Hadron Physics from Superconformal Quantum Mechanics and its Light-Front Holographic Embedding

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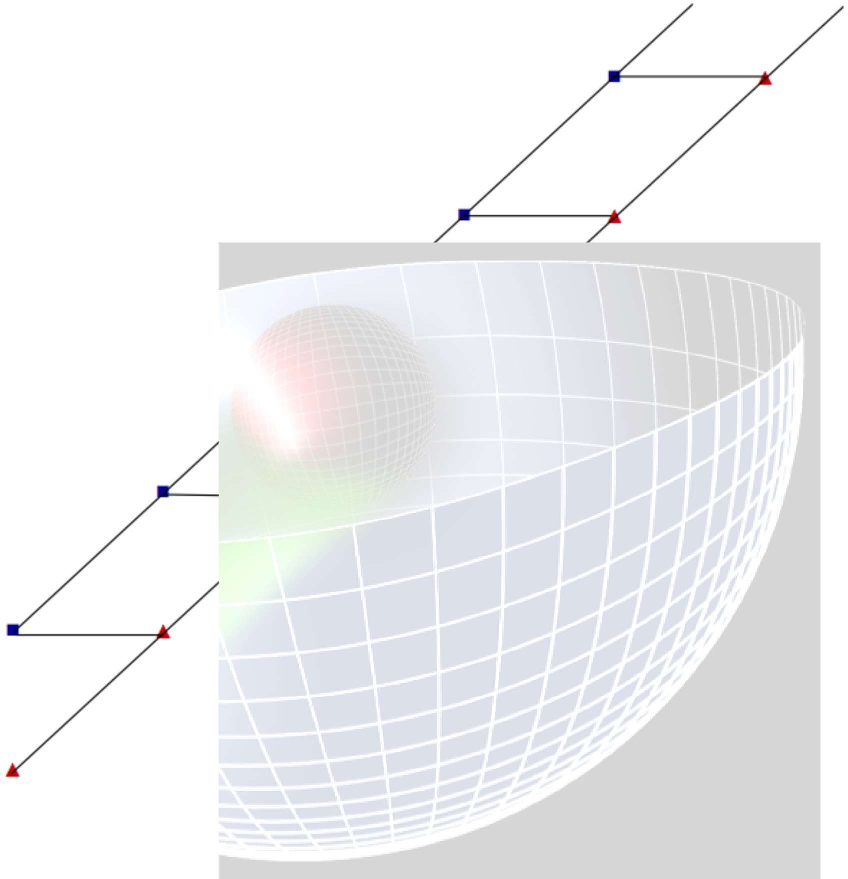


In collaboration with Stan Brodsky, Hans G. Dosch, Alexandre Deur and Cédric Lorce

Quest for a semiclassical approximation to describe bound states in QCD

I. Semiclassical approximation to QCD in the light-front: Reduction of QCD LF Hamiltonian leads to a relativistic LF wave equation, where complexities from strong interactions are incorporated in effective LF potential U

II. Correspondence between equations of motion for arbitrary spin in AdS space and relativistic LF bound-state equations in physical space-time: Embedding of LF wave equations in AdS leads to extension of LF potential U to arbitrary spin from conformal symmetry breaking in the AdS action

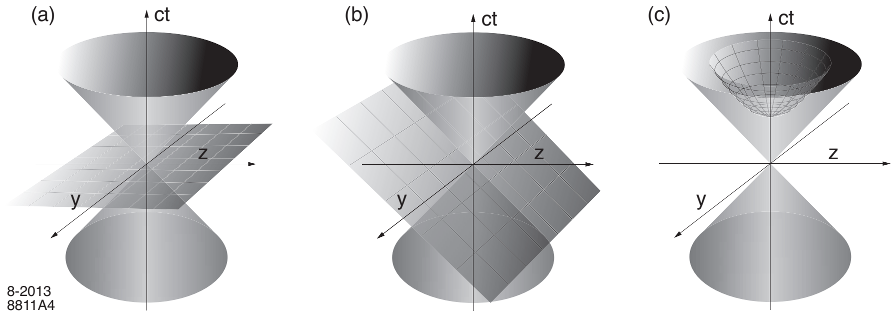
III. Construction of LF potential U: Since the LF semiclassical approach leads to a one-dim QFT, it is natural to extend conformal and superconformal QM to the light front since it gives important insights into the confinement mechanism, the emergence of a mass scale and baryon-meson SUSY

Outline of this talk

(1) Semiclassical approximation to QCD in the light front

Dirac forms of relativistic dynamics [Dirac (1949)]

- Poincaré generators P^μ and $M^{\mu\nu}$ separated into kinematical and dynamical
- Kinematical generators act along initial hypersurface and contain no interactions
- Dynamical generators are responsible for evolution of the system and depend on the interactions
- Each front has its Hamiltonian and evolve with a different time, but results computed in any front should be identical (different parameterizations of space-time)



- *Instant form*: initial surface defined by $x^0 = 0$: $\quad P^0, \ \mathbf{K}$ dynamical, $\ \mathbf{P}, \ \mathbf{J}$ kinematical
- *Front form*: initial surface tangent to the light cone $x^+ = x^0 + x^3 = 0 \quad$ ($P^\pm = P^0 \pm P^3$)

$$P^-, \ J^x, \ J^y \ \text{dynamical} \quad P^+, \ \mathbf{P}_\perp, \ J^3, \ \mathbf{K} \ \text{kinematical}$$

- *Point form*: initial surface is the hyperboloid $x^2 = \kappa^2 > 0, \ x^0 > 0$: $\ P^\mu$ dynamical, $\ M^{\mu\nu}$ kinematical

Effective QCD LF bound-state equation

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Express the hadron 4-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields $\psi_+ = \Lambda_\pm \psi$ and $\mathbf{A}_\perp$ $\left(\Lambda_\pm = \gamma^0\gamma^\pm\right)$ quantized in null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned} P^- &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+\,\gamma^+ \frac{\left(i\nabla_\perp\right)^2 + m^2}{i\partial^+}\psi_+ + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+ \\ \mathbf{P}_\perp &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp \psi_+ \end{aligned}$$

- Construct LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ from mass-shell relation

$$P^2|\psi(P)\rangle = M^2|\psi(P)\rangle$$

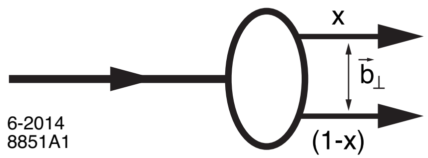
- Simple structure of LF vacuum allows definition of partonic content of hadron in terms of wavefunctions: Retain quantum-mechanical probabilistic interpretation of hadronic states

- Factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

with invariant transverse impact variable

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$



- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $(L = L^z)$

$$M^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where effective potential U includes all interactions, including those from higher Fock states

LF Hamiltonian equation $P_\mu P^\mu |\psi\rangle = M^2 |\psi\rangle$ is a LF wave equation for ϕ

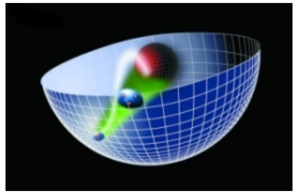
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

- Critical value $L = 0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian

- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time

(2) Embedding higher-spin wave equations in AdS space

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- Why is AdS space important? AdS$_5$ is a 5-dim space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space $ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$ with the group of isometries of AdS$_5$
- Integer spin-J in AdS conveniently described by tensor field $\Phi_{N_1 \dots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x\, dz \sqrt{|g|}\, e^{\varphi(z)}\, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \dots N_J}\, D_{M'} \Phi_{N_1' \dots N_J'} - \mu^2_{\mathit{eff}}(z)\, \Phi^*_{N_1 \dots N_J}\, \Phi_{N_1' \dots N_J'} \Big)$$

  D_M is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ effectively breaks conformality

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable z

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \qquad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\nu_1\cdots\nu_J} = \Phi_J(z)\epsilon_{\nu_1\cdots\nu_J}(P)$

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J = M^2\Phi_J$$

with

$$(\mu\, R)^2 = (\mu_{\mathit{eff}}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

and the kinematical constraints to eliminate the lower spin states $J-1, J-2, \cdots$

$$\eta^{\mu\nu} P_\mu\, \epsilon_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\, \epsilon_{\mu\nu\nu_3\cdots\nu_J} = 0$$

- Kinematical constrains in the LF imply that μ must be a constant

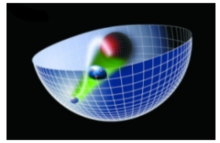
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Light-front mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

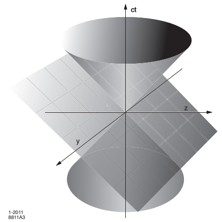
- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2}\, \phi_J(z)$ and $z \to \zeta$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = M^2\Phi_J(z)$$



we find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$



with

$$U(\zeta) = \tfrac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2z}\varphi'(\zeta)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons
- Effective LF confining potential $U(\zeta)$ corresponds to the IR modification of AdS space
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

(3) Conformal quantum mechanics and light-front dynamics: Mesons

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in 1-dim effective QFT the conformal symmetry of 4-dim QCD Lagrangian in the limit of massless quarks: Conformal QM [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

- Conformal Hamiltonian:

$$H = \tfrac{1}{2}\left(p^2 + \frac{g}{x^2}\right)$$

g dimensionless: Casimir operator of the representation

- Schrödinger picture: $p = -i\partial_x$

$$H = \tfrac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right)$$

- QM evolution

$$H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

H is one of the generators of the conformal group *Conf*(R^1). The two additional generators

- Dilatation generator: $D = -\frac{1}{4}\left(px + xp\right)$
- Generator of special conformal transformations: $K = \frac{1}{2}x^2$

are are also invariants of the one-dim conformal QFT

- H, D and K close the conformal algebra

$$[H,D]=iH, \quad [H,K]=2iD, \quad [K,D]=-iK$$

- dAFF construct a new generator G as a superposition of the 3 generators of *Conf*(R^1)

$$G=uH+vD+wK$$

and introduce new time variable τ

$$d\tau=\frac{dt}{u+vt+wt^2}$$

- Find usual quantum mechanical evolution for time τ

$$G|\psi(\tau)\rangle=i\frac{d}{d\tau}|\psi(\tau)\rangle \qquad H|\psi(t)\rangle=i\frac{d}{dt}|\psi(t)\rangle$$

$$G=\frac{1}{2}u\left(-\frac{d^2}{dx^2}+\frac{g}{x^2}\right)+\frac{i}{4}v\left(x\,\frac{d}{dx}+\frac{d}{dx}\,x\right)+\frac{1}{2}wx^2.$$

- Operator G is compact for $4uw-v^2>0$, but action remains conformal invariant !
- Emergence of scale: Since the generators of *Conf*(R^1) $\sim$ $SO(2,1)$ have different dimensions a scale appears in the new Hamiltonian G which according to dAFF may play a fundamental role (One of the generators of $SO(2,1)$ is compact)

Connection to light-front dynamics

- Compare the dAFF Hamiltonian G

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

with the LF Hamiltonian H_{LF}

$$H_{LF} = -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)$$

and identify dAFF variable x with LF invariant variable ζ

- Choose $u = 2, \quad v = 0$
- Casimir operator from LF kinematical constraints: $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2$ fixes the LF potential and the dilaton profile in the dual gravity theory

$$U \sim \lambda^2\zeta^2, \qquad \varphi = \lambda z^2$$

- Effective LF for potential for arbitrary integer-spin from $U(\zeta) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2z}\varphi'(\zeta)$

$$U = \lambda^2\zeta^2 + 2\lambda(J-1)$$

Meson spectrum

- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

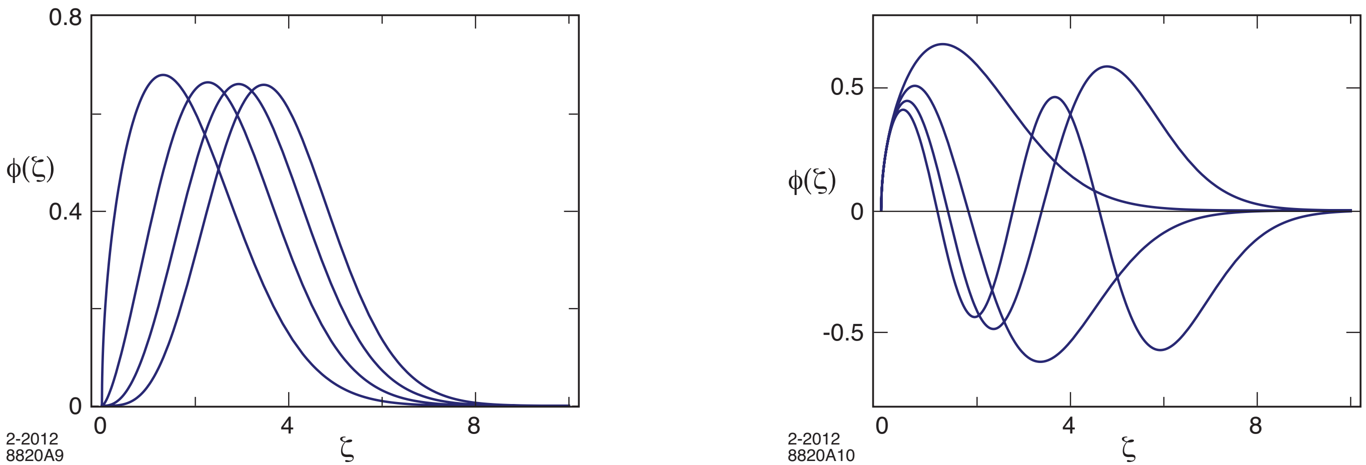
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\,\phi^2(z) = 1$

$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-|\lambda|\zeta^2/2}L_n^L(|\lambda|\zeta^2)$$

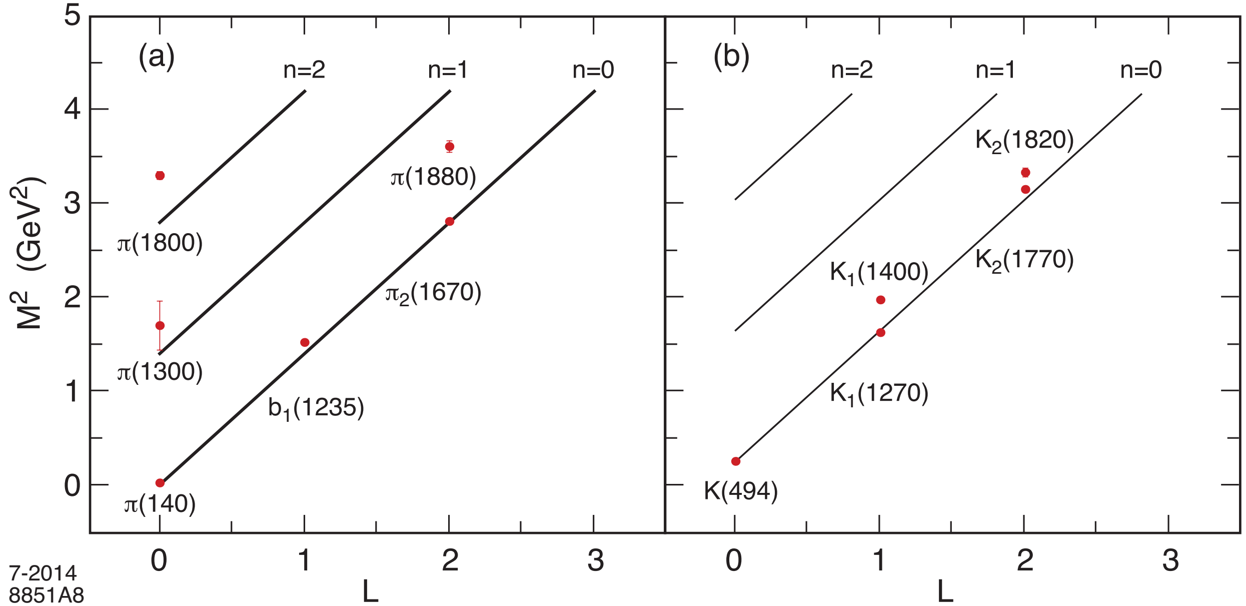
- Eigenvalues for $\lambda > 0$

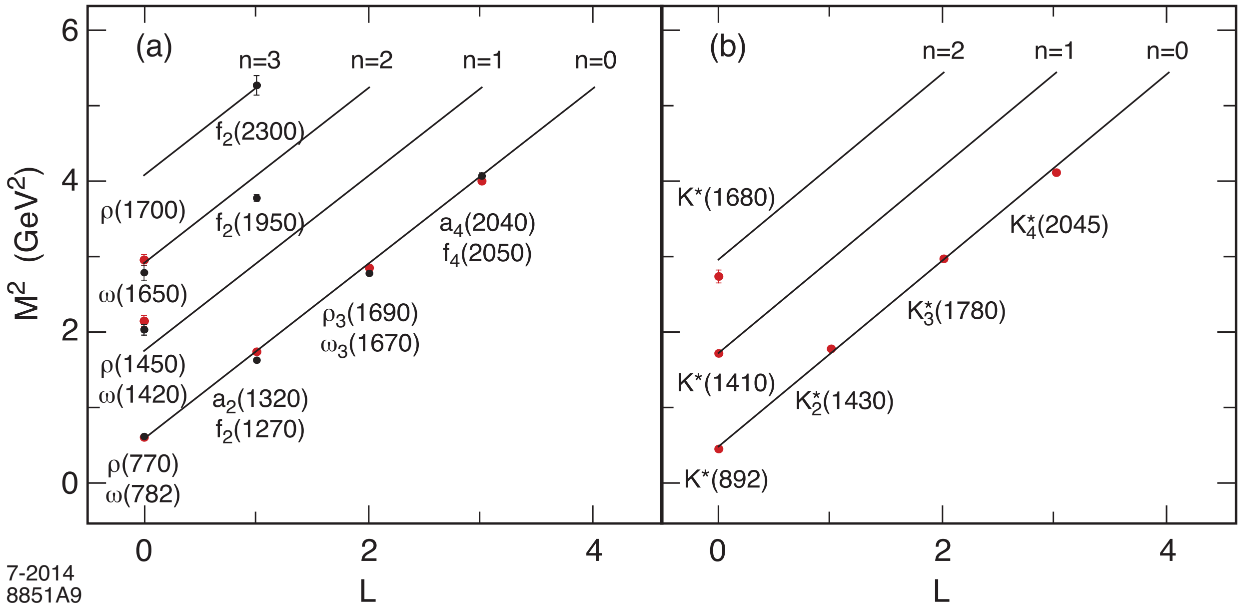
$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

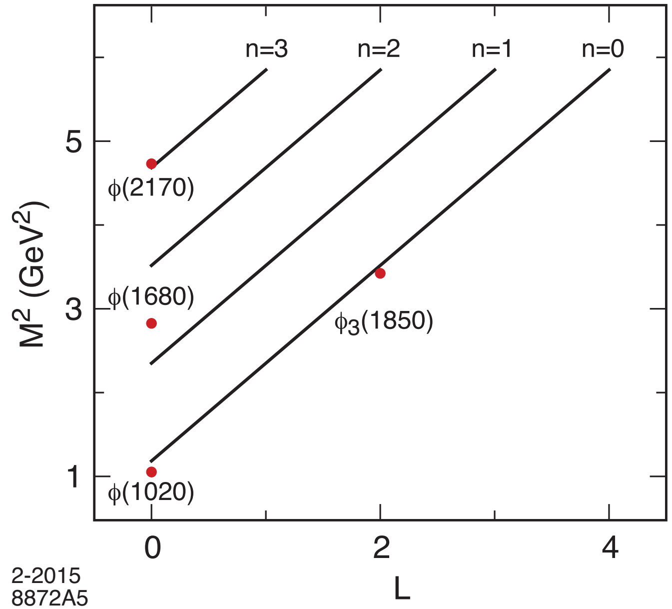
- $\lambda < 0$ incompatible with LF constituent interpretation



Orbital and radial LF wavefunctions for mesons







Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)

Note: Three relevant points . . .

- A linear potential $V_{\rm eff}$ in the *instant form* implies a quadratic potential $U_{\rm eff}$ in the *front form* at large distances $\longrightarrow$ Regge trajectories

$$U_{\rm eff} = V_{\rm eff}^2 + 2\sqrt{p^2 + m_q^2}\, V_{\rm eff} + 2\, V_{\rm eff} \sqrt{p^2 + m_q^2}$$

[A. P. Trawiński, S. D. Glazek, S. J. Brodsky, GdT, H. G. Dosch, PRD **90**, 074017 (2014)]

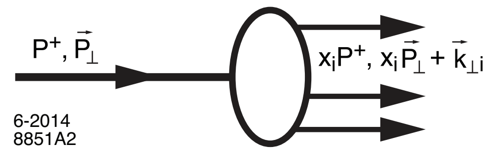
- Results are easily extended to light quarks

[S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)

$$\Delta M^2_{m_q, m_{\overline{q}}} = \frac{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)} \left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}}$$

- For n partons invariant LF variable ζ is [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

$$\zeta = \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big|$$



where x_j and x are longitudinal momentum fractions of quark j in the spectator cluster and of the active quark (LF cluster decomposition)

(4) Superconformal quantum mechanics and light-front dynamics: Baryons

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

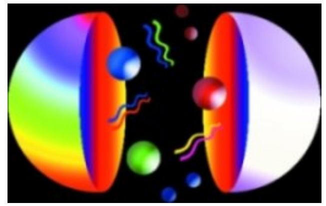
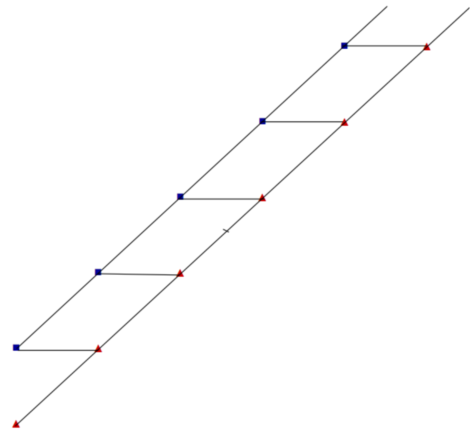


Image credit: N. Evans

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:



$$\tfrac{1}{2}\{Q, Q^\dagger\} = H$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

$$[Q, H] = [Q^\dagger, H] = 0$$

  Note: Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E, but for a zero eigenvalue we can have the trivial solution $|E = 0\rangle = 0$

- A simple realization is

$$Q = \chi\,(ip + W)\,, \qquad Q^\dagger = \chi^\dagger\,(-ip + W)$$

  where χ and $\chi^\dagger$ are spinor operators with $\{\chi, \chi^\dagger\} = 1$ and W is the superpotential

- Following F&R consider a 1-dim QFT invariant under conformal and supersymmetric transformations $\big(W = f/x,\, f$ dimensionless $\big)$
- Conformal graded-Lie algebra has in addition to Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to generator of conformal transformations $K \sim \{S, S^\dagger\}$

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned}
\tfrac{1}{2}\{Q, Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S, S^\dagger\} = K \\
\tfrac{1}{2}\{Q, S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\
\tfrac{1}{2}\{Q^\dagger, S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD
\end{aligned}$$

where the operators

$$\begin{aligned}
H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\
D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\
K &= \tfrac{1}{2}x^2
\end{aligned}$$

satisfy the conformal algebra

- Following F&R define a fermionic generator R, a linear combination of the generators Q and S

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and consider the new generator $G = \frac{1}{2}\{R, R^\dagger\}$ which also closes under the graded algebra $sl(1/1)$

$$\frac{1}{2}\{R, R^\dagger\} = G \qquad \frac{1}{2}\{Q, Q^\dagger\} = H$$

$$\{R, R\} = \{R^\dagger, R^\dagger\} = 0 \qquad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

$$[R, H] = [R^\dagger, H] = 0 \qquad [Q, H] = [Q^\dagger, H] = 0$$

- New QM evolution operator

$$G = uH + wK + \tfrac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

is compact for $uw > 0$: Emergence of a scale since Q and S have different units

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients u and w are fixed to $u = 2$ and $w = 2\lambda^2$

Nucleon Spectrum

- In 2×2 block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

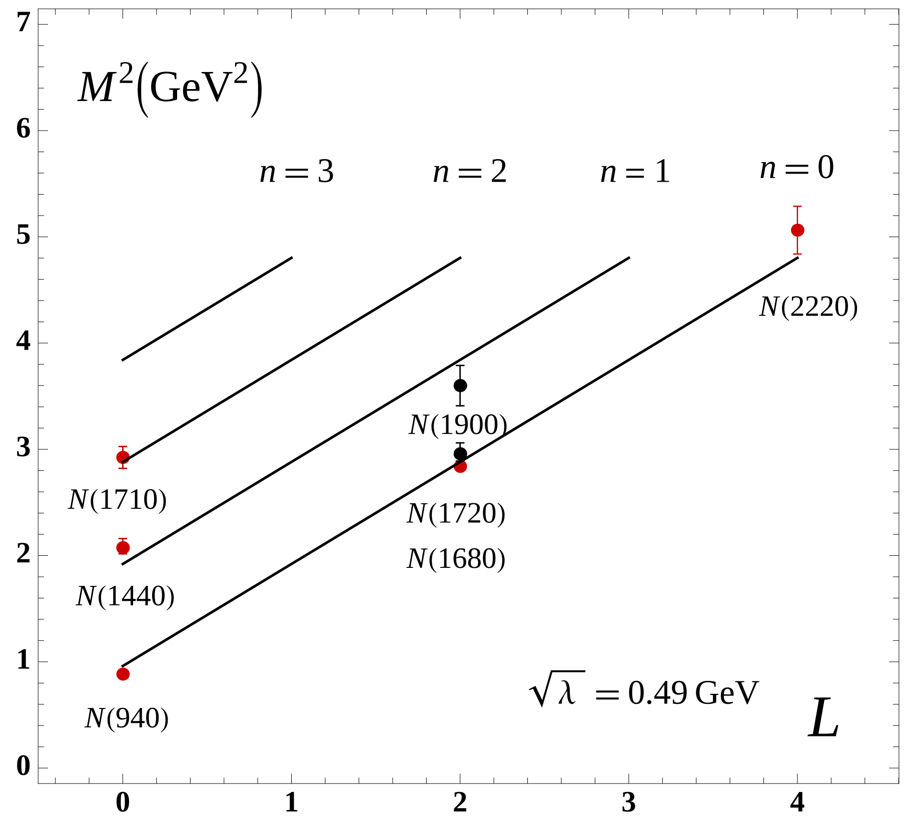
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu}(\lambda\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

- Lowest possible state $n = 0$ and $\nu = 0$
- Orbital excitations $\nu = 0, 1, 2 \cdots = L$
- L is the relative LF angular momentum between the active quark and spectator cluster



(5) Superconformal meson-baryon symmetry

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Meson}} \\ \phi_{\text{Baryon}} \end{pmatrix}$$

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

- Extend superconformal QM to relate bound-state equations for mesons and baryons
- From superconformal Hamiltonian $G = \{R_\lambda^\dagger, R_\lambda\}$ obtain bound-state equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Baryon}} = M^2 \phi_{\text{Baryon}}$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Meson}} = M^2 \phi_{\text{Meson}}$$

- Compare with LFWE for nucleon and pion:

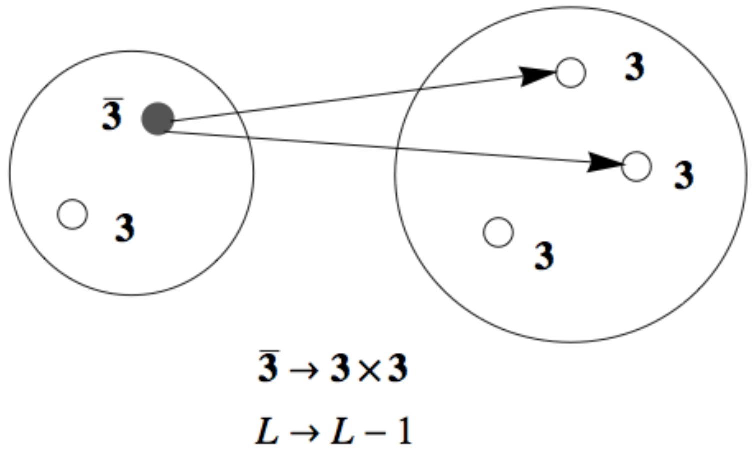
$$\lambda = \lambda_M = \lambda_B, \quad f = L_B + \tfrac{1}{2} = L_M - \tfrac{1}{2}$$

$$\Rightarrow \quad \boxed{L_M = L_B + 1}$$

- Also $R^\dagger|M, L\rangle = |B, L-1\rangle, \quad R^\dagger|M, L=0\rangle = 0$

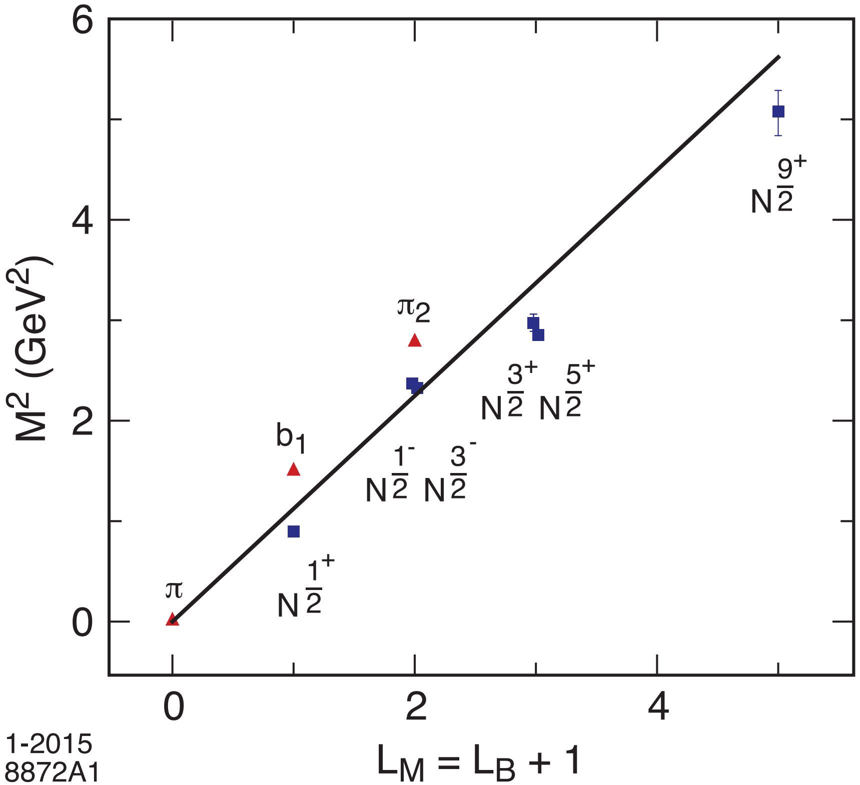
  Special role of the pion as a unique state of zero energy

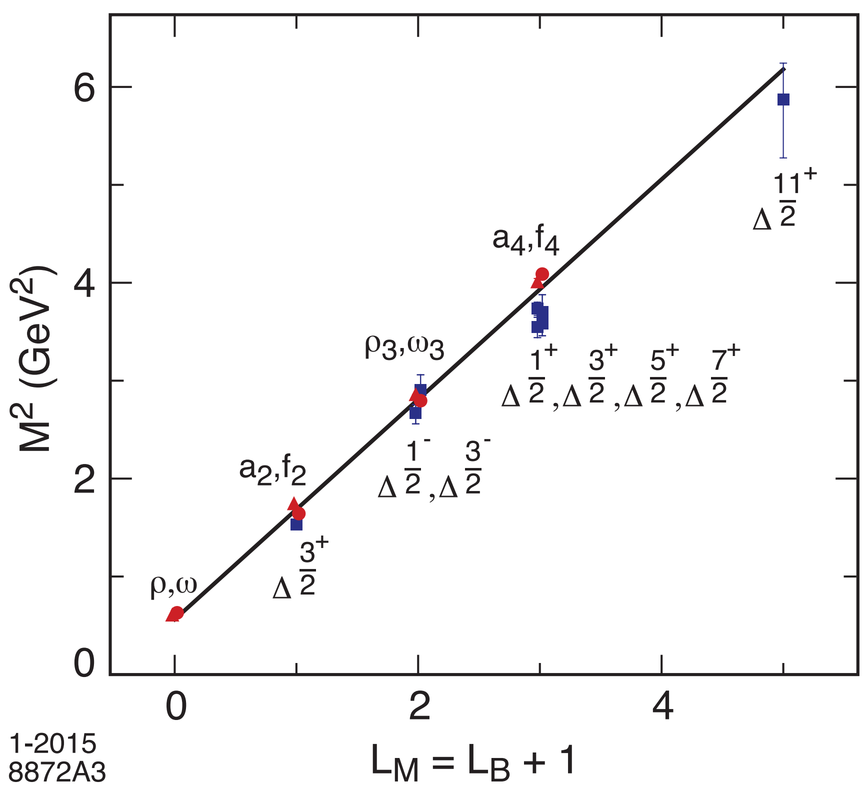
- Emerging dynamical SUSY from SU(3) color



- Superconformal spin-dependent Hamiltonian to describe mesons and baryons (chiral limit)

$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\,\mathbf{I}\,s} \qquad s = 0, 1$$



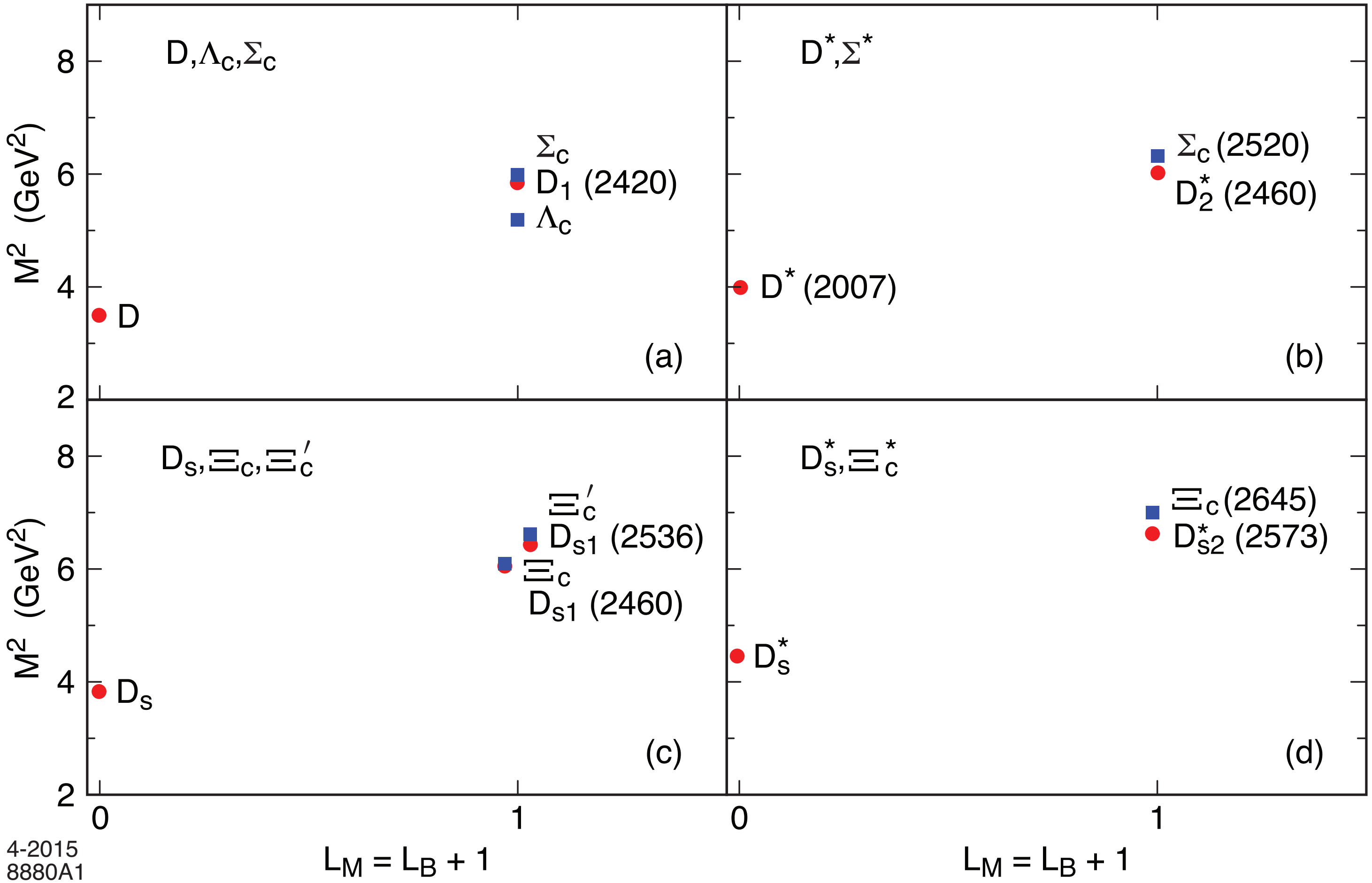


Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

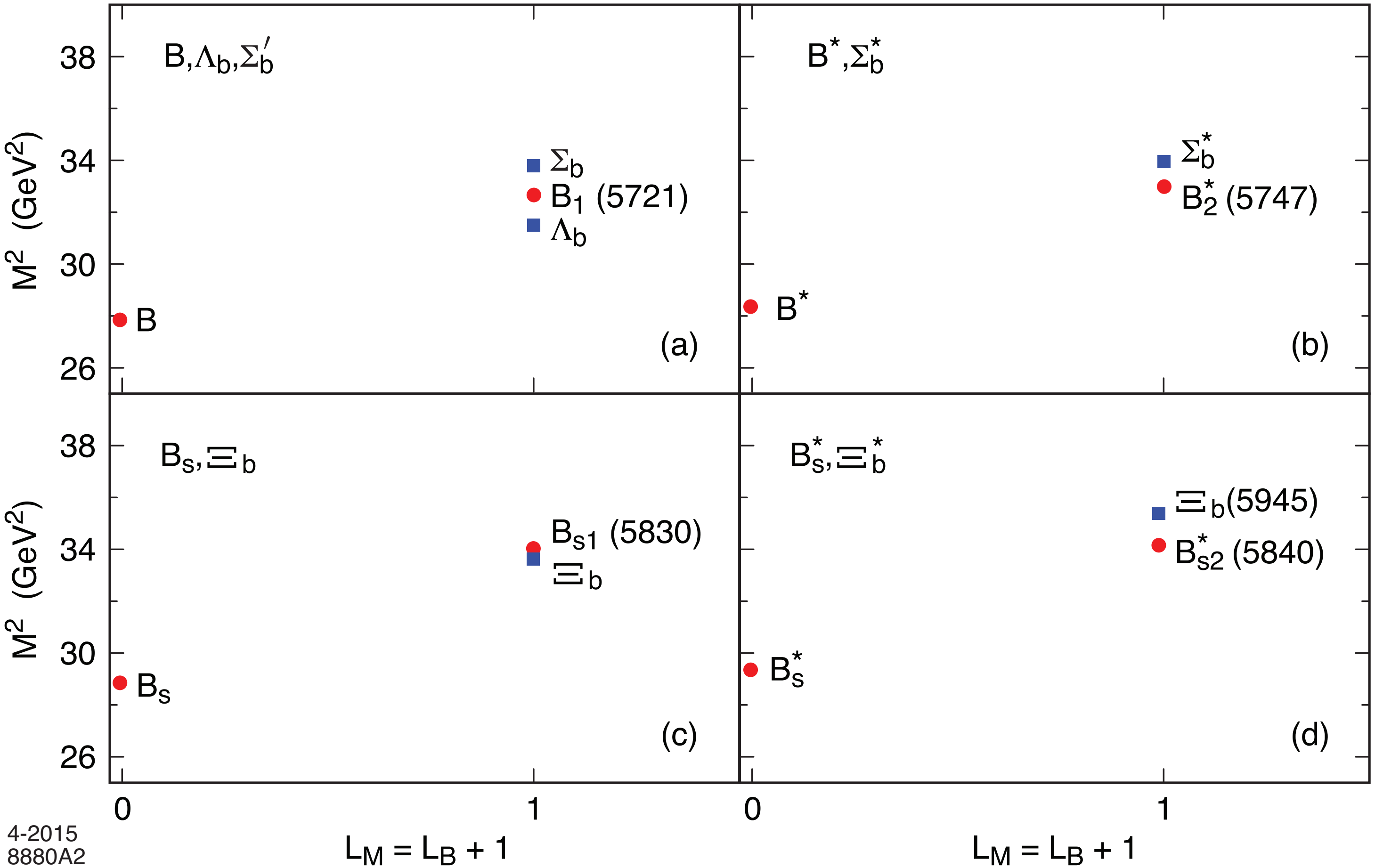
Supersymmetry across the light and heavy-light hadronic spectrum

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **92**, 074010 (2015)]

- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Supersymmetric relations between mesons and baryons with charm



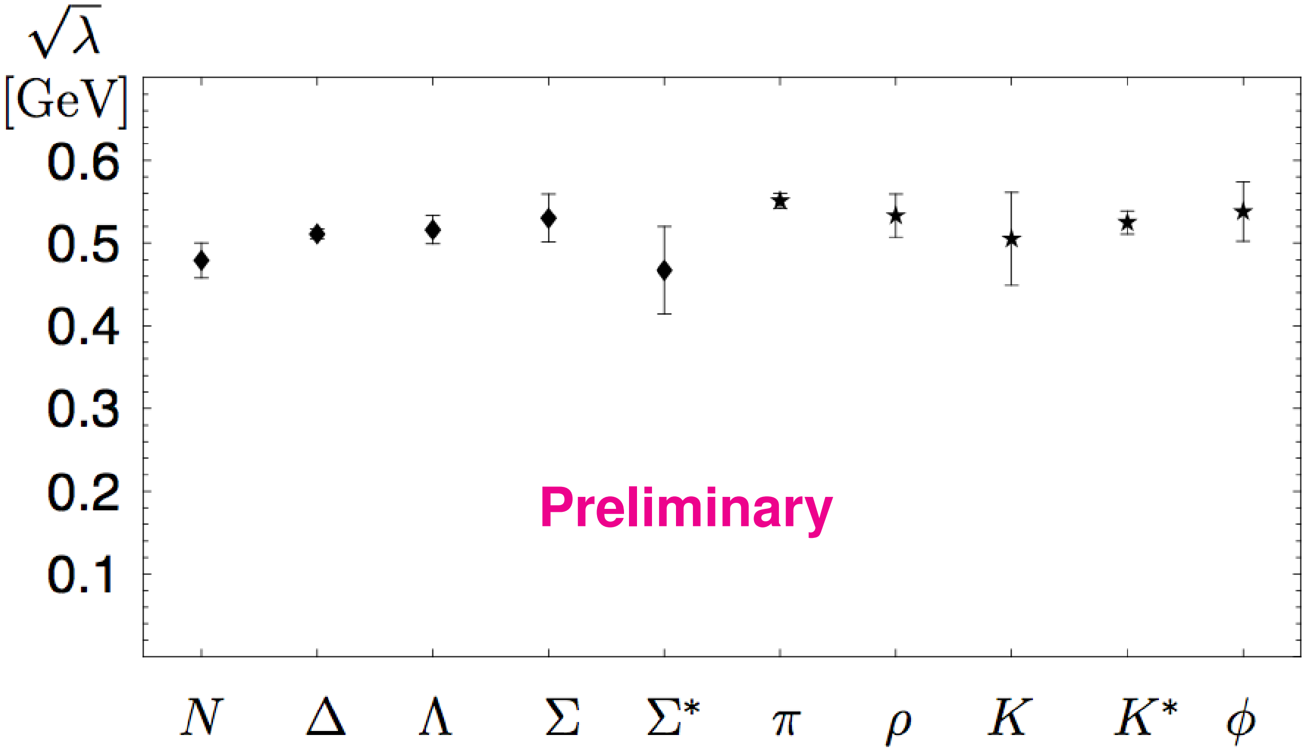
Supersymmetric relations between mesons and baryons with beauty

- Dynamical Supersymmetry: Ex. nuclear supersymmetry F. Iachelo (1980)

  [Recent review: R. Bijker, A. Frank and J. Barea, Rev. Mex. Fiz. **55**, 30 (2009)]

- Hadronic supersymmetry introduced by H. Miyazawa (1966)

- How good is the semiclassical approximation based on superconformal QM and its LF holographic embedding? [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé]



Best fit for the hadronic scale $\sqrt{\lambda}$ from the different sectors including radial and orbital excitations

(6) Light-front holographic cluster decomposition and form factors

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé] **Work in progress**

- LF Holographic FF $F_{\tau=N}(Q^2)$ expressed as the $N-1$ product of poles for twist $\tau = N$

  S. J. Brodsky and GdT, PRD **77**, 056007 (2008)

$$
\begin{aligned}
F_{\tau=2}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)} \\
F_{\tau=3}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)} \\
& \cdots \\
F_{\tau=N}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1+\frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}
\end{aligned}
$$

- Spectral formula

$$
M_{\rho^n}^2 \to 4\lambda\,(n+1/2)
$$

- Cluster decomposition in terms of twist $\tau = 2$ FFs !

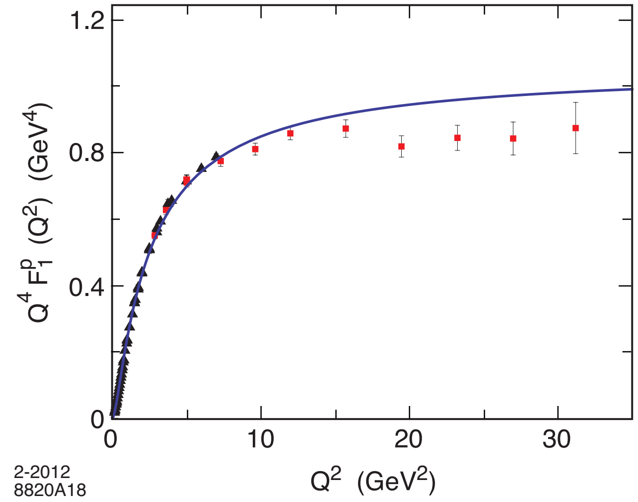
$$
F_{\tau=N}(Q^2) = F_{\tau=2}\left(Q^2\right) F_{\tau=2}\left(\frac{1}{3}Q^2\right)\cdots F_{\tau=2}\left(\frac{1}{2N-3}Q^2\right)
$$

- Example: Dirac proton FF F_1^p

  in terms of the pion form factor F_π:

$$F_1^p(Q^2) = F_\pi\left(Q^2\right) F_\pi\left(\frac{1}{3}Q^2\right)$$

(equivalent to $\tau = 3$ FF)



- But . . . we know that higher Fock components are required.

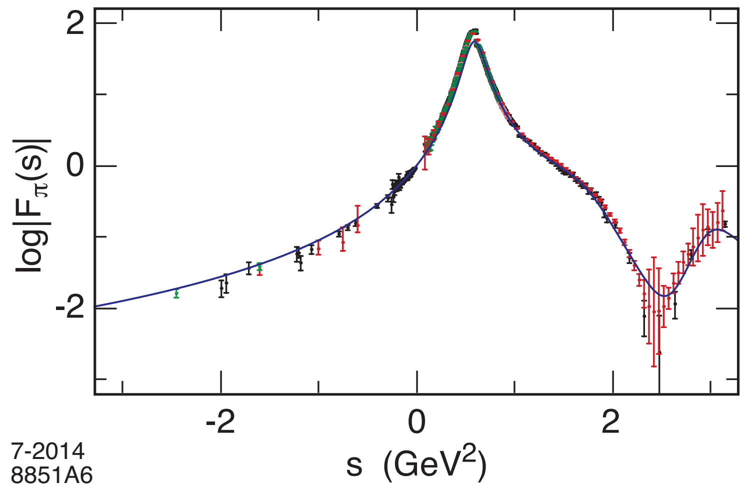
  Example time-like pion FF:

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

$$F_\pi(q^2) = (1-\gamma)F_{\tau=2}(q^2) + \gamma F_{\tau=4}(q^2)$$

$$P_{q\bar{q}q\bar{q}} = 12.5\%$$

S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, PR **584**, 1 (2015)



(7) Infrared behavior of the strong coupling in light-front holography

[S. J. Brodsky, GdT and A. Deur, PRD **81** (2010) 096010]

[A. Deur, S. J. Brodsky and GdT, PLB **750**, 528 (2015)

- Effective coupling $\alpha_{g_1} = g_1^2/4\pi$ defined from and observable: g_1 scheme from Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx \, g_1^{p-n}(x, Q^2)$$

- Infrared behavior of strong coupling in holographic QCD from two-dimensional Fourier transform of the LF transverse coupling:

$$\alpha_{g_1}^{AdS}(Q) = \pi \exp\left(-Q^2/4\lambda\right)$$

- Large Q-dependence of α_s is computed from the pQCD β series:

$$Q^2 d\alpha_s/dQ^2 = \beta(Q) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \cdots)$$

  where coefficients β_i are known up to β_3 in $\overline{MS}$ scheme:

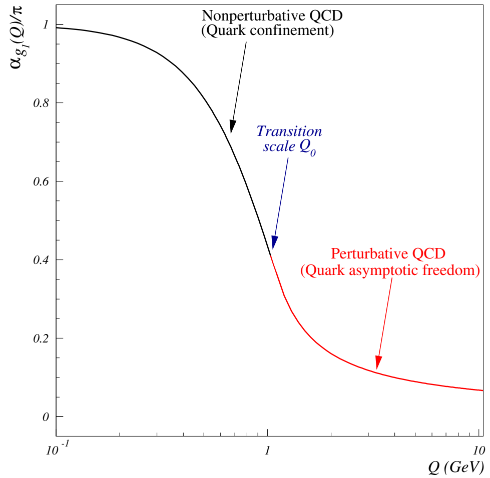
- $\alpha_{g_1}^{pQCD}(Q)$ expressed as a perturbative expansion in $\alpha_{\overline{MS}}(Q)$:

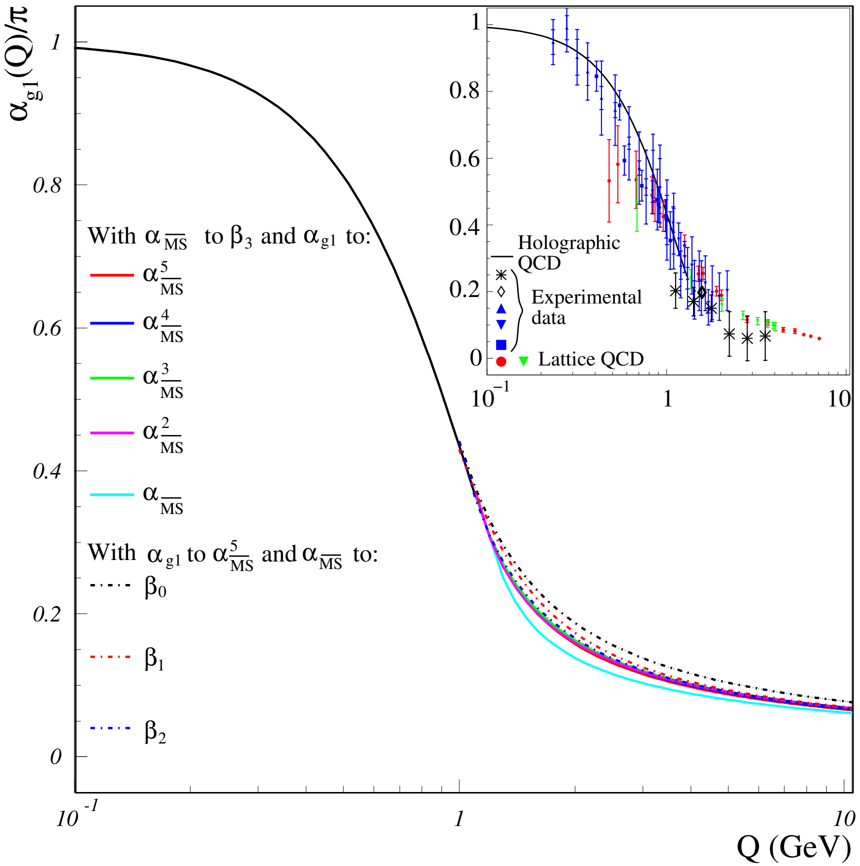
$$\alpha_{g_1}^{pQCD}(Q) = \pi \left[\alpha_{\overline{MS}}/\pi + a_1 \left(\alpha_{\overline{MS}}/\pi\right)^2 + a_2 \left(\alpha_{\overline{MS}}/\pi\right)^3 + \cdots\right]$$

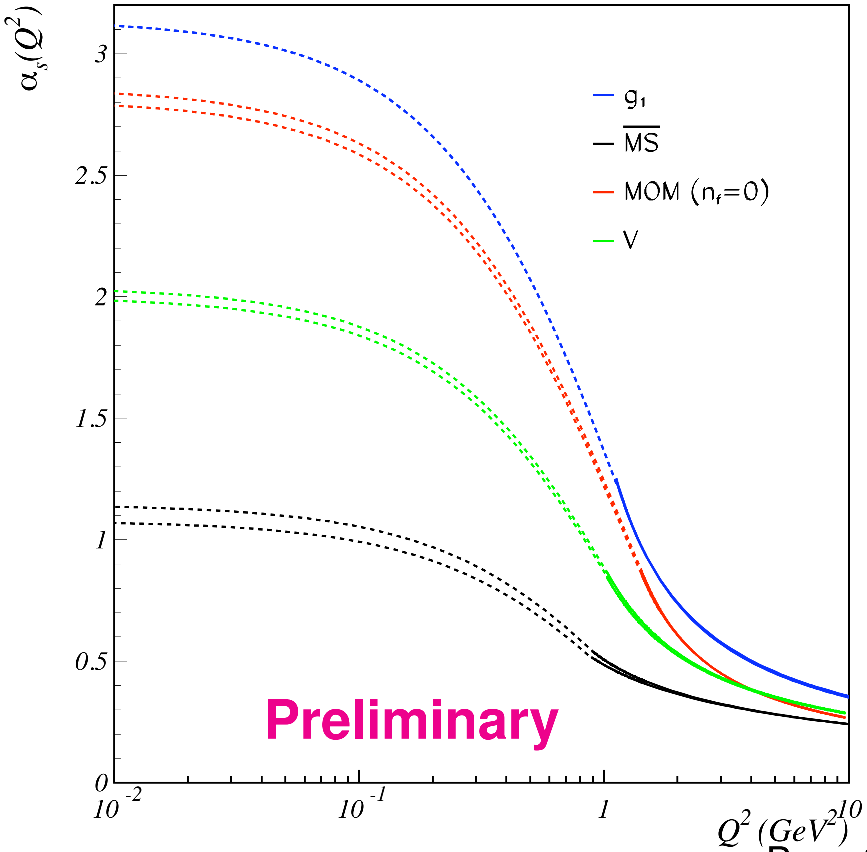
  The coefficients a_i are known up to order a_3

- Λ_{QCD} and Q_0 from matching perturbative and nonperturbative regimes:
  $\Lambda_{\overline{MS}} = 0.341 \pm 0.032$ GeV
  (PDG: $\Lambda_{\overline{MS}} = 0.340 \pm 0.008$ GeV)
  $Q_0^2 = 1.25 \pm 0.19$ GeV^2

- Scheme dependence?







Thanks !

For a review: S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)