Towards 3 particle correlations in the Color Glass Condensate framework

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IN COLLABORATION WITH

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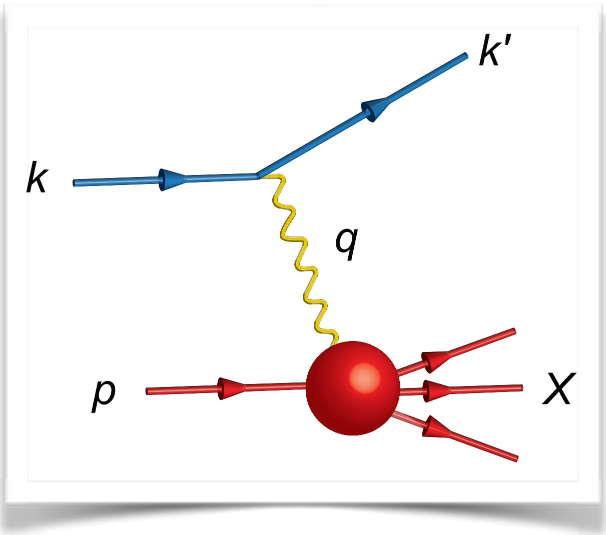
QCD Challenges at the LHC: from pp to AA

(Taxco, 18.-22. Jan. 2016)

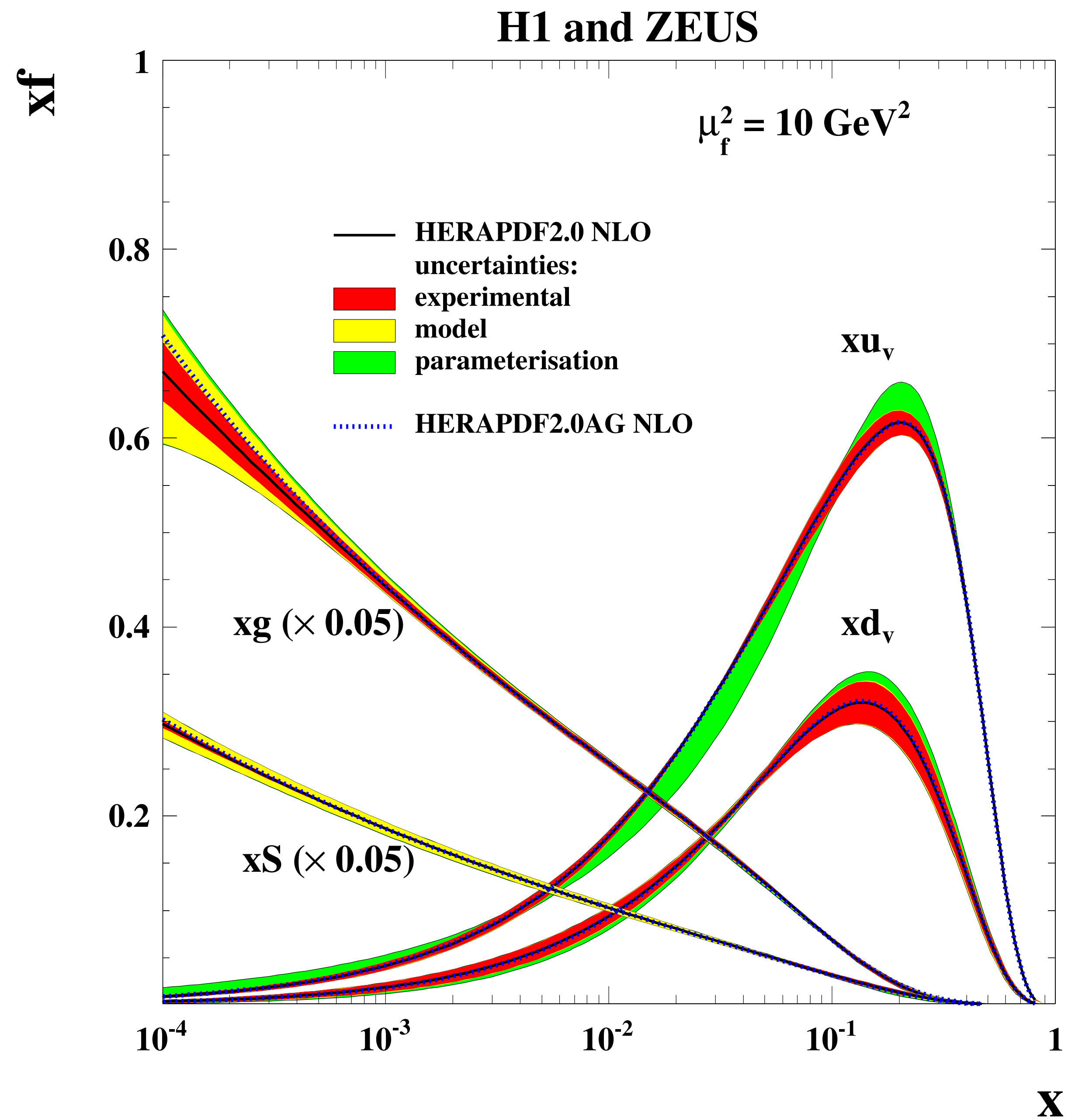
DIS at HERA: parton Distribution functions



HERA collider (92-07): Deep Inelastic Scattering (DIS) of of electrons on protons

Photon virtuality
$Q^2 = -q^2$

Bjorken $x = \dfrac{Q^2}{2p \cdot q}$



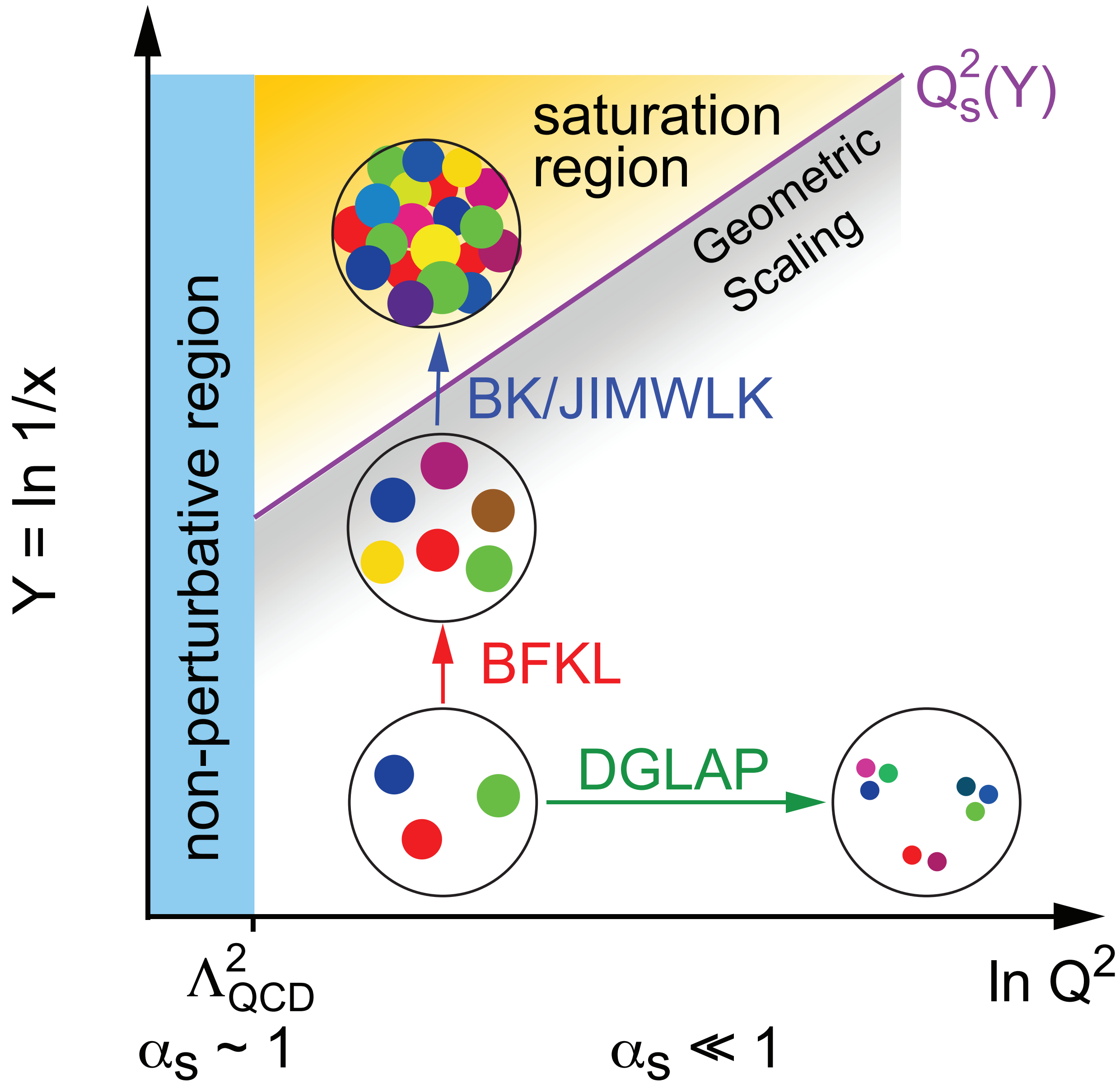
gluon g(x) and sea-quark S(x) distribution like powers ~ $x^{-\lambda}$ for $x \rightarrow 0$

→ invalidates probability interpretation if continued forever (integral over x diverges)

→ at some x, new QCD dynamics must set in

The proton at high energies: saturation

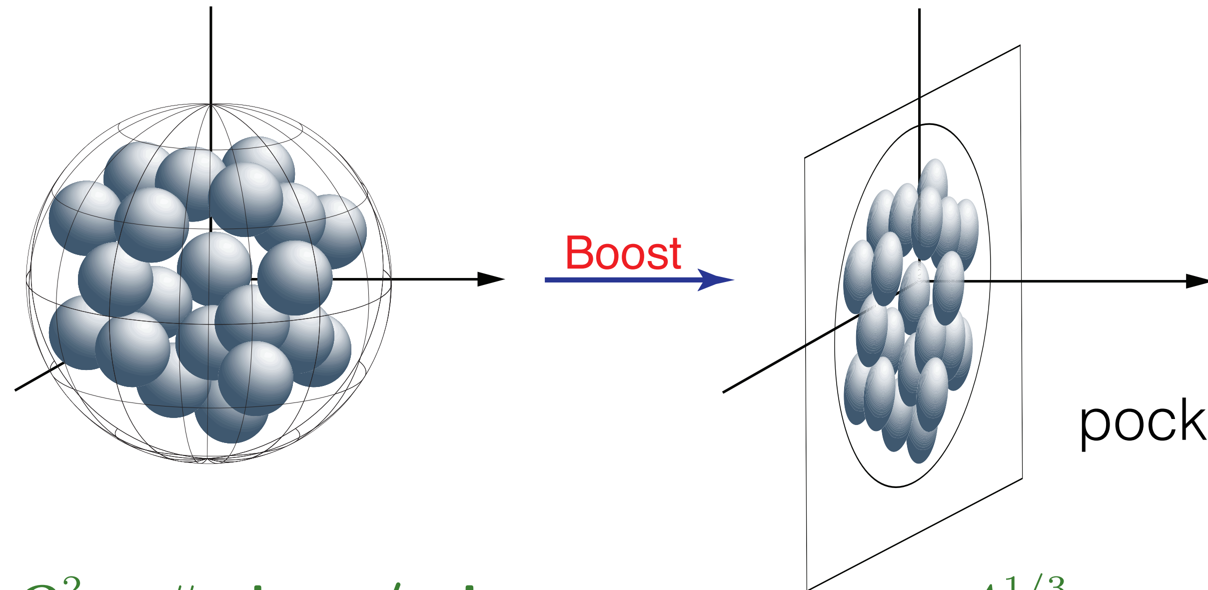
theory considerations:



- effective finite size $1/Q$ of partons at finite Q^2
- at some $x \ll 1$, partons 'overlap' $=$ recominbation effects
- turning it around: system is characterized by saturation scale Q_s
- grows with energy $Q_s \sim x^{-\Delta}$, $\Delta > 0$ & can reach in principle perturbative values $Q_s > 1$GeV

High gluon densities & heavy ions

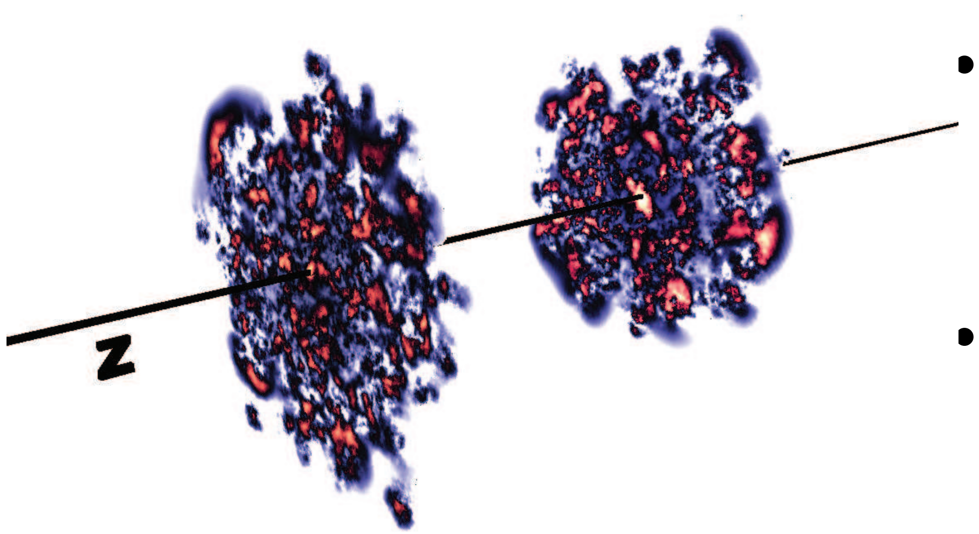
Expect those effects to be even more enhanced in boosted nuclei:



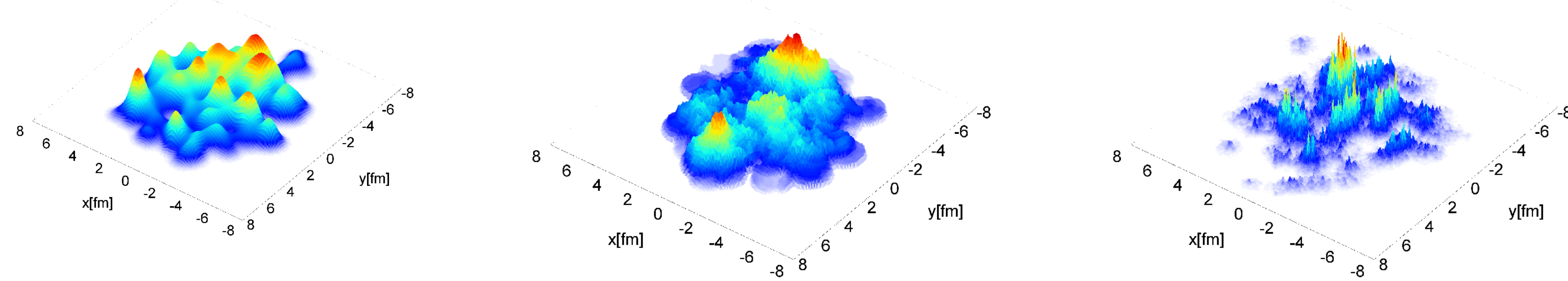
pocket formula:

$x_{eff}(A) = x_{Bjorken}/A$

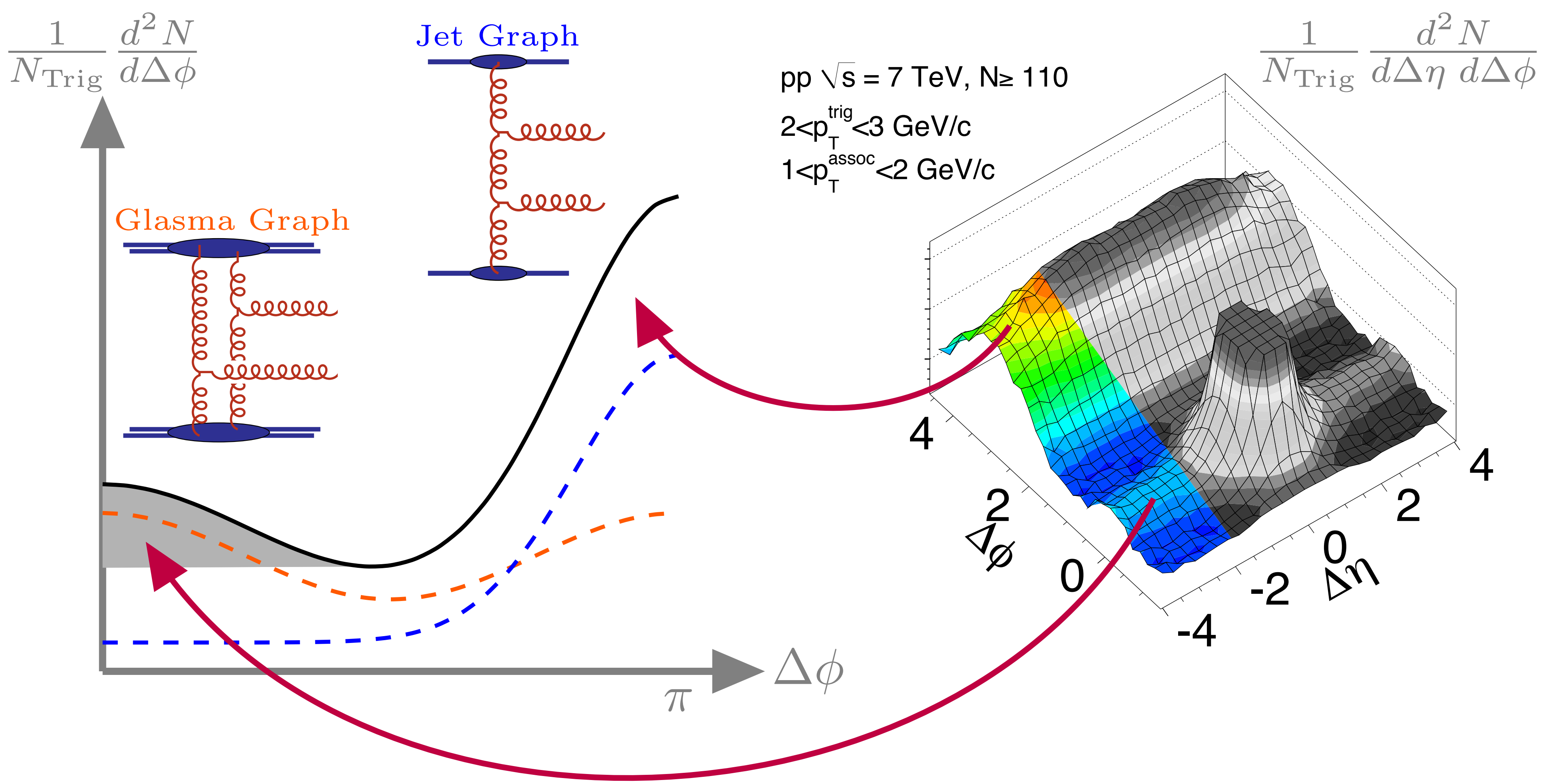
$Q_s^2 \sim \#$ **gluons/unit transverse area** $\sim A^{1/3}$



- Believed: heavy ion collisions at RHIC, LHC = collisions of two Color Glass Condensate
- but what are the correct initial conditions?

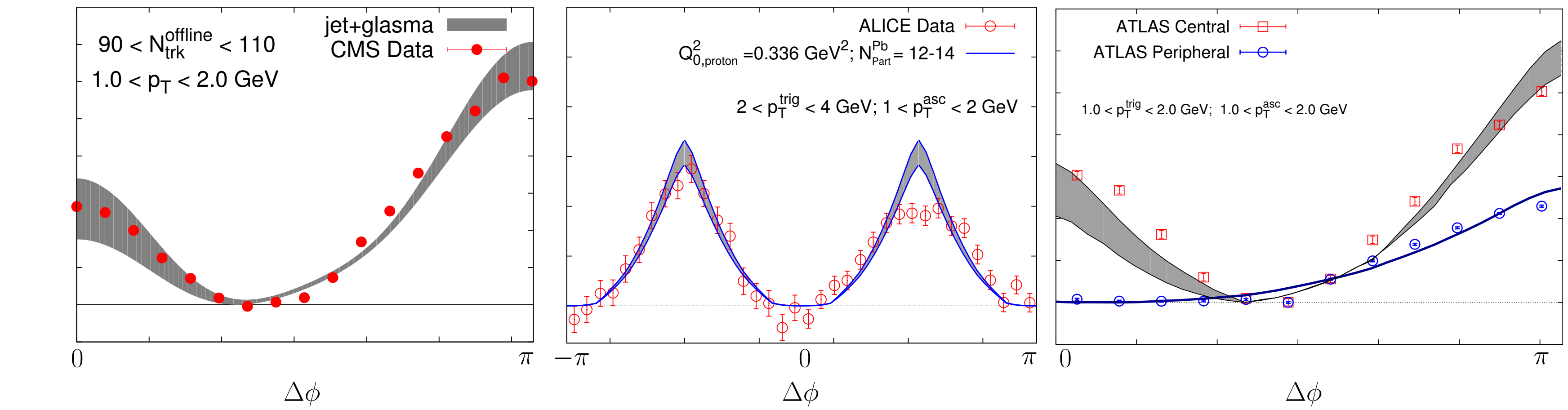


CGC and long-range rapidity correlations in high multiplicity events

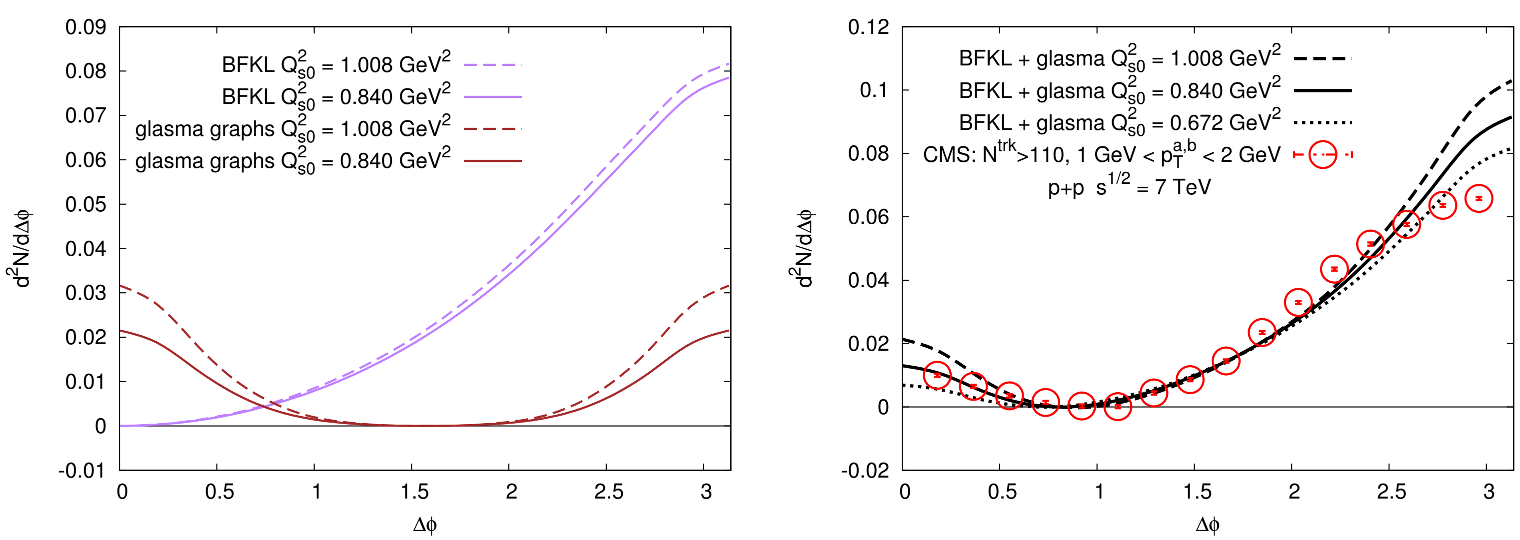


- high multiplicities → screening of color charges introduces → saturation scale
- high & saturated gluon densities (HERA fit with modified initial saturation scale, higher correlators from "Gaussian/dilute approximation")
- take limit $p_T/Q_S \ll 1$, 2 contributions: "glasma" and "jet" graph
  AA: glasma dominates, pp, pA also jet graph (α_S suppressed)

CGC & Ridges [Dusling, Venugopalan, Phys.Rev. D87 (2013) 9, 094034; 5, 051502]



pPb - data

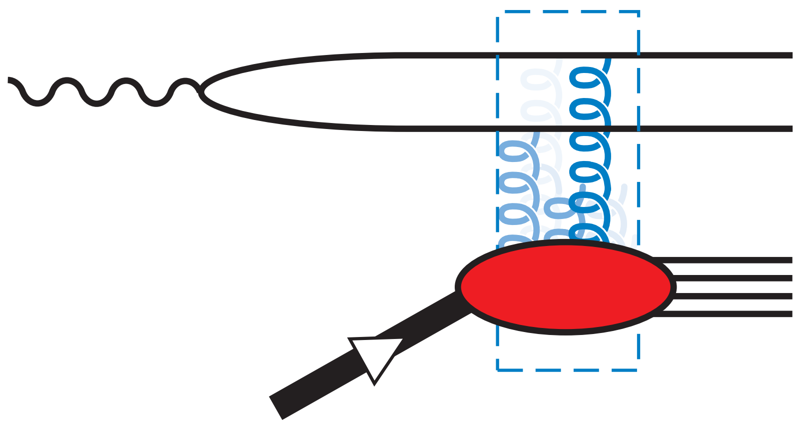


pp - data

Fig. 33. Long range ($2 \leq \Delta\eta \leq 4$) per-trigger yield of charged hadrons as a function of $\Delta\phi$ for p-p collisions at $\sqrt{s} = 7$ TeV. Data points are from the CMS collaboration. The curves show the results for $Q_0^2(x = 10^{-2}) = 0.840$ GeV^2 and $Q_0^2(x = 10^{-2}) = 1.008$ GeV^2.

works rather good, some say too good …

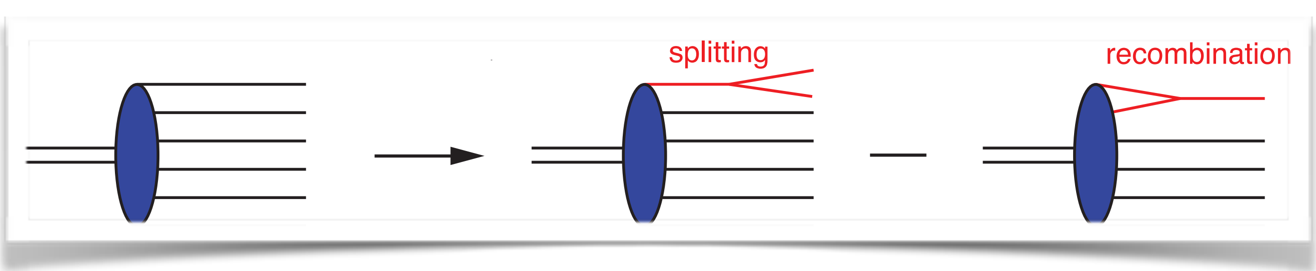
What do we know really about saturated gluons? — DIS on a proton at HERA

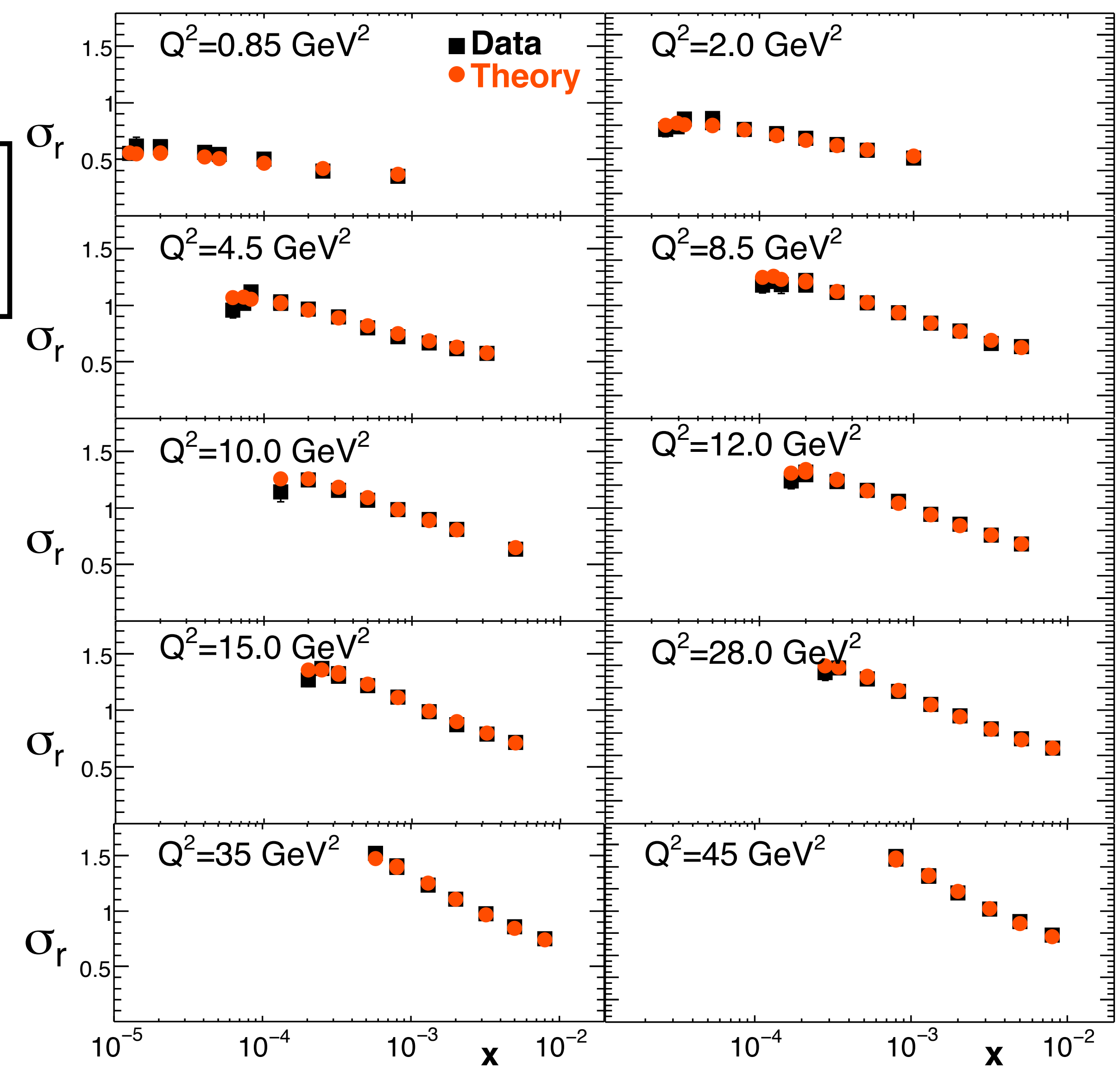


factorisation into photon wave function ψ ($\gamma^* \to$ qqbar) & color dipole $\mathcal{N}$ (~dense gluon field)

$$\sigma_{L,T}^{\gamma^* A}(x,Q^2) = 2\sum_f \int d^2\boldsymbol{b}\, d^2\boldsymbol{r} \int_0^1 dz \left|\psi_{L,T}^{(f)}(r,z;Q^2)\right|^2 \mathcal{N}(x,\boldsymbol{r},\boldsymbol{b})$$

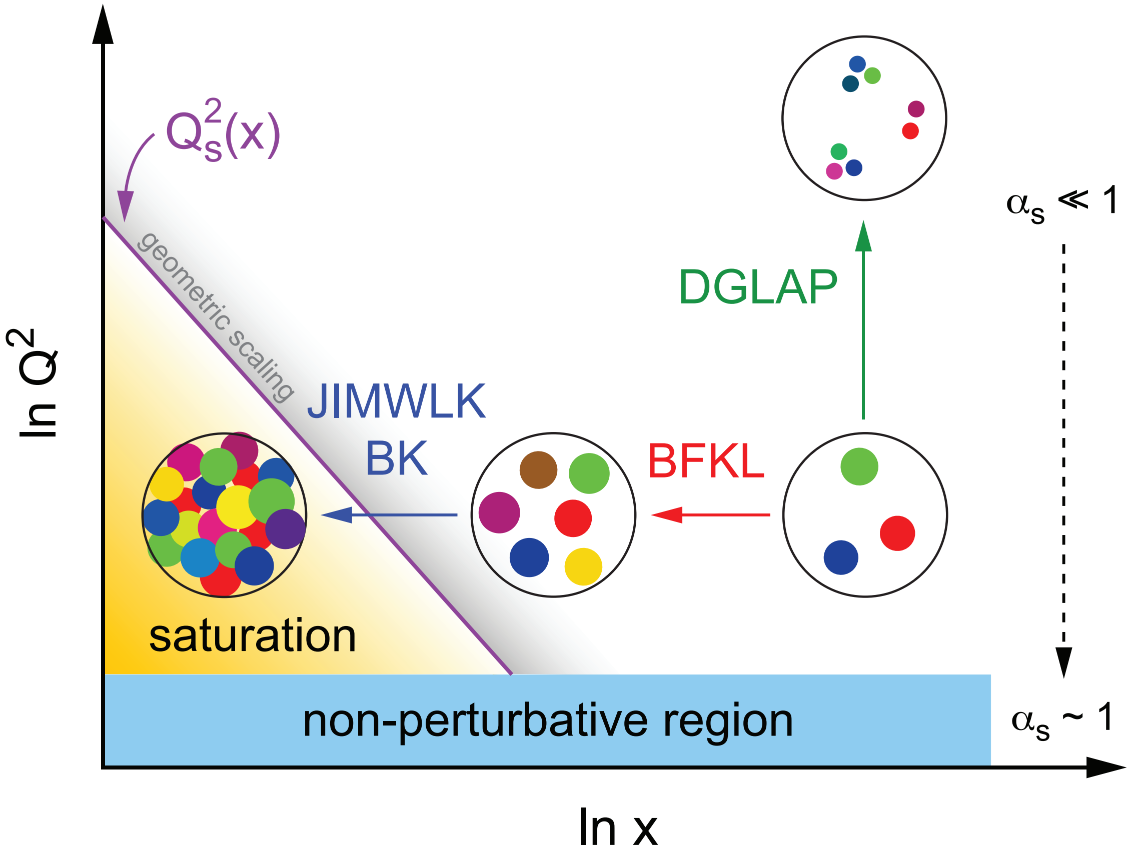
color dipole $\mathcal{N}$: all information about gluon distribution + follows non-linear evolution in ln(1/x) [JIMWLK or BK]



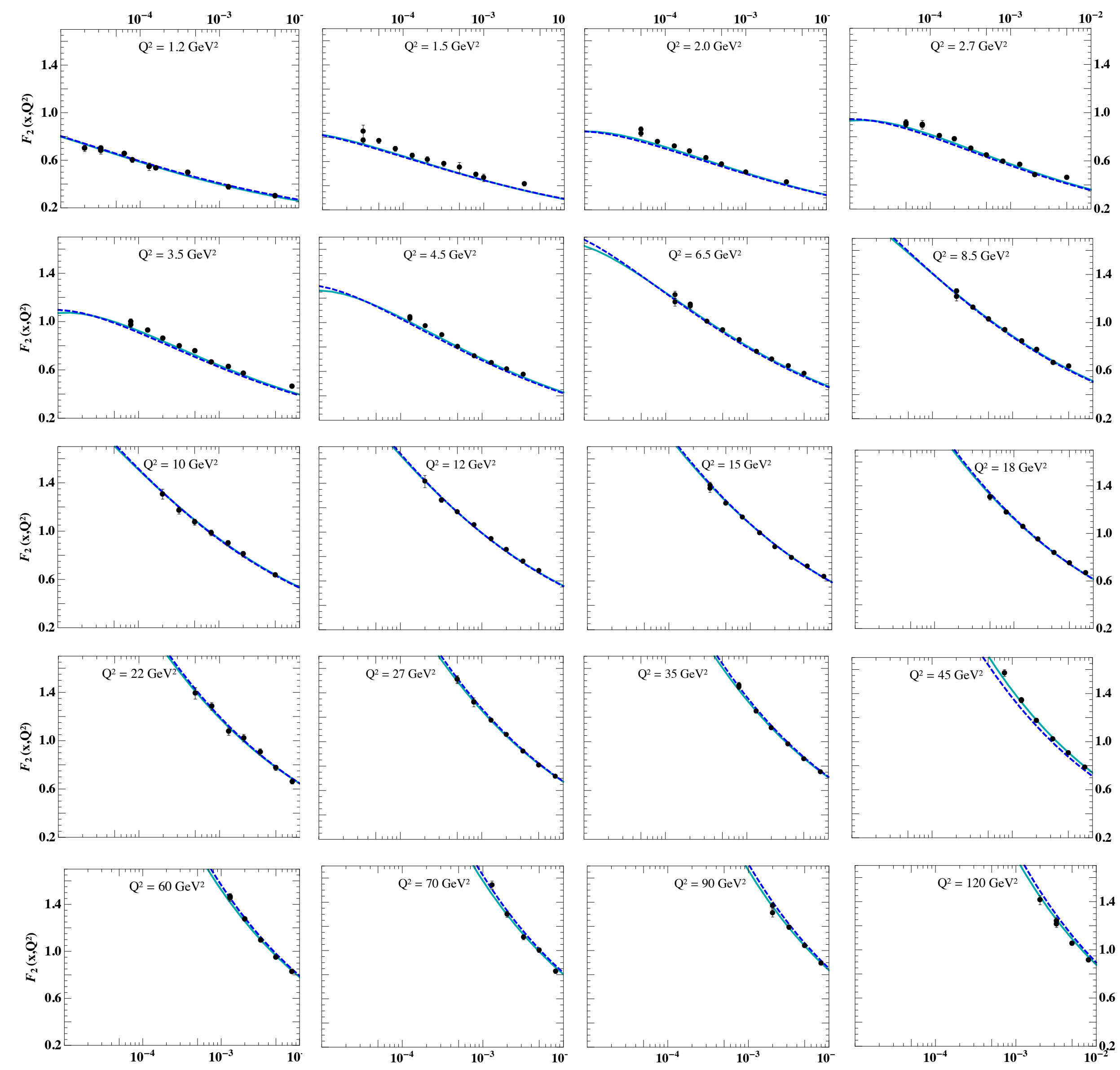


☑ achieve a good description of combined (= high precision!) HERA data through rcBK fit

[Albacete, Armesto, Milhano, Quiroga, Salgado, EPJ C71 (2011) 1705]



But …data also described by pdf-fits (=DGLAP) — intrinsically dilute (virtual photon interacts with single quark, gluon)



… and also (collinear improved) NLO BFKL evolution can fit data

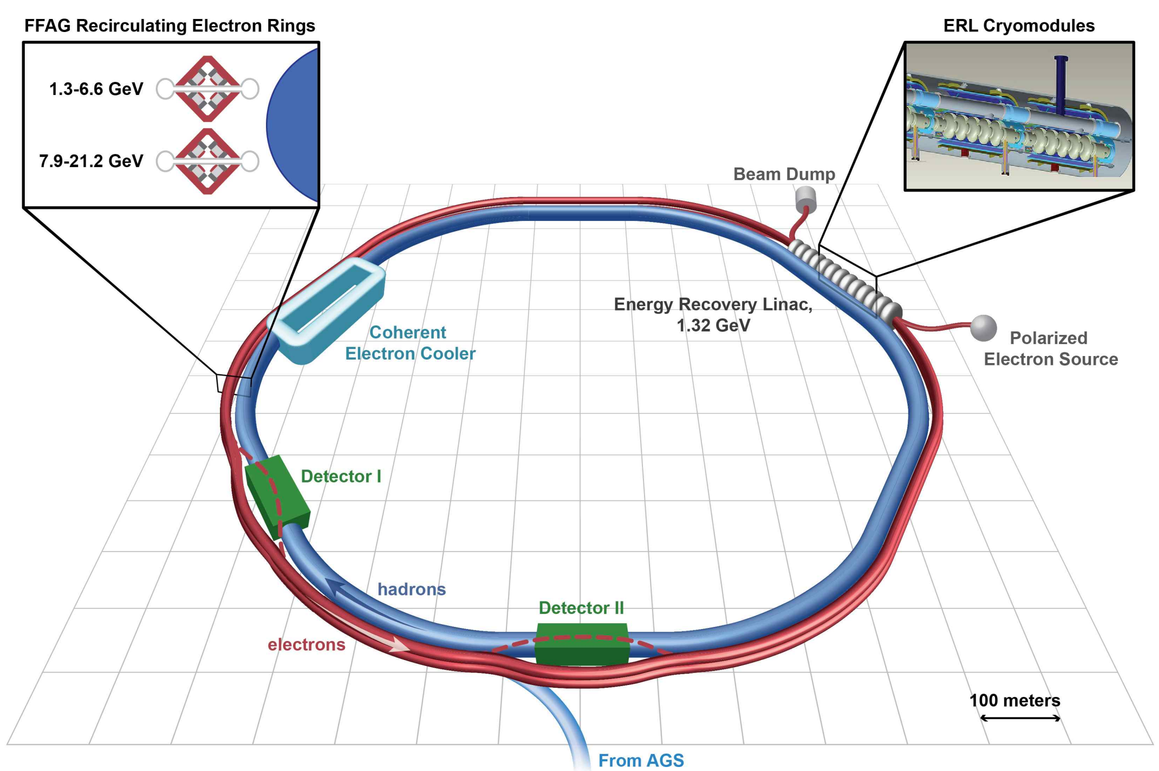
[MH, Salas, Sabio Vera; PRD 87 (2013) 7, 076005]

What we know and what we don't know

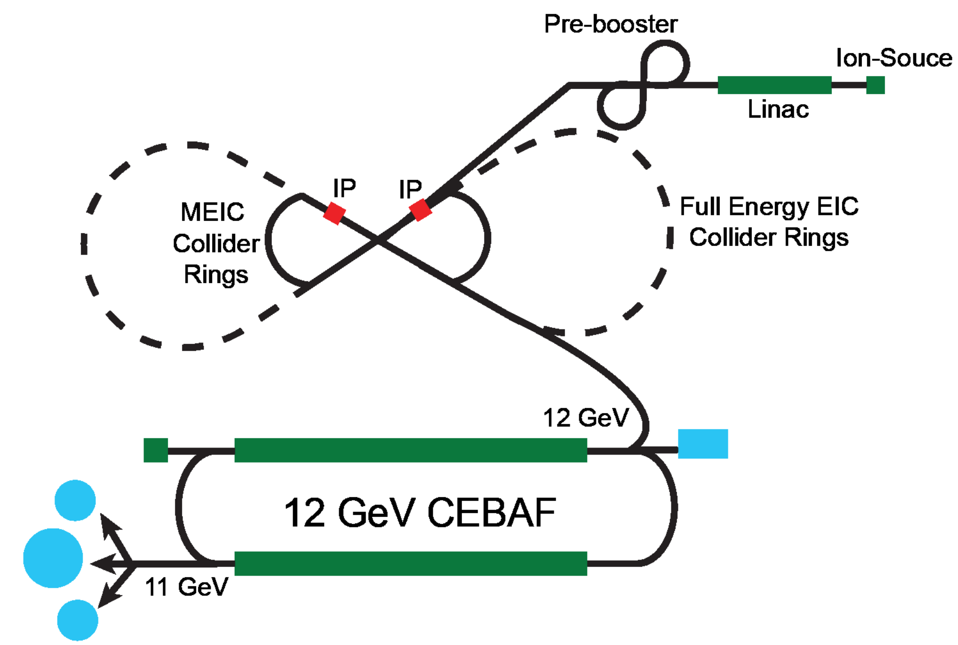
- extracted saturation scales at HERA not so large (0.75-2 GeV^2) + DGLAP fits initial conditions at small Q^2
- description of HERA data by saturation AND DGLAP not really a contradiction, but also not yet definite proof for saturation, cannot claim complete control
- can use HERA fits (*e.g.* rcBK) in *pA*, *AA*, high multiplicity events through scaling of (initial) saturation scale $Q_s(A) = Q_s^{HERA} \cdot A^{1/3}$, but rely on assumptions/arguments
- in general: initial conditions not controlled on the level of accuracy as *e.g.* in pp through conventional pdfs

A collider to search for a definite Answer:

the world's first eA collider: will allow to probe heavy nuclei at small x
(using 16GeV electrons on 100GeV/u ions)



Brookhaven National Laboratory: supplement RHIC with Electron Recovery Linac (eRHIC)



Jefferson Lab: supplement CEBAF with hadron accelerator (MEIC)

2015: endorsed by Nuclear Science Advisory Committee (NSAC) As highest priority for new Facility construction in US Nuclear Science Long Range plan

+ plans for LHeC etc.

Tasks for theory…

so far:

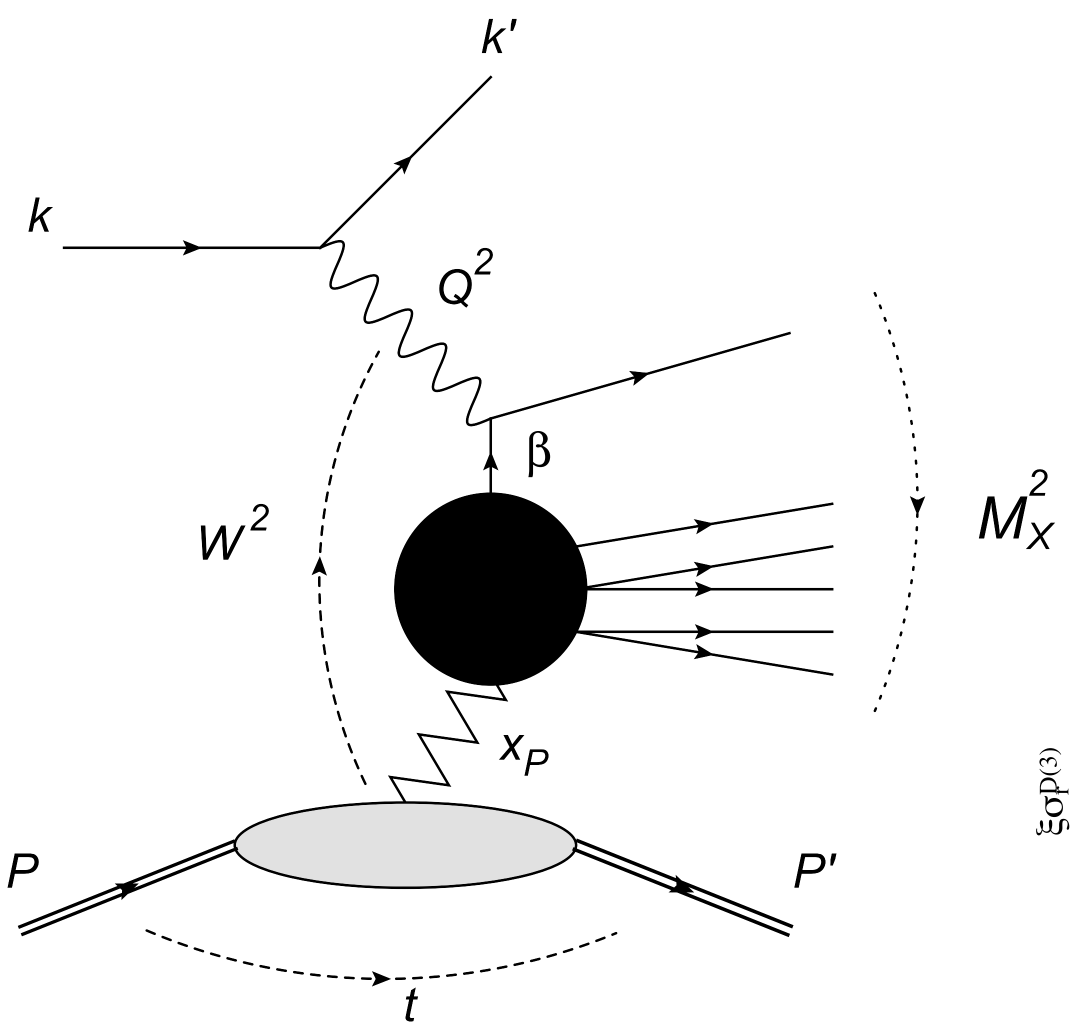
- still rely often on models (even though an sophisticated level) such as IPsat, bCGC → x-dependence = assumption + fit
- fits with evolution (rcBK): LO BK + running coupling corrections, coefficients at LO, with a few NLO exceptions (inclusive DIS, single inclusive jet in pA)

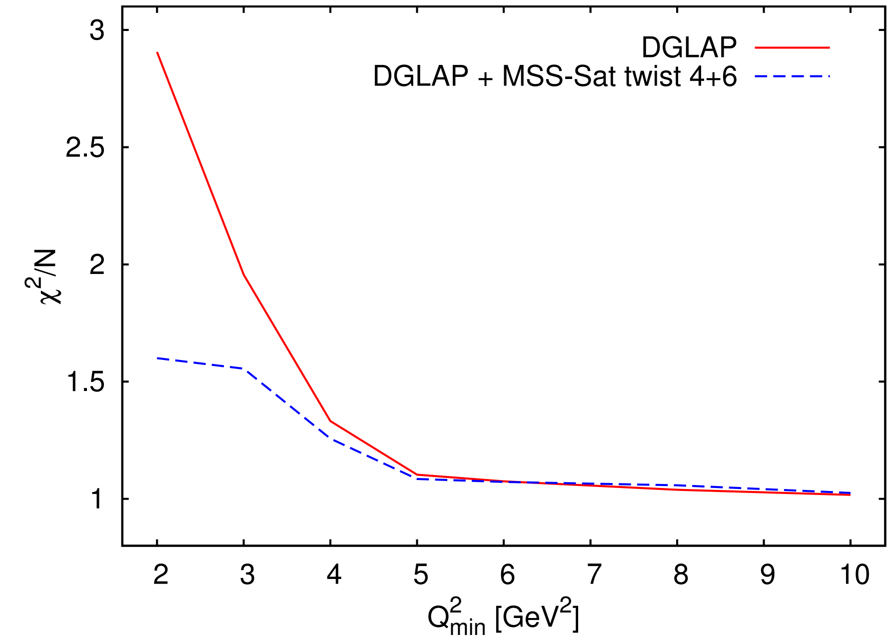
recent progress:

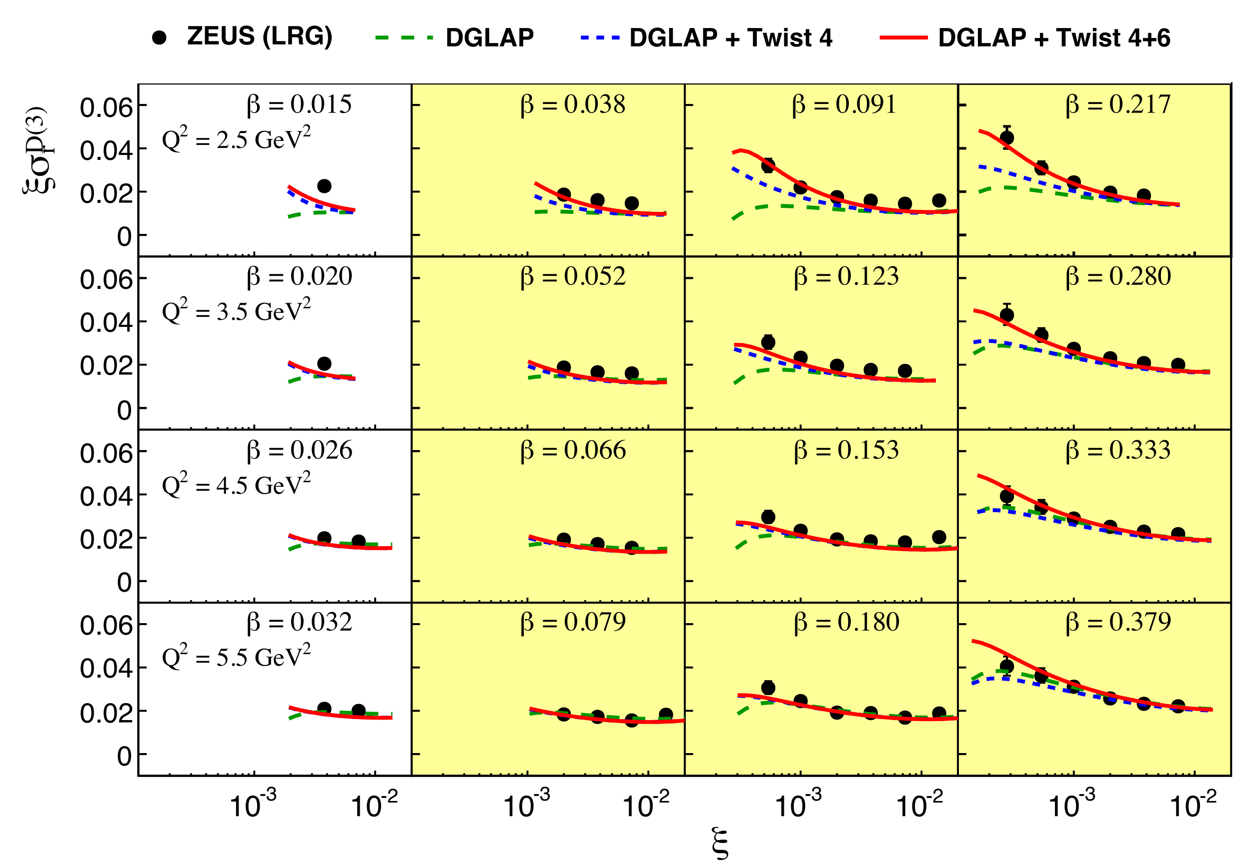
- NLO corrections for evolution [Balitsky, Chirilli; PRD 88 (2013) 111501, PRD 77 (2008) 014019]; [Kovner,Lublinsky, Mulian; PRD 89 (2014) 6, 061704] known & studied + resummed & used for first HERA fit [Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos, PLB750 (2015) 643]

missing:
→ NLO corrections for coefficients of exclusive observables
— provide strongest constraints on saturation

Example 1: Diffractive DIS at HERA



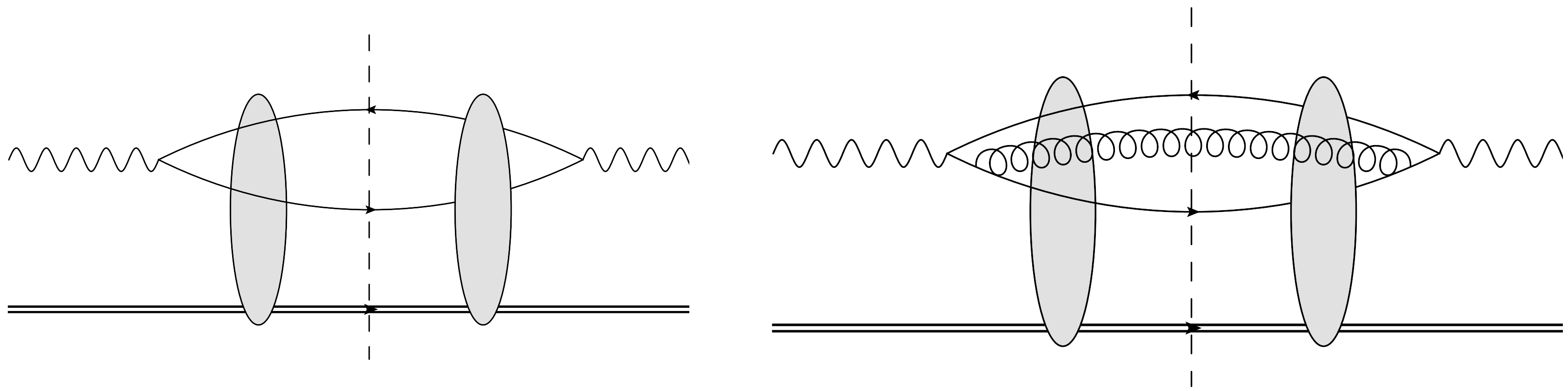




higher twist effects at small Q^2 as signal for saturation

[Motyka, Slominski, Sadzikowski, Phys.Rev. D86 (2012) 111501]

Theoretical Limitations

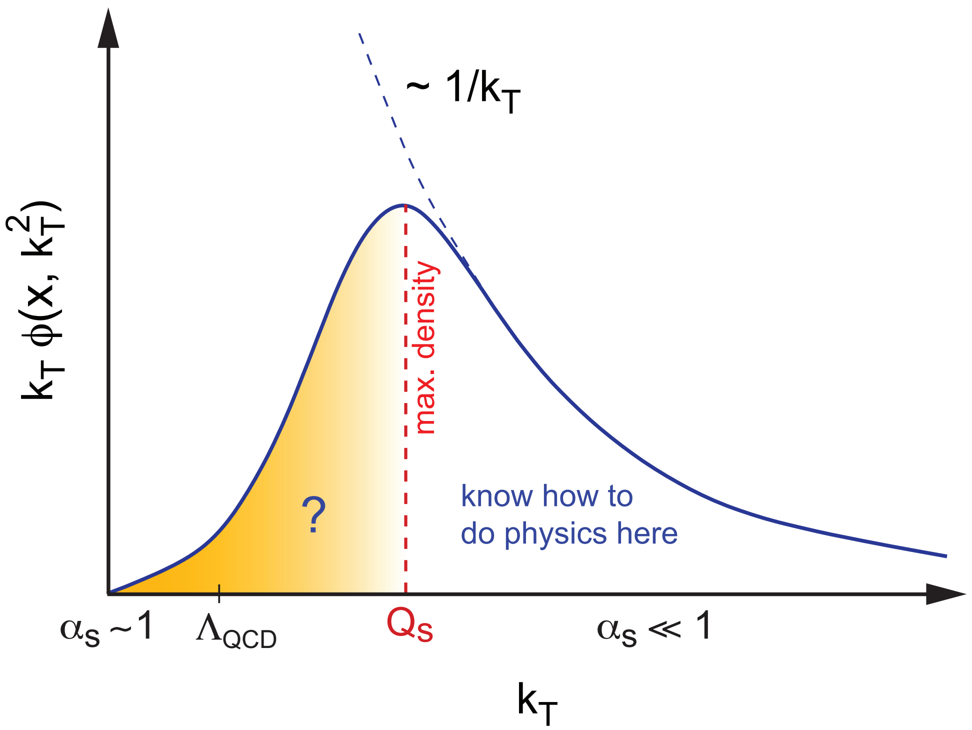


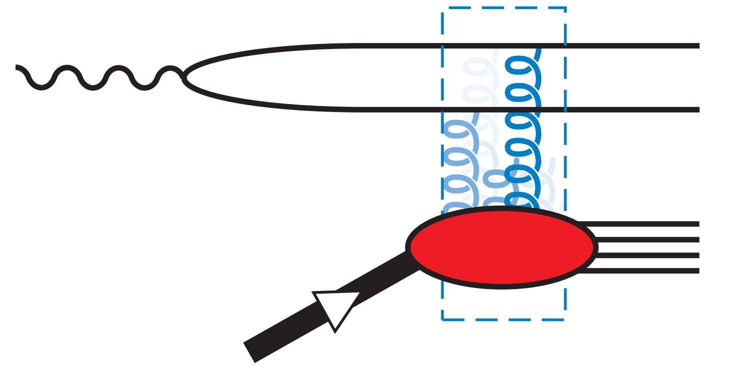
- large M_X requires **$q\bar{q}g$** → also **$q\bar{q}$** at 1-loop since inclusive — so far modelled using eikonal approximation [C. Marquet, Phys. Rev. D76, 094017 (2007)]

- color dipole (=target interaction): truncation to certain twist of GBW model

- motivated through "reggeization" in pQCD, but arbitrariness remains …

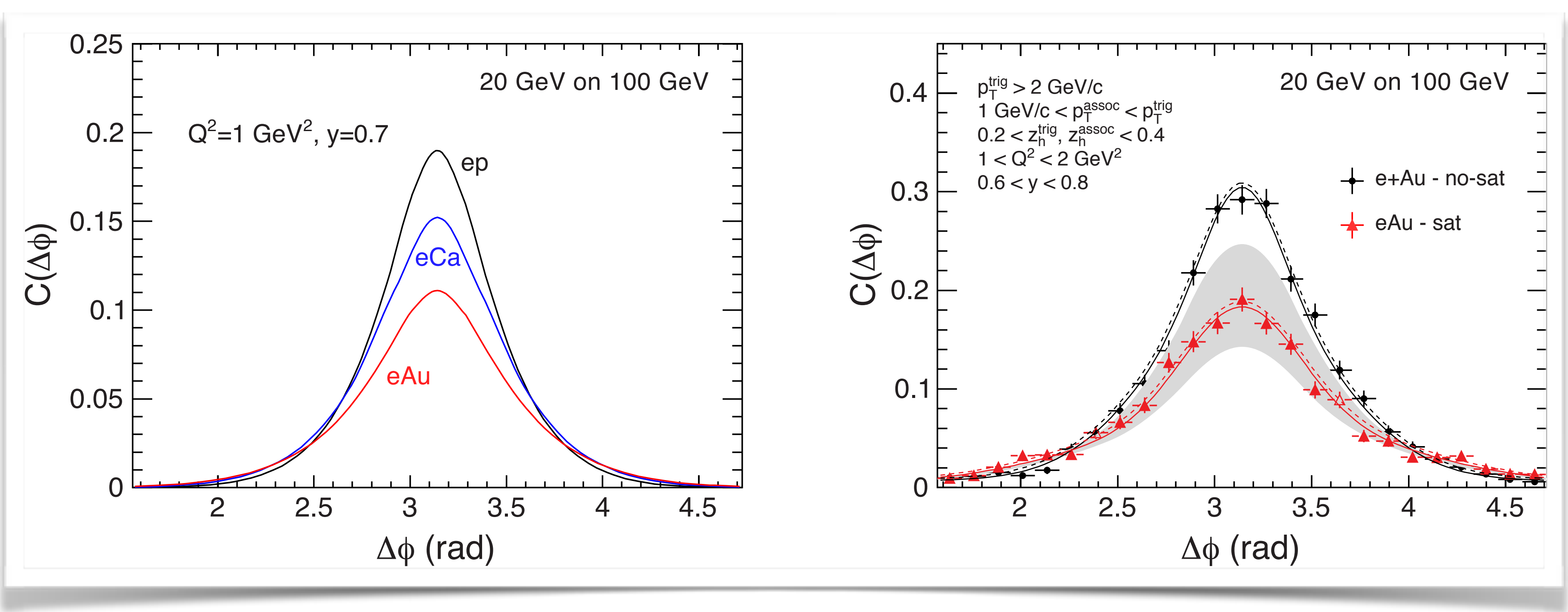
A popular observable in the EIC program:
Di-Hadron De-correlation in DIS





collinear factorization (dilute pQCD):
gluon k_T peaked at k_T=0
- expect di-hadrons back-to-back

Saturation (CGC):
gluon k_T peaked at saturation scale
- expect de-correlated di-hadrons



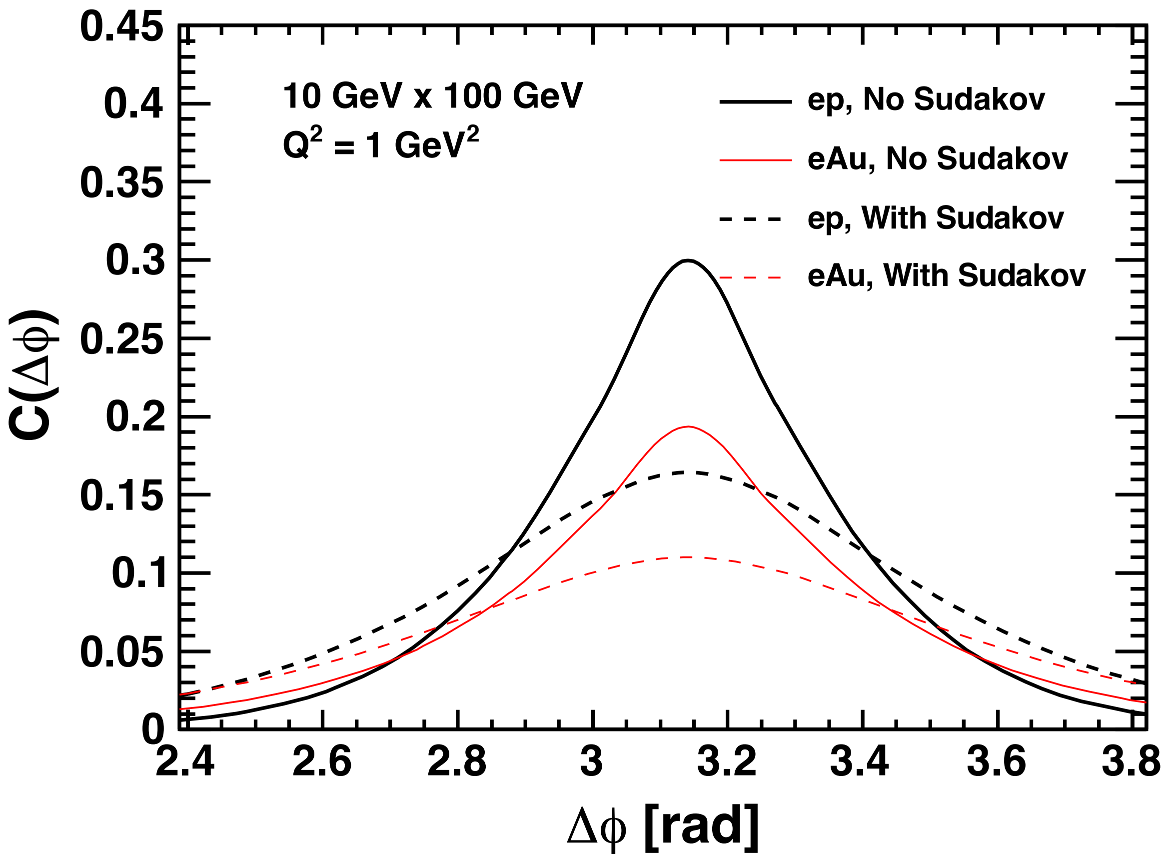
Potential limitations

- also here the NLO corrections are missing ($q\bar{q}g$ + $q\bar{q}$ at 1-loop)
- soft radiative corrections have been evaluated at leading order
  [Mueller, Xiao, Yuan, Phys.Rev. D88 (2013) 11, 114010]

comparison of *ep* and *eA* shows at first clear signal ….

…. but Sudakov factors have a big effect ….

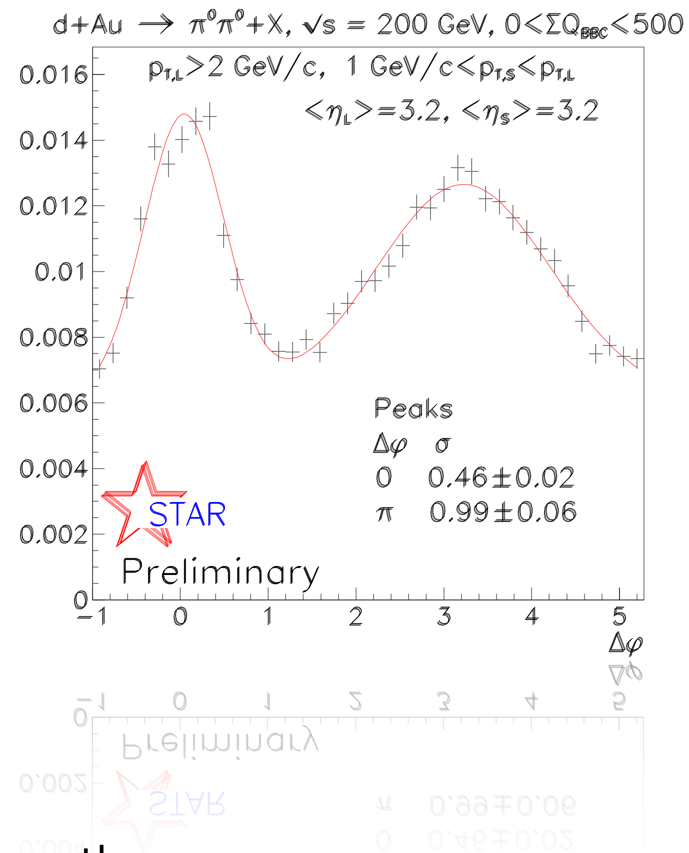
…. signal remains, but inclusion of higher order corrections necessary for precise distinction of different approaches

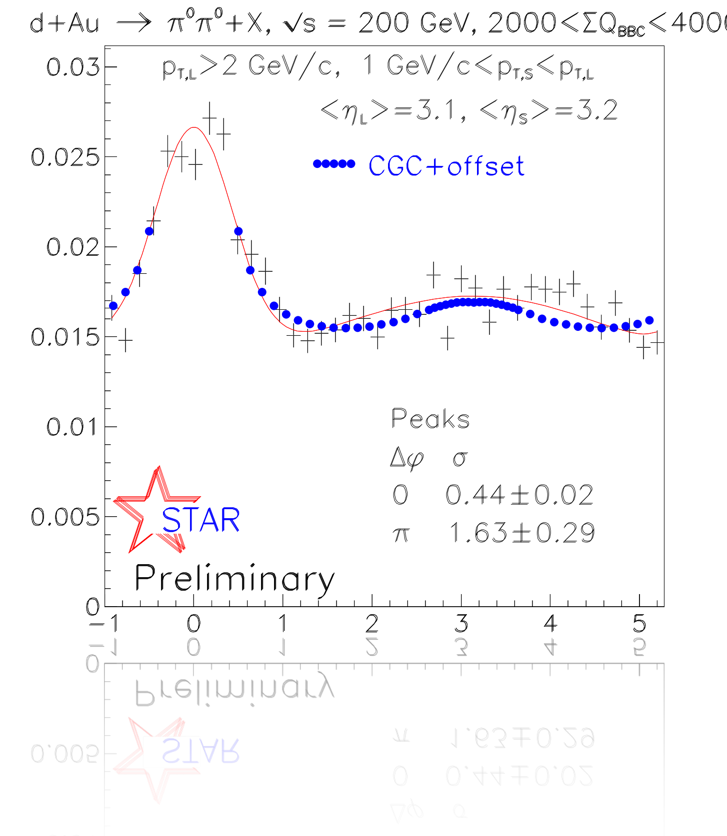


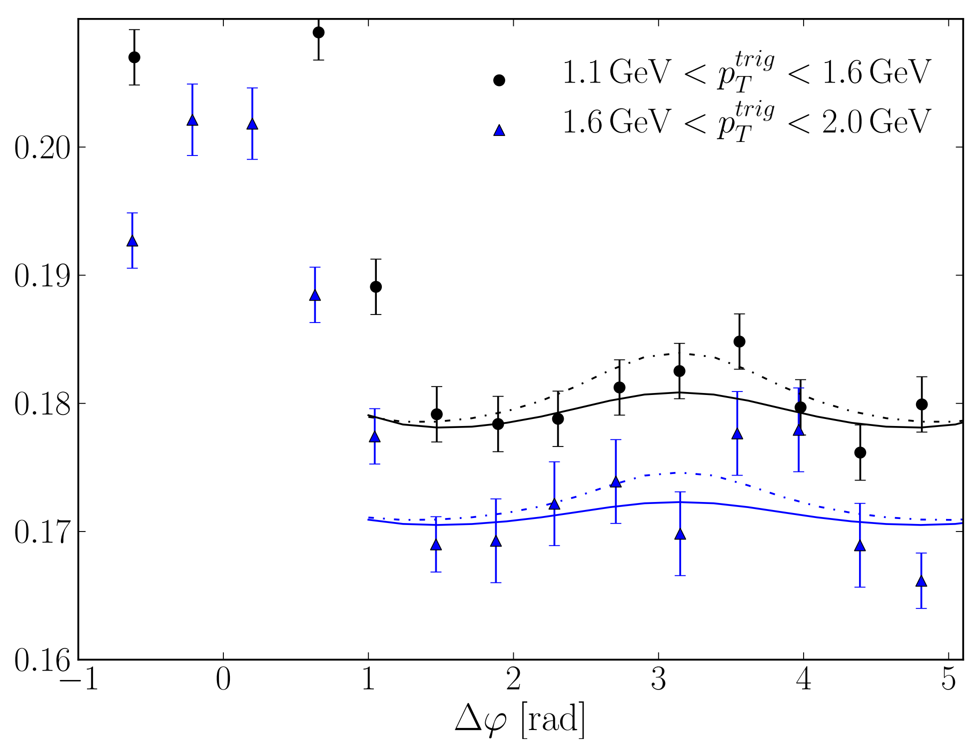
[Zheng,Aschenauer, Lee, Xiao, PRD89 (2014)7, 074037]

CGC and d-Au collisions at RHIC

signal in *d-Au* collisions at RHIC:
depletion of away side peak in central collisions described by CGC



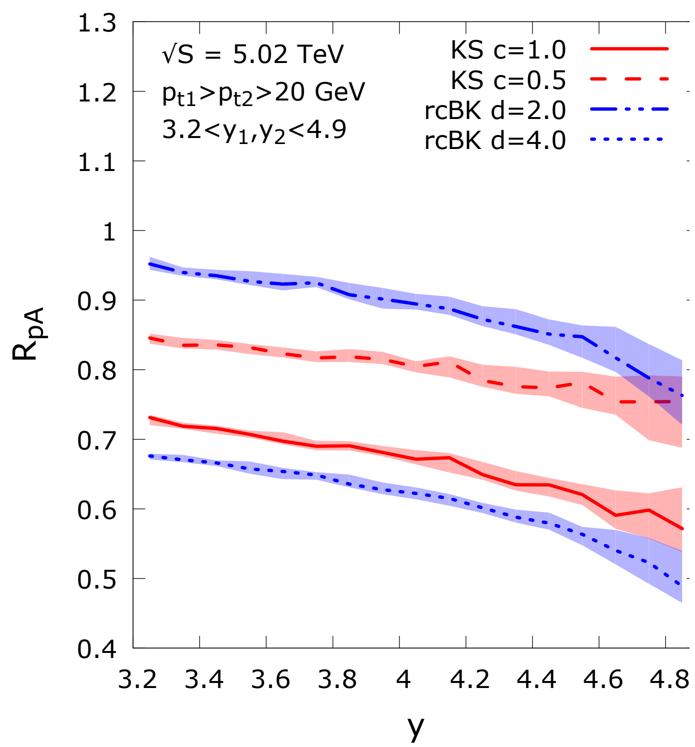


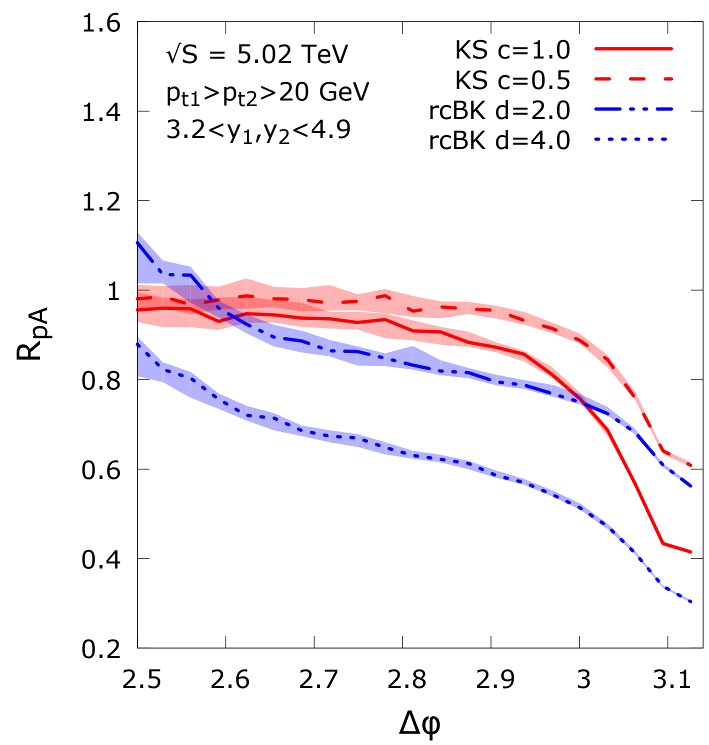


theory:
involves higher correlator (‘quadrupole’, not only dipole) — state-of-the art: calculate in Gaussian/ dilute approximation from dipole **[Lappi, Mantysaari, Nucl.Phys. A908 (2013) 51-72]**

π^0 azimuthal correlation compared to the PHENIX *d-Au* result (0.5GeV<p^{ass}<0.75 GeV, 3<y_1,y_2<3.8).
solid line: Q_{S0}^2 = 1.51 GeV^2,
dashed line: Q_{S0}^2 = 0.72 GeV^2

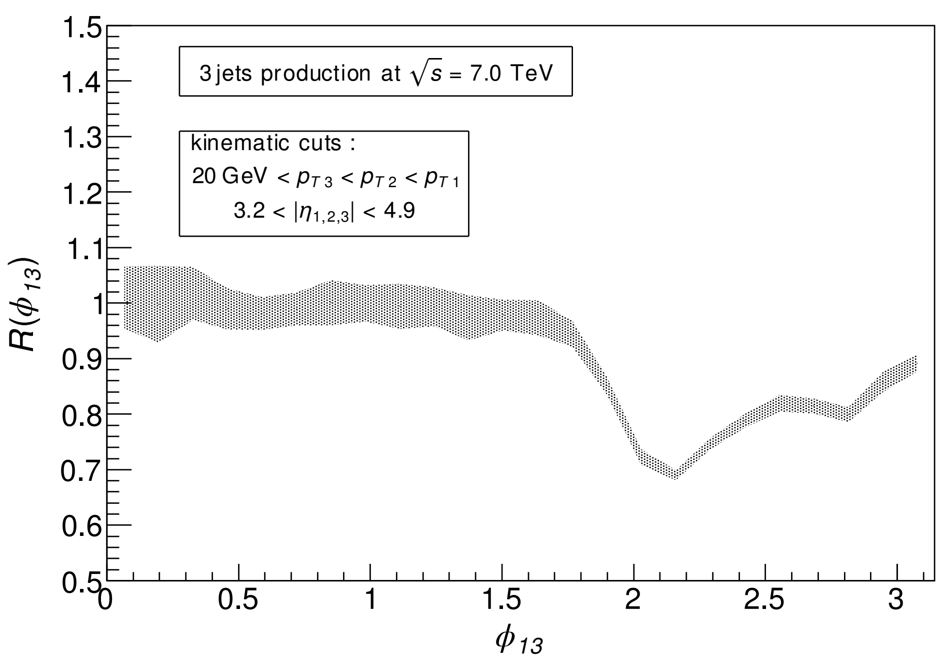
2 & 3 forward jets in *pPb*@LHC

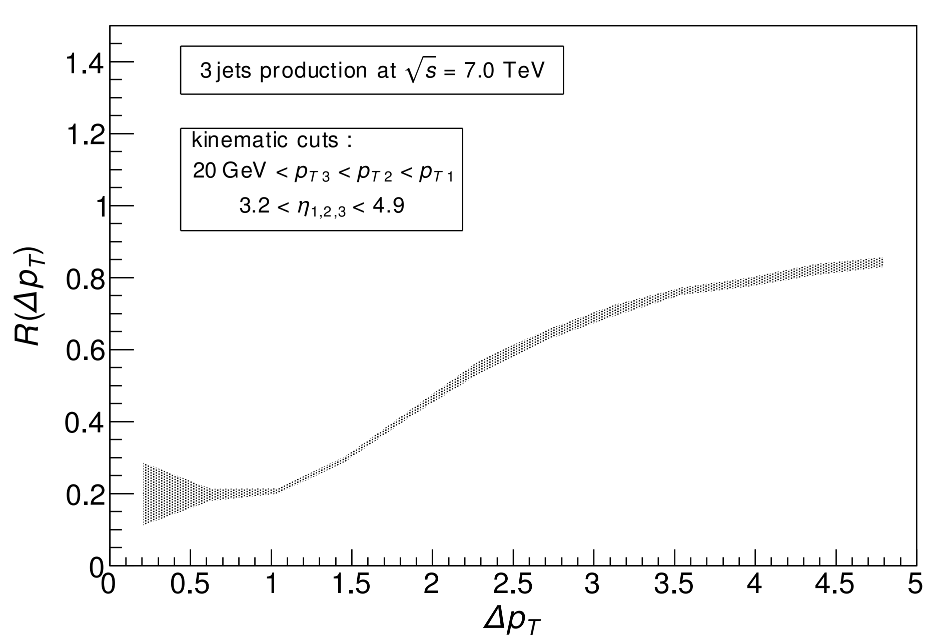




$$R_{\mathrm{pA}} = \frac{\dfrac{d\sigma^{p+A}}{d\mathcal{O}}}{A\,\dfrac{d\sigma^{p+p}}{d\mathcal{O}}}\,.$$

[v. Hameren, Kotko, Kutak, Marquet, Sapeta, Phys.Rev. D89 (2014) 9, 094014]

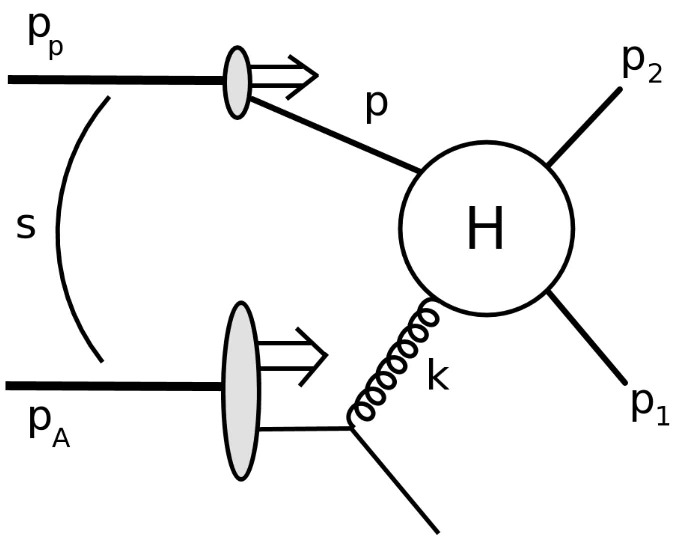




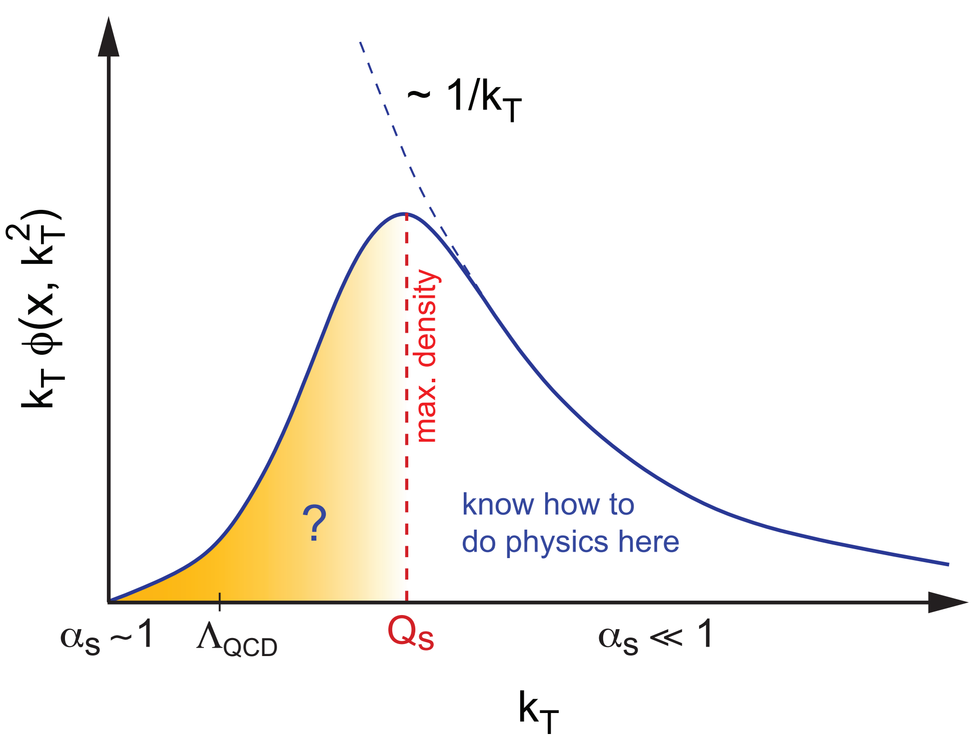
$$\Delta p_T = |\vec{p}_{T\,1} + \vec{p}_{T\,2} + \vec{p}_{T\,3}|$$

[v. Hameren, Kotko, Kutak, Phys.Rev. D88 (2013) 094001]

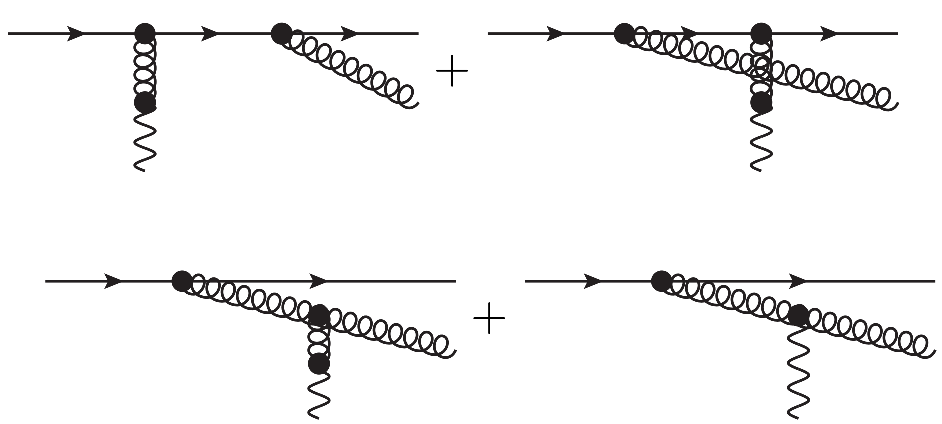
Theory description: use dilute approximation



gluon distribution obeys BK evolution



- use hybrid formalism: proton through collinear pdfs, Pb saturated gluon
- dilute expansion $|p_{1t}+p_{2t}| \gg Q_S$
  (2 jets: complete LO matrix element known in principle, 3 jets: unknown)
- hard process: only single scattering with glue field, saturation through k_T dependence



the presented studies have certain limitations

- uncontrolled higher order corrections (only LO in α_S)
- dilute expansion $p_{1t}+p_{2t}|\gg Q_S$
  (=probe the tail of saturation, but appropriate in certain kinematics)

need to increase theory precision for establishing saturation + extracting gluon distributions (important for precision at EIC but also LHC, HERA analysis)

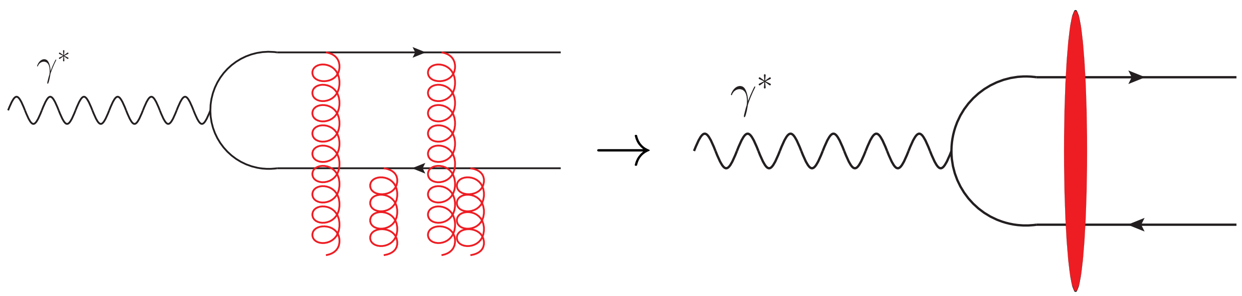
our project: calculate (NEW: NLO from momentum space)

A. tri-particle production at LO (new for DIS, pA 1st complete)
   expect more stringent tests of CGC through more complex final state

B. di-particle production at NLO (3 partons a subset!)
   reduce uncertainties + possibly identify overlap region between collinear factorisation and saturation physics

As a first step: limit to DIS (electron-nucleus i.e. γ^*A collisions)
but derive important general results on the way
→ first step for future *pPb* calculation in “hybrid-”formalism

Theory: quarks, gluons in the presence of high gluon densities

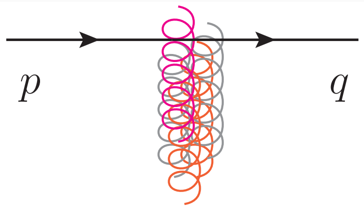


$x \to 0$: a single interaction with a strong & Lorentz contracted gluon field

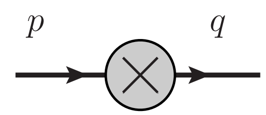
- propagation of quarks, gluons in presence of a strong ~1/g background gluon field $A^{+,a}(z^-, \boldsymbol{z}) = \alpha^a(\boldsymbol{z})\delta(z^-)$
- target=background field: used to build gluon distributions
- technically: use factorisation of QCD amplitudes in high energy limit (= x →0 limit)

Theory: Propagators in background field

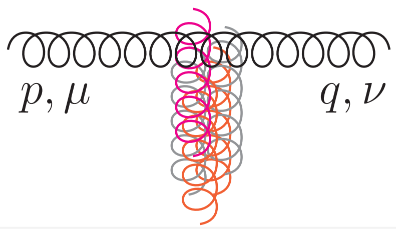
use light-cone gauge, with $k^- = n^- \cdot k$, $(n^-)^2 = 0$, $n^- \sim$ target momentum



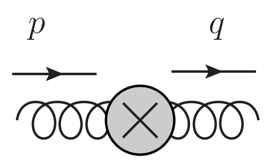
$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$



$$\tilde{S}_F^{(0)}(q)$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



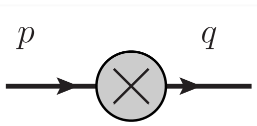
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$

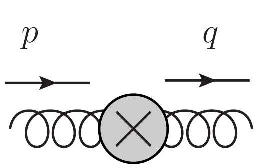
[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], …

interaction with the background field:



$$= 2\pi\delta(p^- - q^-)\not{n}^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left\{\theta(p^-)[V(\boldsymbol{z}) - 1] - \theta(-p^-)[V^\dagger(\boldsymbol{z}) - 1]\right\}$$



$$= -2\pi\delta(p^- - q^-)2p^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

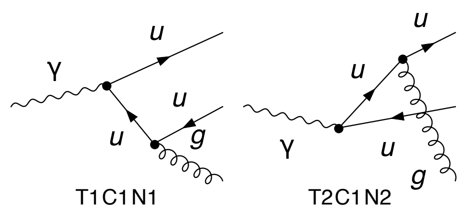
$$\cdot \left\{\theta(p^-)[U(\boldsymbol{z}) - 1] - \theta(-p^-)[U^\dagger(\boldsymbol{z}) - 1]\right\}$$

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) t^c$$

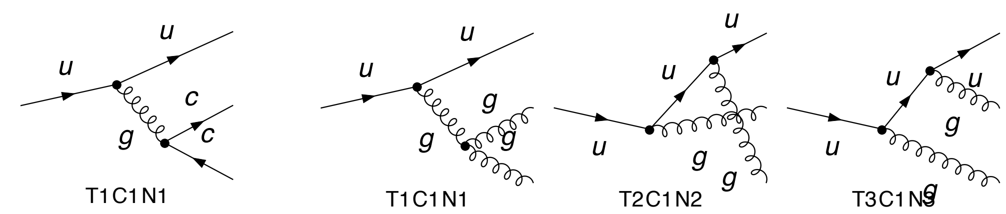
$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) T^c$$

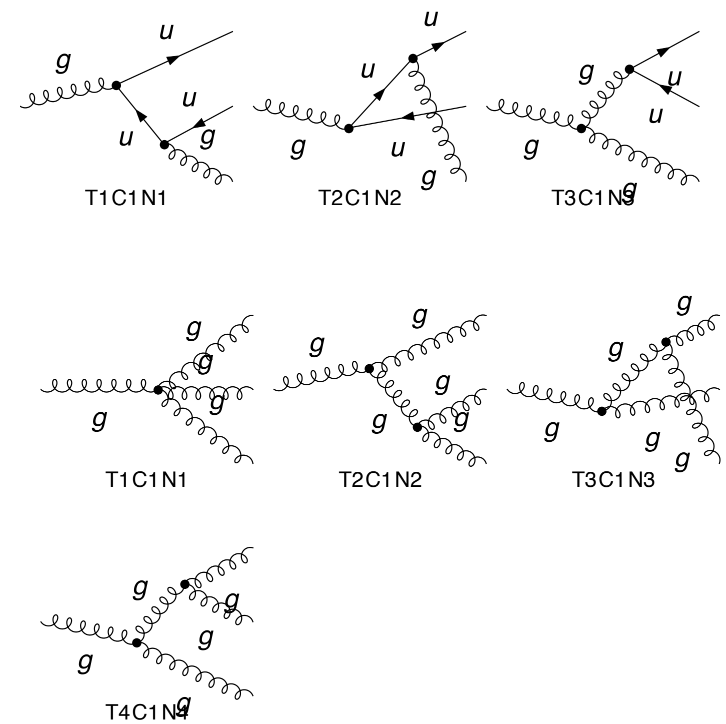
strong background field resummed into path ordered exponentials (Wilson lines) in contrast to dilute expansion: every line interacts with dense gluon field

Difference between DIS and LHC calculation: 3 parton production



DIS: $\gamma^* \to 3$ partons



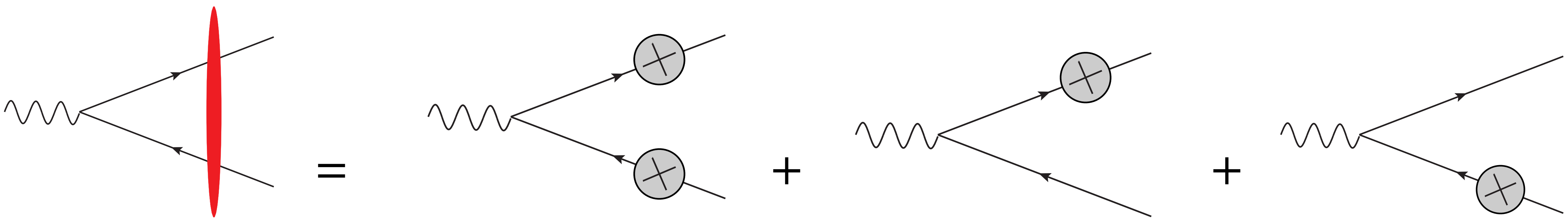


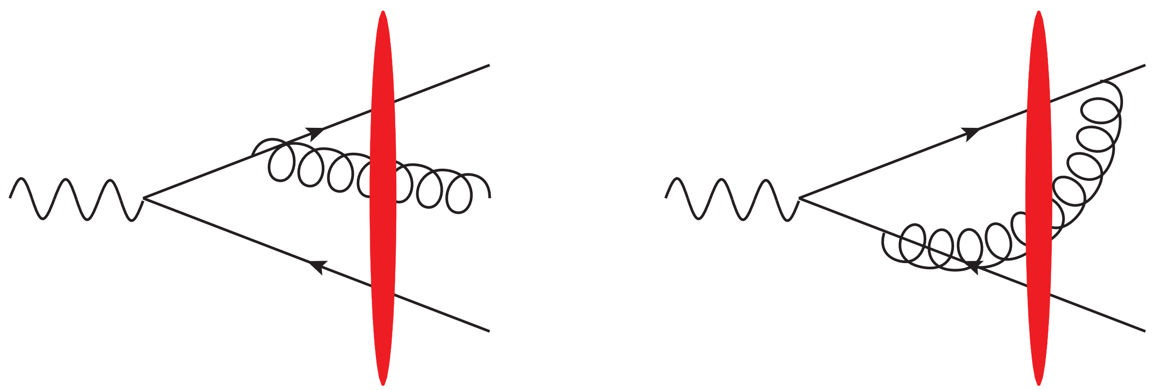
LHC:
q,g$\to$3 partons

- Feynman diagrams do not yet contain interaction with background field: each internal & each external coloured line to be split into 2 terms (-1)
- DIS the preferred playground for theory developments

1 extra parton — can cause a lot of work! (even for DIS process)

di-hadrons at LO: paper & pencil calculation *e.g.* [Gelis, Jalilian-Marian,PRD67, 074019 (2003)]





each line & each final state splits into two terms (free + interaction)
→ real NLO: 16 diagrams (amp. level)
→ virtual NLO: 32 diagrams (amp. level)

on X-sec. level: up to 16 Gamma matrices in a single Dirac trace
→ 15! = 1307674368000 individual terms (not all non-zero though)

- necessary to achieve (potential) cancelations of diagrams BEFORE evaluation
- require automatization of calculation (= use of Computer algebra codes)

Reduce # of Diagrams

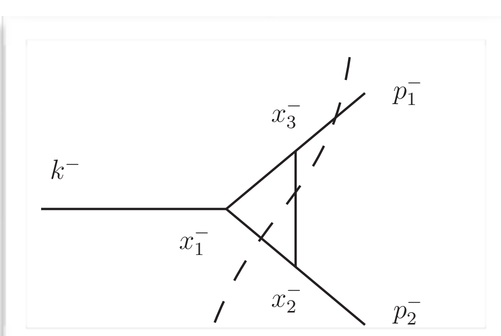
Configuration space: cuts at $x^-=0$

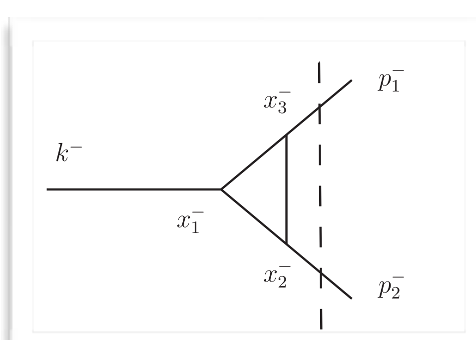
- diagrams to configuration space → momentum delta function as integral at each vertex + four momentum integral at each internal internal line
- Feynman propagator in configuration space

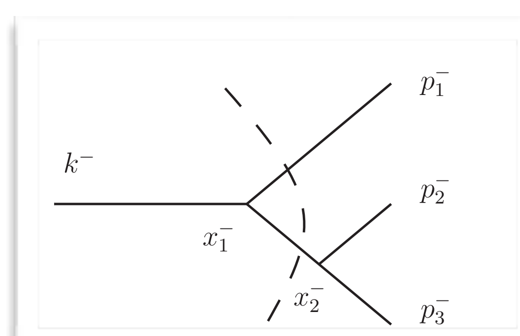
$$\Delta_F^{(0)}(x) = \int \frac{d^dp}{(2\pi)^d} \frac{i \cdot e^{-ip\cdot x}}{p^2 - m^2 + i0} = \int \frac{dp^+}{(2\pi)} \int \frac{dp^- d^{d-2}\boldsymbol{p}}{(2\pi)^{d-1}} \frac{e^{-ip^- x^+ + i\boldsymbol{p}\cdot\boldsymbol{x}}}{2p^-} \cdot \frac{i \cdot e^{-ip^+ x^-}}{p^+ - \frac{\boldsymbol{p}^2 + m^2 - i0}{2p^-}}$$

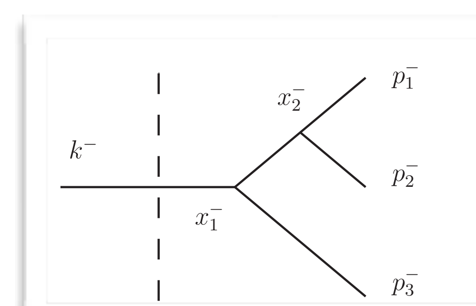
$$= \int \frac{dp^- d^{d-2}\boldsymbol{p}}{(2\pi)^{d-1}} \frac{e^{-ipx}}{2p^-} \left[\theta(p^-)\theta(x^-) - \theta(-p^-)\theta(-x^-)\right]_{p^+ = \frac{\boldsymbol{p}^2+m^2}{2p^-}}$$

- divide x_i^- integral $\int_{-\infty}^{\infty} dx_i^- \to \int_{-\infty}^{0} dx_i^- + \int_{0}^{\infty} dx_i^-$ → each of our diagrams cut by a line separating positive & negative light-cone time
- s-channel kinematics [$k^- = p_1^- + p_2^- + \ldots$, all positive] → only s-channel type cuts possible (~vertical cuts)

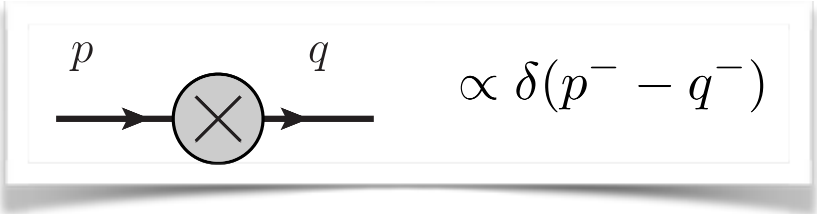








Configuration space can help

- recall: 

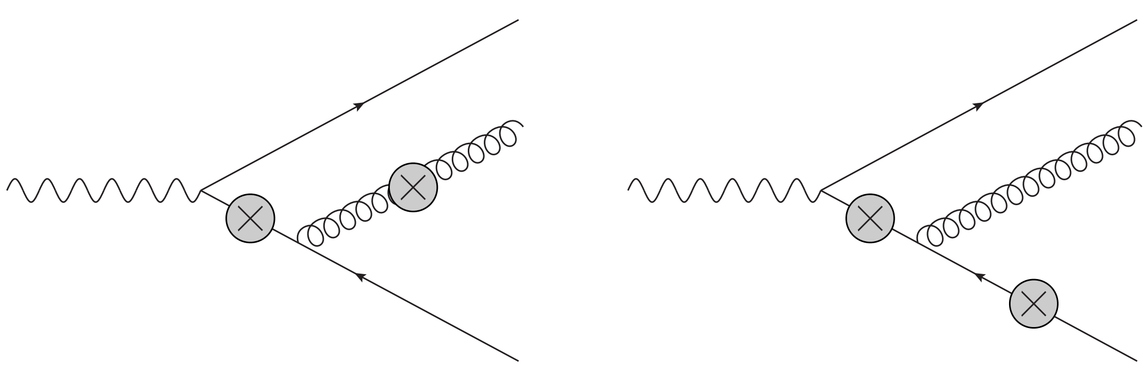
i.e. minus momentum flow not altered through interaction

- recall: interaction placed at slice z^-=0

$$A^{+,a}(z^-, \boldsymbol{z}) = \alpha^a(\boldsymbol{z})\delta(z^-)$$

→ interaction must be always placed at a z^-=0 cut of the diagram.
Note: this applies equally to configuration and momentum space

- evaluates already sum of a large fraction of diagrams (~50%) to zero

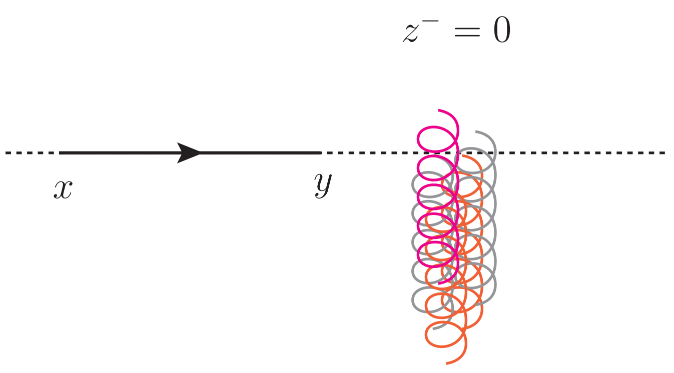


forbidden configurations:
cannot be accommodated by
vertical (s-channel type) cut

Can we Do better? …. more constraints

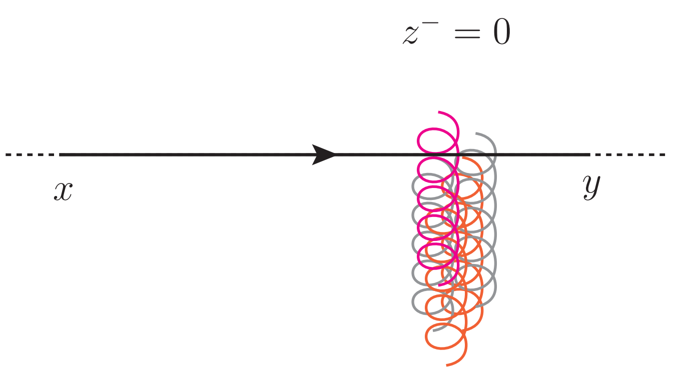
consider complete configuration space propagator (free + interacting part)

$$S_F(x,y) = \int \frac{d^dp}{(2\pi)^d}\frac{d^dq}{(2\pi)^d} e^{-ipx}\left[\tilde{S}_F^{(0)}(p)(2\pi)^d\delta^{(d)}(p-q) + \tilde{S}_F^{(0)}(p)\tau_F(p,q)\tilde{S}_F^{(0)}(q)\right]e^{iqy}$$



obtain free propagation for

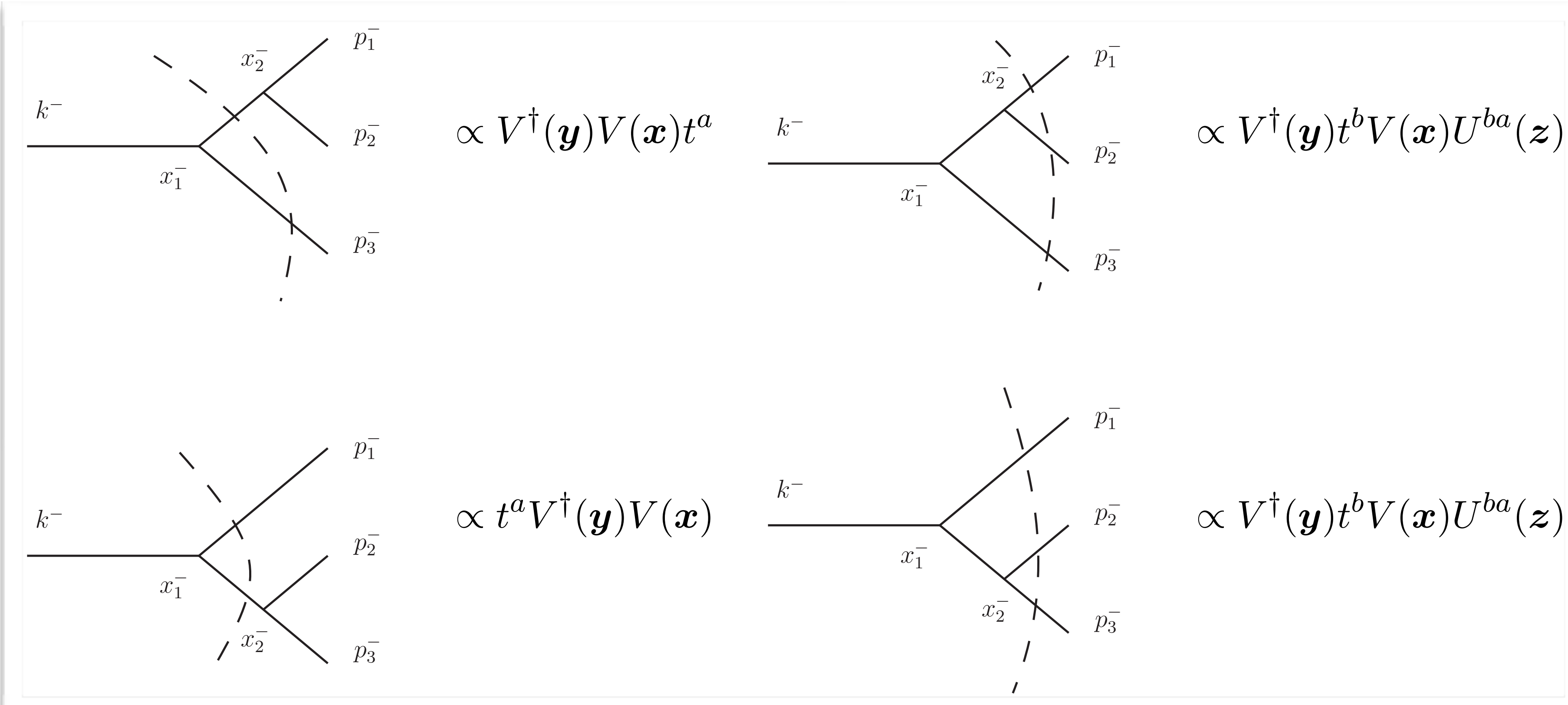
- $x^-, y^- < 0$ ("before interaction")
- $x^-, y^- > 0$ ("after interaction")



propagator proportional to complete Wilson line V (fermion) or U (gluon) if we cross cut at light-cone time 0

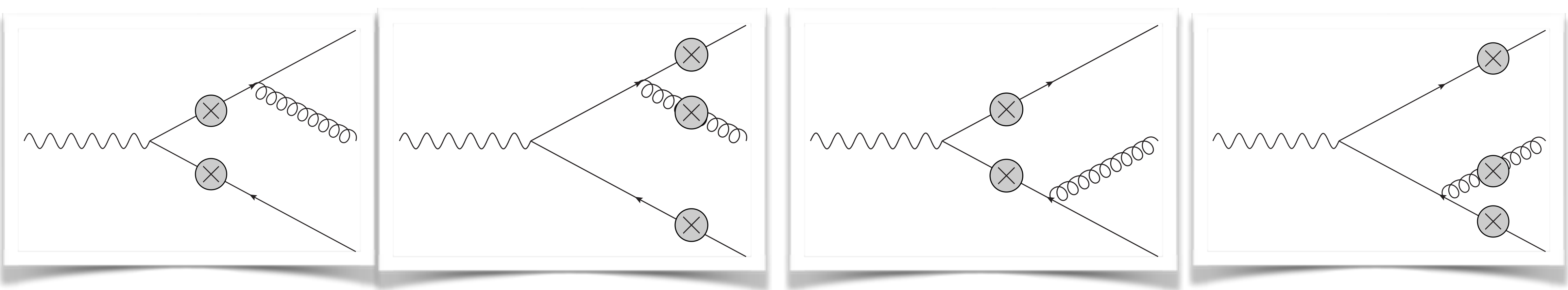
- no direct translation to momentum space
  adding free propagation & interaction→ mixing of different mom. space diagrams
- but strong constraints on the structure of the full result

Configuration Space predicts which Operators have non-zero coefficients



momentum space: necessary coefficients from only 4 (instead of 16) diagrams

(cancelation of all other contributions verified by explicit calculations)



virtual corrections: similar result,

necessary coefficients from 8 (instead of 32) diagrams

Structure of Wilson correlators

for 3 particle production in DIS

Wilson lines build correlators = different gluon distributions

(in general more than one)

- *e.g.* inclusive DIS at LO: target interaction through color dipole

$$\mathcal{N}(\boldsymbol{r},\boldsymbol{b}) = \frac{1}{N_c}\mathrm{Tr}\Big(1 - V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\Big) \qquad \boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y} \qquad \boldsymbol{b} = \frac{1}{2}(\boldsymbol{x}+\boldsymbol{y})$$

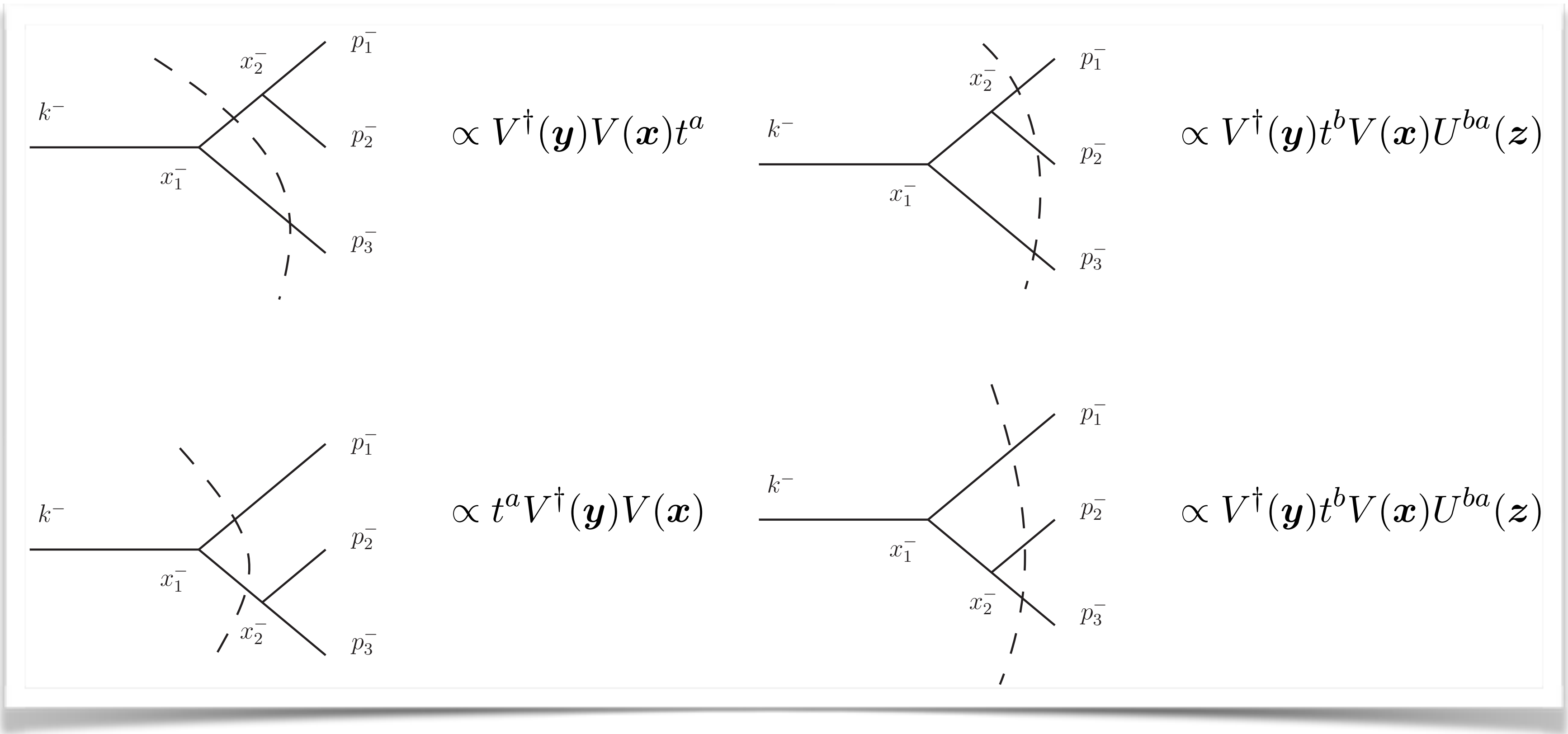
- 2 parton final state: new correlator — the quadrupole

$$\mathcal{N}^{(4)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4) = \frac{1}{N_c}\mathrm{Tr}\Big(1 - V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\Big)$$

- for large N_c at most quadrupoles in n-particle production; finite N_c n-particle $\triangleq n$ correlators

[Dominguez, Marquet, Stasto, Xiao; Phys.Rev. D87 (2013) 034007] isolate Wilson line & color generators of amplitudes

+ square them (Mathematica)



+ express adjoint Wilson lines in terms of fundamental

$$U^{ab}(z_t) = \mathrm{tr}\left[t^aV(z_t)t^bV^\dagger(z_t)\right]$$

+ make use of Fiery identities

$$\mathrm{tr}\left[t^aAt^aB\right] = \frac{1}{2}\mathrm{tr}\left[A\right]\mathrm{tr}\left[B\right] - \frac{1}{2N_c}\mathrm{tr}\left[AB\right]$$

$$\mathrm{tr}\left[t^aA\right]\mathrm{tr}\left[t^aB\right] = \frac{1}{2}\mathrm{tr}\left[AB\right] - \frac{1}{2N_c}\mathrm{tr}\left[A\right]\mathrm{tr}\left[B\right]$$

$$
\begin{aligned}
\mathrm{tr}\,[W_1W_1^*] &= \frac{\left(N_c^2-1\right)S_Q(x_t,x'_t,y'_t,y_t)}{2N_c}\\
\mathrm{tr}\,[W_1W_2^*] &= \frac{1}{4}\left(S_D(z'_t,x'_t)S_Q(x_t,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_1W_3^*] &= \frac{1}{2}\left(S_D(x_t,y)S_D(y'_t,x'_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_1W_4^*] &= \frac{1}{4}\left(S_D(z'_t,x'_t)S_Q(x_t,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_2W_1^*] &= \frac{1}{4}\left(S_D(x_t,z)S_Q(z_t,x'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_2W_2^*] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t)S_Q(z,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_2W_3^*] &= \frac{1}{4}\left(S_D(z,y_t)S_Q(x_t,x'_t,y'_t,z)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_2W_4^*] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z)S_Q(z_t,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_3W_1^*] &= \frac{1}{2}\left(S_D(x_t,y_t)S_D(y'_t,x'_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_3W_2^*] &= \frac{1}{4}\left(S_D(y'_t,z'_t)S_Q(x_t,x'_t,z'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_3W_3^*] &= \frac{\left(N_c^2-1\right)S_Q(x_t,x'_t,y'_t,y_t)}{2N_c}\\
\mathrm{tr}\,[W_3W_4^*] &= \frac{1}{4}\left(S_D(y'_t,z'_t)S_Q(x_t,x'_t,z'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
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\mathrm{tr}\,[W_4W_2^*] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t)S_Q(z,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_4W_3^*] &= \frac{1}{4}\left(S_D(z,y_t)S_Q(x_t,x'_t,y'_t,z)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)\\
\mathrm{tr}\,[W_4W_4^*] &= \frac{1}{8}\left(S_Q(x_t,x'_t,z'_t,z_t)S_Q(z_t,z'_t,y'_t,y_t)-\frac{S_Q(x_t,x'_t,y'_t,y_t)}{N_c}\right)
\end{aligned}
$$

DIS: dipole and quadrupole sufficient even at finite N_C

altogether 7 independent terms

with

$$S_D(x_{1,t},x_{2,t}) = \mathrm{tr}\left[V(x_{1,t})V^\dagger(x_{2,t})\right]$$

$$S_Q(x_{1,t},x_{2,t},x_{3,t},x_{4,t}) = \mathrm{tr}\left[V(x_{1,t})V^\dagger(x_{2,t})V(x_{3,t})V^\dagger(x_{4,t})\right]$$

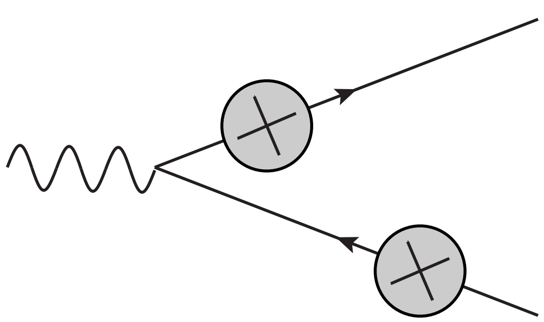
Loop integrals

something slightly strange:

Loop Integrals also for Real corrections

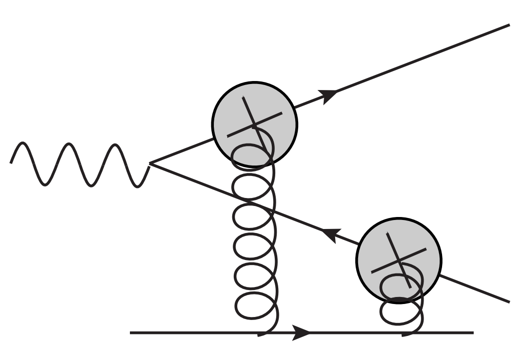
technical reason:

- momentum space amplitudes obtained from field correlators during LSZ reduction procedure
- integration over coordinates at vertices yields delta functions which evaluate momentum integrals trivially
- here: coordinate dependence of background field → some of the delta functions absent



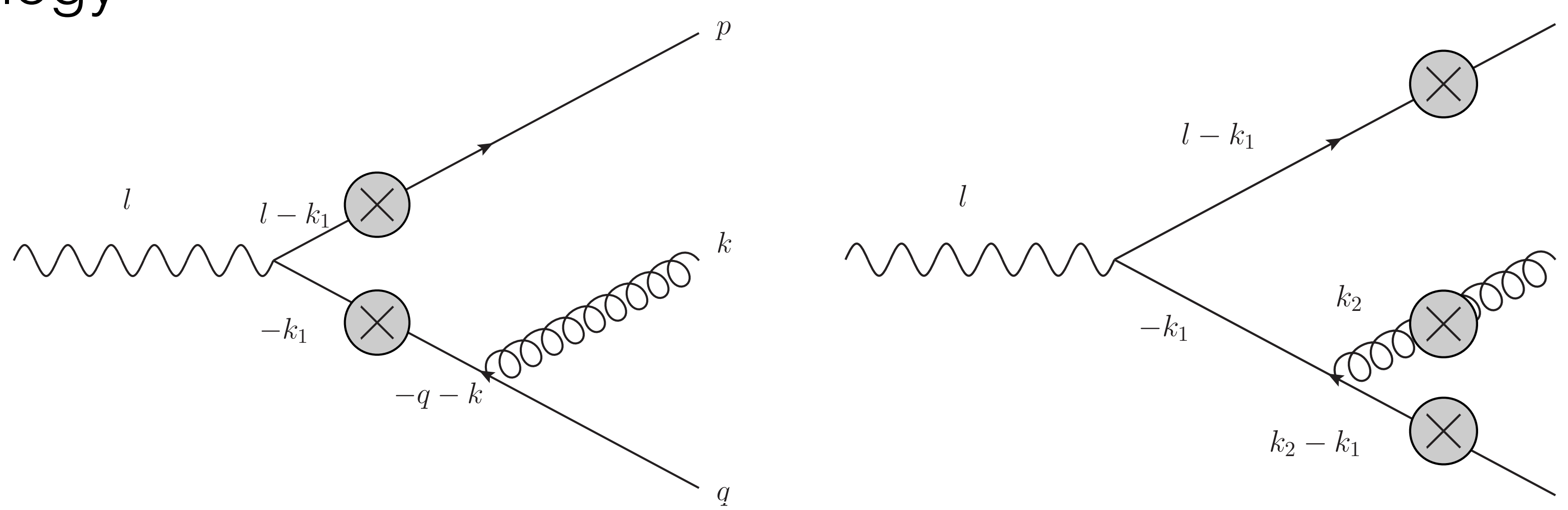
intuitive picture:

background field = t-channel gluons interacting with the target
→ naturally provide a loop which is factorized & (partially) absorbed into the projectile in the high energy limit



3 particle production:

a 1-loop and a 2-loop topology



k_1 and k_2 are loop momenta
new complication: exponentials/Fourier factors

conventional: *e.g.* k_1^+ integration by taking residues, then transverse integrals
particular for 2 loop case: complicated transverse integrals

developed a new technique

- ★ complete exponential factors to 4 dimensions
- ★ evaluate integral using “standard” momentum space techniques

a 1-loop example:

$$I(p_1,p_2) = \int \frac{d^dk_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l-k_1)^2]} e^{ix_t(\cdot k_{1,t}-p_{1,t})} e^{-iy_t\cdot(k_{1,t}+p_{2,t})} (2\pi)^2 \delta(p_1^- - k_1^-)\delta(l^- - k_1^- - p_2^-)$$

start with integral which contains

- delta functions
- transverse exponential factors

$$I(p_1,p_2) = 2\pi\delta(l^- - p_1^- - p_2^-) e^{-iy_t\cdot(p_{1,t}+p_{2,t})} \int dr^+ \int dr^- \delta(r^+) \int \frac{d^dk_1}{i\pi^{d/2}} \frac{1}{[k_1^2][(l-k_1)^2]} e^{ir\cdot k_1}$$

- introduce relative coordinate r=x-y
- represent delta function by integral
- introduce dummy integral over r+

→obtain 4(d) dimensional integral

next step:

- Schwinger/**α**-parameters
- complete square in exponent, Wick rotation, Gauss integral
- reconstruct delta functions to evaluate (some) of the **α**-parameter integrals

$$\left(\frac{i}{k^2-m^2+i0}\right)^\lambda = \frac{1}{\Gamma(\lambda)} \int_0^\infty d\alpha\, \alpha^{\lambda-1} e^{i\alpha(k^2-m^2+i0)}$$

to facilitate these steps for 2, 3 loops (virtual!): “developed” Mathematica package ARepCGC; implements necessary text-book methods **[V. Smirnov, Springer 2006]**

Complete result in terms of 2 functions

$$f_{(a)}(\bar{Q}^2, -r^2) = \int_0^\infty d\lambda \lambda^{a-1} e^{-\lambda \bar{Q}^2} e^{\frac{r^2}{4\lambda}} = 2^{1-a} \left(\frac{-r^2}{\bar{Q}^2}\right)^{a/2} K_a\left(\sqrt{\bar{Q}^2(-r^2)}\right)$$

$$h_{(a,b)}(\bar{Q}^2, r_1^2, r_3^2) = \int_0^\infty d\alpha \alpha^{a-1} e^{-\alpha \bar{Q}^2} e^{\frac{r_1^2}{4\alpha}} \cdot \int_0^{\bar{\rho}} d\rho \rho^{b-1} e^{\frac{r_3^2}{4\alpha\rho}}$$

$$= 2^{1-a} \int_0^{\bar{\rho}} d\rho \rho^{b-a/2-1} \left(\frac{-\rho r_1^2 - r_3^2}{\bar{Q}^2}\right)^{a/2} K_a\left[\sqrt{\bar{Q}^2 \cdot \left(-r_1^2 - \frac{r_3^2}{\rho}\right)}\right]$$

- $K_a(x)$ modified Bessel function of 2nd kind (Macdonald function)
- require $f_{(a)}$ for *a=0,-1* and $h_{\{a,b\}}$ for *a=0,-1, -2* and *b = 0, -1*
- further reduction possible due to integration by parts identities
- $h_{\{a,b\}}$ can be directly evaluated for b=-1; general case into infinite sum over Bessel functions;
  numerics: keeping integral might be most stable
- massive case trivial as long one accepts one remaining integration for $h_{\{a,b\}}$

From Gamma matrices to cross-sections

Dirac traces from Computer Algebra Codes

- possible to express elements of Dirac trace in terms of scalar, vector and rank 2 tensor integrals

- Evaluation requires use of computer algebra codes;
use 2 implementations:
FORM **[Vermaseren, math-ph/0010025]** &
Mathematica packages FeynCalc and FormLink

- result (3 partons) as coefficients of "basis"-functions $f_{(a)}$ and $h_{(a,b)}$; result lengthy (~100kB), but manageable

- currently working on further simplification through integration by parts relation between basis function (work in progress)

Next step: complete NLO corrections

- integrate one of the produced particles → additional divergences
  - rapidity divergence: JIMWLK evolution of dipoles & quadrupoles (and their products)
  - high M_X diffraction: require extension of JIMWLK to exclusive reactions **[Hentschinski, Weigert, Schäfer, Phys.Rev. D73 (2006) 051501]**
  - soft singularities cancel between real & virtual
- for $\gamma^* \to hh + X$: final state collinear divergences: fragmentation functions
- for $q \to jj + X$ etc: initial state collinear divergences: parton distribution functions + need to take care of potential soft factors

work in progress

related work:
[Boussarie, Grabovsky, Szymanowski, Wallon, JHEP1409, 026 (2014)]
[Balitsky, Chirilli, PRD83 (2011) 031502, PRD88 (2013) 111501]
[Beuf, PRD85, (2012) 034039]

Summary

- CGC is a systematic approach to high gluon densities in high energy collisions — used to fit a wealth of data (ep, pp, pA, AA)
- LO CGC works (sometimes too) well; qualitative/semi-quantitative description of data requires NLO
- to arrive at a precise picture of saturated gluon densities we need precision — both experiment and theory
- Di-jet/-hadron angular correlations offer a unique probe of the CGC (both *eA* and *pA*)
- Tri-jet/-hadron should be even more discriminatory

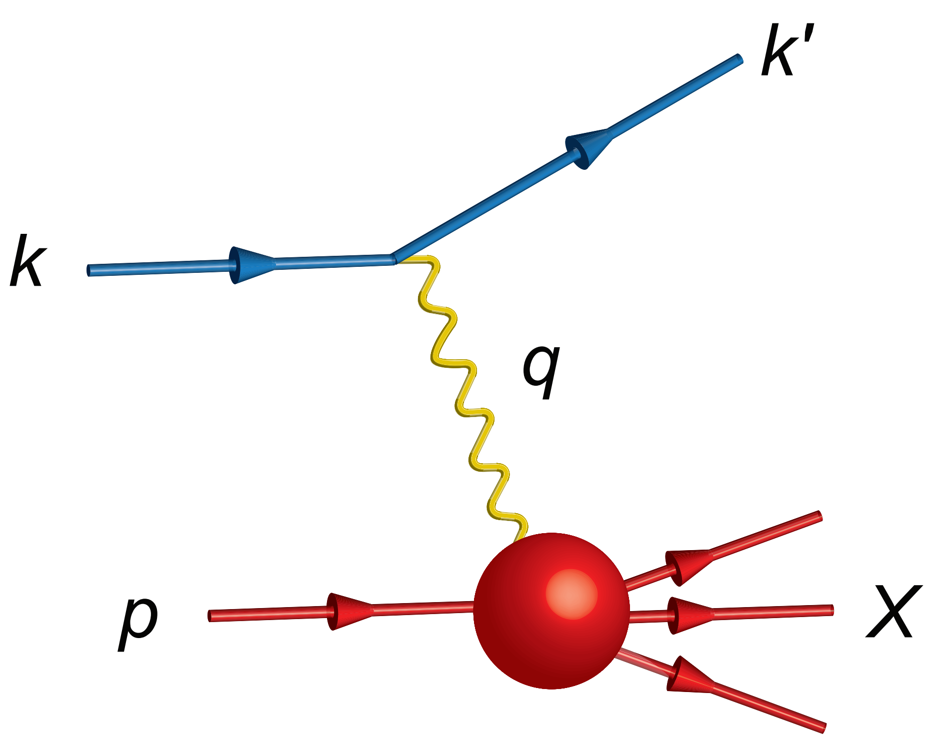
Summary

- developed techniques (diagram reduction, integrals) - might have been available before, but never been exploited in a systematic way for this kind of calculation
- proof of concept for NLO momentum space calculation advantage: benefit from standard techniques for higher orders in QCD
  (important: soft- and collinear singularities!)
- concentrate on DIS, but results (integrals, codes) extends beyond →3-jets, NLO correction for saturation/ CGC observables in *e.g.* pA at RHIC/LHC

Danke!

Electron-nucleus/-on scattering

- knowldege of scattering enery + nucleon mass + measure scattered electron ➡ control kinematics



Photon virtuality
$Q^2 = -q^2$

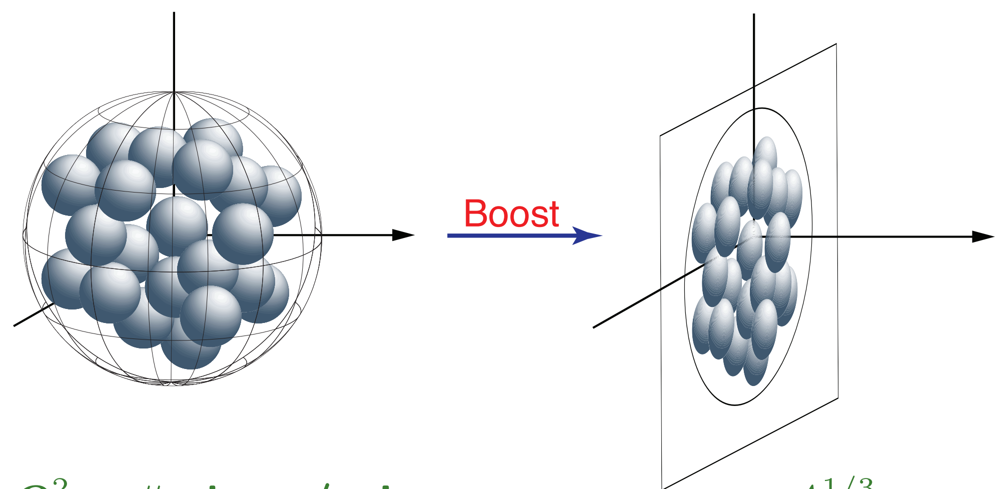
Resolution
$\lambda \sim \frac{1}{Q}$

Mass of system X
$$W = (p+q)^2$$
$$= M_N^2 + 2p \cdot q - Q^2$$

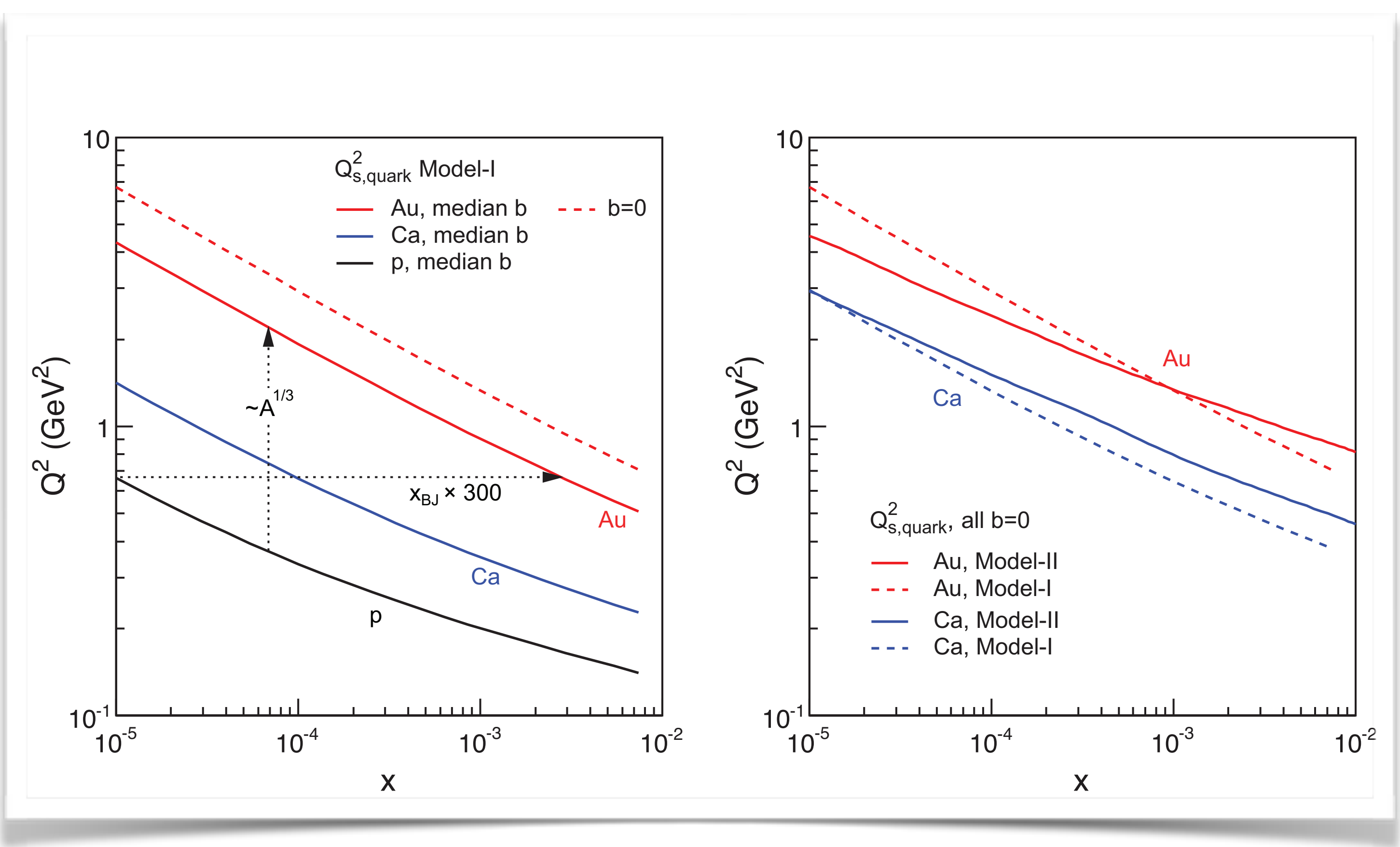
Bjorken $\boldsymbol{x} = \dfrac{Q^2}{2p \cdot q}$

Inelasticity $y = \dfrac{2p \cdot q}{2p \cdot k}$

Expect those effects to be even more enhanced in boosted nuclei:



$Q_s^2 \sim \#$ **gluons/unit transverse area** $\sim A^{1/3}$

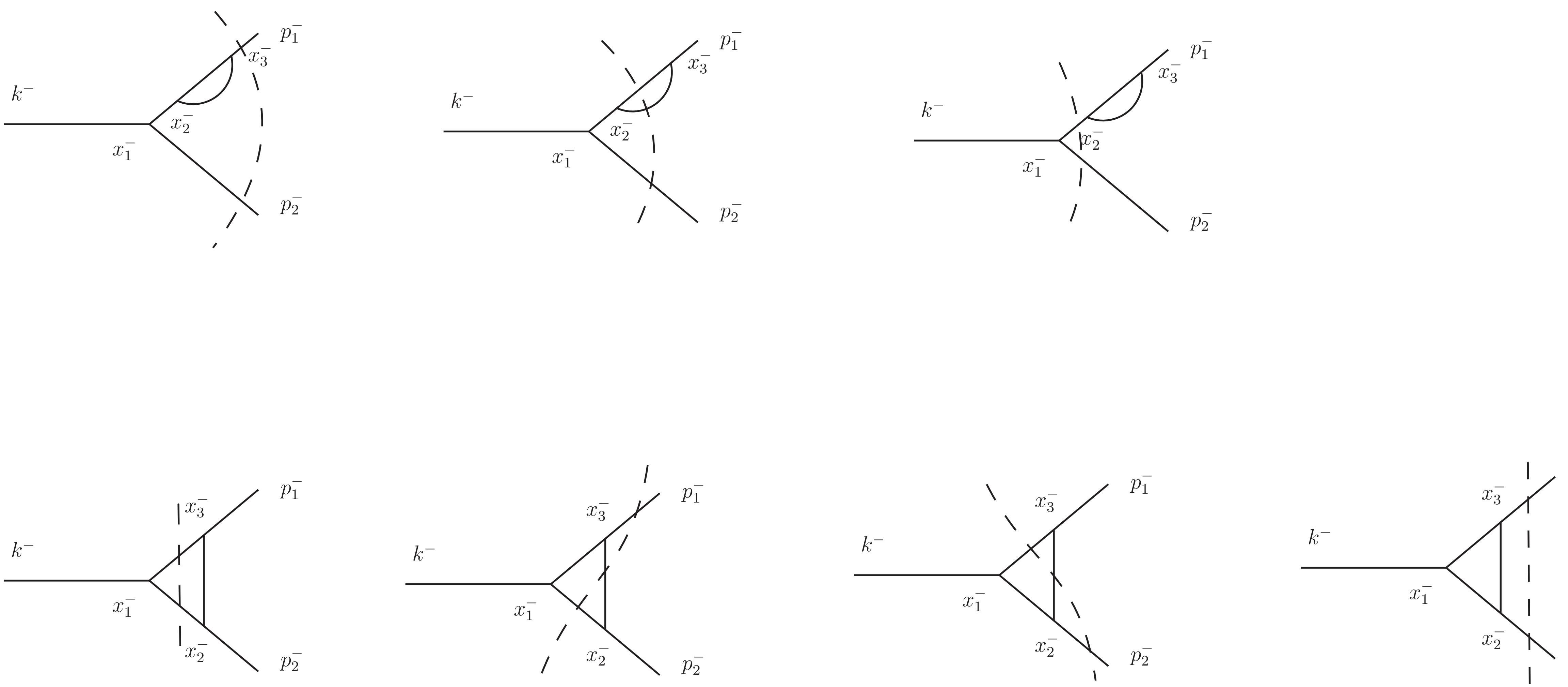


momentum vs. configuration space

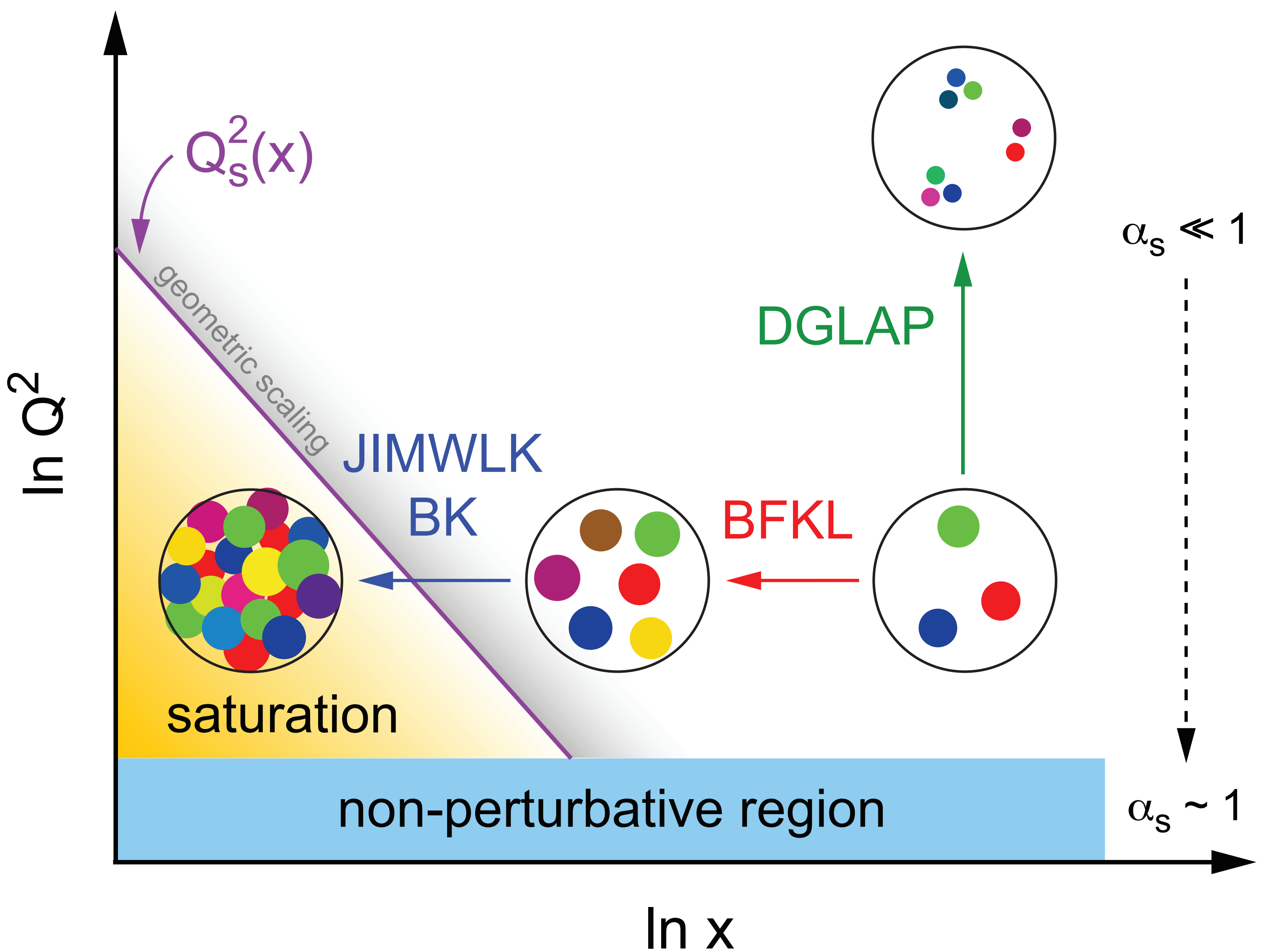
	conventional pQCD (make use of	inclusion of finite masses (charm mass!)	intuition: interaction at t=0 with Lorentz
momentum space	well explored	complication, but doable	lose intuitive picture at first -> large # of
configuration space	poorly explored	very difficult	many diagrams automatically zero

our approach:
work in momentum space, but exploit relation to configuration space to set a large fraction of all diagrams to zero

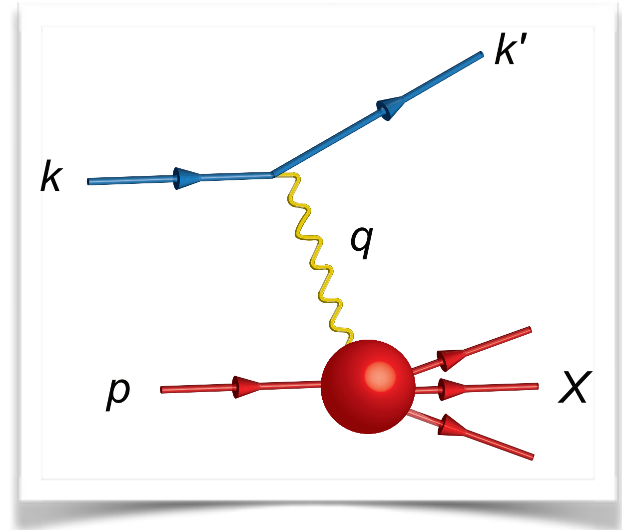
the lC-Time Slice x^{-}=0: ‘cuts’ through diagrams



Searching for saturation effects



Theory predictions for high & saturated gluon densities



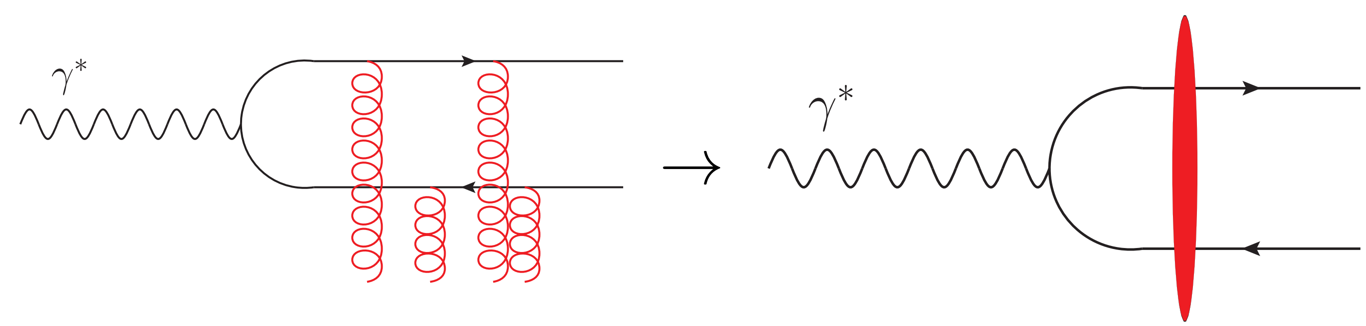
$x = Q^2/2p\cdot q \to 0$ limit corresponds to perturbative

high energy limit $2p\cdot q \to \infty$ for fixed resolution Q^2

- make use of factorisation of cross-sections in the high energy limit
- allows to resum interaction of quarks & gluons with strong gluon field to all orders in the strong coupling→resummation of finite density effects
- DIS X-sec. as convolution of "photon wave function" (process-dependent) and "color dipole factor" (universal, resums ln1/x)

$$\sigma_{L,T}^{\gamma^* A}(x,Q^2) = 2\sum_f \int d^2\boldsymbol{b}\,d^2\boldsymbol{r}\int_0^1 dz\,\left|\psi_{L,T}^{(f)}(r,z;Q^2)\right|^2 \mathcal{N}(x,\boldsymbol{r},\boldsymbol{b})$$

- physical picture: virtual photon splits into color dipole (quark-antiquark pair) which interacts with Lorentz contracted target field



$$A^{+,a}(z^-,\boldsymbol{z}) = \alpha^a(\boldsymbol{z})\delta(z^-)$$

$x \to 0$: a single interaction with a strong & Lorentz contracted gluon field