The dynamical recombination probabilities among two mesons with/without tetraquark configurations

Genaro Toledo
IFUNAM

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Outline

- Motivation
- Two non-interacting mesons
- Four-body system
- Variational approach
- Results:
  - Two mesons, tetraquark and their mixing
  - Dynamical Recombination Probabilities
- Conclusions

Motivation

Multi-quark systems can have important implications in the phenomena we observe in nature, from an enhanced spectroscopy to quark recombination effects.

Recent experimental results provide strong evidence on the formation of four quark states.

LHCb Collab. (R. Aaij et al.), Phys. Rev. Lett. 112, 222002 (2014).

$e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$ $\quad D^{*+}\bar{D}^{*0}$ $\quad Z_c^\pm(4025)$

S. Choi et al. (Belle Collaboration), Phys. Rev. Lett. 100, 142001 (2008)

Z.Q. Liu et al. (Belle Collaboration), Phys. Rev. Lett. 110, 252002 (2013)

M. Ablikim et al. (BES III Collaboration), Phys. Rev. Lett. 110, 252001 (2013).

$e^+e^- \to \pi^+\pi^- J/\psi$ $\quad \pi^\pm J/\psi$ $\quad$ **Zc(3900)**

Motivation

Since the early years of the quark model, theoretical studies have been performed to inquiry on the existence and stability of the tetraquark as an isolated object

How its mixing with a meson state can help us to understand the observed spectroscopy of states like the σ meson.

Less attention has been paid to the features of the tetraquark formation as two mesons are forced to approach each other as it could happen in a hadron-hadron collision or, when the four quarks are produced very close in space as in the WW decay, which eventually freeze out to two mesons.

$$e^+e^- \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4$$

Perturbative

Non-perturbative fragmentation hadronic final stage

Motivation

Can they turn into a tetraquark or mixed state?

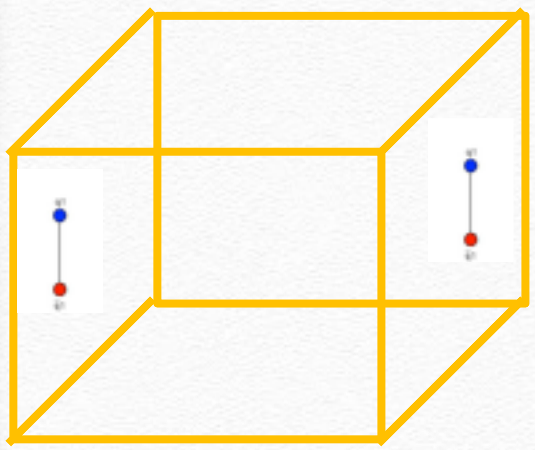
How their properties reflect such modification?

In the present work we address these questions by considering a system composed of:

two identical quarks (qq) and

two identical anti-quarks (QQ)

Two Non-interacting Mesons



Meson: quark-antiquark state interacting via a linear potential

$$\left[\frac{P_1^2}{2m_1}+\frac{P_2^2}{2m_2}+kr\right]\Psi\left(\vec{r}_1,\vec{r}_2\right)=E\ \Psi\left(\vec{r}_1,\vec{r}_2\right).$$

Exact solution for the wave function and energy of a single meson

$$\rho_n(r)=\frac{1}{r}\mathrm{Ai}\left(r\left[2\mu k\right]^{1/3}-\xi_n\right).$$

$$E_n=\left[\frac{k^2}{2\mu}\right]^{1/3}|\xi_n|$$

Variational approach to the wave function

$$F_\lambda(r)=\sqrt{\frac{3\lambda^2}{2\pi}}e^{-\lambda r^{3/2}}.$$

$$\lambda_0=\frac{2\sqrt{k\mu}}{3},$$

$$E_0=\frac{3^{5/3}k\Gamma\left(\frac{8}{3}\right)}{2^{7/3}(k\mu)^{1/3}}.$$

Meson Mean square radius

$$\langle r_M^2 \rangle \equiv \left\langle \sum_{i=1}^{2} \left(\vec{r}_i - \vec{R} \right)^2 \right\rangle$$

$$\langle r_M^2 \rangle_0 = \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \frac{3^{4/3}}{4} \Gamma\left(\frac{10}{3}\right),$$

m_2/m_1	λ_0	E_0	E_{exact}	$\langle r_M^2 \rangle_0$
1	0.4714	2.3472	2.33811	1.50255
1.44643	0.5125	2.2197	2.21106	1.38885
4.6131	0.6043	1.9889	1.98118	1.52605
14.0774	0.6441	1.9061	1.8987	1.73656

Meson optimal variational parameter and energy, the exact energy and mean square radius for different mass ratios respect to the lightest u mass (k=m_1=1 units)

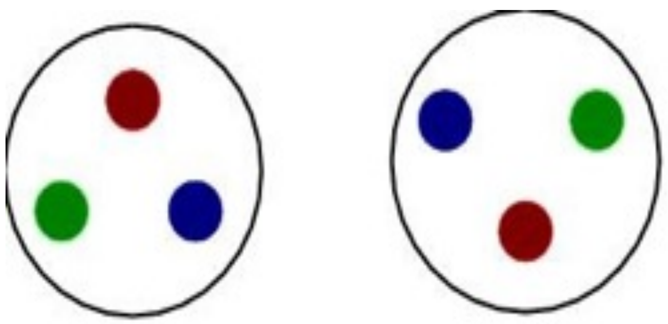
Dynamical Recombination: The string-flip model

Horowitz, Moniz, Negele 80's J. Piekarewicz, GTS

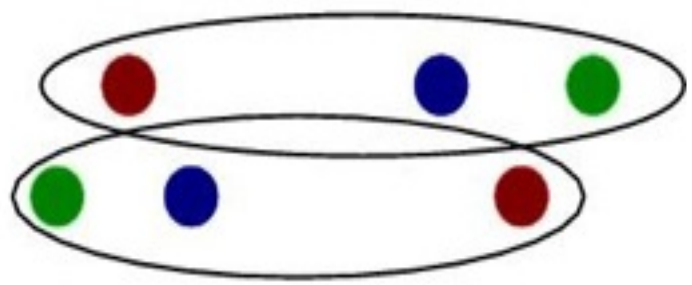
Quarks dof , Colors: red, blue, green, Flavors: u, d, s, c

Many-body potential

- Gluon flux tubes producing a minimal configuration of the system.

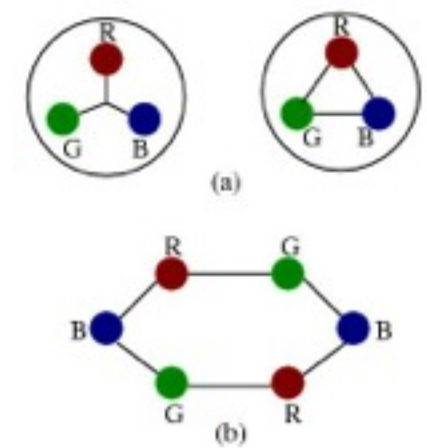


Optimal grouping



Non-optimal grouping

- Color combinations to built singlets.



Increasing size clustering

Optimal two-colors pairing potential

Ex. red and blue quarks

(Similar for color-anticolor)

$$V_{RB} = \min_P \sum_{i=1}^{A} v(\mathbf{r}_{iR}, P(\mathbf{r}_{iB})) = \min_P \sum_{i=1}^{A} \frac{1}{2} k(\mathbf{r}_{iR} - \mathbf{r}_{jB})^2$$

The quarks interact by a potential and form color singlet clusters.

Vbaryon = VRB + VRG + VGB

Vmeson = VRR + VGG + VBB

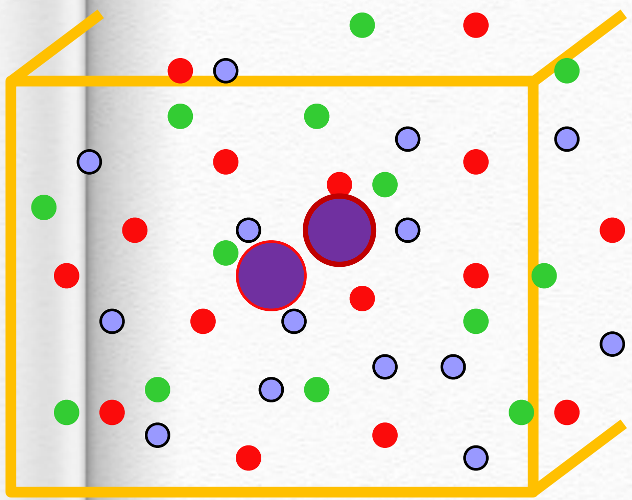
Property	The model
Confinement	Yes
Cluster separability	Yes
Exchange symmetry	Yes
qbar-q production	No
Low density limit (isolated hadrons)	Yes
High density limit (free Fermi gas of quarks)	Yes

The inclusion of interactions between the quarks provides a dynamical picture of the system evolution from low to high energy density.

Color screening

G. Toledo & J. Piekarewicz PRC 70, 3526(04)

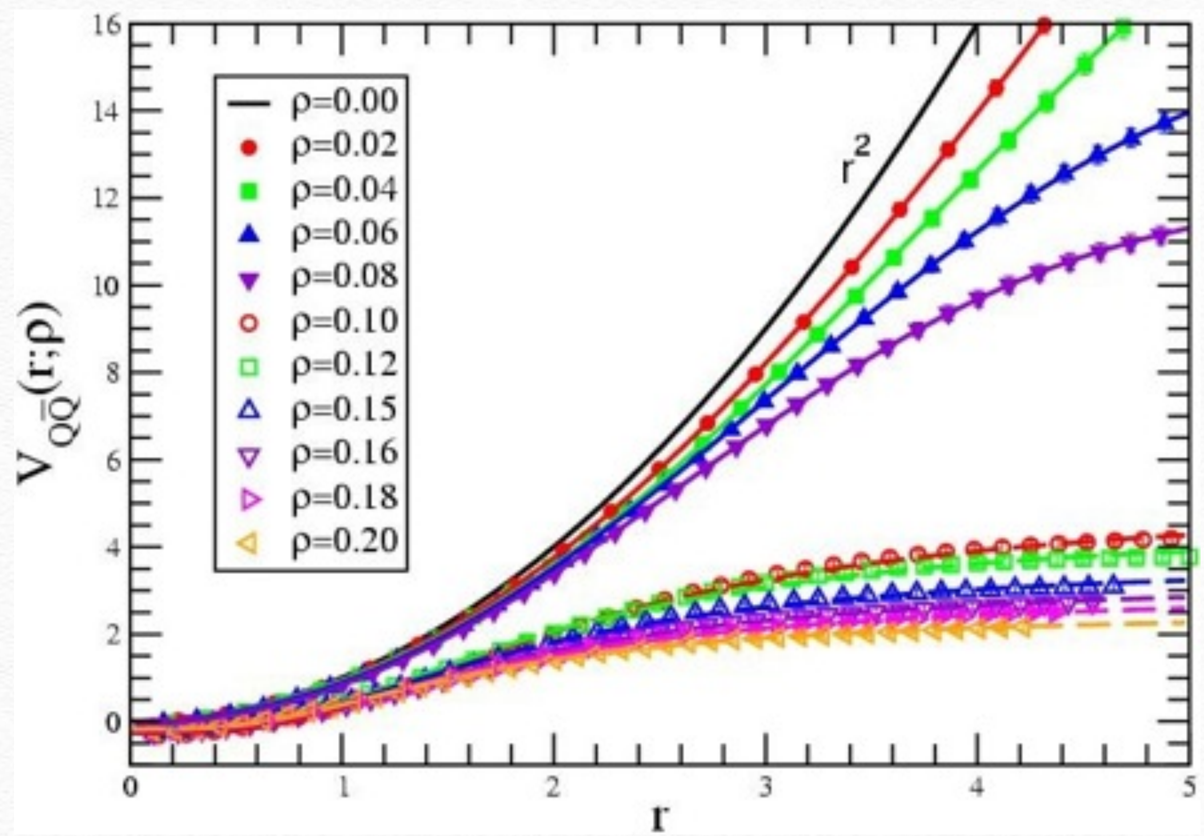
Matsui and H. Satz Suppression PLB 178, 416(86)
J. Rafelski et al Enhancement by recombination PRC 63, 054905(01)

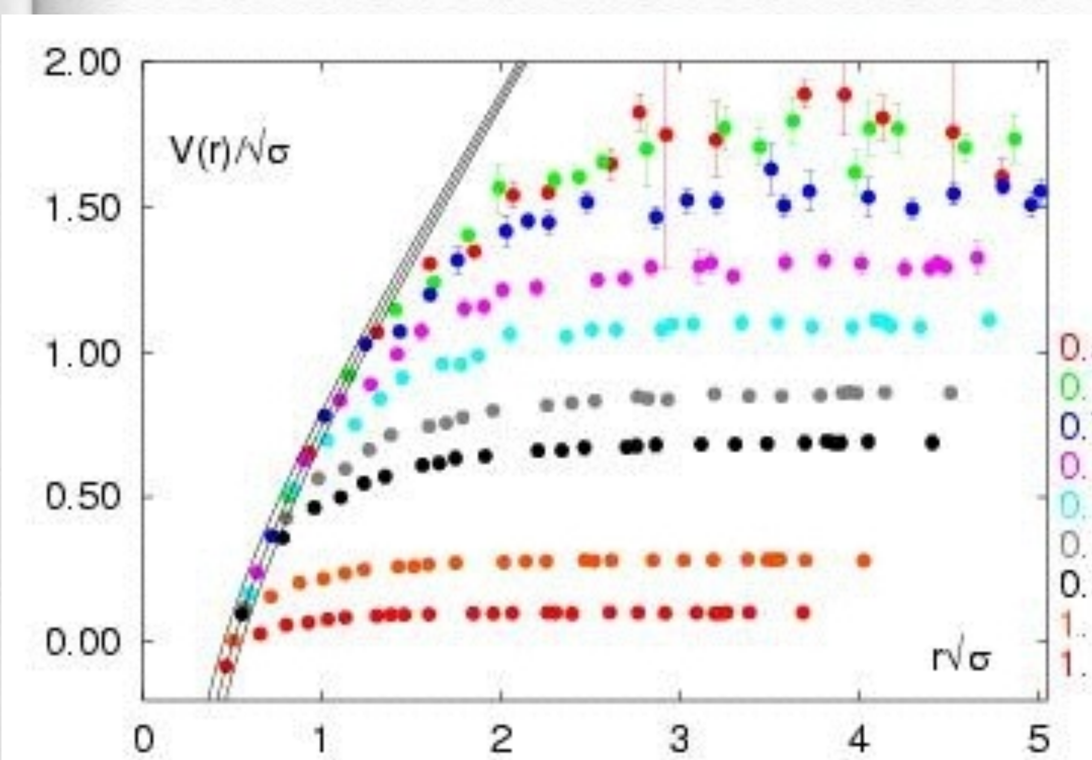


We can compute the potential of a heavy-quark pair by the change on the system potential energy due to their appearence at a relative distance r

$$V_{QQ}(r)=<V>_{A+1}(r)- <V>_{A}$$

quark-antiquark potential at zero temperature and finite baryon density





Lattice QCD result at finite temperature and zero baryon density F. Karsch Nucl. Phys B (2001), hep-lat/0312037

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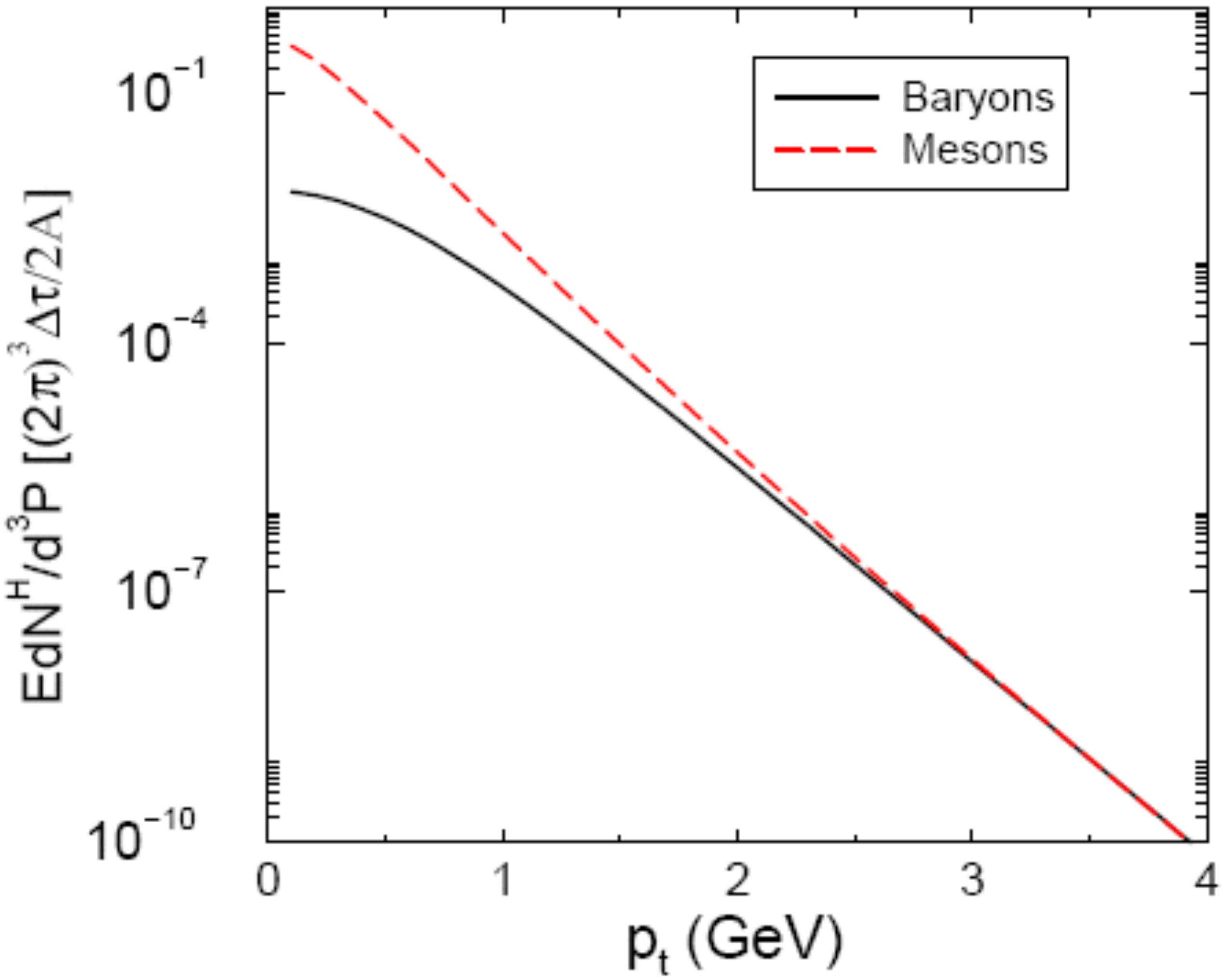
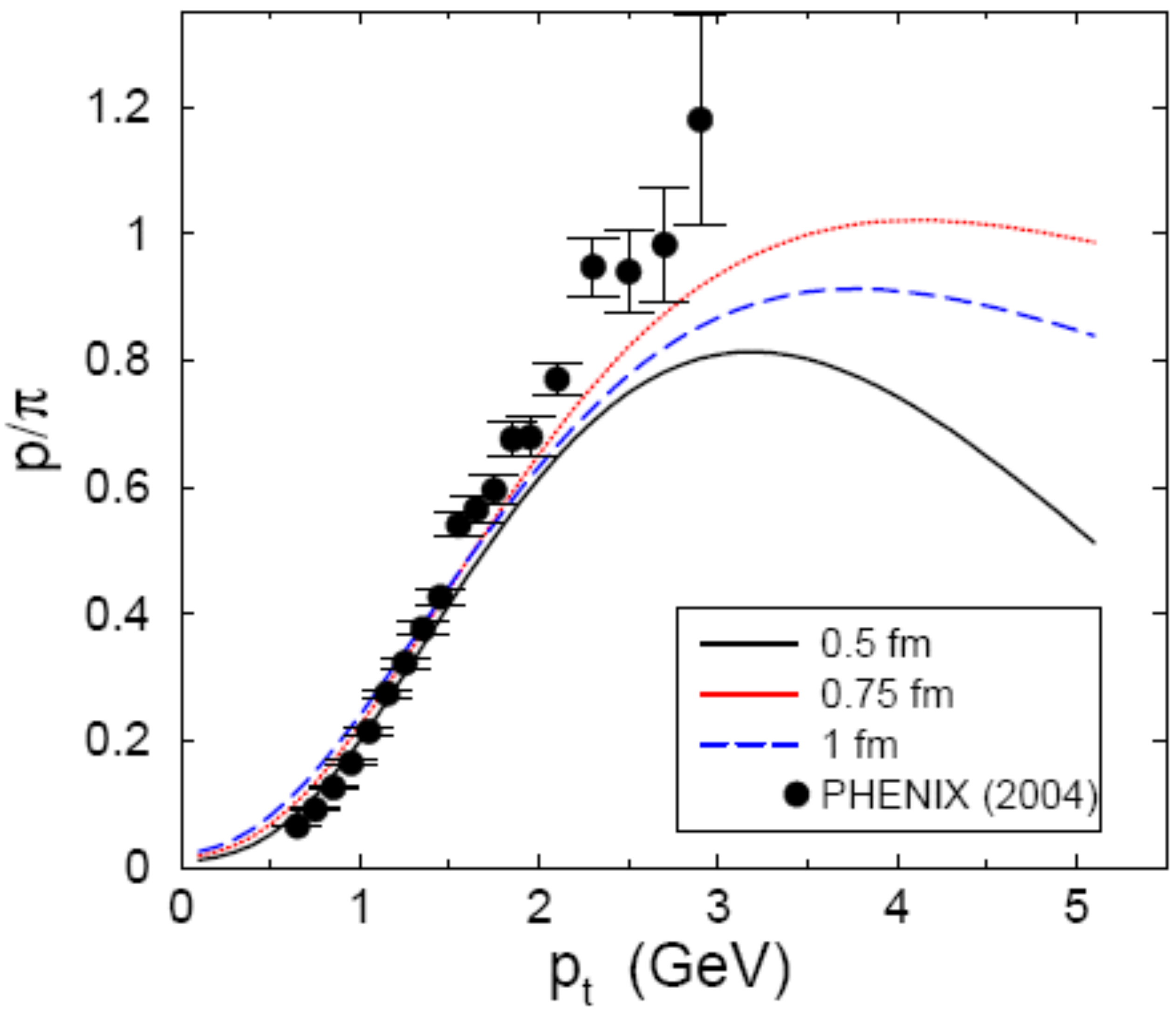
Proton to pion ratio from dynamical recombination in RHIC

PRC 77 044901(08), A. Ayala, M. Martínez, G. Paic, G. Toledo

Momentum distributions



p/π ratio prediction



Several initial evolution times
and same final freeze-out τ_f =3.5fm

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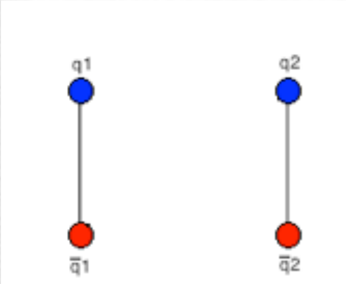
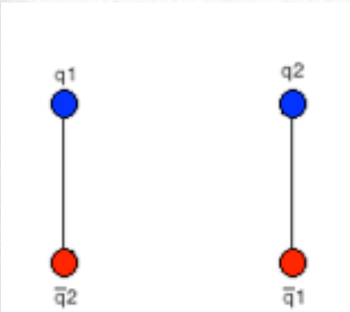
Four-body system

Many-body potential

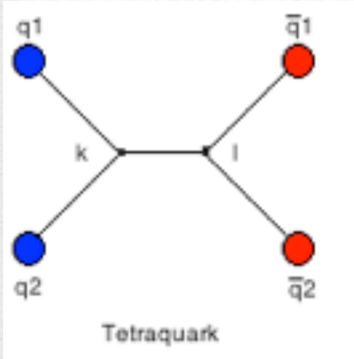


$$V_{m1} = V(\vec{r}_1, \vec{r}_3) + V(\vec{r}_2, \vec{r}_4)$$

Quark exchange among mesons
(meson-meson interaction)



$$V_{m2} = V(\vec{r}_1, \vec{r}_4) + V(\vec{r}_2, \vec{r}_3).$$



$$V_{4Q} = \sum_{i=1}^{2} V(\vec{k}, \vec{r}_i) + \sum_{j=3}^{4} V(\vec{l}, \vec{r}_j) + V(\vec{k}, \vec{l}).$$

Tetraquark configuration

k and **l** vectors minimize the total length to connect the four particles. Numerical determination is invoked after any single particle changes position.

$$V = min(V_{m1}, V_{m2}, V_{4Q})$$

F. Lenz et al., Annals of Phys. 170 65(1986).

Variational approach

Variational wave function

Variational parameter

$$\Psi_\lambda = \Phi_{FG}\left(e^{-\lambda Q}\right),$$

FG: Fermi correlations among identical quarks

J. Carlson and V. R. Pandharipande Phys. Rev. D 43 1652(1991).

J. Vijande, A. Valcarce, J. M. Richard, Phys. Rev. D 76 114013 (2007); ibid 87 034040(2013).

$$Q_{m_1} = r_{13}^{3/2} + r_{24}^{3/2}, \qquad Q_{m_2} = r_{14}^{3/2} + r_{23}^{3/2} \qquad Q_{4Q} = r_{1k}^{3/2} + r_{2k}^{3/2} + r_{kl}^{3/2} + r_{3l}^{3/2} + r_{4l}^{3/2},$$

Energy evaluation

$$H = \sum_{i=1}^{4} \frac{P_i^2}{2m_i} + V.$$

$$\langle H\rangle_\lambda = T_{FG} + \langle W\rangle_\lambda + \langle V\rangle_\lambda,$$

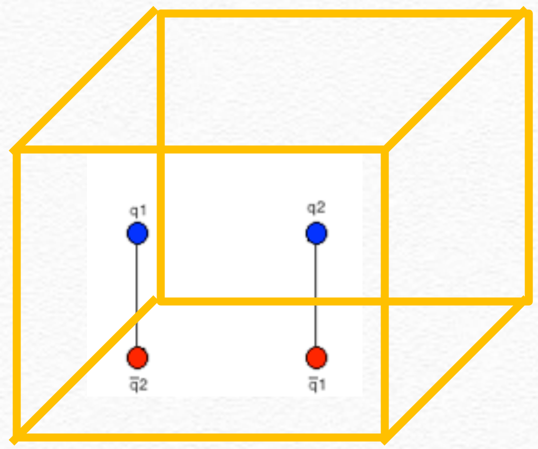
$$W = \sum_{i=1}^{4} \frac{\lambda^2}{2m_i} \sum_j \left[\partial_j Q\right]^2,$$

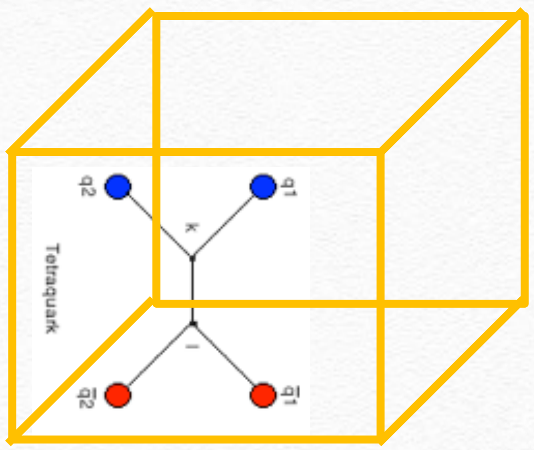
$$\frac{\partial \langle H\rangle_\lambda}{\partial\lambda} = 0$$

C. J. Horowitz, E. J. Moniz and John W. Negele, Phys. Rev. D 31 1689(1985). M. Oka and C. J. Horowitz, Phys. Rev. D 31 2773(1985) G. Toledo Sanchez and J. Piekarewicz, Phys. Rev. C 65, 045208 (2001); ibid 70, 035206 (2004).

Four-body system

Flipping among color singlets configurations





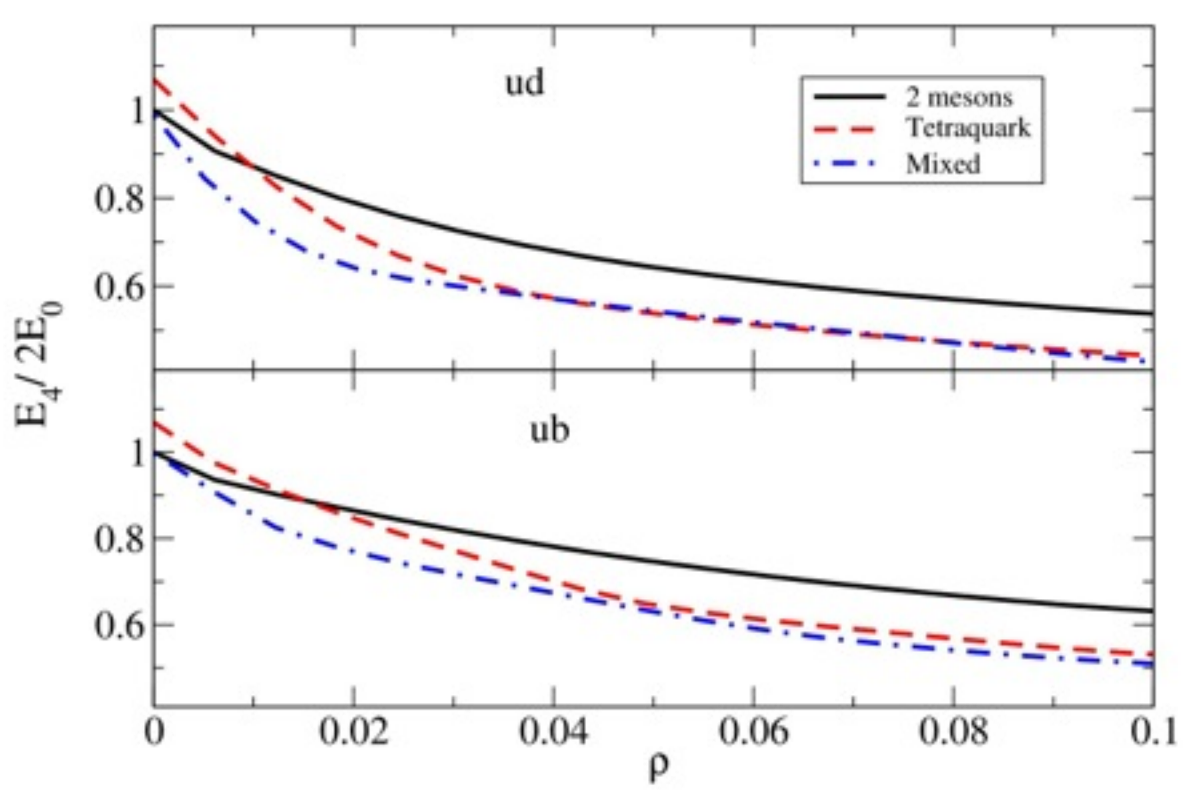
Results

We performed a MC simulation to study three cases:

Two mesons interacting via quark exchange
Tetraquark
Mixed

We define a particle density parameter as a measure of the inter-particle separation: $\varrho = N/V = 4/L^3$
N: number of particles, V: box volume.

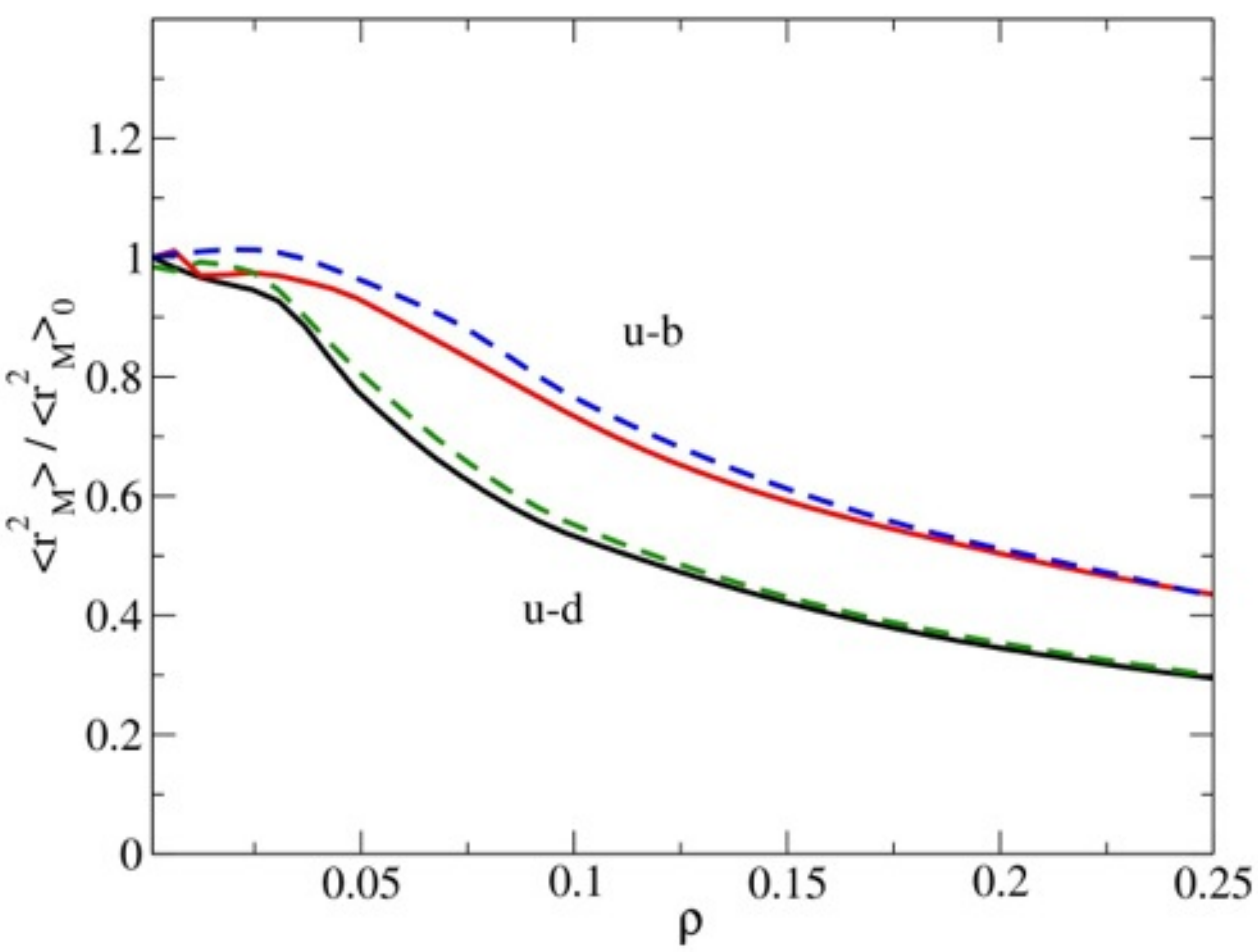
Energy evolution



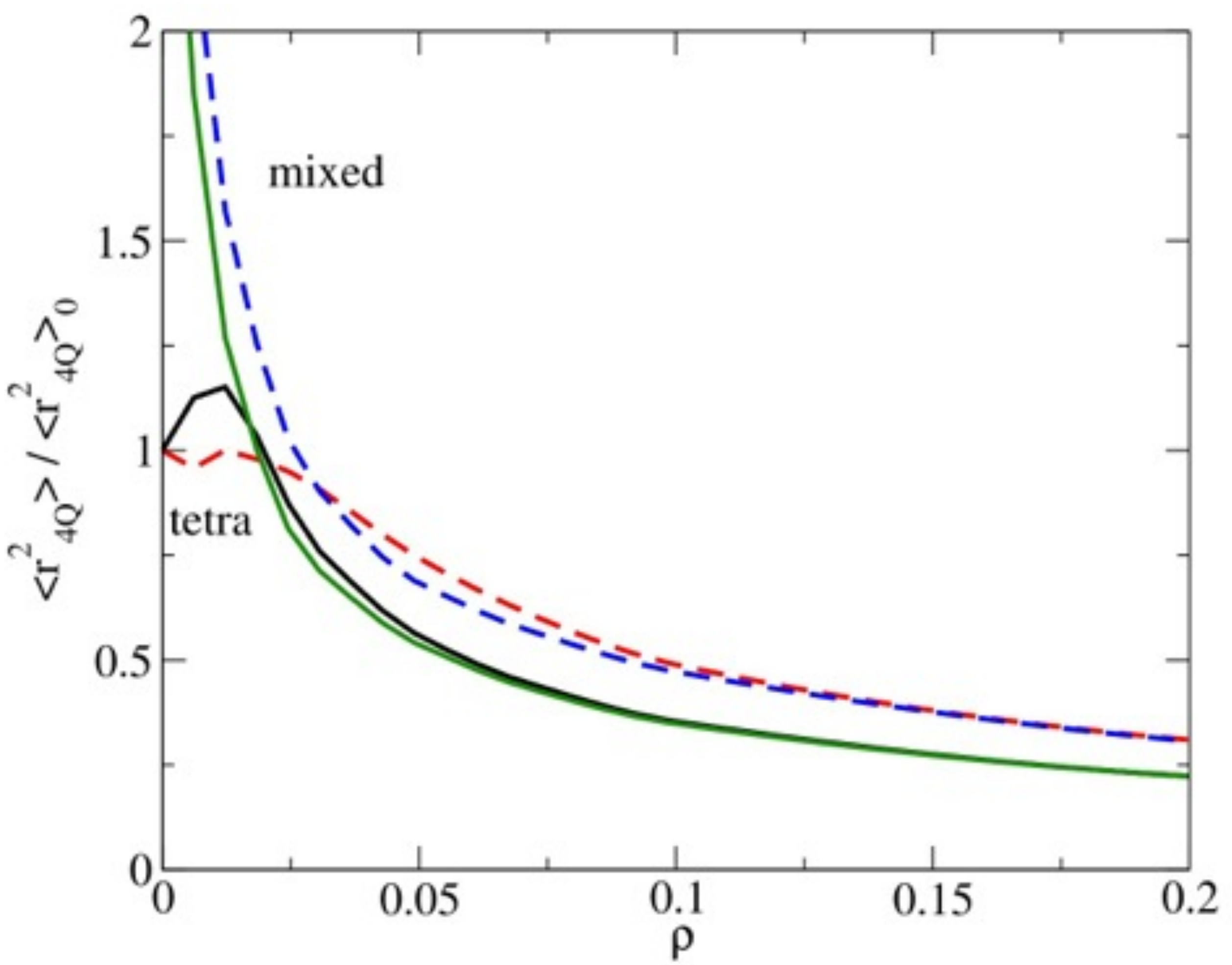
m_2/m_1	$2E_0$	E_{4Q_0}	$\langle r^2_{4Q} \rangle_0$
1	4.6944	5.02 ± 0.02	13.9 ± 0.1
1.44643	4.4394	4.77 ± 0.02	13.2 ± 0.2
4.6131	3.9778	4.24 ± 0.02	12.5 ± 0.3
14.0774	3.8122	4.07 ± 0.02	13.1 ± 0.2

Tetraquark case. Zero density limit. Energy and Mean Square radius, compared to two mesons for several mass ratios

Mean square radius



Meson

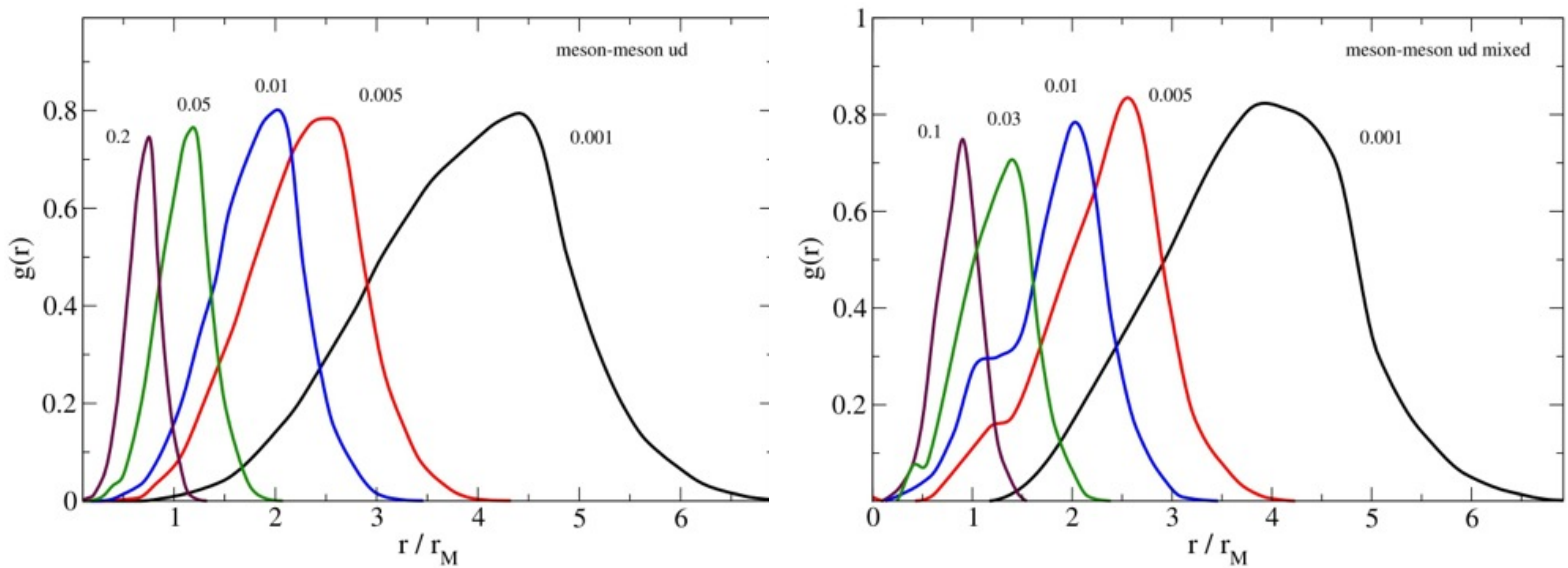


Tetraquark

Normalized to the zero density limit

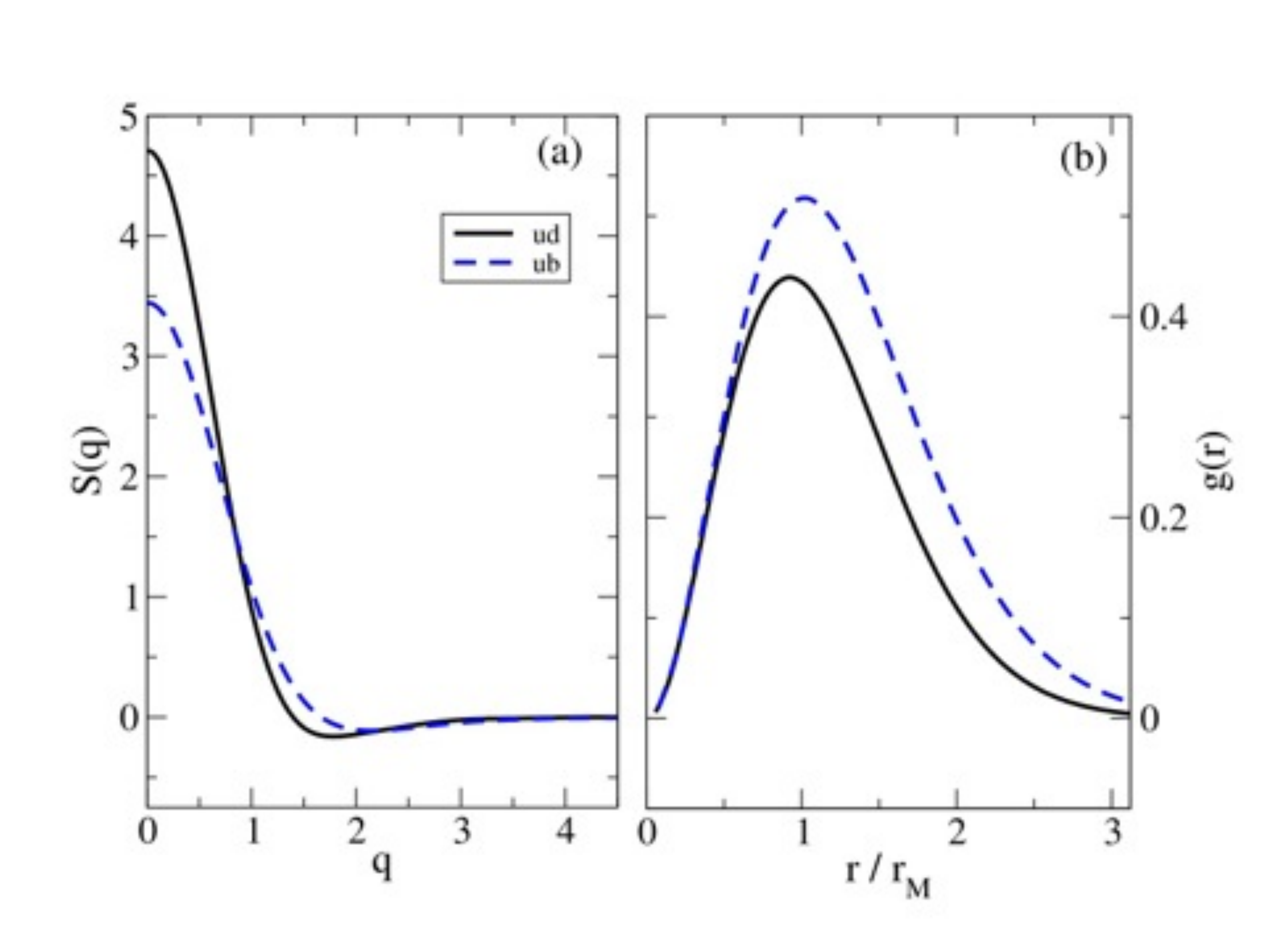
Two particle correlation function

$$g(r) \equiv \frac{V}{4\pi r^2 N^2} \left\langle \sum_{i<j=1}^{N} \delta\left(\vec{r} - \vec{r}_{ij}\right) \right\rangle$$



Meson- Meson correlation function is modified by the tetraquark presence

Di-quark



Di-quark correlation function parametrization

$$g(r)_{q-q} = A_0 r^2 e^{-r^{A_2}/A_1^2}$$

A0=0.64, A1=1.24, A2=1.51

Samuel H. Blitz and Richard F. Lebed, Phys. Rev. D 91 094025(2015). Pedro Bicudo and Marc Wagner, Phys. Rev. D 87, 114511 (2013).

The static structure factor S(q) can be obtained as the Fourier transform of the correlation function g(r)

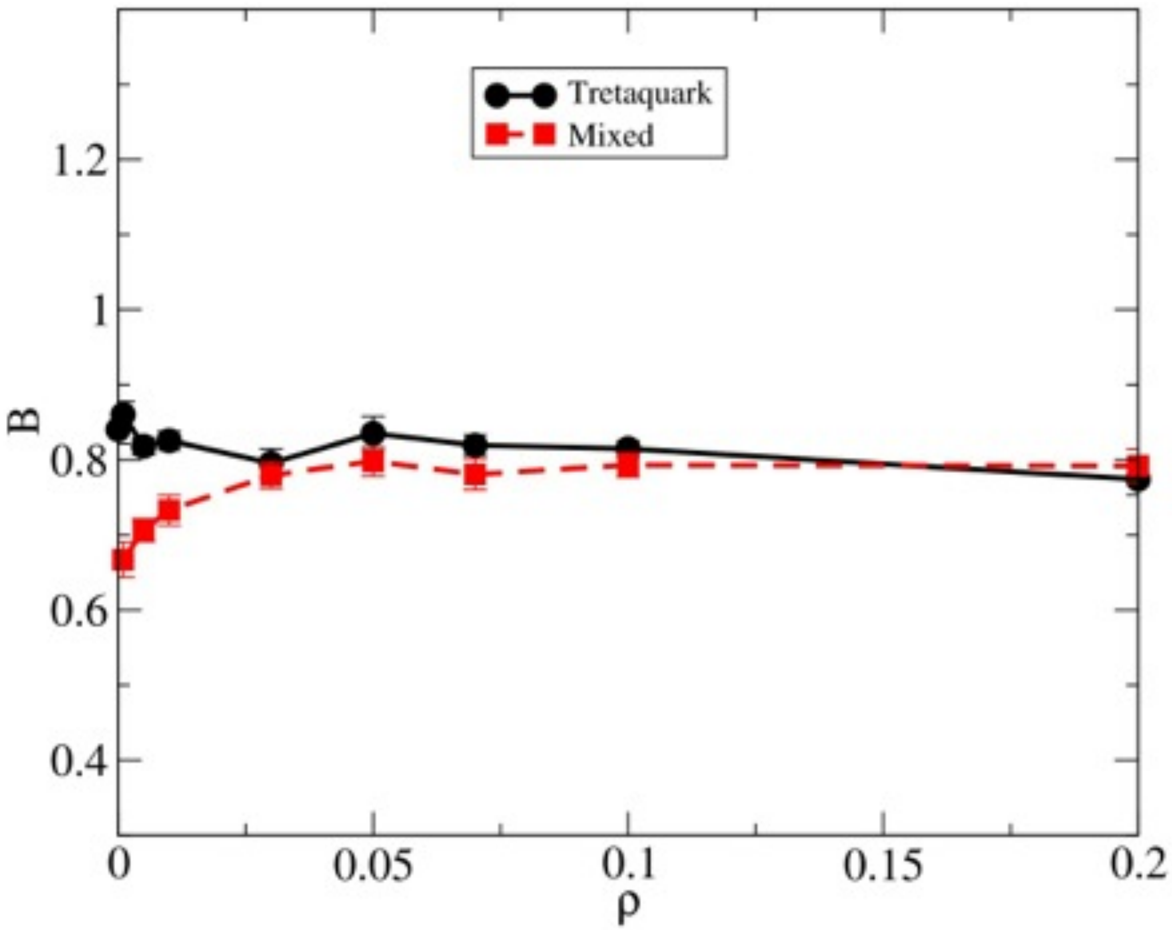
$$S(\mathbf{q}) = 1 + \frac{N}{V} \int d^3r \; g(\mathbf{r}) \; e^{-i\mathbf{q}\cdot\mathbf{r}}.$$

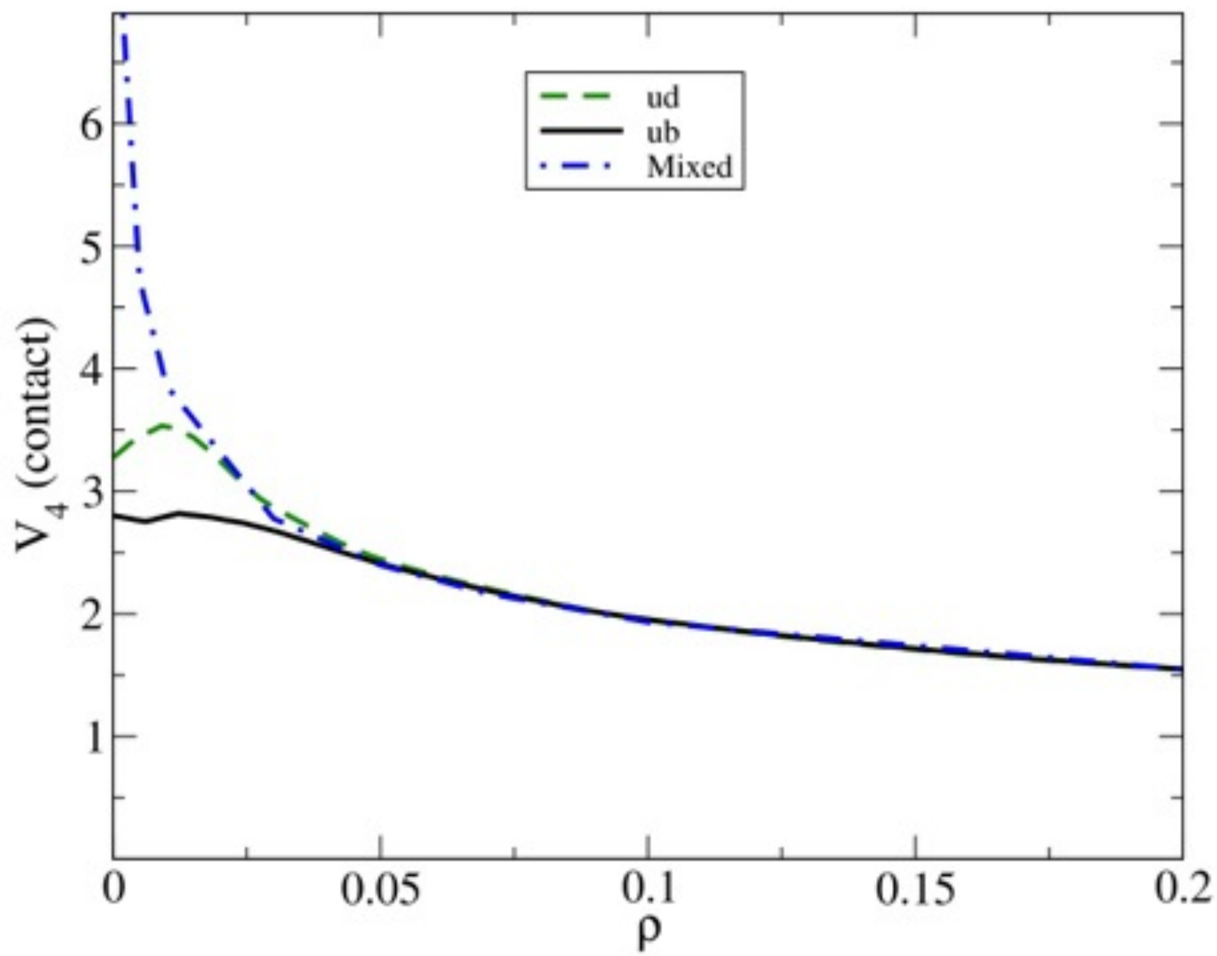
Tetraquark Potential

$$V(R) = R_0 + BR \qquad R \equiv \sqrt{\sum r_{ij}^2}$$

Linear behaviour

Effective Contact

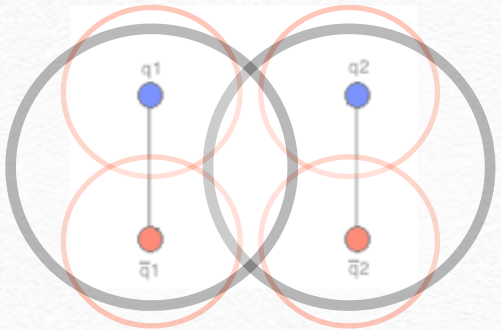




Recombination effective scenarios

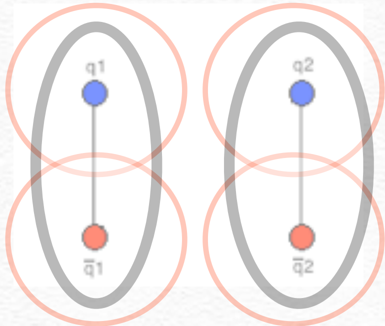
Scenario I: Spherical Volumes

The probability for a reconnection to occur is taken to be proportional to the overlap of the spherical colour sources



Scenario II: Elongated bags

The recoupling probability is taken to be proportional to the integrated overlap between cylinders.

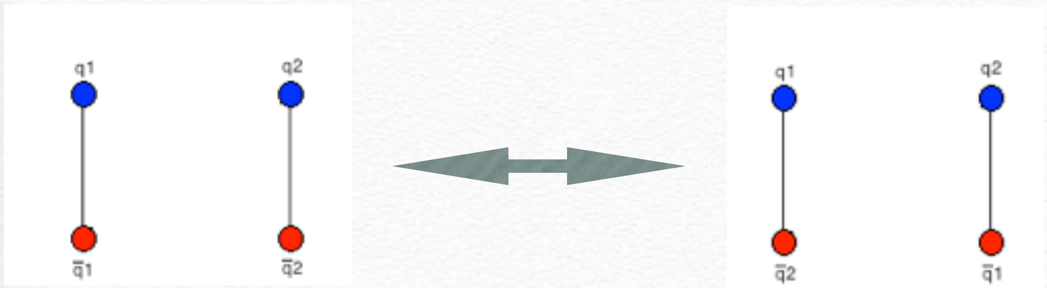


G. Abbiendi et al, (The OPAL Collaboration), Eur. Phys. J. C. 45 307(2006).

T. Sjostrand and V. A. Khoze, Z. Phys.C 62 281(1994); Phys. Rev. Lett. 72 28(1994).

Dynamical recombination

Scenario A. We define the recombination probability for the **two mesons** case by

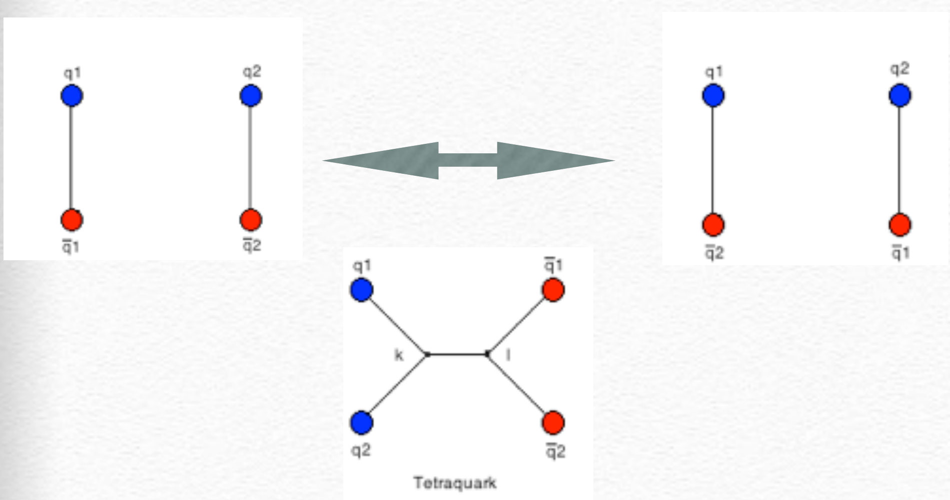


$$Pr_{2m} = \frac{N(V_{m1} \leftrightarrow V_{m2})}{N(V_{m1}) + N(V_{m2})}$$

where $N(V_i \rightarrow V_j)$ denotes the number of flippings from the i to the j configuration,

and $N(V_i)$ is the number of times the system visits the i configuration.

Scenario B. If we consider the **tetraquark** configuration as another possible configuration contributing to the recombination, the probability becomes:



$$\begin{aligned} Pr_{mix} = \; & [N(V_{m1} \leftrightarrow V_{m2}) + N(V_{m1} \leftrightarrow V_{4Q}) + \\ & N(V_{m2} \leftrightarrow V_{4Q}) + N(V_{4Q} \rightarrow V_{4Q})] \, / \\ & [N(V_{m1}) + N(V_{m2}) + N(V_{4Q})] \end{aligned}$$

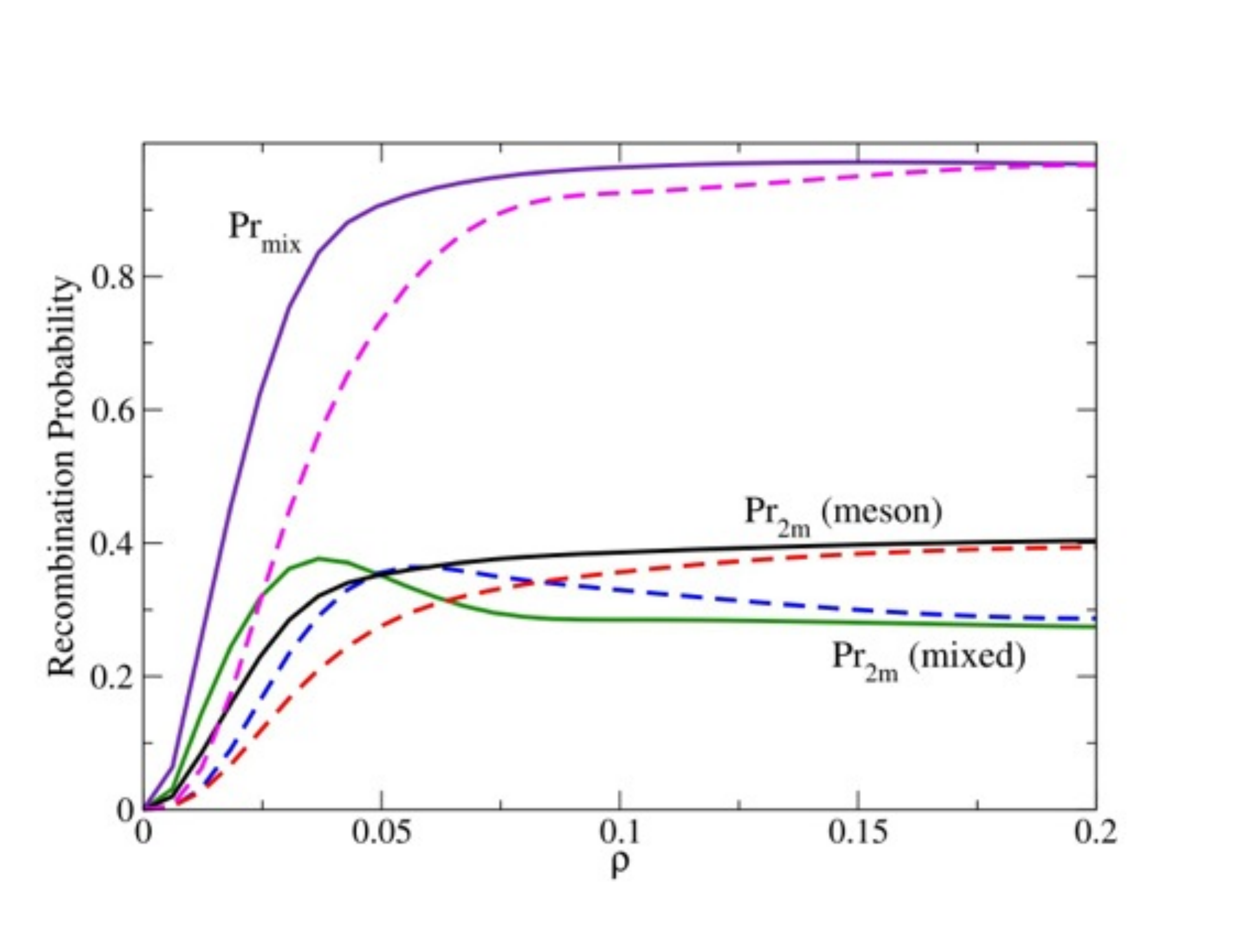
G. Abbiendi et al, (The OPAL Collaboration), Eur. Phys. J. C. 45 307(2006).

T. Sjostrand and V. A. Khoze, Z. Phys.C 62 281(1994); Phys. Rev. Lett. 72 28(1994).

Dynamical recombination

$$Pr_{2m} = \frac{N(V_{m1} \leftrightarrow V_{m2})}{N(V_{m1}) + N(V_{m2})}$$

$$Pr_{mix} = [N(V_{m1} \leftrightarrow V_{m2}) + N(V_{m1} \leftrightarrow V_{4Q}) + N(V_{m2} \leftrightarrow V_{4Q}) + N(V_{4Q} \to V_{4Q})] / [N(V_{m1}) + N(V_{m2}) + N(V_{4Q})]$$



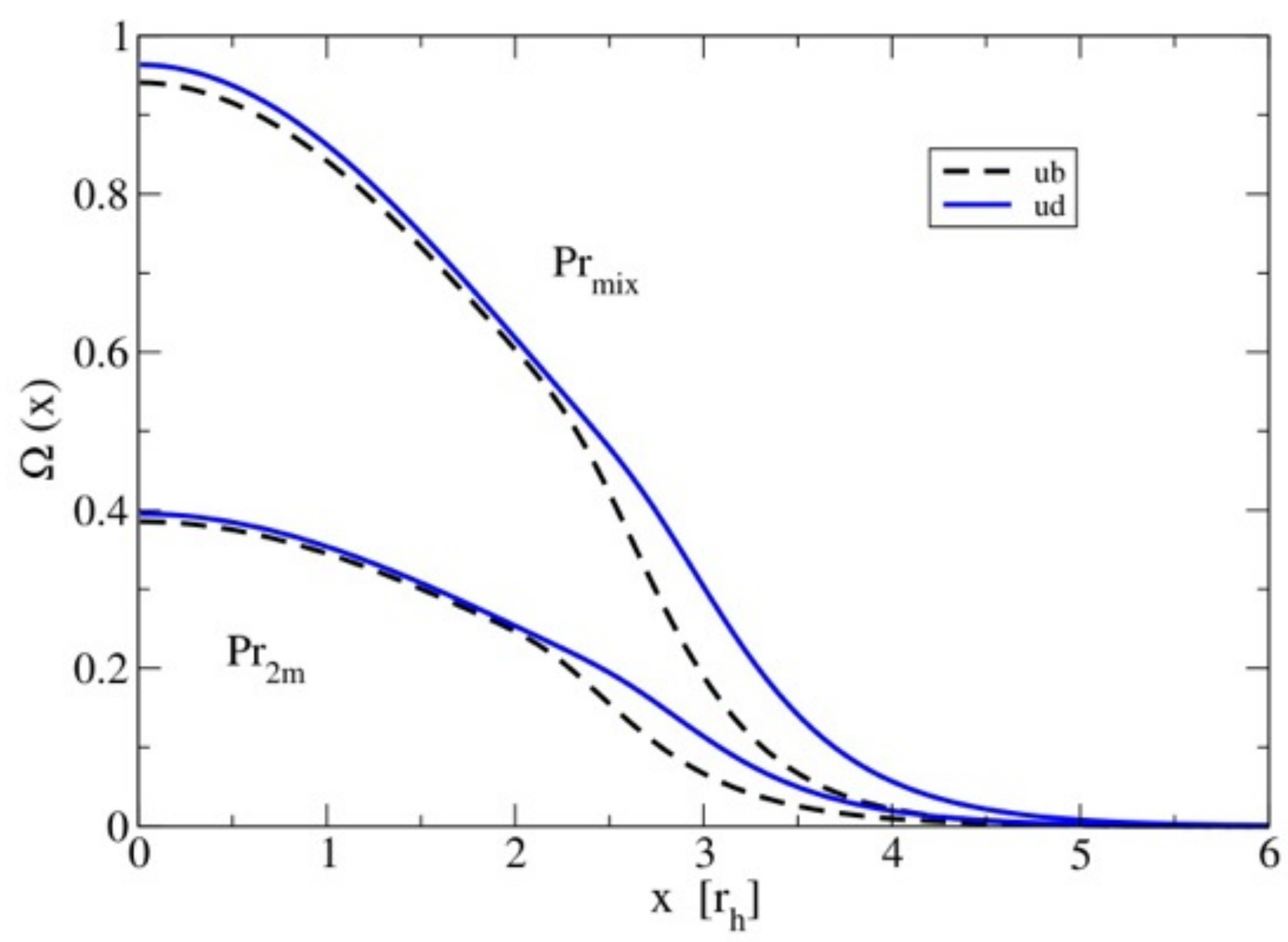
Dynamical Recombination

A qualitative estimate of this effect can be exhibited considering a simple expansion model, such that we can evolve the system along the density profile. We can define the color strength function

$$\Omega(x) \equiv P_{frag}(t)Pr(x)$$

where Pfrag(t) is the probability that the system has not yet fragmented, with $\tau_{frag} \approx 3r_h$ the proper lifetime, taken as three times the meson radius r_h.

$$P_{frag}(t) = exp(-t^2/\tau^2_{frag})$$



Conclusions

We have performed a MC simulation considering three possible structures: two mesons, tetraquark and mixed configurations

We determined wether it is energetically more favorable to form a tetraquark or two mesons and the mixing among them, as a function of the particle density.

We have shown that there is a modification in the meson-meson correlation function by the presence of the tetraquark state at intermediate densities

A parameterization was found for the diquark , which is useful to compute additional static properties, in particular we computed the diquark static structure factor.

We did track the dynamical flipping among configurations and determined the recombination probability evolution as a function of the particle density. We have shown that the probability is largely affected when considering the tetraquark as an intermediate recombination state.

Thanks !

Variational parameter