(P)}}(\widehat{W}(\lambda)) = e^{-\frac{1}{4}\Omega(\lambda, J^{(P)}\lambda)}, \quad \lambda := (\varphi, \pi)$$

$$\Phi(t, \vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[\frac{(1+i)}{2} q_{\vec{k}}(t) + \frac{(1-i)}{2} q_{-\vec{k}}(t) \right] e^{i\vec{k}\cdot\vec{x}}$$

$$\Pi(t, \vec{x}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \left[\frac{(1+i)}{2} p_{\vec{k}}(t) + \frac{(1-i)}{2} p_{-\vec{k}}(t) \right] e^{i\vec{k}\cdot\vec{x}}$$

$$\omega(\widehat{W}(\{q_{\vec{k}}, p_{\vec{k}}\})) = e^{-\frac{1}{2}\sum_{\vec{k}} \Omega^{(\vec{k})}((q_{\vec{k}}, p_{\vec{k}}), J_{\vec{k}}(\tilde{q}_{\vec{k}}, \tilde{p}_{\vec{k}}))} = \prod_{\vec{k}} e^{-\frac{1}{2}\left(\frac{1}{\omega_{\vec{k}}} q_{\vec{k}}^2 + \omega_{\vec{k}} p_{\vec{k}}^2\right)}.$$

$$\omega_{\varphi-pol}(\widehat{W}(\{q_{\vec{k}},p_{\vec{k}}\})) = \prod_{\vec{k}} \delta_{q_{\vec{k}},0} = \delta_{\{q_{\vec{k}}\},0} = \delta_{\varphi,0},$$

$$\omega_{\pi-pol}(\widehat{W}(\{q_{\vec{k}},p_{\vec{k}}\})) = \prod_{\vec{k}} \delta_{p_{\vec{k}},0} = \delta_{\{p_{\vec{k}}\},0} = \delta_{\pi,0}.$$

$\varphi-$polarization

$$\omega_{\varphi-pol}(\widehat{W}(\varphi,\pi)) = \delta_{\varphi,0} \qquad \Rightarrow$$

$$\omega_{\varphi-pol}(\widehat{U}(\pi)) = 1, \omega_{\varphi-pol}(\widehat{V}(\varphi)) = \delta_{\varphi,0},$$

$\pi-$polarization

$$\omega_{\pi-pol}(\widehat{W}(\varphi,\pi)) = \delta_{\pi,0} \qquad \Rightarrow$$

$$\omega_{\pi-pol}(\widehat{U}(\pi)) = \delta_{\pi,0}, \quad \omega_{\pi-pol}(\widehat{V}(\varphi)) = 1.$$

Consider a foliation of $\mathbb{R}^4$ given by e_t. A diffeomorphism $f \in \text{Diff}(\mathbb{R}^4)$ defines a new foliation $e'_{t'} := f \circ e_t$.

$$
\begin{aligned}
\varphi(t;\vec{x}) \mapsto \varphi'(t';\vec{x}') &= \varphi(t;\vec{x}) = \varphi(t(t';\vec{x}');\vec{x}(t';\vec{x}')), \\
\pi(t;\vec{x}) \mapsto \pi'(t';\vec{x}') &= \frac{\sqrt{q'}}{N'}\left[\left(\frac{\partial t}{\partial t'} - N'^a \frac{\partial t}{\partial x'^a}\right)\pi(t;\vec{x}) + \right. \\
&\left. + \left(\frac{\partial \vec{x}}{\partial t'} - N'^a \frac{\partial \vec{x}}{\partial x'^a}\right)\cdot \vec{\nabla}\varphi(t;\vec{x})\right].
\end{aligned}
$$

Polymer Field Theory (PFT)

Diffeomorphism acting on the labels of $\mathcal{W}$

$$\varphi(t;\vec{x}) = \tilde{A}\,\varphi(0;\vec{x}(\Delta t';\vec{x}')) + \tilde{B}\,\pi(0;\vec{x}(t';\vec{x}')).$$
$$\pi(t;\vec{x}) = \tilde{C}\,\varphi(0;\vec{x}(t';\vec{x}')) + \tilde{D}\,\pi(0;\vec{x}(t';\vec{x}')),$$

$$\tilde{\Theta} := -\Delta + m^2, \quad \tilde{A} := \cos\left[t\sqrt{\tilde{\Theta}}\right], \quad \tilde{B} := \tilde{\Theta}^{-\frac{1}{2}} \sin\left[t\sqrt{\tilde{\Theta}}\right],$$
$$\tilde{C} := -\left(\frac{\partial t}{\partial t'}\right)\tilde{\Theta}^{\frac{1}{2}} \sin\left[t\sqrt{\tilde{\Theta}}\right] + \left(\frac{\partial \vec{x}}{\partial t'}\right)\frac{\partial}{\partial \vec{x}} \cos\left[t\sqrt{\tilde{\Theta}}\right],$$
$$\tilde{D} := \left(\frac{\partial t}{\partial t'}\right)\cos\left[t\sqrt{\tilde{\Theta}}\right] + \left(\frac{\partial \vec{x}}{\partial t'}\right)\frac{\partial}{\partial \vec{x}}\tilde{\Theta}^{-\frac{1}{2}} \sin\left[t\sqrt{\tilde{\Theta}}\right],$$

Polymer Field Theory (PFT)

Automorphism generated by diffeomorphisms

The group Diff($\mathbb{R}^4$) induces an automorphism α_f in the algebra $\mathcal{W}$. The automorphism $\alpha_f : \mathcal{W} \to \mathcal{W}$; $W(\varphi_0; \pi_0) \mapsto W(\varphi'; \pi')$ can be defined as

$$W(\varphi'; \pi') :=$$

$$= W\left(\widehat{A}\varphi_0 + \widehat{B}\pi_0; \frac{\sqrt{q'}}{N'}\left[\left(\frac{\partial t}{\partial t'} - N'^a \frac{\partial t}{\partial x'^a}\right)\widehat{C} + \left(\frac{\partial x^b}{\partial t'} - N'^a \frac{\partial x^b}{\partial x'^a}\right)\partial_b \widehat{A}\right]\varphi_0 + \right.$$

$$\left. + \frac{\sqrt{q'}}{N'}\left[\left(\frac{\partial t}{\partial t'} - N'^a \frac{\partial t}{\partial x'^a}\right)\widehat{D} + \left(\frac{\partial x^b}{\partial t'} - N'^a \frac{\partial x^b}{\partial x'^a}\right)\partial_b \widehat{B}\right]\pi_0\right),$$

Only $t' = t$, $\vec{x}' = \vec{x}'(\vec{x})$ (tangential diffeomorphisms) are such that

$$\omega_{\varphi-pol}(\alpha_f(W(\varphi_0, \pi_0))) = \omega_{\varphi-pol}(W(\varphi_0, \pi_0))$$

$$\omega_{\pi-pol}(\alpha_f(W(\varphi_0, \pi_0))) = \omega_{\pi-pol}(W(\varphi_0, \pi_0))$$

Symmetry group

- The subgroup $\text{Diff}^T(\mathbb{R}^4)$ of tangential diffeomorphisms preserve the value of the algebraic states $\omega_{\varphi-pol}$ and $\omega_{\pi-pol}$.
- $\text{Diff}^T(\mathbb{R}^4)$ is the symmetry group of the PFT.
- The Lorentz boosts are excluded from the symmetries of the PFT: (*i*) $\text{Diff}(\mathbb{R}^4) \supset \mathcal{P} \supset \mathcal{E}$ (*ii*) $\mathcal{E} \subset \text{Diff}^{(T)}(\mathbb{R}^4) \subset \text{Diff}(\mathbb{R}^4)$.

$$\omega_{J^{(P)}}(\widehat{U}(f)) = e^{-\frac{1}{4}\int d^3x\, f(-\Delta+m^2)^{-1/2} f} = \langle\overline{\Psi}_0|\,\widehat{U}(f)\Psi_0\rangle,$$

$$\langle\Psi_0|\,\widehat{U}(f)\Psi_0\rangle = \int_{\mathcal{S}'^{(0)}} d\mu_G(\varphi')\overline{\Psi}_0\,\widehat{U}(f)\Psi_0, \qquad \mathcal{H}_\varphi = L^2(\mathcal{S}'^{(0)}, d\mu_G)$$

$$(\widehat{U}(f)\cdot\Psi)[\varphi'] = e^{-i\int_{\mathbb{R}^3} d^3\vec{x}\,\varphi' f}\Psi[\varphi'],$$

$$d\mu_G(\varphi') = e^{-\int_{t_0} d^3\vec{x}\,\varphi'(-\Delta+m^2)^{1/2}\varphi'}\mathcal{D}\varphi',$$

$$(\widehat{\pi}[g]\cdot\Psi)[\varphi'] = -i\int_{t_0} d^3\vec{x}\left[g\frac{\delta}{\delta\varphi'} - \varphi'(-\Delta+m^2)^{1/2}g\right]\Psi[\varphi'].$$

Polymer Field Theory (PFT)

Viqar construction of PFT Hilbert space

$$\omega_{\varphi-pol}(\widehat{W}(\{q_{\vec{k}}, p_{\vec{k}}\})) = \prod_{\vec{k}} \delta_{q_{\vec{k}},0} = \delta_{\{q_{\vec{k}}\},0} = \delta_{\varphi,0},$$

$$\omega_{\pi-pol}(\widehat{W}(\{q_{\vec{k}}, p_{\vec{k}}\})) = \prod_{\vec{k}} \delta_{p_{\vec{k}},0} = \delta_{\{p_{\vec{k}}\},0} = \delta_{\pi,0}.$$

Reducible representation and the good' one?

$$\mathcal{H}_{\varphi-pol} = \prod_{\vec{k}} \mathcal{H}^{(\vec{k})}_{q-poly}, \qquad \mathcal{H}^{(\vec{k})}_{q-poly} = L^2(\mathbb{R}_d, d\mu_c),$$

$$\mathcal{H}_{\pi-pol} = \prod_{\vec{k}} \mathcal{H}^{(\vec{k})}_{p-poly}, \qquad \mathcal{H}^{(\vec{k})}_{p-poly} = L^2(\overline{\mathbb{R}}, d\mu_{Bohr}).$$

Select a different abelian subalgebra within the Weyl algebra denoted by $\mathcal{V}$ and with generators of the form

$$V_g(\pi) := W(g,0) = e^{-i\int_{t_0} d^3\vec{x} g\pi}.$$

Apply Gel'fand spectral theory:

$$\breve{\cdot} : \mathcal{V} \to \breve{\mathcal{V}}, v \mapsto \breve{v} := C(X(v)), \qquad X \in \Delta(\mathcal{V}), \qquad \breve{\mathcal{V}} = C(\Delta(\mathcal{V}))$$

$$V_g \in \breve{\mathcal{V}}, \qquad \mathcal{H}_{PF} = L^2(\Delta(\mathcal{V}), d\mu_{PF}), \qquad (\widehat{V}_g\Psi)[X] := X(V_g)\Psi[X],$$

$d\mu_{PF}$ is a regular Borel measure on $\Delta(\mathcal{V})$ and is given by

$$\int_{\Delta(\mathcal{V})} d\mu_{PF}(X) X(V_g(\pi)) = \omega(\breve{V}_g(\pi)) = \delta_{g,0}.$$

PF is thus given in the momentum polarization and $\widehat{\pi}$ does not exist.

Polymer Field Theory (PFT)

Fock representation: momentum polarization

$$\omega_{J(P)}(\widehat{V}(g)) = e^{-\frac{1}{4}\int d^3x\, g(-\Delta+m^2)^{1/2} g} = \langle \overline{\Psi}_0 |\, \widehat{V}(g)\Psi_0\rangle,$$

$$\langle \Psi_0 |\, \widehat{V}(g)\Psi_0\rangle = \int_{\mathcal{S}'(1)} d\mu_G(\pi')\overline{\Psi}_0\, \widehat{V}(g)\Psi_0, \qquad \mathcal{H}_\pi = L^2(\mathcal{S}'^{(1)}, d\mu_G)$$

$$(\widehat{V}(g)\cdot\Psi)[\pi'] = e^{-i\int_{\mathbb{R}^3} d^3\vec{x}\,\pi' g}\Psi[\pi'],$$

$$d\mu_G(\pi') = e^{-\int_{t_0} d^3\vec{x}\,\pi'(-\Delta+m^2)^{-1/2}\pi'}\mathcal{D}\pi',$$

$$(\widehat{\varphi}[f]\cdot\Psi)[\pi'] = i\int_{t_0} d^3\vec{x}\,\left[f\frac{\delta}{\delta\pi'} - i\pi'(-\Delta+m^2)^{-1/2} f\right]\Psi[\pi'].$$

- First, the amplitude $\langle\Psi_0|\widehat{V}(g)\Psi_0\rangle$ in the Fock representation gives $\int_{\mathcal{S}'(1)} d\mu_G \overline{\Psi}_0 \left(\widehat{V}(g)\Psi_0\right) = \omega_{J(P)}(\widehat{V}(g)) = e^{-\frac{1}{4}\int_{t_0} d^3\vec{x} g(-\Delta+m^2)^{1/2} g}$.
- The same value is obtained within the momentum polarization as $\omega^2_{J(P)}(\widehat{V}(g))\langle\Psi_0|e^{i\int_{t_0}\widehat{\varphi}[-i(-\Delta+m^2)^{1/2}g]}|\Psi_0\rangle$
- The condition $\widehat{P} := \widehat{V}(g) - \omega^2_{J(P)}(\widehat{V}(g))e^{i\int_{t_0}\widehat{\varphi}[-i(-\Delta+m^2)^{1/2}g]} \approx 0$ is only satisfied by the vacuum state Ψ_0.

How can we recover the vacuum state of the Fock representation from the PF representation? (M. Varadarajan analysis)

$$\mathcal{D} \subset \mathcal{H}_{PF} \subset \mathcal{D}^*, \qquad (\mathcal{D}^*, ||\cdot||_P) = \mathcal{H}_\pi$$

$$\langle \Omega | \widehat{P} | N_g \rangle = 0, \quad \rightarrow \quad \Omega[v] = \sum_g e^{-\frac{1}{2}\int_{t=0} g(-\Delta + m^2)^{1/2} g} \overline{V}_g[v]$$

$$(\mathcal{V}\Omega)[v] \in \mathcal{L}^* \subset \mathcal{D}^*, \qquad (\mathcal{L}^*, \langle \cdot \rangle_P) \approx \mathcal{H}_\pi$$

$$\langle \widehat{V}_{g_1} \Omega[v] | \widehat{V}_{g_2} \Omega[v] \rangle_P := \omega_{J^P}(\widehat{V}_{g_2 - g_1})$$

We can use $(\mathcal{L}^*, \langle \cdot \rangle_P)$ to calculate amplitudes.

Conclusions

- PFT is a non-regular representation of the Weyl algebra of the real scalar field.
- No unitarily equivalent to the Fock-Schrödinger representation.
- The symmetry group is the group formed with the tangential diffeomorphisms.
- The Poincarè condition picks out a unique element Ω in the dual space $\mathcal{D}^*$.
- The abelian algebra $\mathcal{V}$ induces a dense set $\mathcal{L}^*$. This space can be completed with the norm induced by the product $\langle \cdot \rangle_P$. The resulting Hilbert space is equivalent to the Fock representation in the momentum polarization.

Questions

- How the Boosts emerge from this construction?
- How the complement space $Diff^T(\mathbb{R}^4)/\mathcal{E}$ is removed in this construction?
- Which is the relation with the Loops Representation?, can PF representation be seen as an intermediate representation between Loops Representation and the Fock representation?

Thanks

Consider the space $\mathcal{S}'^{(1)}(\mathbb{R}^3)$ as the topological dual of $\mathcal{S}^{(0)}(\mathbb{R}^3)$.

$$\begin{aligned} \Gamma' &= \mathcal{S}^{(0)}(\mathbb{R}^3) \times \mathcal{S}'^{(1)}(\mathbb{R}^3) \in (\varphi, \pi'), \\ \tilde{F}_{\tilde{\lambda}}[(\varphi,\pi)] &= \tilde{F}_{(g,f')}[(\varphi,\pi)] := \int_{\mathbb{R}^3} d^3\vec{x}(f'\varphi - g\pi), \\ \Omega^{(1)}\left(\tilde{F}_{\tilde{\lambda}}, \tilde{F}_{\tilde{\mu}}\right) &= \Omega^{(1)}\left(\tilde{F}_{(g_1,f_1')}, \tilde{F}_{(g_2,f_2')}\right) := \int_{\mathbb{R}^3} d^3\vec{x}\,(f_1' g_2 - g_1 f_2'). \end{aligned}$$

PL employs $\tilde{F}_{\tilde{\lambda}}$ with $f' := \sum_j f_j \delta(\vec{x} - \vec{x}_j)$, where $f_j \in \mathbb{R}$, $\vec{x}_j \in V$ and V is a graph in $\mathbb{R}^3$ and recall $\tilde{\mathcal{S}}^{(1)} \in \mathcal{S}'^{(1)}$

$$f'(\vec{x}) = \sum_j f_j \delta(\vec{x} - \vec{x}_j),$$

The Weyl algebra generated with elements $W(g, f')$ and multiplication and involution given by

$$\begin{aligned} W(g_1, f_1') \cdot W(g_2, f_2') &= e^{\frac{i}{2}\Omega^{(1)}((g_1,f_1');(g_2,f_2'))} W(g_1 + g_2, f_1' + f_2'), \\ W^*(g, f') &= W(-g, -f'). \end{aligned}$$

$$U_{f'}(\varphi) = U_{V,\vec{f}}(\varphi) := e^{i \int_{\mathbb{R}^3} d^3\vec{x}\, f'(\vec{x})\varphi(\vec{x})} = e^{i \sum_j f_j \varphi(\vec{x}_j)}.$$

$$\omega_{PL}(U_{V,\vec{f}}) = \delta_{\vec{f},\vec{0}}, \qquad \mathcal{H}_{PL} := L^2(\Delta(\tilde{\mathcal{U}}), d\mu_{PL}),$$

where $\Delta(\tilde{\mathcal{U}})$ is the Gel'fand spectrum of $\tilde{\mathcal{U}}$ and $d\mu_{PL}$ is a regular Borel measure defined in the spectrum.

$$(\widehat{U}(V, \vec{f})\Psi)[X] := X[U(V, \vec{f})]\Psi[X], \tag{1}$$

where $X \in \Delta(\tilde{\mathcal{U}})$ and $X[U(V, \vec{f})] := e^{i \int_{t_0} d^3\vec{x}\, f' \varphi}$.

$$(\widehat{\pi}[g] \cdot U_{f'})(\varphi) = \left(\int_{t_0} d^3\vec{x} g f' \right) U_{f'}(\varphi). \quad (2)$$

$$\mathcal{H}_{Fock} \approx \mathcal{H}_{r-Fock} \Leftrightarrow \mathcal{H}_{PL}$$

r es un parámetro introducido para LLENAR los vertex set V y establecer un vínculo entre las holonomías puntuales y las holonomías DESPARRAMADAS. Las holonomías DESPARRAMADAS no están bien definidas en $\mathcal{H}_{Fock}$ y si en $\mathcal{H}_{r-Fock}$. La relación entre ambos espacios de Fock es que las álgebras son isomorfas pero los espacios de Hilbert tienen distinto espacio de configuración cuántico.