An Application of the Non-Extensive approach: the Soft+Hard model at various energies

G.G. Barnaföldi & T.S. Biró & K. Ürmössy & P. Ván

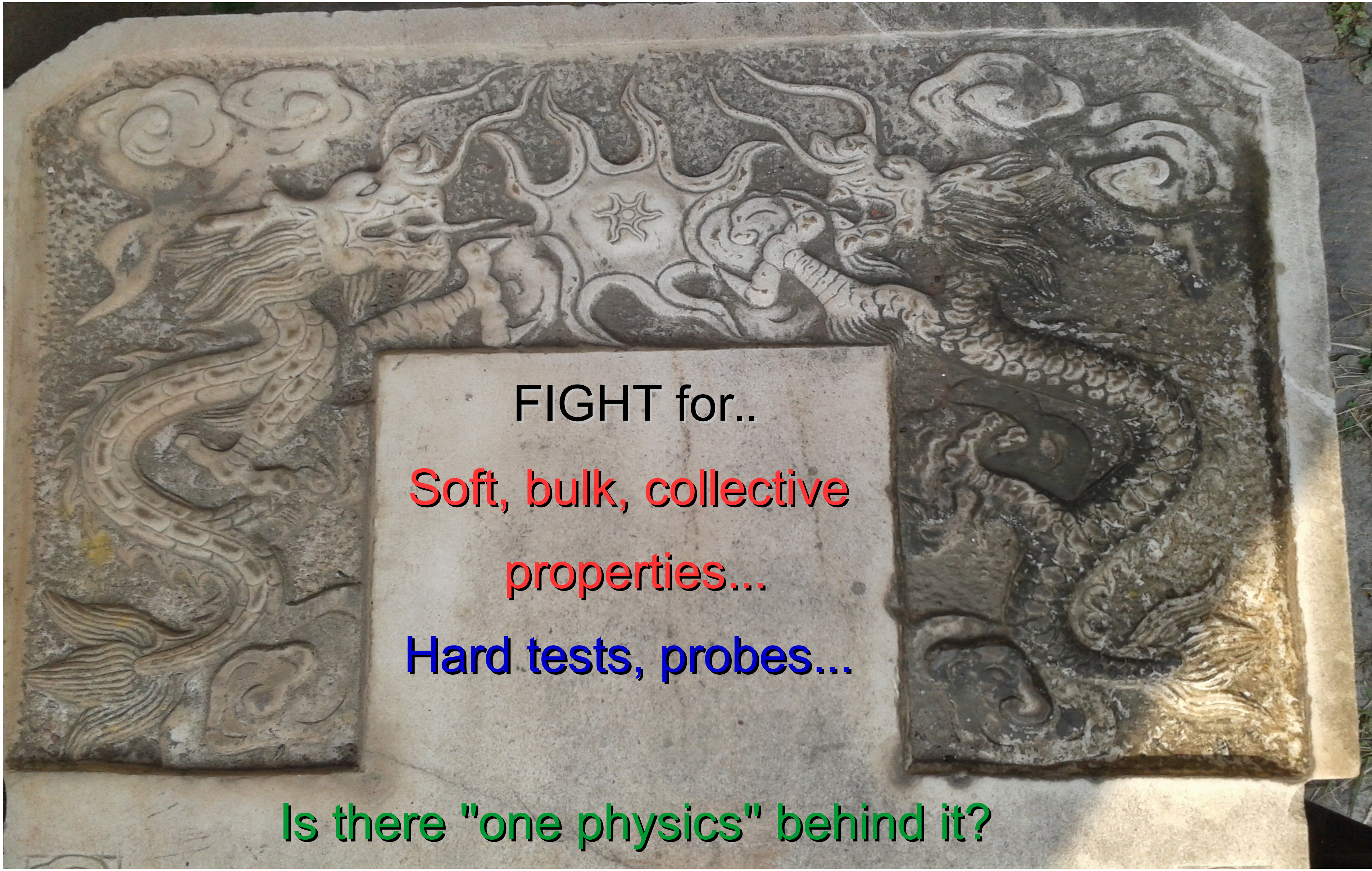
Wigner RCP of the Hungarian Academy of Sciences

Approach: Eur. Phys. J. A49 (2013) 110, Physica A 392 (2013) 3132

Application: J.Phys.CS 612 (2015) 012048 arXiv:1405.3963, 1501.02352, 1501.05959

QCD Challenges at the LHC, Taxco, Mexico, 21st January 2016

I N T R O...


FIGHT for..
Soft, bulk, collective
properties...
Hard tests, probes...
Is there "one physics" behind it?

OUTLINE

- Motivation...
  - Is there physics behind the parameters of FFs?
  - How about the p_T power of the tail?
  - Can we understand an experimental parameter, T, which we use to fit to low the p_T spectra?

- For 'hard' guys: Derivation of the parameter q

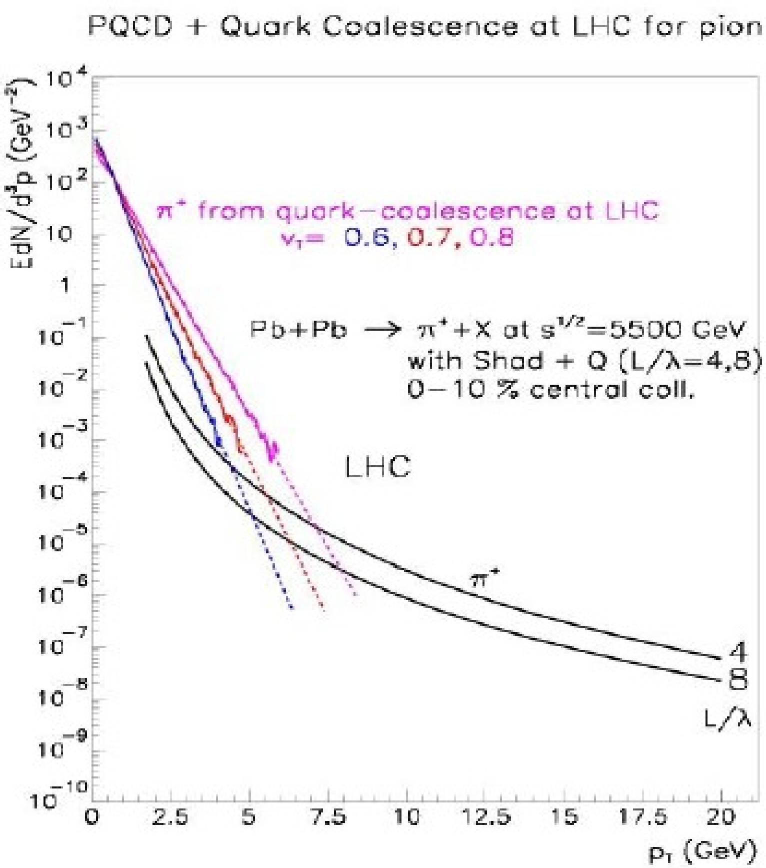
  - The phyiscal meaning of the 'mysterious q' by deriving Tsallis/Rényi-like entropies from the first principles

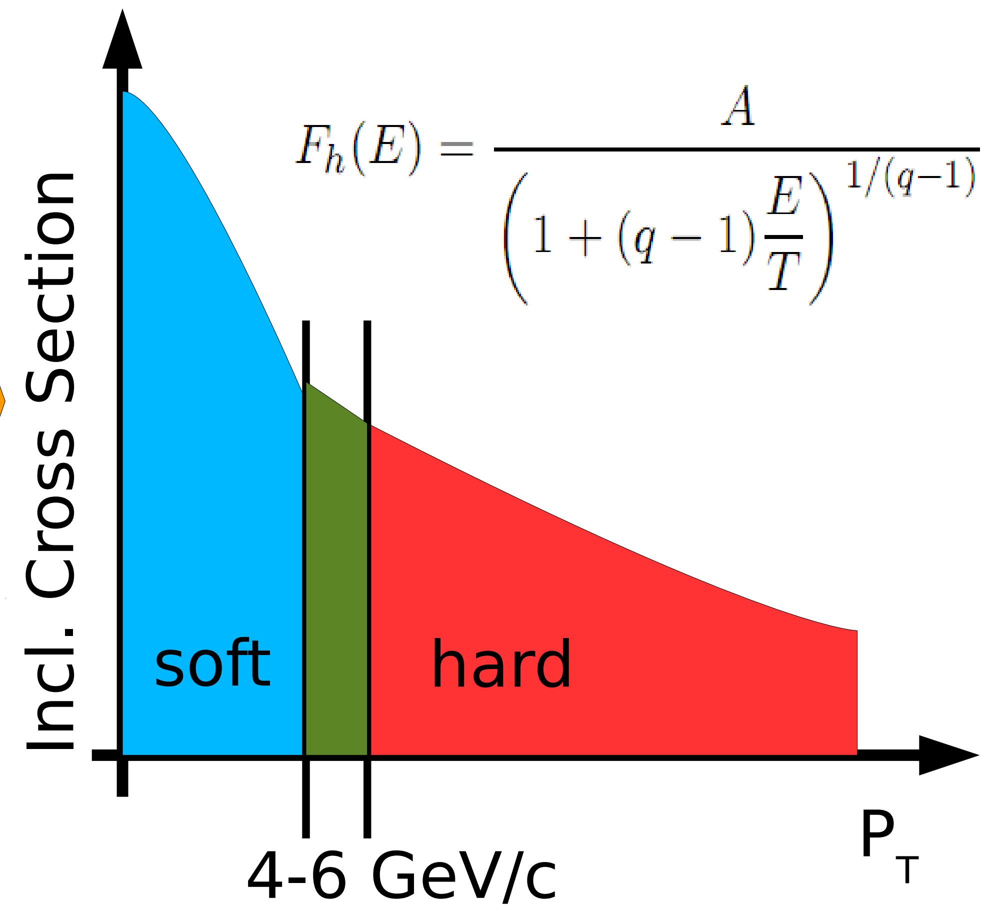
- For 'soft' guys: What can be the parameter T?

An application: a simple Bag model to get QGP temperature

MOTIVATION

- Simplest and best fit to hadron spectra at low-p_T & high-p_T





P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

What is the physical meaning of these 'q' and 'T' parameters?

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The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$S_{12} = S_1 + S_2$$
$$E_{12} = E_2 + E_2$$

$\Rightarrow$ $S_B = -\sum_i p_i \ln p_i$

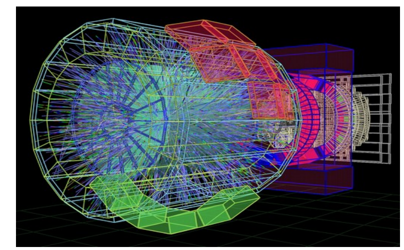
The non-extensive statistical approach

- Extensive Boltzmann – Gibbs statistics

$$\begin{aligned} S_{12} &= S_1 + S_2 \\ E_{12} &= E_2 + E_2 \end{aligned} \quad \longrightarrow \quad S_B = -\sum_i p_i \ln p_i$$

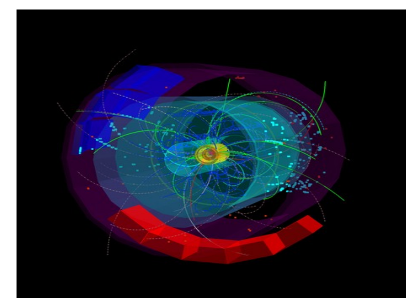
- Non-extensivity → generalized entropy

$$\begin{aligned} \hat{L}_{12}(S_{12}) &= \hat{L}_1(S_1) + \hat{L}_2(S_2) \\ L_{12}(E_{12}) &= L_1(E_1) + L_2(E_2) \end{aligned} \quad \longrightarrow \quad S_T = \frac{1}{1-q} \sum_i (p_i^q - p_i)$$



- Tsallis entropy

$$S_{12} = S_1 + S_2 + (q-1)S_1 S_2 \quad \longrightarrow \quad \hat{L}(S) = \frac{1}{q-1} \ln(1 + (q-1)S)$$



from here: Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

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Physica A 392 (2013) 3132

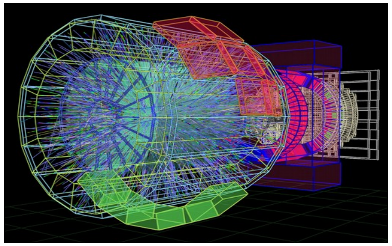
The non-extensive statistical approach

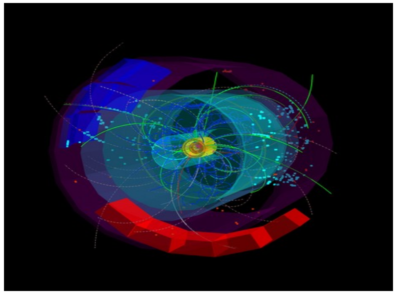
- Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + \boxed{(q-1)}\frac{\varepsilon}{\boxed{T}}\right]^{\boxed{-\frac{1}{q-1}}}$$

$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

$$\frac{1}{T} = \langle S'(E)\rangle$$





Eur. Phys. J. A49 (2013) 110,

Physica A 392 (2013) 3132

The non-extensive statistical approach

- Tsallis – Pareto distribution

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

$$q = \frac{\langle S'(E)^2 + S''(E)\rangle}{\langle S'(E)\rangle^2}$$

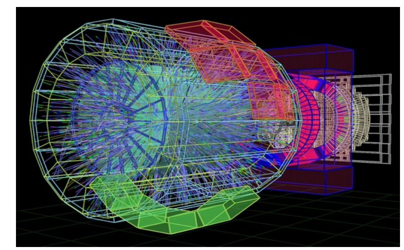
$$\frac{1}{T} = \langle S'(E)\rangle$$

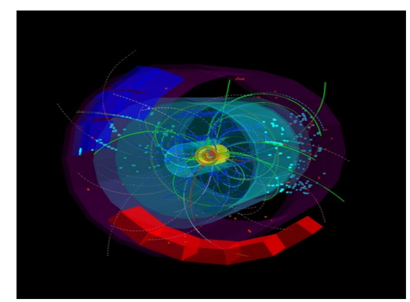
$$q = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2}.$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

$$T = \frac{E}{\langle n\rangle}$$

$$T = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{DT}{1-(q-1)(D+1)}$$





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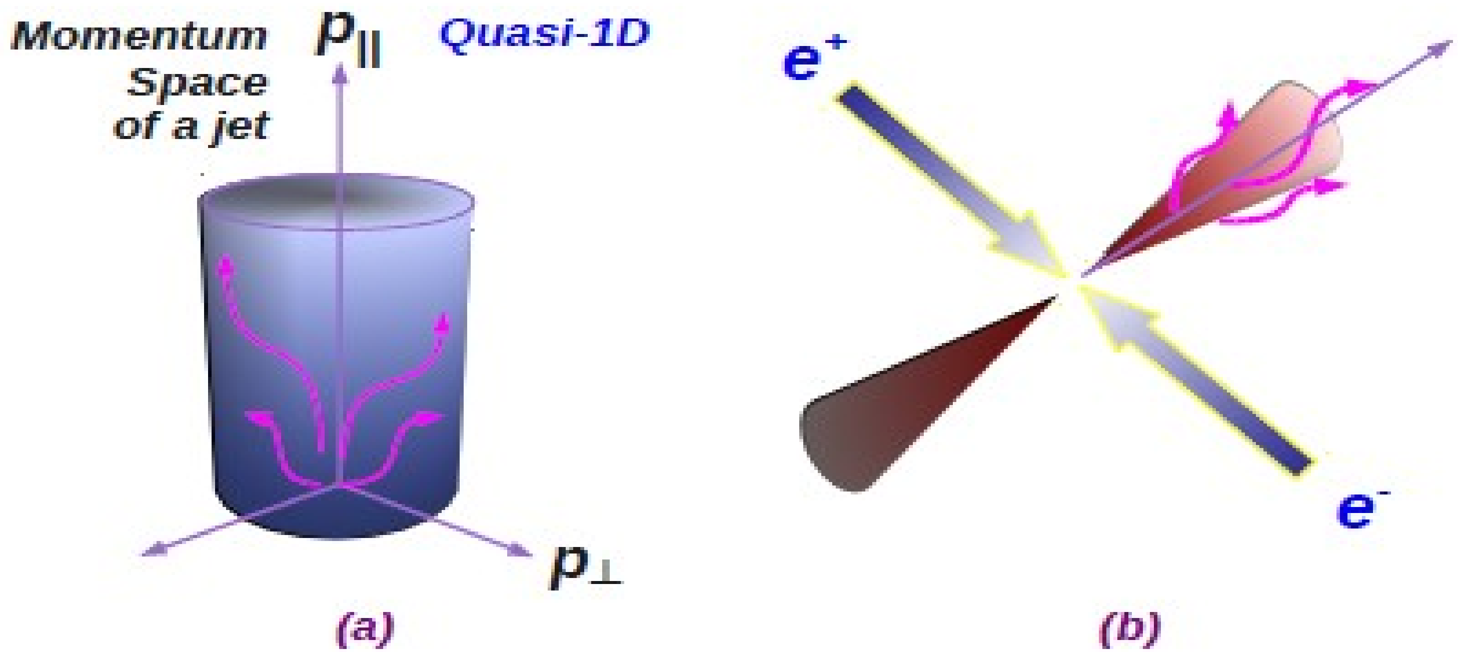
Summary of the theoretical derivation

- Basic principles
  - Keep the standard thermodynamical/statistical formalism
  - In strongly correlated, finite systems, entropy is not additive
  - A generalized, non-extensive entropy can be introduced

- Non-extensive statistical approarch
  - Same formalism, thermodynamical rules
  - Generelized statistics: Tsallis–Pareto, Rényi, etc.
  - New parameter, Tsallis q besides temperature
  - Temperature will be different and connected to q
  - Physical definition/meaning can be derived

Testing the Tsallis-Pareto-like distribution in small systems, like pp or e^+e^-

'Thermodynamics of Jets' in small systems



K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

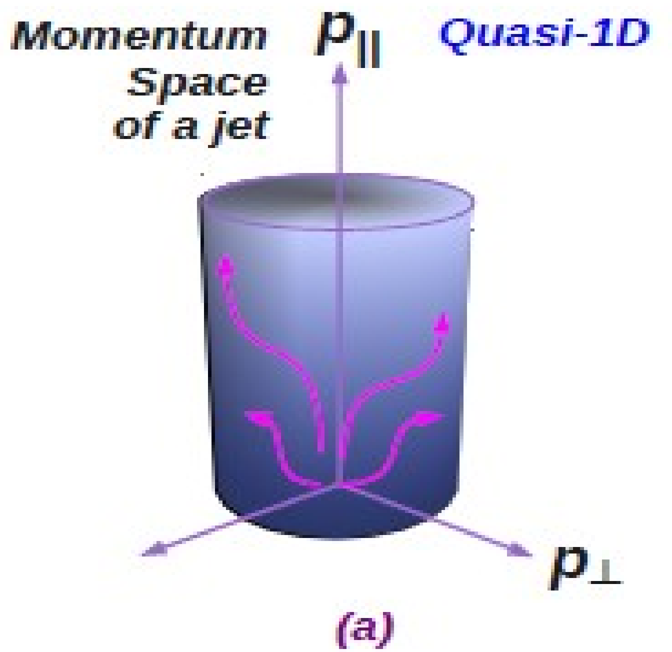
- Microcanonical Jet-Fragmentation in pp at LHC energies:

  Phys. Lett. B701 (2011) 111

- Generalized Tsallis distribution in e^+e^- collisons

  Phys. Lett. B718 (2012) 125

'Thermodynamics of Jets' in small systems



K. Ürmössy, G.G. Barnaföldi, T.S. Bíró:

- Microcanonical Jet-Fragmentation in pp at LHC energies:
  Phys. Lett. B701 (2011) 111

pp: Tsallis–Pareto fits from 0.2-7 TeV

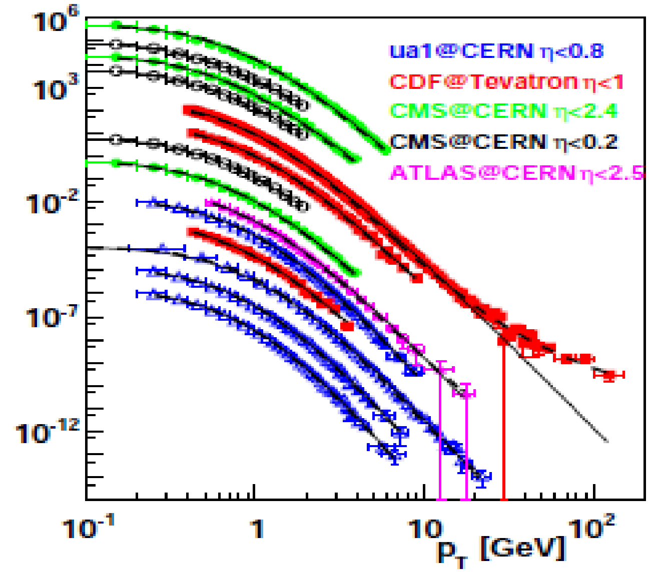
Data used for these pp fits

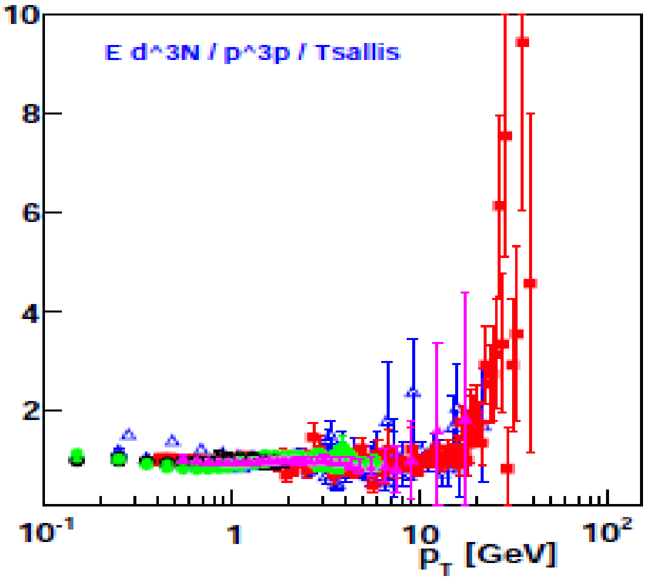
- Khachatryan V et al [CMS] 2010 JHEP02(2010)041, CMS-QCD-10-006, CERN-PH-EP-2010-009, FERMILAB-PUB- 10-170-CMS
- Aaltonen T et al [CDF] 2009 PRD 79 112005
- Adare A et al [PHENIX] 2010 arXiv:1005.3674 [hep-ex]
- Albajar C et al [UAl] 1990 Nucl. Phys. B 335 261
- Bocquet G et al [UAl] 1996 Phys. Lett. B 366 434
- Abe F et al [CDF] 1988 Phys. Rev. Lett. 61 1819
- Aad G et al [ATLAS] 2010 Phys. Lett. B 688 21

Ref: GGB, K. Ürmössy, TS Biró: J.Phys. CS 270 012008 2011

pp: Tsallis–Pareto with evolution in pp

- More TEST: 0.2 - 7 TeV midrapidity data





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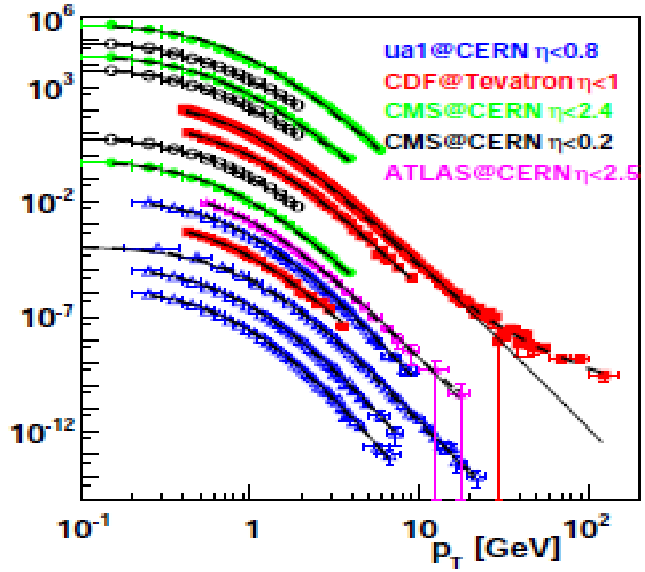
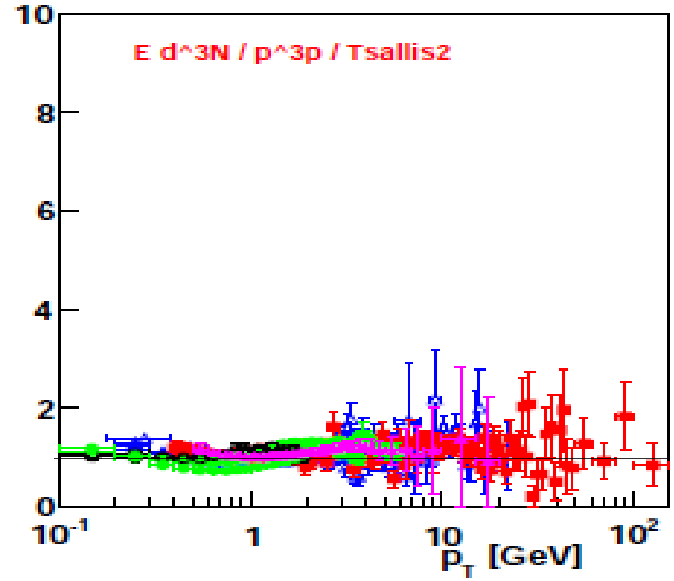
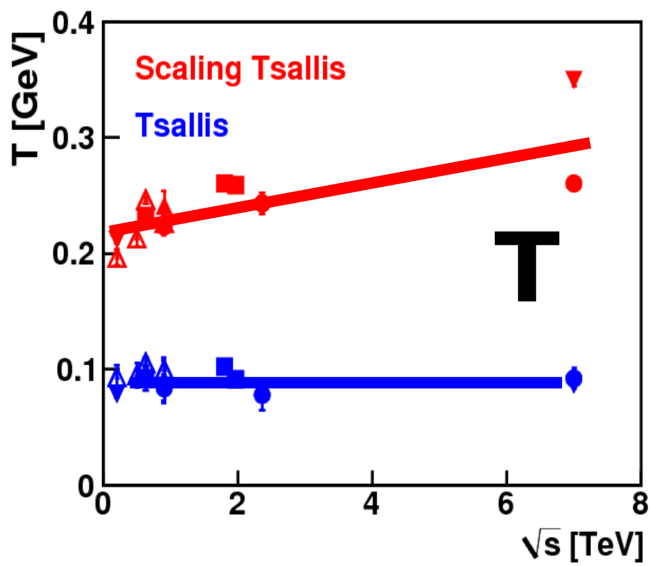
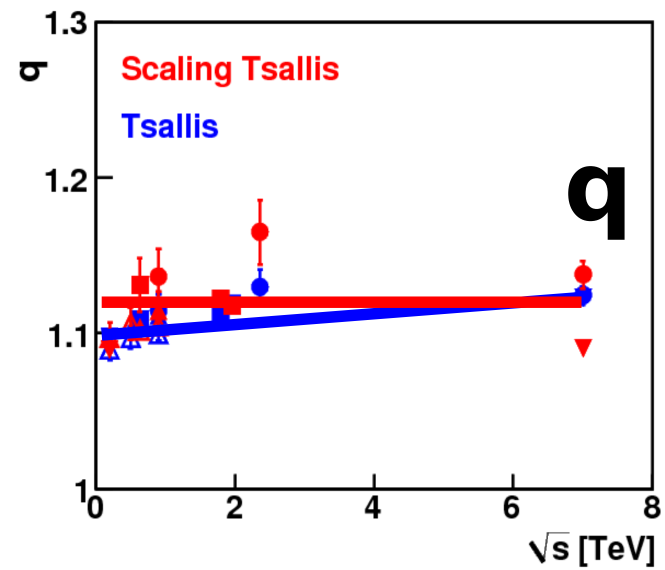




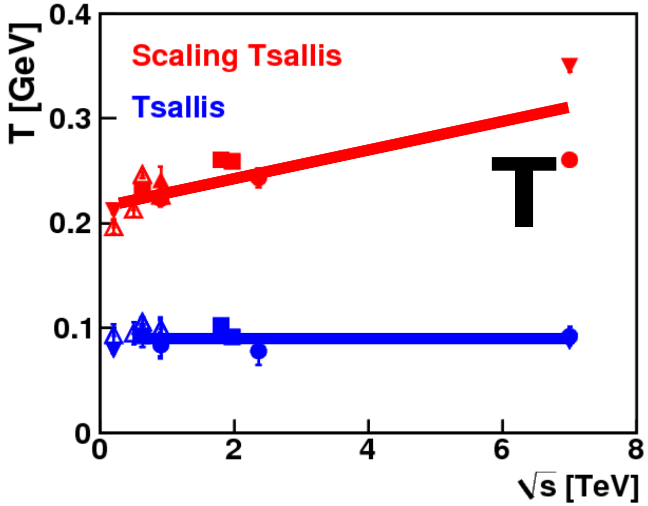
- C.m. Energy dependence of the T & q parameters

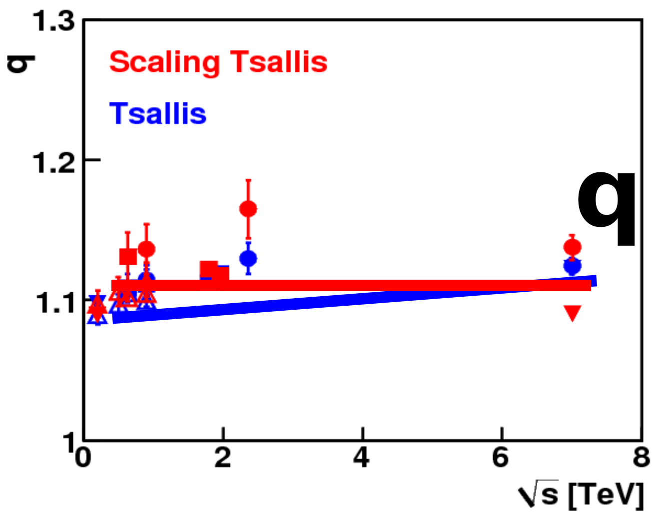


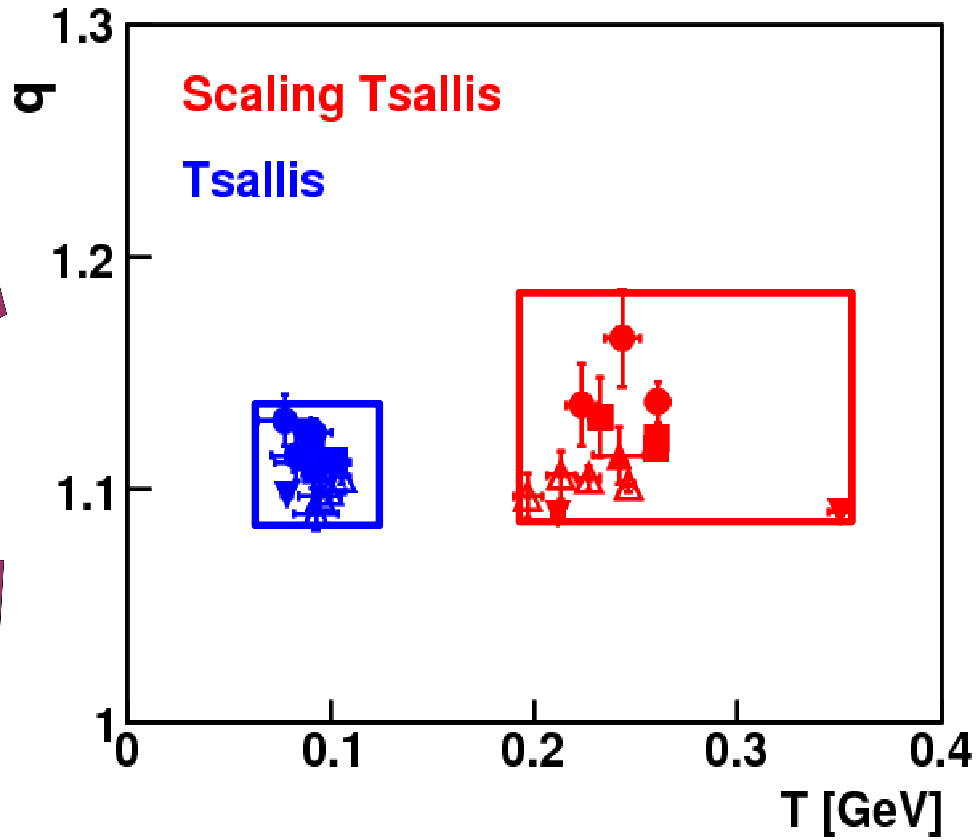


pp: T–q parameter space and evolution

- TEST on various midrapidity pp data @ 0.2-7 TeV

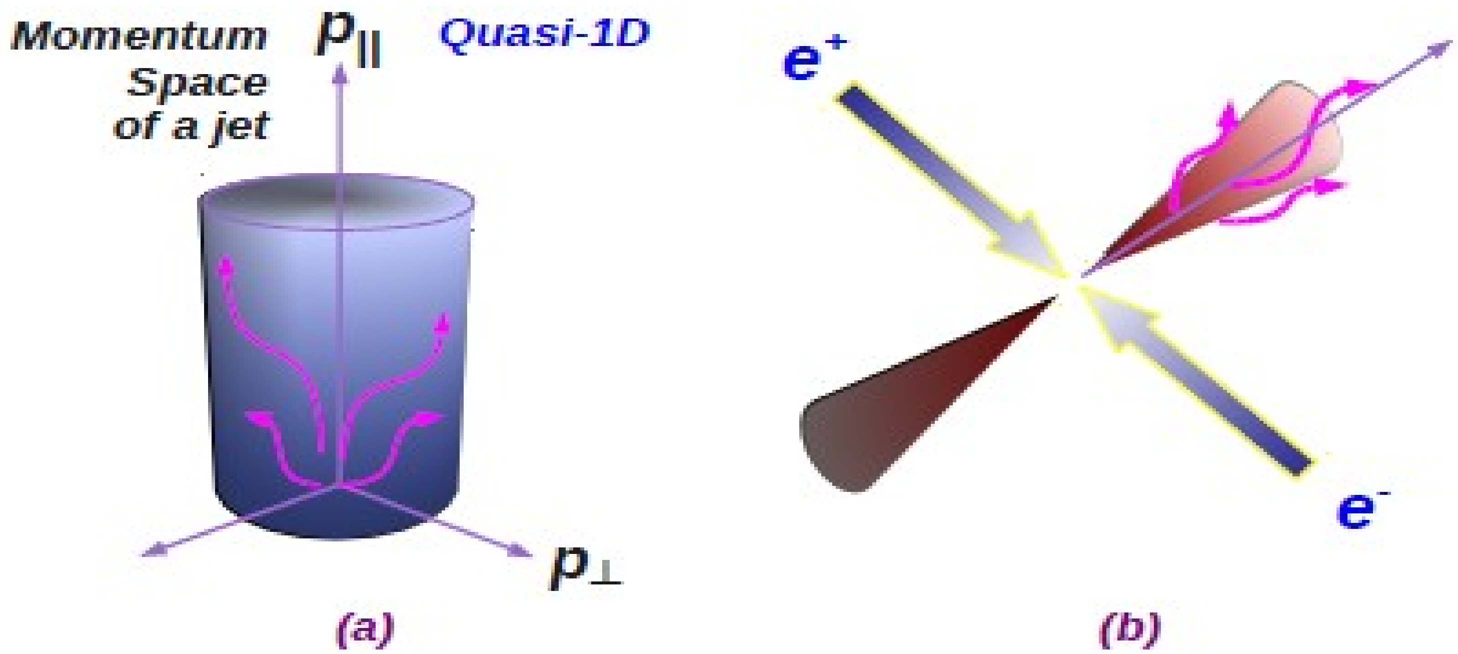






Parameters have a well-defined place in the parameter space...

'Thermodynamics of Jets' in small systems



- Generalized Tsallis distribution in e^+e^- collisons

  Phys. Lett. B718 (2012) 125

ee: Tsallis–Pareto fits from 14-201 GeV

Data used for fits

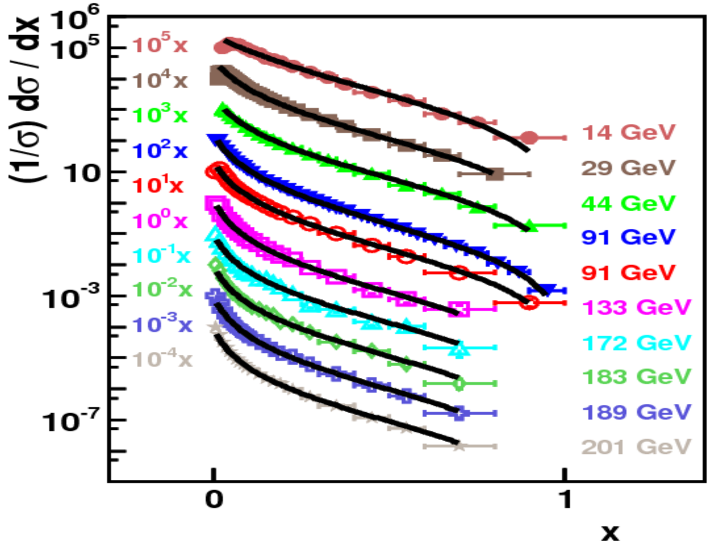
- Braunschweig W et al [TASSO] 1998 Z.Phys. C47 187; 1989 Z.Phys. C45 193
- Aihara H et al [TPC/Two Gamma] 1988 PRL 61 1263
- Abreu P et al [DELPHI] 1993 PL B311 408; 1991 Z.Phys. C50 185
- Akers R et al [OPAL] 1995 Z. Phys. C68 203
- Alexander G. et al [OPAL] 1996 Z.Phys. C72 191, 2000 Eur.Phys.J. C16 185, 2003 Eur.Phys.J. C27 467
- Derrick, M. et al 1986 Phys. Rev. D34 3304
- Zheng, H.W. et al. [AMY] 1990 Phys. Rev. D42 737
- Adeva, B. et al.[L3] 1992 Z.Phys. C55 39
- Acton, P.D. et al. [OPAL] 1992 Z.Phys. C53 539

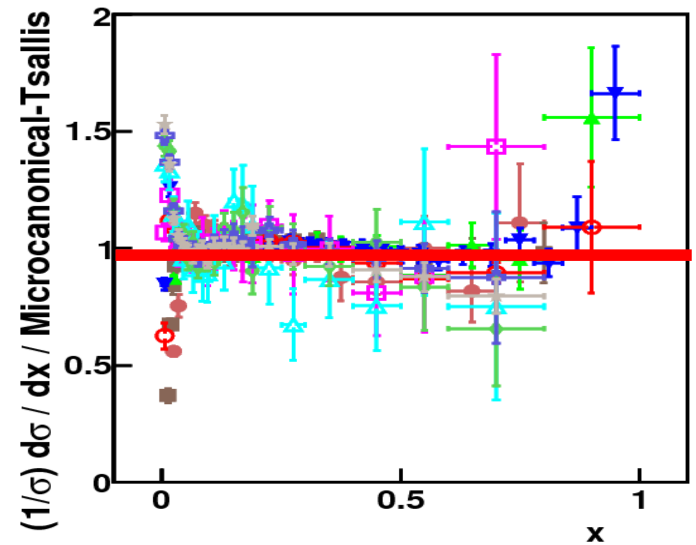
Ref: K. Ürmössy, GGB, TS Biró: arXiv:1101.3023 (2011)

ee: Microcanonical Tsallis – Pareto in e^+e^-

- Micro-canonical Tsallis

$$\frac{A\,x^{D-1}\,(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

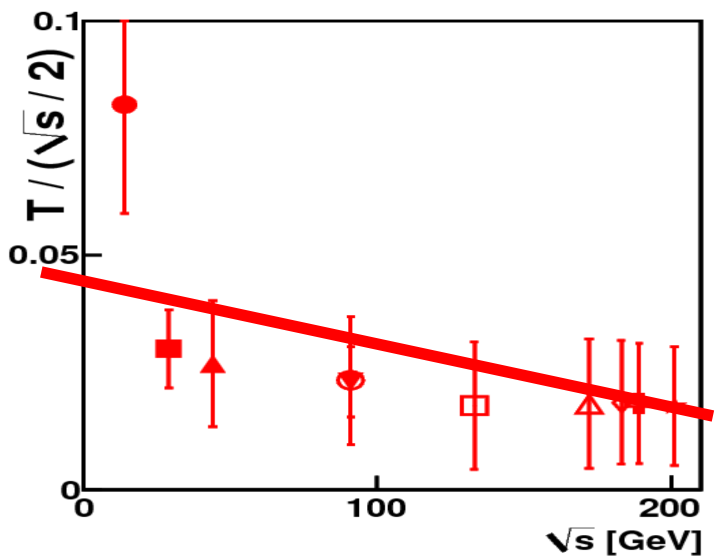


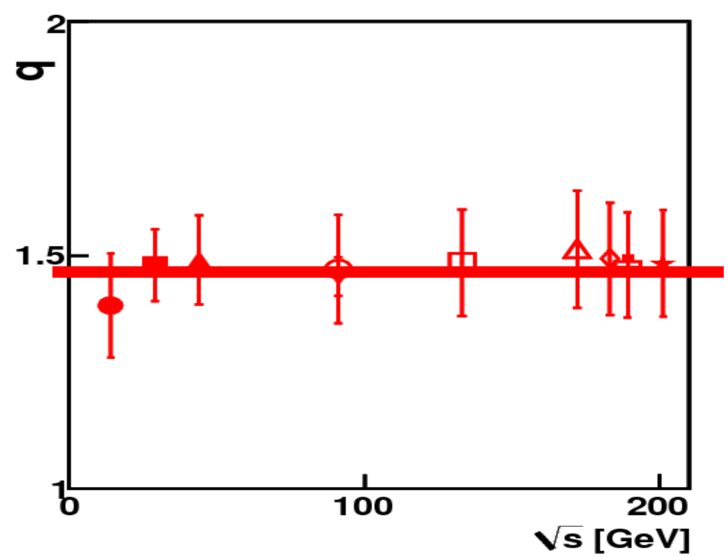


- T smaller
- q=const

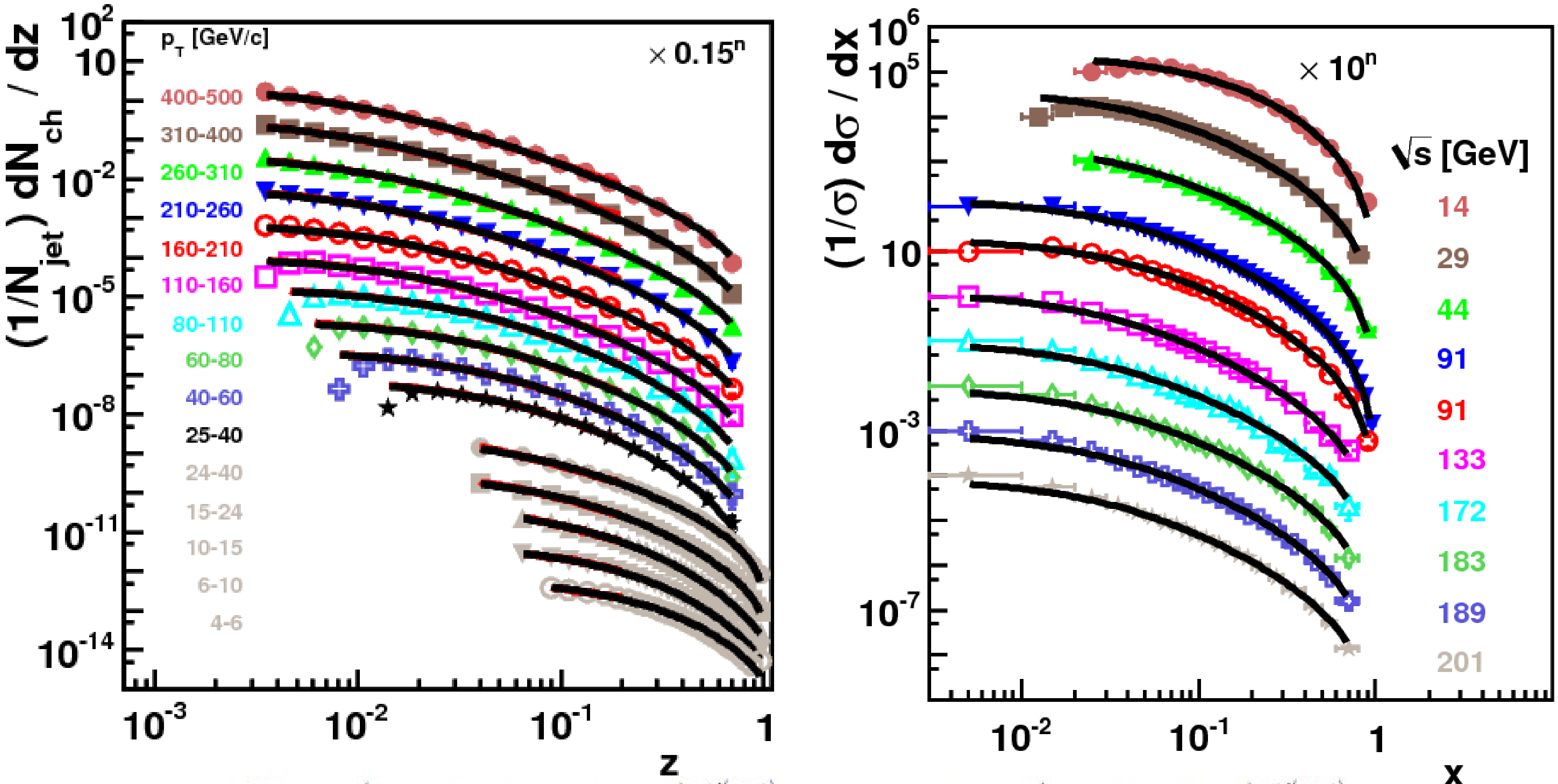
$$q \;=\; 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$





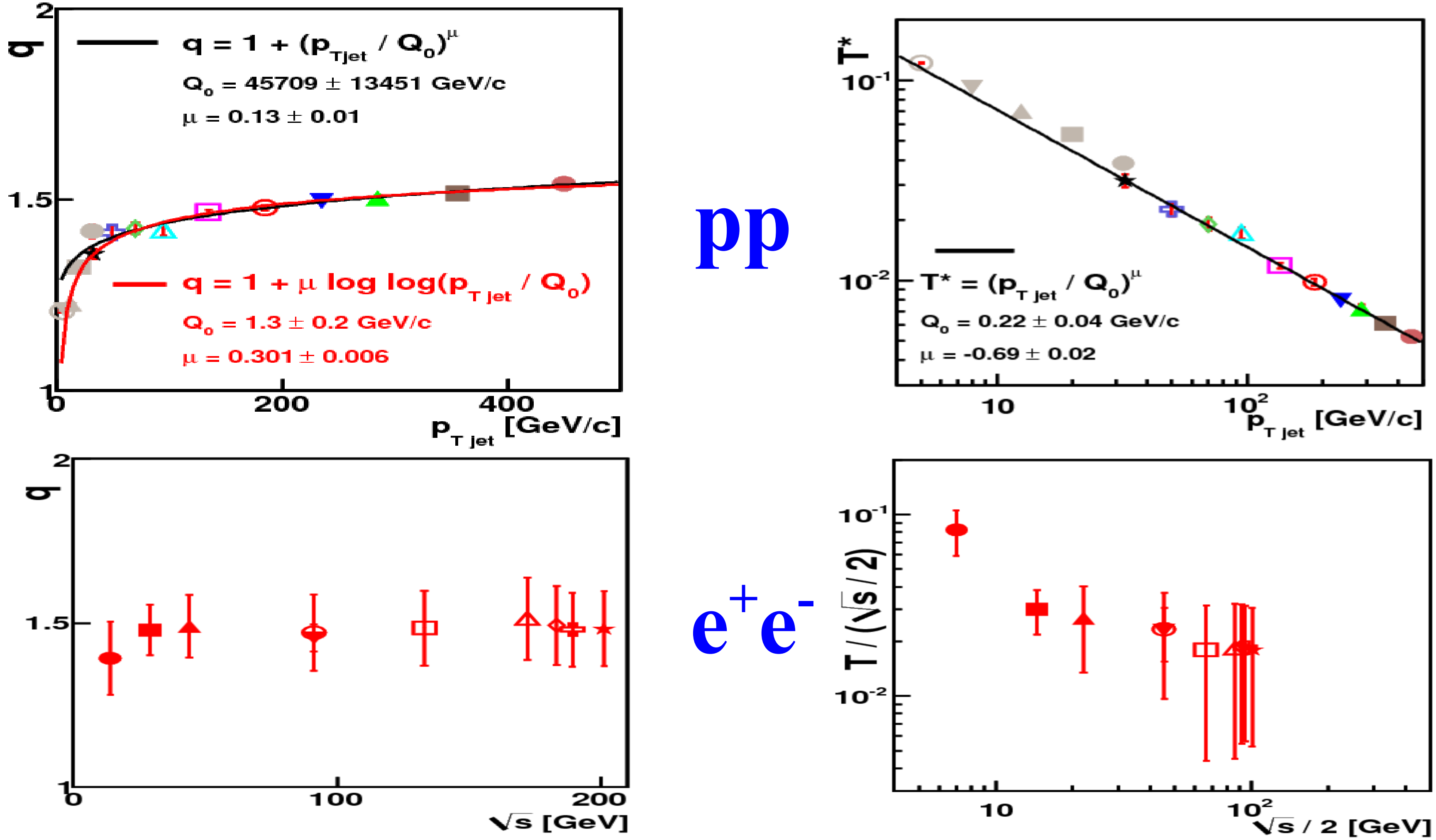
Fits for jet spectra in pp (left) and e^+e^- (right)



$$\frac{d\sigma}{dx} \propto \left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{-1/(q-1)} \Rightarrow \left(1+\frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$

Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

Evolution of parameters q and T in pp & e^+e^-



Ref: K Ürmössy, GGB, TS Biró, PLB 710 (2011) 111, PLB 718 (2012) 125.

What are we measuring in small systems, like pp or e^+e^-?

Hadronization in Parton Model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (*a*) fragment into a hadron (*h*), $D_{h/a}(z,Q^2)$.

DGLAP scale evolution:

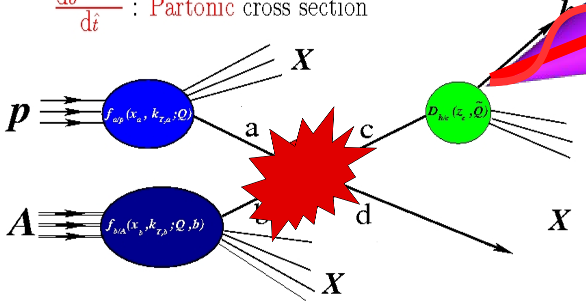
$$\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z},Q^2\right) D_i^h(z,Q^2)$$

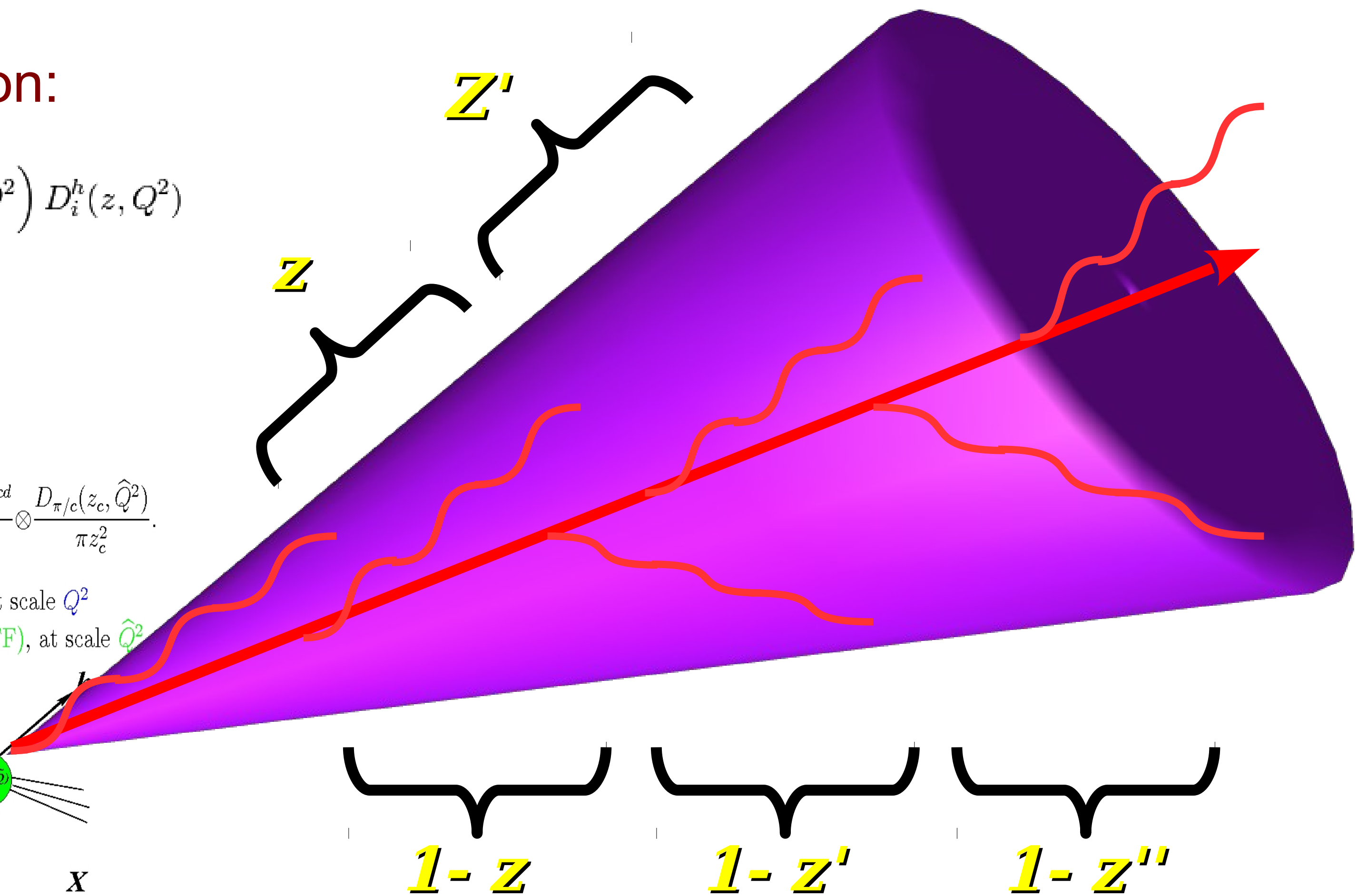
$$E_\pi \frac{\mathrm{d}\sigma_\pi^{pA}}{\mathrm{d}^3p_\pi} \sim f_{a/p}(x_a,Q^2;k_T)\otimes f_{b/A}(x_b,Q^2;k_T,b)\otimes \frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}}\otimes\frac{D_{\pi/c}(z_c,\widehat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a,Q^2;k_T,b)$: Parton Dist. Function (PDF), at scale Q^2

$D_{\pi/c}(z_c,\widehat{Q}^2)$: Fragmentation Function for π (FF), at scale $\widehat{Q}^2$

$\frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}}$: Partonic cross section





Hadronization via associative composition

Program Performed:

1) Search and fit Tsallis-Pareto distribution to data.

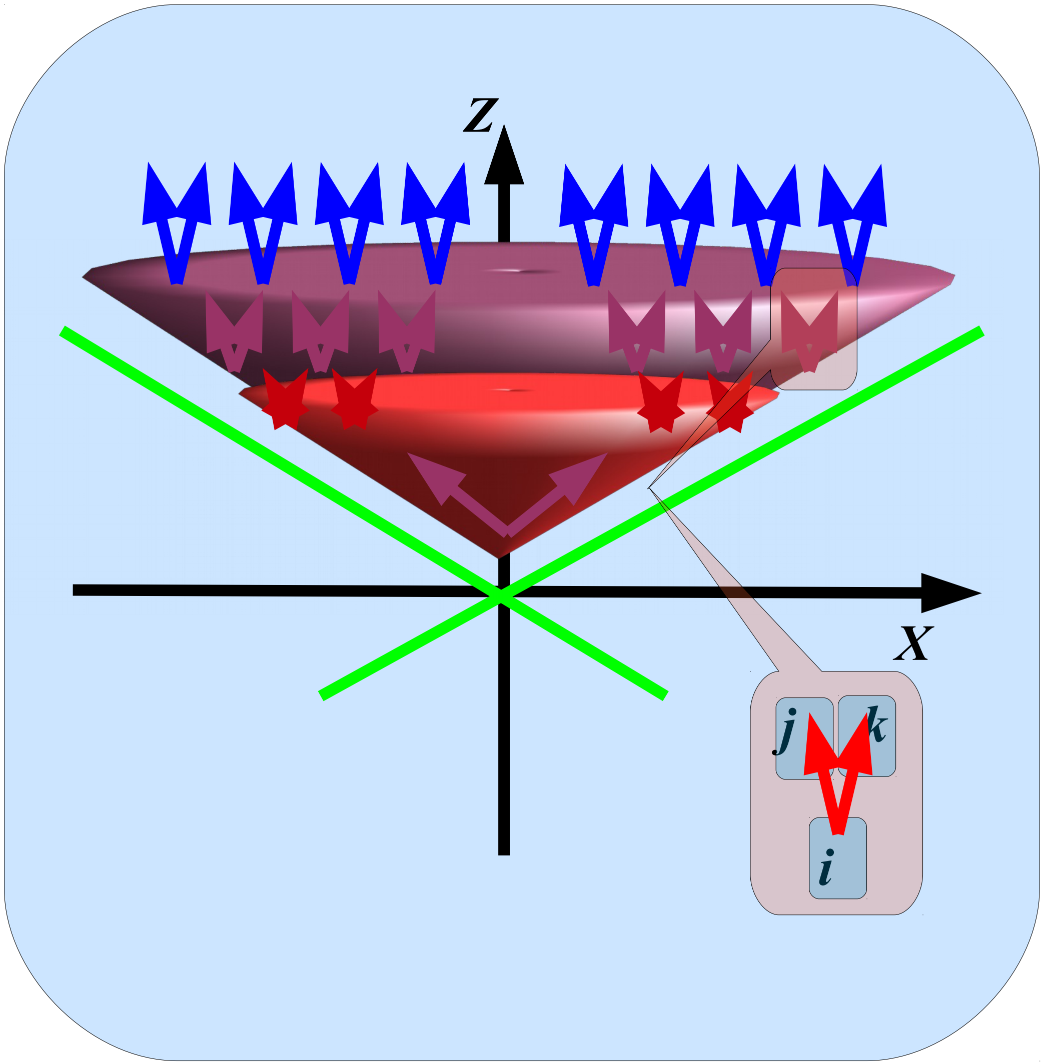
2) Serach for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' $P_{q\,g}$, $P_{g\,g}$

4) Test: can a DGLAP-like evolution equation be obtained?

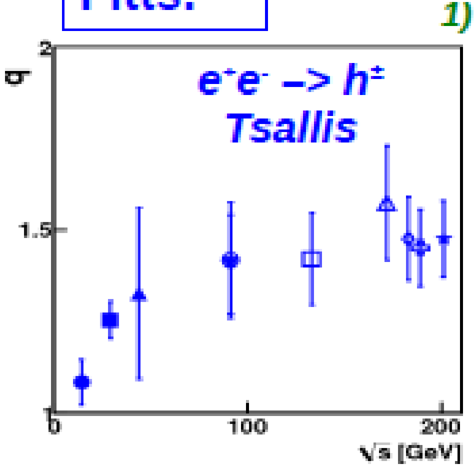
$$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$$

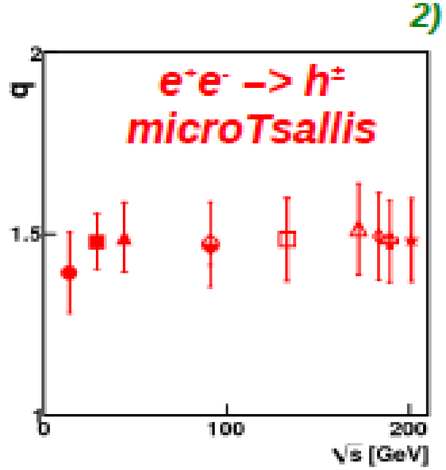
$$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$$

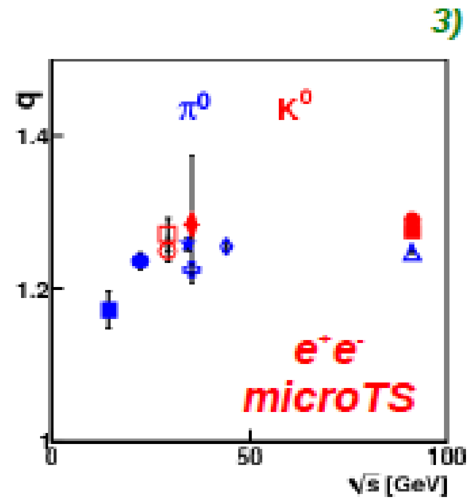


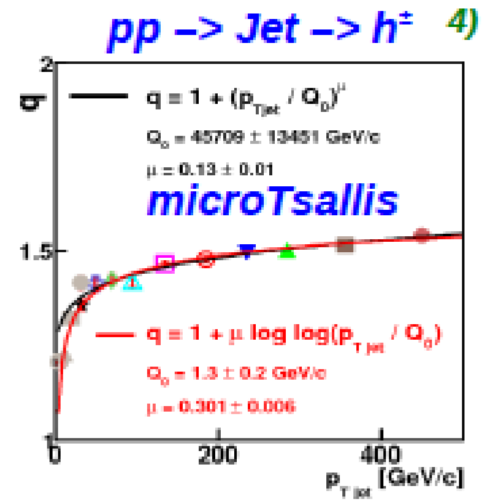
Scale Evolution of the parameter q

Fitts:

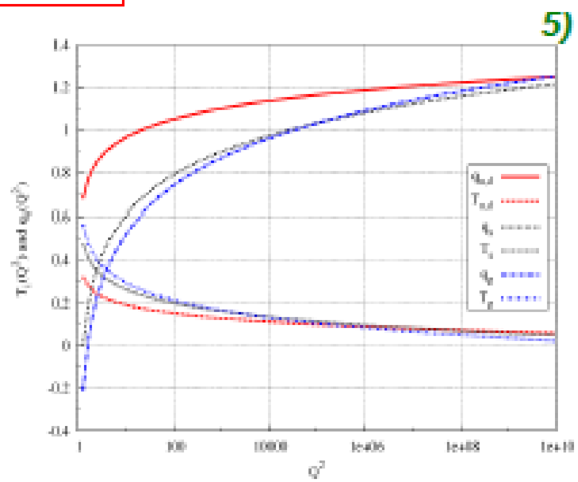








Theory: Scale evolution of *q, T from fits to AKK Frag. Funcs:*



$$D_{p_i}^{\pi^\pm}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$q_i = q_{0i}+q_{1i}\ln(\ln(Q^2))$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

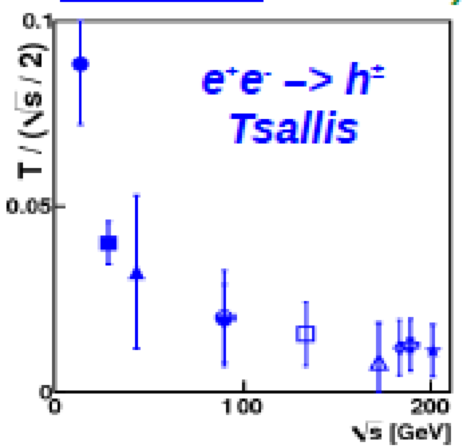
4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf.* C10-05-26.1, p.357-363

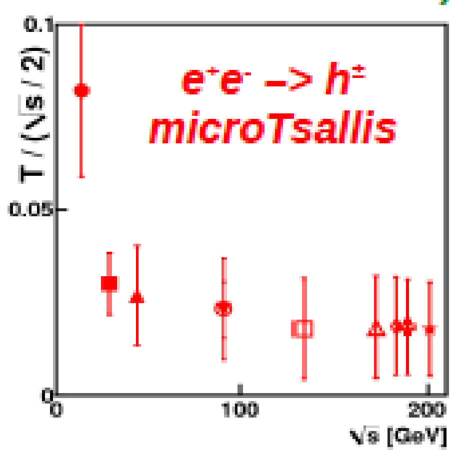
Scale Evolution of the parameter T

Fitts:

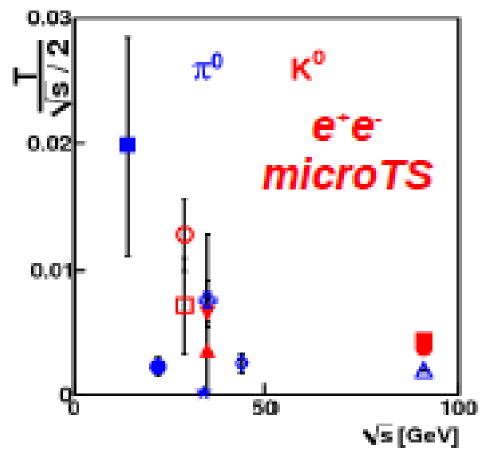
1)



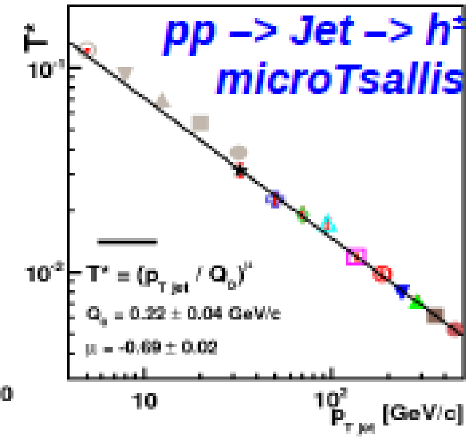
2)



3)

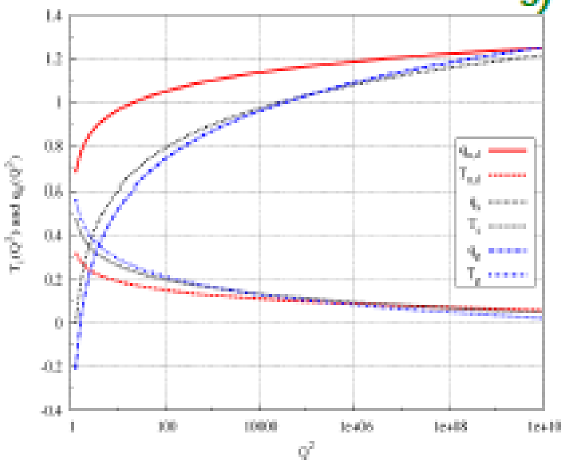


4)



Theory: Scale evolution of *q*, *T from fits to AKK Frag. Funcs:*

5)



$$D^{\pi^+}_{p_i}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$T_i = T_{0i}+T_{1i}\ln(\ln(Q^2))$$

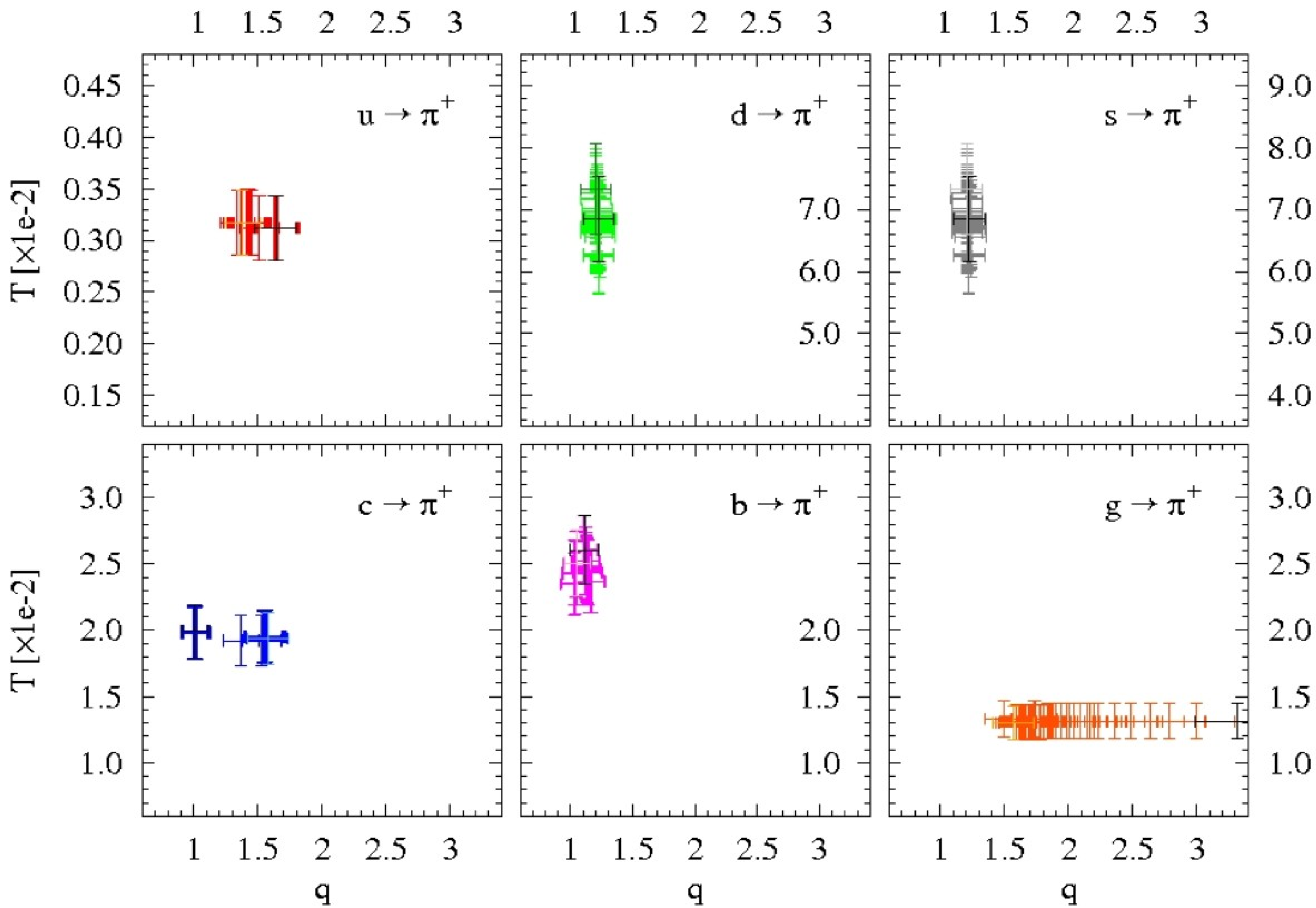
1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

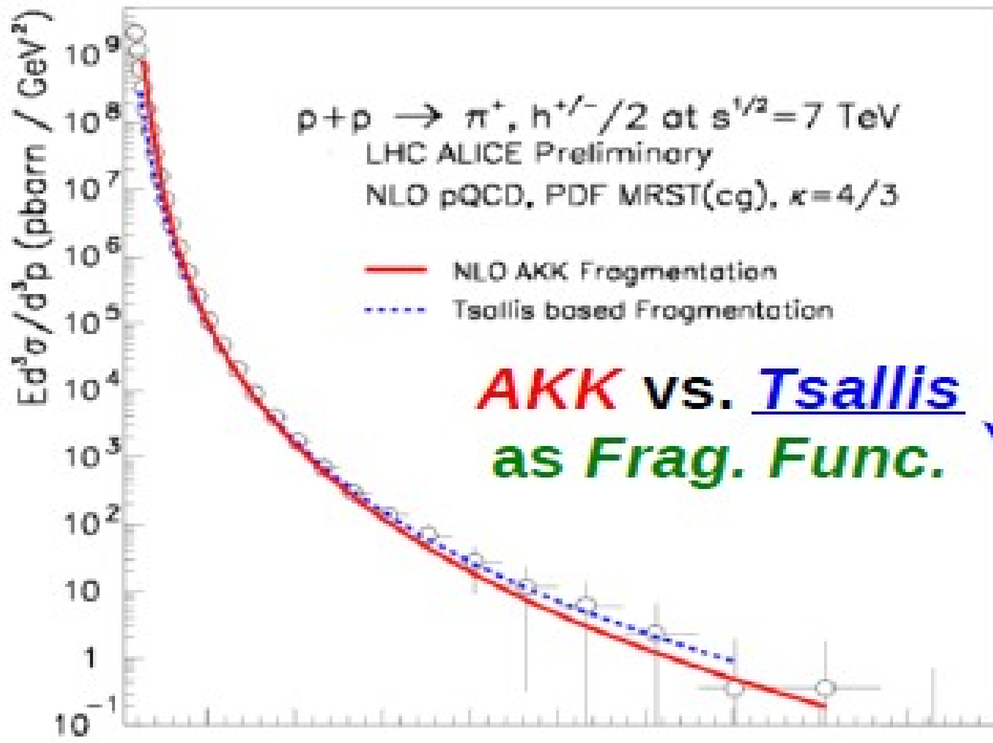
4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf.* C10-05-26.1, p.357-363

Full calculation of fitted FFs with DGLAP



Test of the FF via NLO pQCD code (kTpQCDv20)



$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

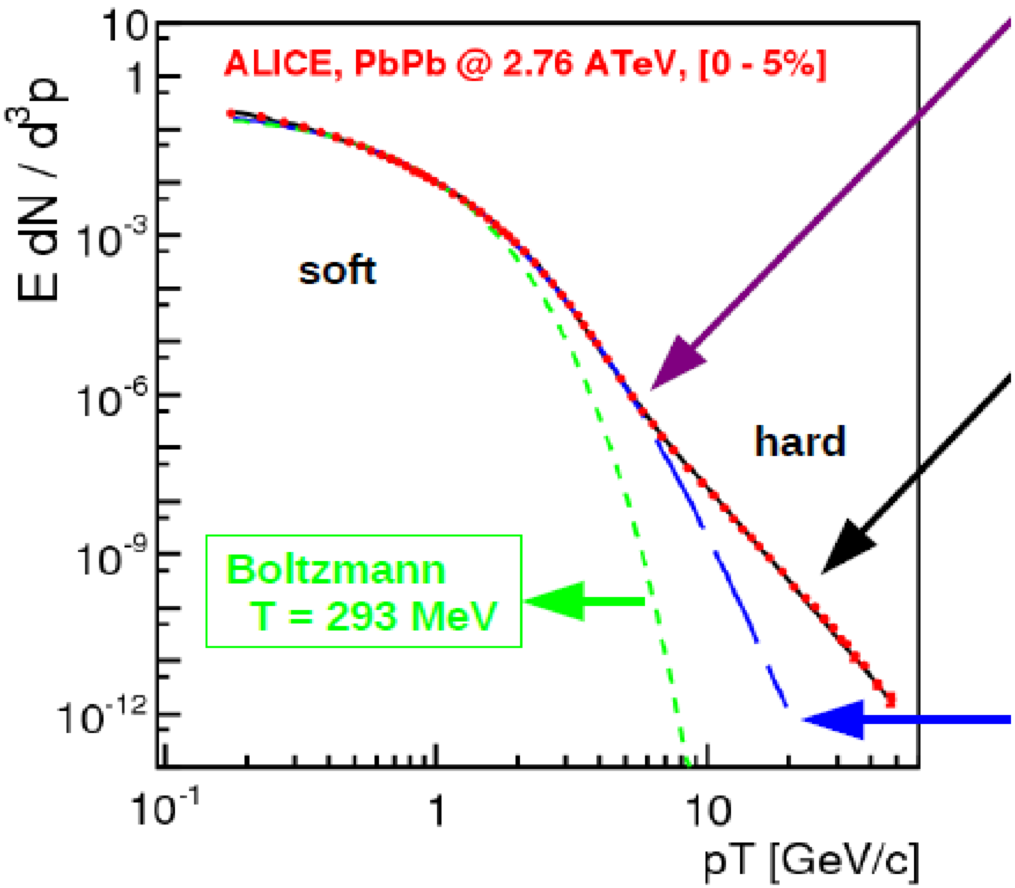
Summary of the small systems

- Based on pp and ee fits
  - Specific thermodynamical model required
  - Microcanonical Tsallis – Pareto distribution seems OK
  - Parameters T & q has energy dependence
  - Assuming parameter evolution, values are well defined
  - Measure of non-extensivity is clear

- The physics origin of the parameters
  - Seems connected to the final state and hadronization
  - Presents similar evolution as DGLAP
  - Reduced parameter values can be obtained

What if, we would apply this for
a bigger system (AA)
where
Boltzmann–Gibbs
use to work?

Measuring non-extensivity in AA collsions

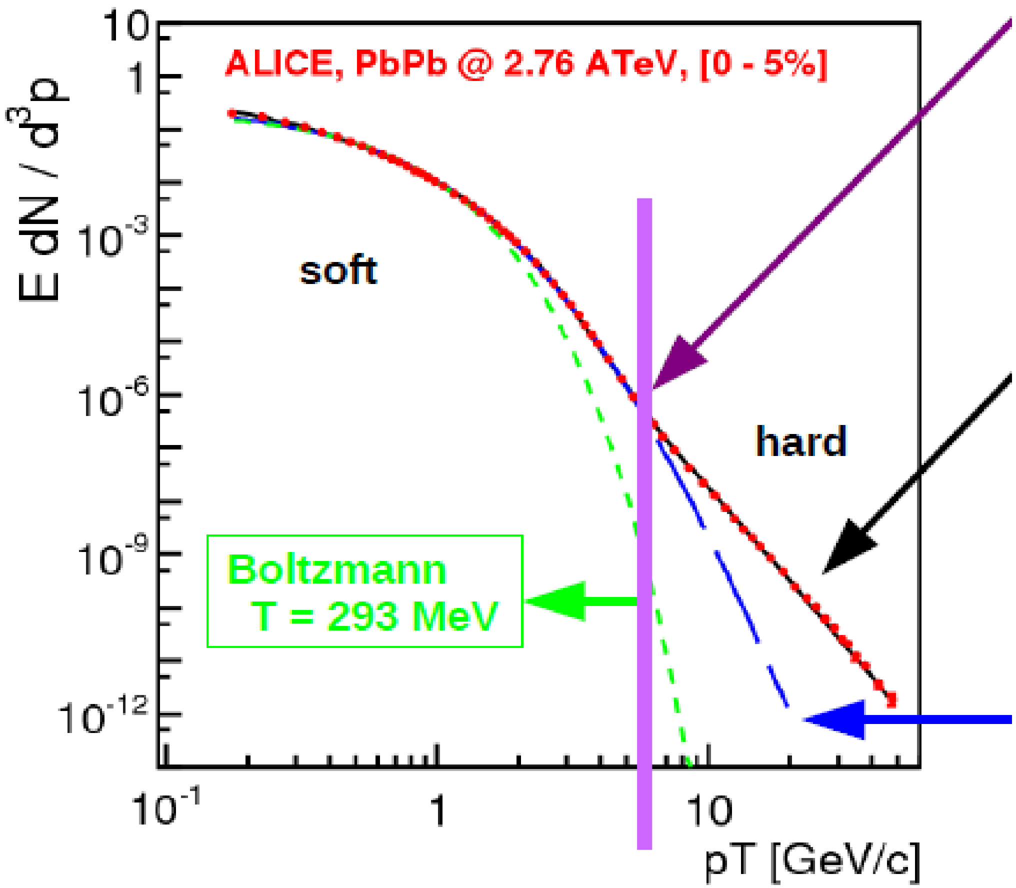


The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

Tsallis spectra with $p^{-13.7}$

Measuring non-extensivity in AA collsions



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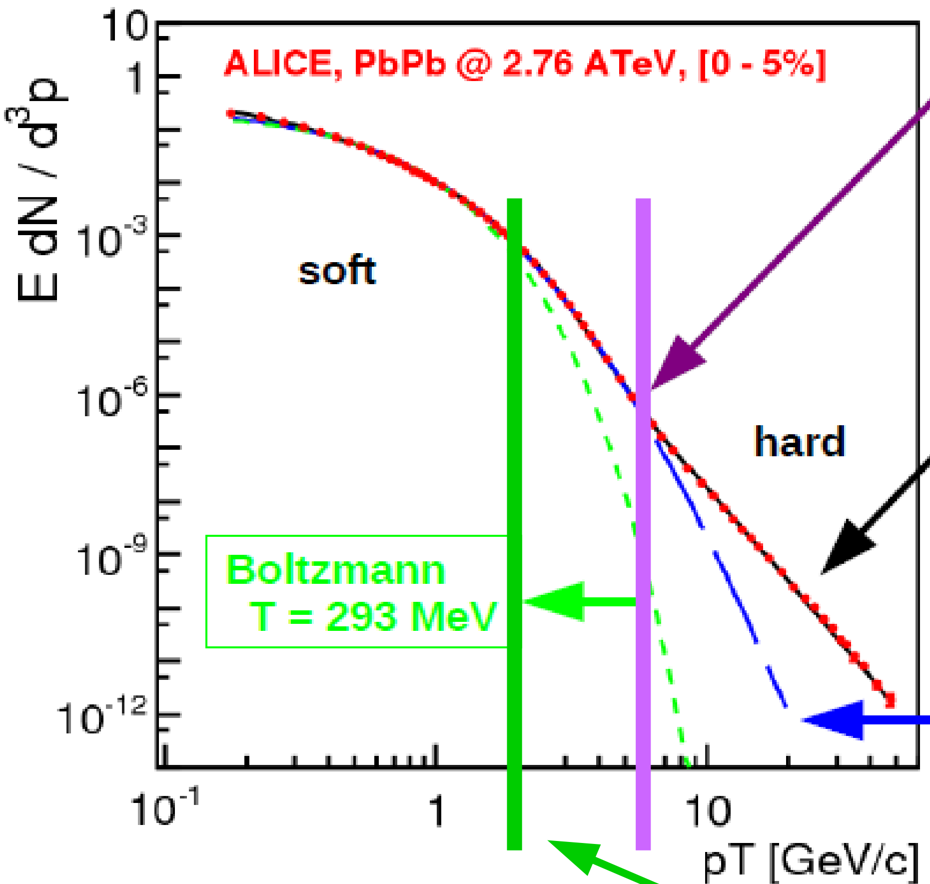

The power of the spectra changes at around 6 GeV/c

Power-law with $p^{-6.08}$ dependence

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Handling soft/hard regime with a new approach, using not only the temperature, T

What is the values for *T* & *q* in AA
a.k.a.
a simple model for the
Quark Gluon Plasma temperature

The temperature (slope) of the system

- Generalizing the Gibbs ensamble for our case:

$$S = -\sum_i P_i \ln P_i \quad \Rightarrow \quad L(S) = \sum_i P_i L(-\ln P_i)$$

- Taking P_i weights of system, E_i , results cut power law:

$$P_i = \left(Z^{1-q} + (1-q)\frac{\beta}{q} E_i \right)^{\frac{1}{q-1}} = \frac{1}{Z}\left(1 + \frac{Z^{-1/C}\, e^{S/C}}{C-1} \frac{E_i}{T} \right)^{-C}$$

- Partition sum is related to Tsallis entropy, $L(S_1)$ and E_1

$$\ln_q Z := C\left(Z^{1/C} - 1 \right) = L(S_1) - \frac{1}{1-1/C} \beta E_1$$

- In $C \rightarrow \infty$ limit, the inverse log slope of the energy distribution:

$$\boxed{T_{\rm slope}(E_i) = \left(-\frac{d}{dE_i} \ln P_i \right)^{-1} = T_0 + E_i/C, \quad \text{with} \quad T_0 = T e^{-S/C} Z^{1/C} (1 - 1/C)}$$

Experimental data fits by $T_{slope}(E)$

- Taking the $T_{slope}(E)$ fit using $$T_{\mathrm{slope}}(E_i) = \left(-\frac{d}{dE_i}\ln P_i\right)^{-1} = T_0 + E_i/C.$$
- Fitted data
  - RHIC@200GeV AuAu: T_0= 48 MeV, C=4.5

    T.S. Biró, K. Ürmössy, Zs. Schram: JPG36 064044 (2009)
    T.S. Biró, K. Ürmössy: JPG37, 0940027 (2010),
    K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

  - ALICE@900GeV pp: T_0= 55 MeV, C=8

    J. Cleymans, D. Worku: JPG39, 025006 (2012)

The obtained values are surprizingly low!!! Why????

- Findings: *K*=2 (mesons) and *K*=3 (baryons)

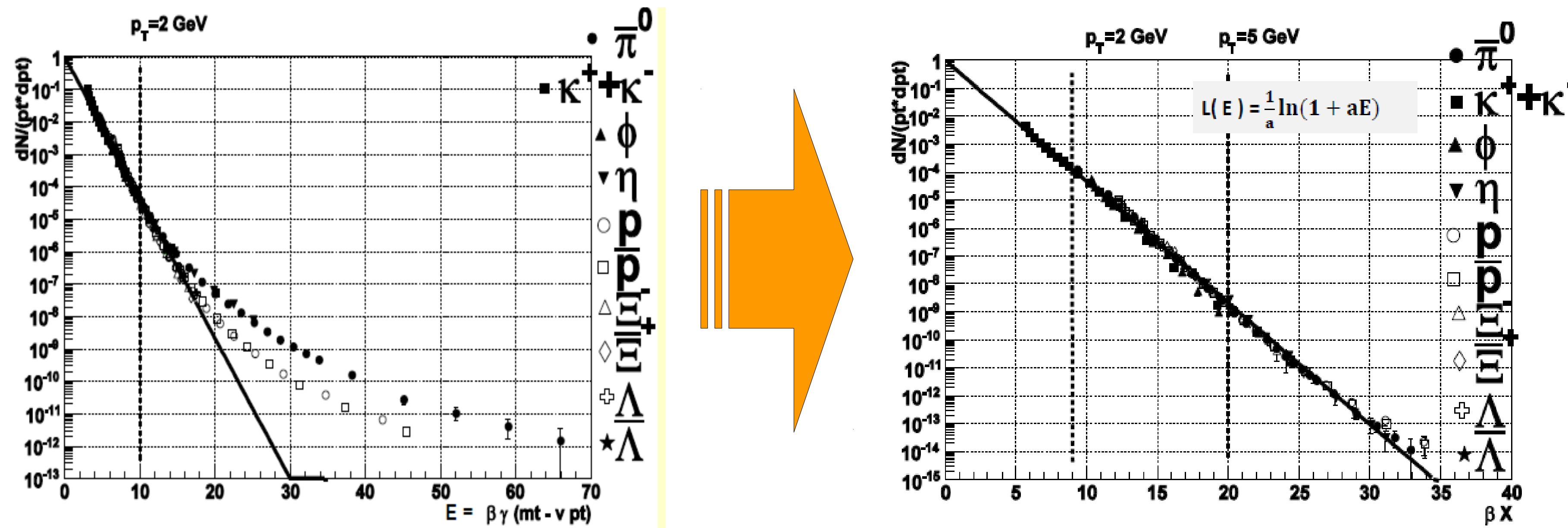
$$P_{\mathrm{hadron}}(E) = P_i^K(E/K)$$ and $$T_{\mathrm{slope}}^{\mathrm{hadron}}(E) = T_{\mathrm{slope}}^{\mathrm{quark}}(E/K)$$

Experimental data fits by $T_{slope}(E)$

- Findings: *K=2* (mesons) and *K=3* (baryons)

$$P_{\text{hadron}}(E) = P_i^K(E/K) \quad \text{and} \quad T^{\text{hadron}}_{\text{slope}}(E) = T^{\text{quark}}_{\text{slope}}(E/K)$$

This finding is coming from the scaling of the PID-spectra...



T.S.Biró, K.Ürmössy, JPhysG 36, 064044, 2009

Simple thermal model for heavy-ion collisions

- Test of T_0 in physical models, in a finite termostats,

  small subsystem: $\lim_{C\to\infty} T_0 = T_1$ and $T_1 = 1/\beta_1 = T e^{-S/C}$

- Taking Stefan-Boltzmann in a bag, with a fix volume, *V* and bag constant, *B*

$$E/V = \sigma T^4 + B \qquad p = \tfrac{1}{3}\sigma T^4 - B \qquad S = \tfrac{4}{3}\sigma V T^3$$

- The heat capacity is:

$$C = \frac{dE}{dT} = 4\sigma V T^3 + \left(\sigma T^4 + B\right)\frac{dV}{dT}$$

Simple thermal model for heavy-ion collisions

- Let's discuss some specific cases:

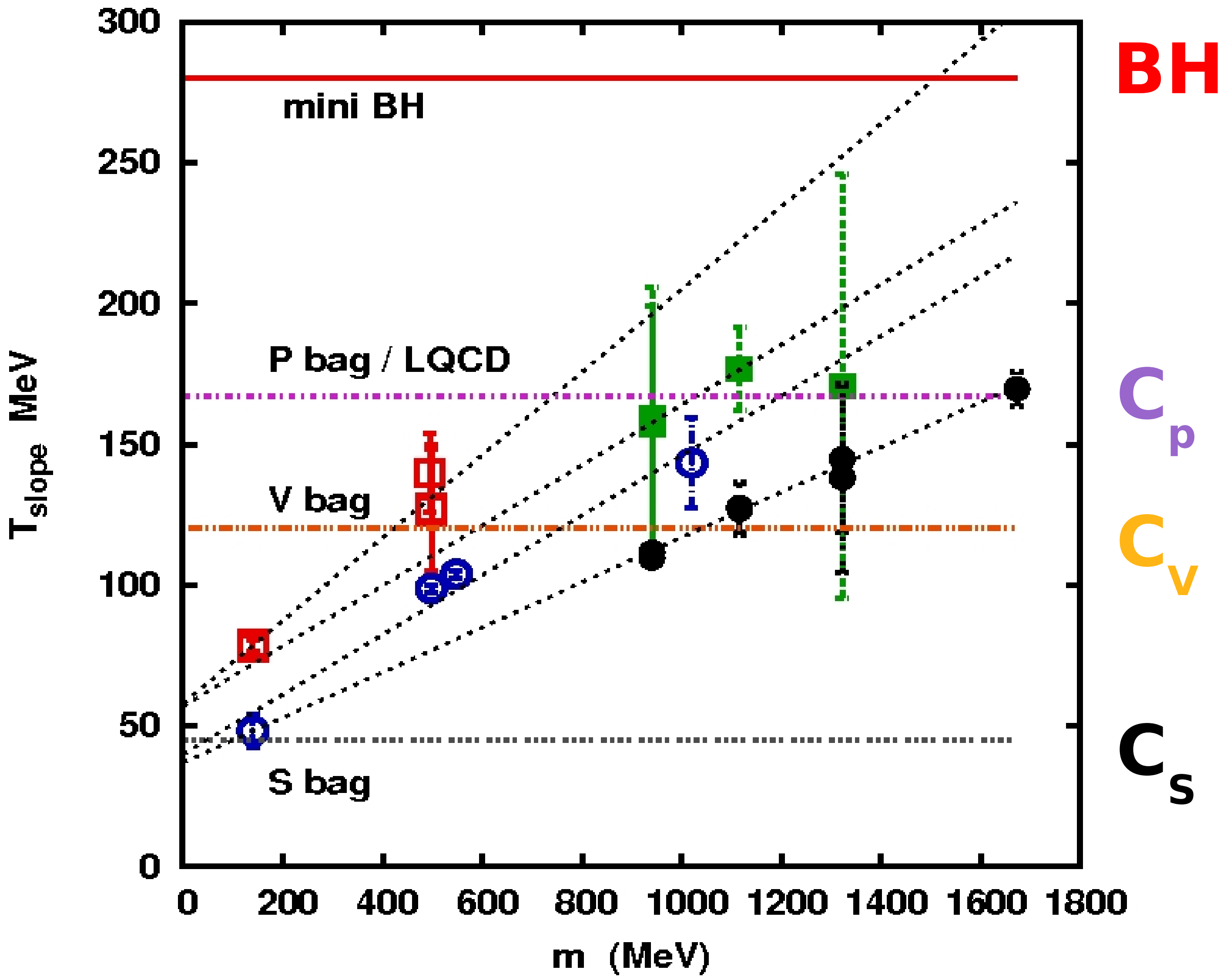
	Heat capacity	Subsystem's T	Note
C_V	$C_V = 4\sigma V T^3 = 3S$	$T_{1V} = T\mathrm{e}^{-1/3}$	
C_p	$C_p = \infty$	$T_{1P} = T$	
C_S	$C_S = 3S(1 - T_*^4/T^4)/4$	$T_{1S} \leq T\mathrm{e}^{-4/3}$	$C_S \leq 3S/4$
BH	$C = -2S$	$T_1 = T\mathrm{e}^{1/2}$	

- Taking the lattice QCD value T=167 MeV, T_{slopes} are:

$$T_{1P} = T = 167 \text{ MeV}, T_{1V} = T\mathrm{e}^{-1/3} \approx 120 \text{ MeV and } T_{1S} \leq T\mathrm{e}^{-4/3} \approx 45 \text{ MeV}$$

for Tsallis distribution of valence quarks

The temperature slope for different models



TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

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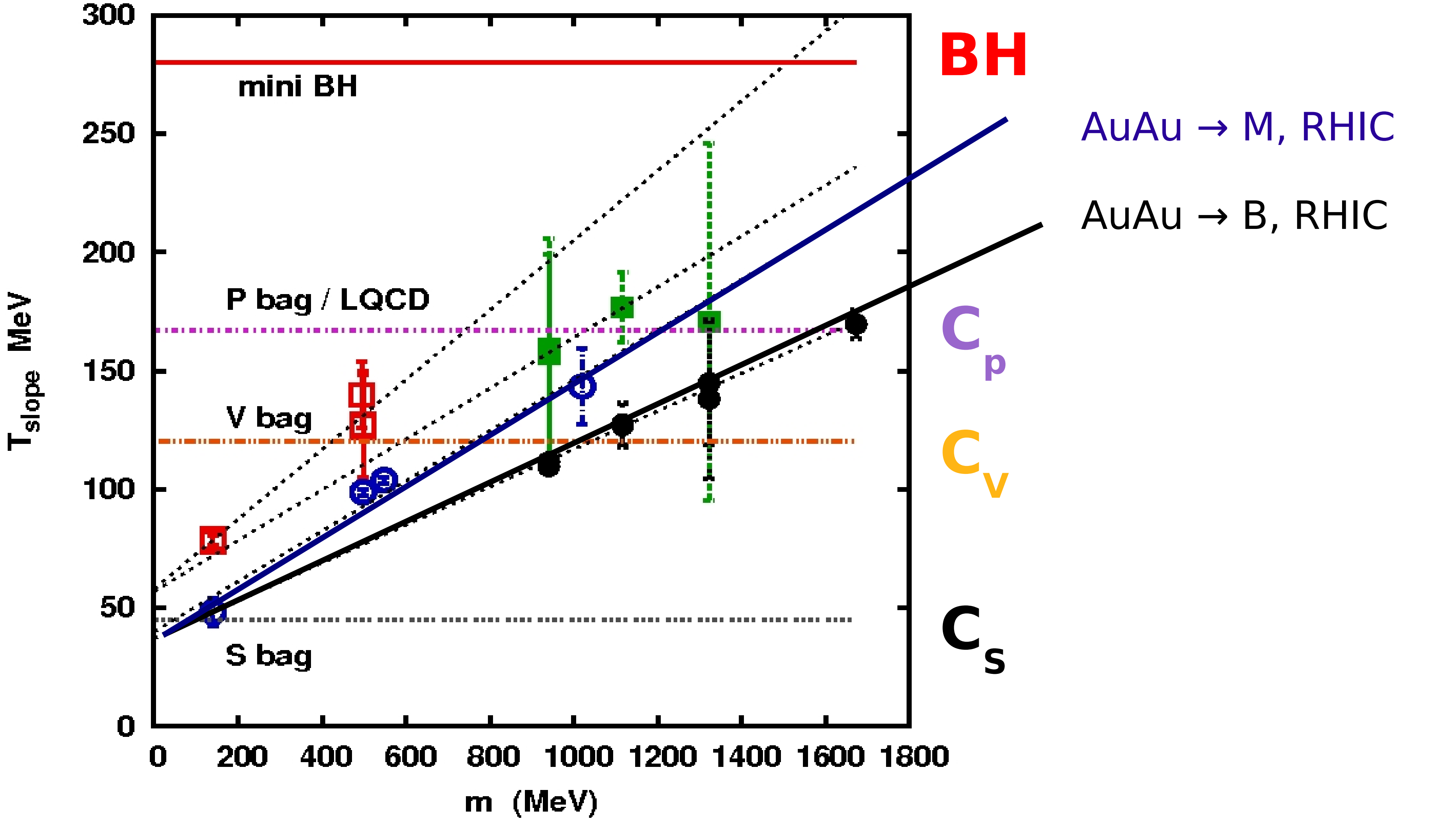

TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

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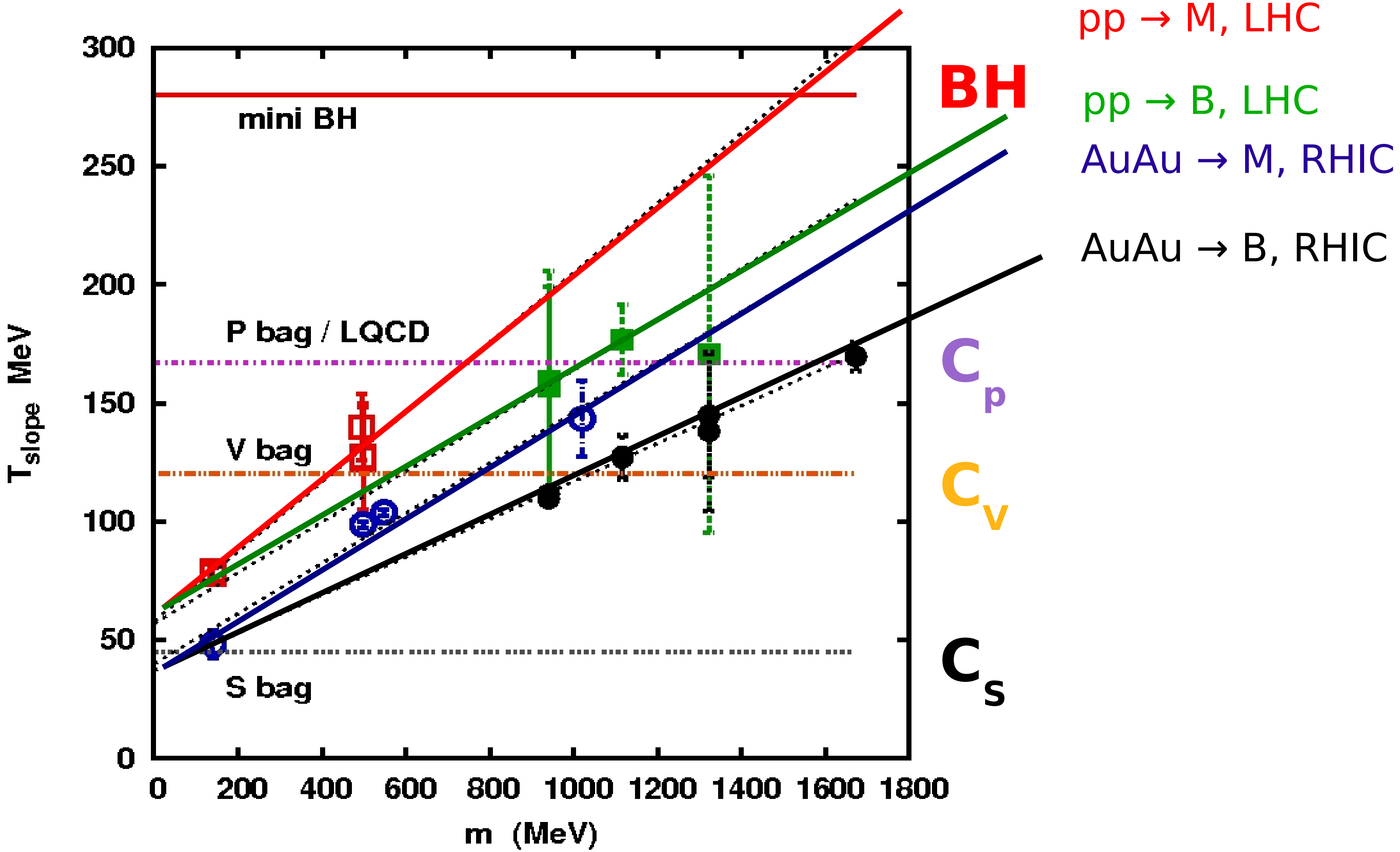

TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

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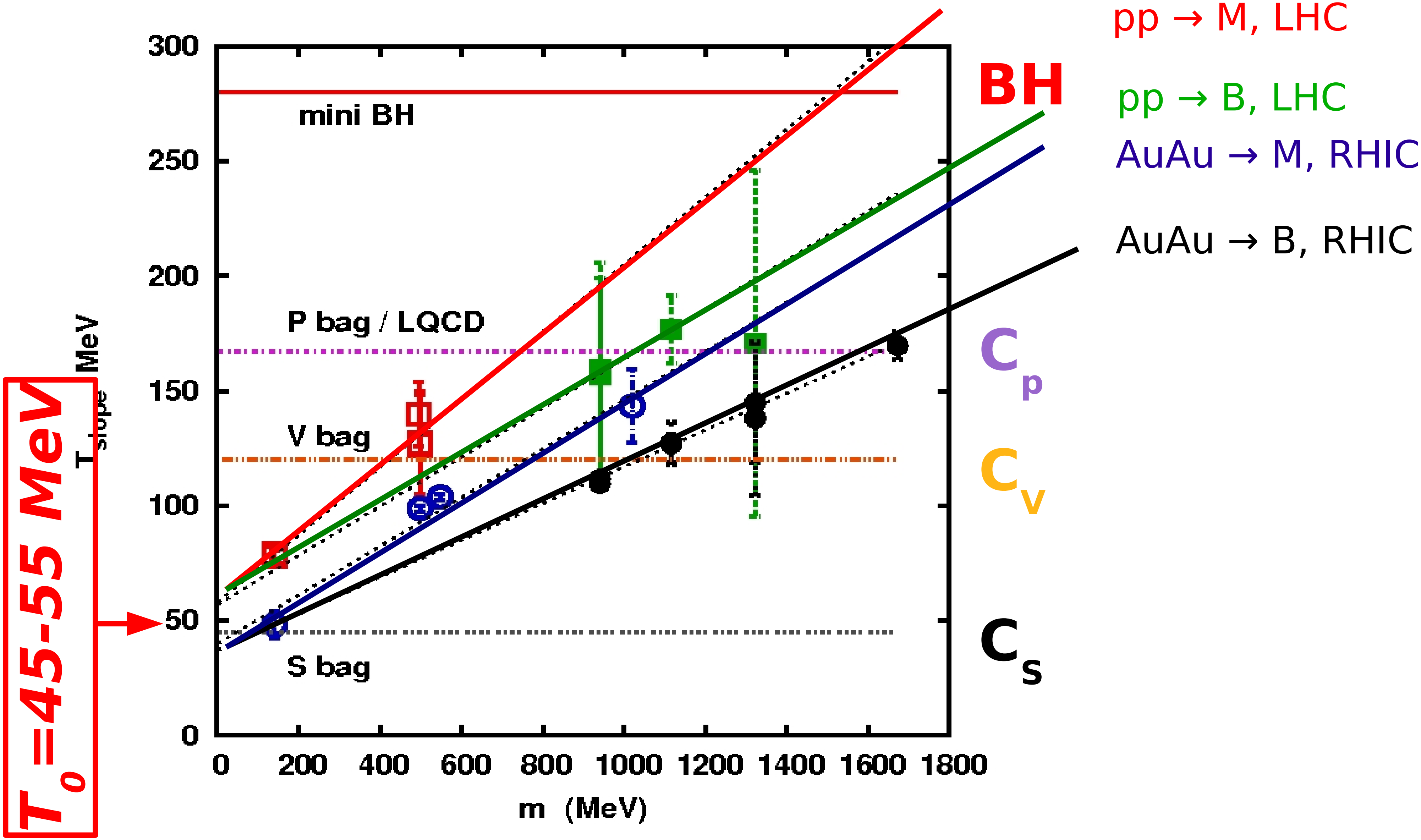

TS Biró, GGB, P. Ván, EPJ A49 (2013) 110

In case of good high-p_T spectra
both
Soft and Hard 'components'
can be tested in parallel

The soft + hard model

- Simplest approximation: soft ('bulk') + hard ('jet') contribution

$$p^0 \frac{dN}{d^3\mathbf{p}} = p^0 \frac{dN}{d^3\mathbf{p}}^{\text{hard}} + p^0 \frac{dN}{d^3\mathbf{p}}^{\text{soft}}$$

arXiv:1405.3963, 1501.02352, 1501.05959

J.Phys.CS 612 (2015) 012048

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- Identified hadron spectra is given by double Tsallis–Pareto:

$$\left.\frac{dN}{2\pi p_T dp_T dy}\right|_{y=0} = f_{hard} + f_{soft} \qquad f_i = A_i \left[1 + \frac{(q_i - 1)}{T_i}[\gamma_i(m_T - v_i p_T) - m]\right]^{-1/(q_i - 1)}$$

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in where parameters are given by

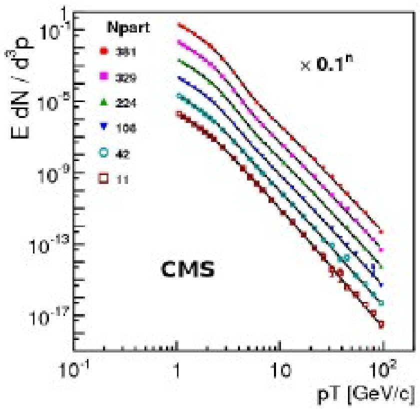
- Lorentz factor $\gamma_i = 1/\sqrt{1-v_i^2}$
- Transverse mass $m_T = \sqrt{p_T^2 + m^2}$
- Doppler temperature $T_i^{Dopp} = T_i\sqrt{\frac{1+v_i}{1-v_i}}$

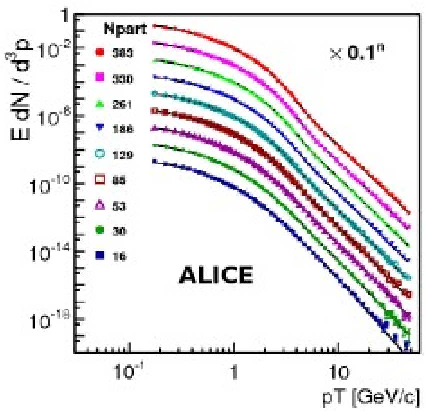
- Finally we assume N_{part} scaling for the parameters

$$\begin{aligned} q_i &= q_{2,i} + \mu_i \ln(N_{part}/2) \\ T_i^{Dopp} &= T_{1,i} + \tau_i \ln(N_{part}) \,. \end{aligned}$$

arXiv:1405.3963, 1501.02352, 1501.05959
J.Phys.CS 612 (2015) 012048

Fit of pp and PbPb (centra/peripheral) data





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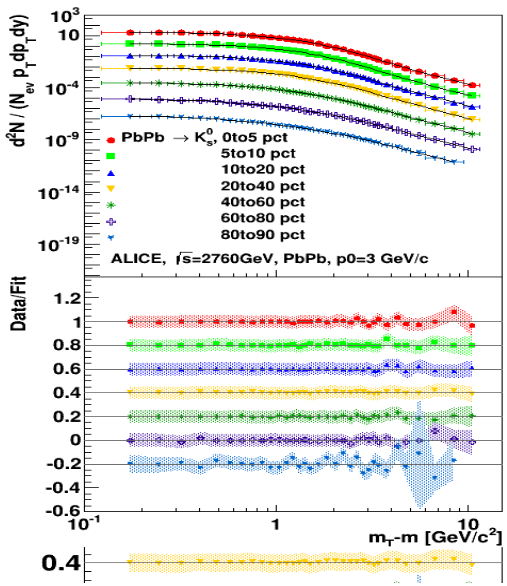

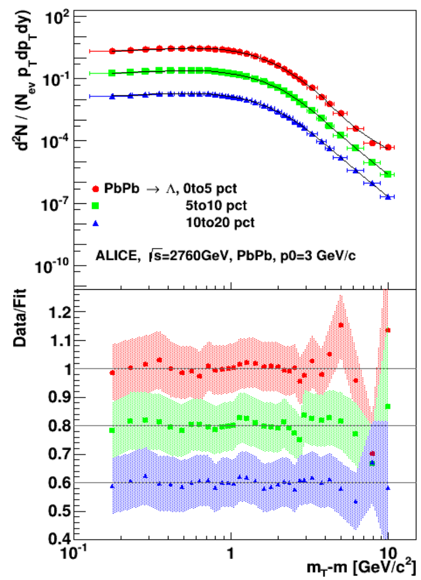

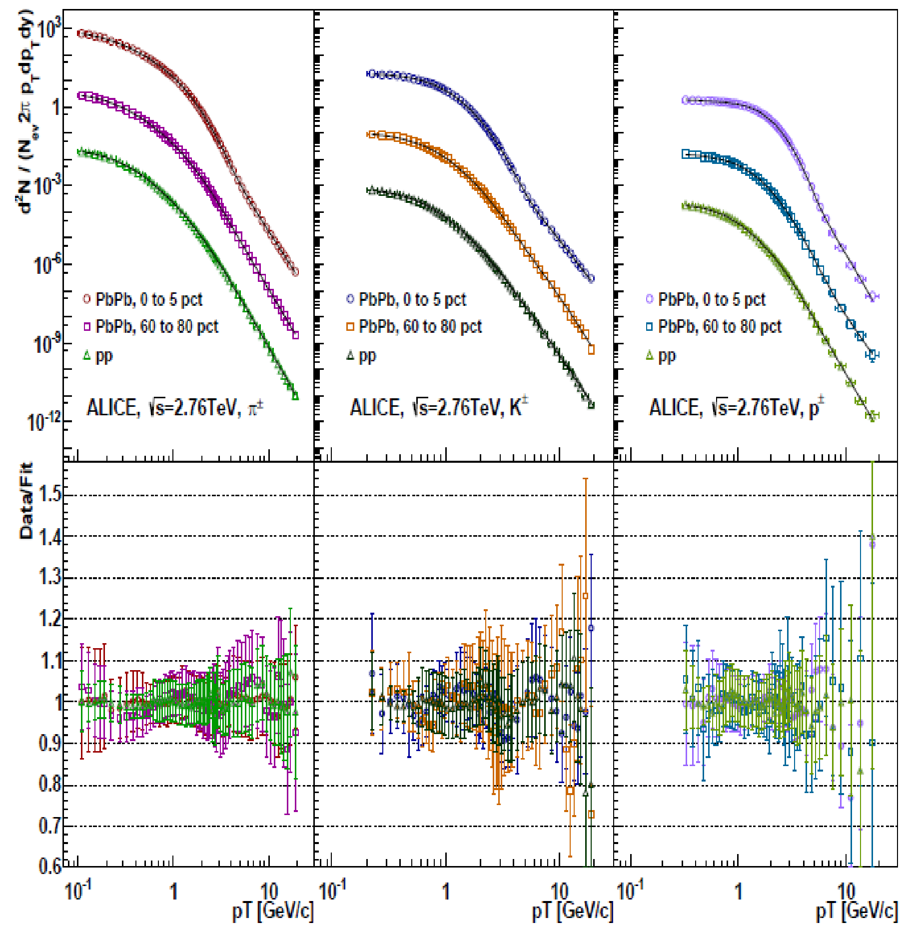

Parameters of the soft+hard model

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	$q_{2,soft}$	$q_{2,hard}$	μ_{soft}	μ_{hard}
CMS	1.058 $\pm$ 0.025	1.136 $\pm$ 0.001	-0.008 $\pm$ 0.005	0.005 $\pm$ 0.0003
ALICE	1.074 $\pm$ 0.018	1.131 $\pm$ 0.002	-0.009 $\pm$ 0.004	0.006 $\pm$ 0.0006
PHENIX	1.073 $\pm$ 0.016	1.100 $\pm$ 0.002	-0.005 $\pm$ 0.004	0.000 $\pm$ 0.0006

	T_1^{soft} [MeV]	T_1^{hard} [MeV]	τ_{soft} [MeV]	τ_{hard} [MeV]
CMS	310 $\pm$ 20	126 $\pm$ 5	9.9 $\pm$ 3.7	5.3 $\pm$ 0.8
ALICE	266 $\pm$ 16	194 $\pm$ 2	11.5 $\pm$ 2.9	-12.5 $\pm$ 0.5
PHENIX	165 $\pm$ 26	192 $\pm$ 20	9.3 $\pm$ 5.5	18.7 $\pm$ 4.6

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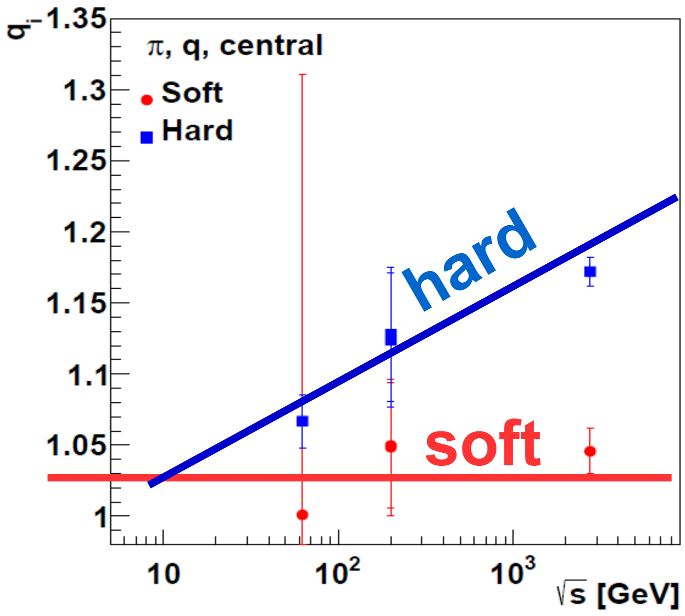
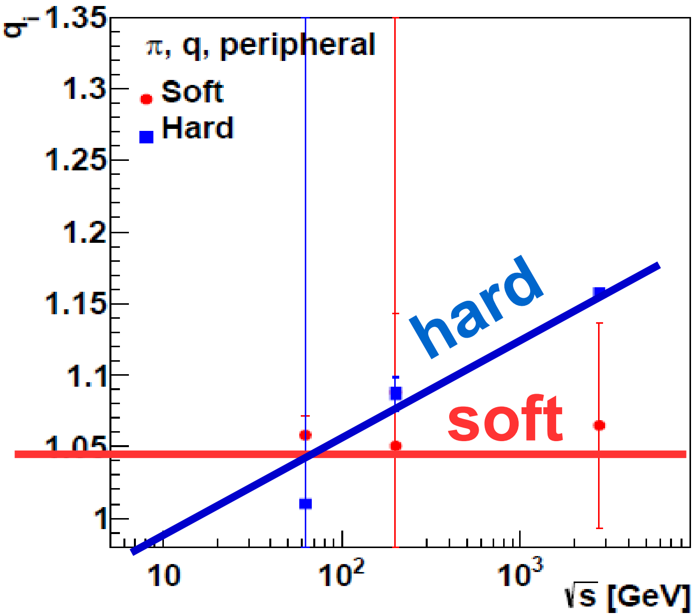
The c.m. energy dependence of *q* & *T*

q **measures non-extensivity**

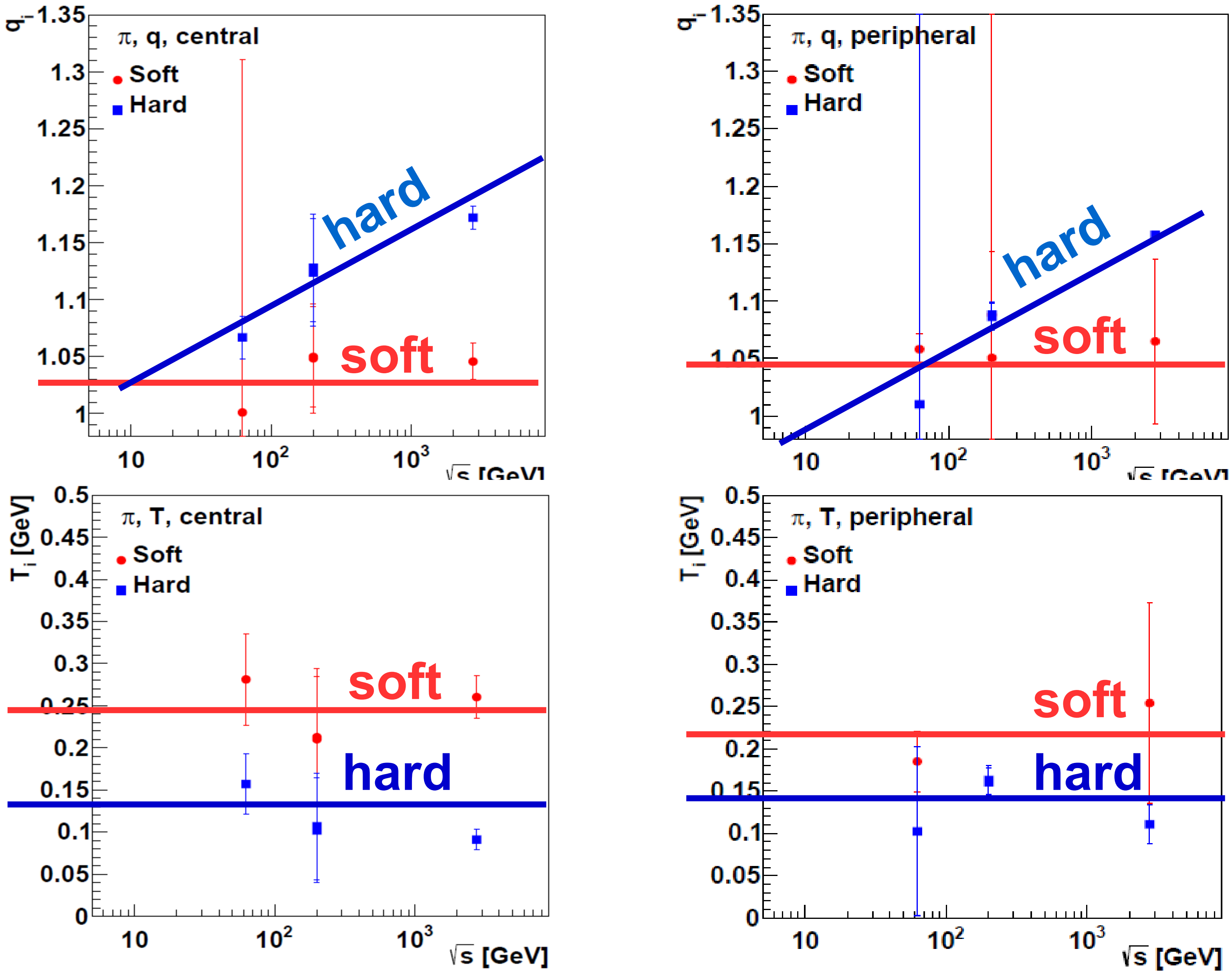




The c.m. energy dependence of *q* & *T*

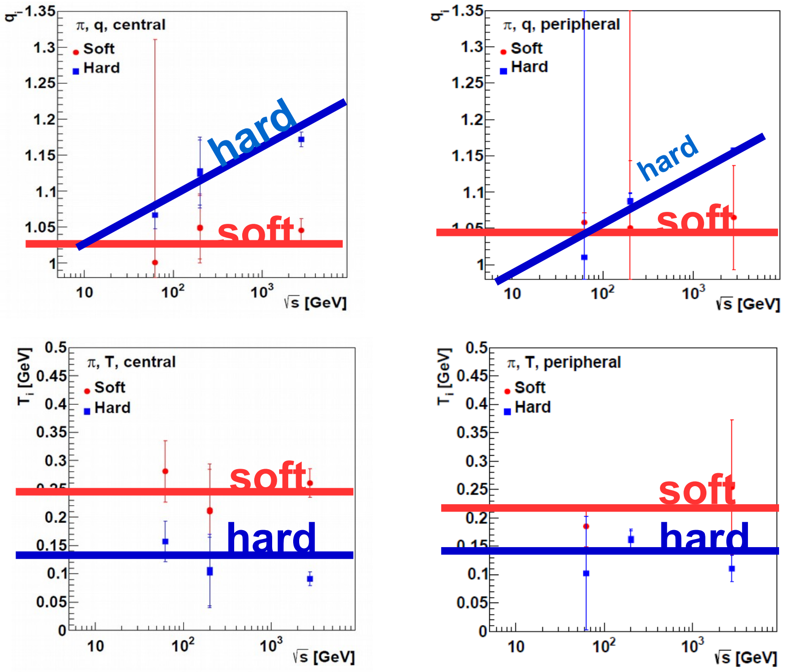
***q* measures non-extensivity**

***T* measures average *E* per multiplicity**



The c.m. energy dependence of *q* & *T*

- Energy dependence
  - Parameter q
    - HARD: clearly increasing
    - SOFT: no relevant change
  - Parameter *T*
    - HARD: central decreasing peripheral const?
      $T_{centr} = T_{periph}$
    - SOFT: similar trend
      T_{centr} ~100 MeV higher



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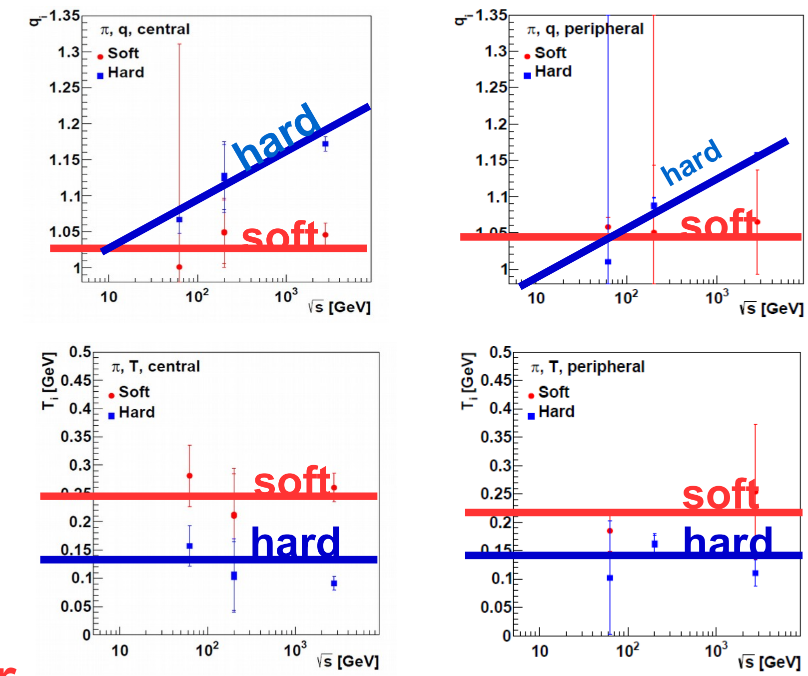

- Energy dependence
  - Parameters q & T present different values for centr./periph.
  - Above RHIC soft is BG-like and hard is more TP-like.

Summary of the large systems (AA)

- Based on AA fits
  - Simple Tsallis–Pareto distribution does NOT fit well
  - Boltzmann–Gibbs + Tsallis–Pareto also NOT the best fit
  - Temperature-like parameter presents smaller value
  - Degrees of freedom (hadronic/partonic) can be specified
  - Mass and Baryon/Meson scaling can be seen

- Measuring non-extensitivity by soft+hard model
  - Double Tsallis–Pareto seems fits well
  - Hard component seems similar as pp and e^+e^-
  - Soft component is Boltzmann–Gibbs-like ($q \rightarrow 1$)

S U M M A R Y

- Non-extensive statistical approach in e^+e^- & pp
  - Obtained Tsallis/Rényi entropies from the first principles.
  - Providing phyiscal meaning of $q=1-1/C + \Delta T^2/T^2$
  - *Boltzmann Gibbs limit* $C \rightarrow \infty$ & $\Delta T^2/T^2 \rightarrow 0$ $(q \rightarrow 1)$,
  - *Tsallis – Pareto fits on spectra in* e^+e^-, *pp*
  - *In connection with FSI and hadronization*

- Application of 'soft+hard' model in AA
  - Simply thermal model results smaller T values
  - Tsallis – Pareto + Exp does not working.
  - Double Tsallis – Pareto measures non-extensitivity
  - SOFT: $q \rightarrow 1$, suggest Boltzmann Gibbs (QGP)
  - HARD: $q > 1.1$, Tsallis – Pareto like
  - Asimuthal anisotropy can be obtained too.

B A C K U P

Related publications..

1. arXiv:1409.5975: Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle

2. arXiv:1405.3963 Disentangling Soft and Hard Hadron Yields in PbPb Collisions at $\sqrt{s_{NN}}$ = 2.76 ATeV

3. arXiv:1405.3813 New Entropy Formula with Fluctuating Reservoir, Physica A (in Print) 2014

4. arXiv:Statistical Power-Law Spectra due to Reservoir Fluctuations

5. arXiv:1209.5963 Nonadditive thermostatistics and thermodynamics, Journal of Physics, Conf. Ser. V394, 012002 (2012)

6. arXiv:1208.2533 Thermodynamic Derivation of the Tsallis and Rényi Entropy Formulas and the Temperature of Quark-Gluon Plasma, EPJ A 49: 110 (2013)

7. arXiv:1204.1508 Microcanonical Jet-fragmentation in proton-proton collisions at LHC Energy, Phys. Lett. B, 28942 (2012)

8. arXiv:1101.3522 Pion Production Via Resonance Decay in a Non-extensive Quark-Gluon Medium with Non-additive Energy Composition Rule

9. arXiv:1101.3023 Generalised Tsallis Statistics in Electron-Positron Collisions, Phys.Lett.B701:111-116,2011

10. arXiv:0802.0381 Pion and Kaon Spectra from Distributed Mass Quark Matter, J.Phys.G35:044012,2008