Twenty-first Century Lattice Gauge Theory:

Consequences of the QCD Lagrangian

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Fermilab and Mexico

• Many contributions in experimental physics to the Fermilab program.

• Theoretical Physics Department offers opportunity for aspiring young theorists: “Latin American Graduate Students”.

  • Six month visit to Fermilab to work with one of us, coordinated by Marcela Carena y López.

  • See http:theory.fnal.gov for details.
Aim of this talk

• Provide a survey of results about QCD, obtained using numerical lattice gauge theory, that are both

  • quantitatively impressive;

  • qualitatively noteworthy.

• Some quoted results have replaced ignorance, guesses, and beliefs with scientific knowledge.

• Others aid the interpretation of experiments or observations in particle physics, nuclear physics, and astrophysics.
Quantum Chromodynamics—QCD

- Modern theory of the strong force: quarks+gluons → hadrons → nuclei.

- A gauge theory, mathematically similar to quantum electrodynamics:

\[
\mathcal{L}_{\text{QED}} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \sum_{\text{charged } l} \bar{\psi}_l (\slashed{D}_l + m_l) \psi_l
\]

\[
\mathcal{D}_l = \gamma^\mu (\partial_\mu + q_l e_0 A_\mu)
\]

\[
\mathcal{L}_{\text{QCD}} = -\frac{1}{2g_0^2} F_{\mu\nu}^a F^{\mu\nu a} - \sum_{\text{colored } f} \bar{\psi}_f^i (\slashed{D} + m_f)_{ij} \psi_f^i
\]

\[
\mathcal{D}_{ij} = \gamma^\mu (\partial_\mu \delta_{ij} + A_{\mu}^{a t_{ij}})
\]

- Now the gauged quantum number is not electric charge, but color.

- SU(3) gauge symmetry: gauge boson “gluon” carries color.

- Laws of Nature.
Color vs. colour

- With SU(3), states with equal amounts of colors red, green, and blue (or equal amounts of cyan, magenta, and yellow) are neutral.

- In vision, light (ink) with equal amounts of colours red, green, and blue (equal amounts of cyan, magenta, and yellow) are white (black) or gray.

- For QCD, I will follow spelling of physicists Greenberg, Gell-Mann, Nambu, …, rather than francophile administrators and secretaries at CERN.
The QCD Lagrangian

- SU(3) gauge symmetry and $1 + n_f + 1$ parameters:

$$L_{\text{QCD}} = \frac{1}{g_0^2} \text{tr}[F_{\mu\nu}F^{\mu\nu}]$$

$$- \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f$$

$$+ \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr}[F_{\mu\nu}F_{\rho\sigma}]$$

- Observable CP violation $\propto \vartheta = \theta - \arg \det m_f$ (if all masses nonvanishing):

  - neutron electric-dipole moment sets limit $\vartheta \approx 10^{-11}$;

  - bafflingly implausible cancellation called the strong CP problem.
Quantum Chromodynamics

• The most perfect theory—asymptotic freedom.

• Triumph of reductionism: quark model $\oplus$ parton model $\oplus$ color = $\text{QCD}$.  

• Multi-scale problem: $m_u, m_s, m_\pi, m_K, \Lambda_{\text{QCD}}, m_c, m_b, m_t; Q^2; a^{-1}; L^{-1}$.  

• Rich in symmetry: C, P, T; chiral symmetry, heavy-quark symmetry.

• Rich in emergent phenomena: hadron masses, chiral symmetry breaking, phase transitions, atomic nuclei ...  

• ... requiring nonperturbative methods (lattice gauge theory) and a full exploitation of symmetries, asymptotic freedom.
Asymptotic Freedom

At short-distances, the force in QCD looks similar to QED:

\[ F(r) = -\frac{4}{3} \frac{\alpha_s(1/r)}{r^2} \]

where the 4/3 is a color factor.

The key difference is that virtual gluons reduce the effective \( \alpha_s \) at short distances.

Verified in experiment.

Relates \( \alpha_s \) to a physical scale, \( \Lambda_{QCD} \).

Politzer, \textit{PRL} \textbf{30} (1973) 1346;
Gross, Wilczek, \textit{PRL} \textbf{30} (1973) 1343

\[ 1/\alpha_s \]

\( \Lambda_{QCD} \)

\( Q \) [GeV]

\( 10^0 \) \( 10^1 \) \( 10^2 \) \( 10^3 \)

ASK & Quigg, \textit{arXiv:1002.5032}
Lattice Gauge Theory

K. Wilson, *PRD* 10 (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].

- Gauge symmetry on a spacetime lattice:
  - mathematically rigorous definition of QCD functional integrals;
  
  \[
  \langle \bullet \rangle = \frac{1}{Z} \int DUD\psi D\bar{\psi} \exp (-S) \]

  - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.

- Lowest-order strong coupling expansion demonstrates confinement.
Nowadays “lattice QCD” usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.

A big computer.

Some compromises:

- finite human lifetime $\Rightarrow$ Wick rotate to Euclidean time: $x^4 = ix^0$;

- finite memory $\Rightarrow$ finite space volume & finite time extent;

- finite CPU power $\Rightarrow$ light quarks heavier than up and down.
Lattice Gauge Theory

\[ \langle \bullet \rangle = \frac{1}{Z} \int [DU D\psi D\bar{\psi} \exp(-S) [\bullet] ] \]

- Infinite continuum: uncountably many d.o.f.

- Infinite lattice: countably many; used to define QFT

- Finite lattice: can evaluate integrals on a computer; dimension \( \sim 10^8 \)
Some Jargon

• QCD observables (quark integrals by hand):

\[ \langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \prod_{f=1}^{n_f} \det(\mathcal{D} + m_f) \exp \left( -S_{\text{gauge}} \right) \]

• Quenched means replace \( \det \) with 1. (Obsolete.)

• Unquenched means not to do that.

• Partially quenched (usually) doesn’t mean “\( n_f \) too small” but \( m_{\text{val}} \neq m_{\text{sea}} \), or even \( \mathcal{D}_{\text{val}} \neq \mathcal{D}_{\text{sea}} \) (“mixed action”).
Some algorithmic issues

e.g., ASK, hep-lat/0205021

• lattice $N_S^3 \times N_4$, spacing $a$

• memory $\propto N_S^3 N_4 = L_S^3 L_4 / a^4$

• $\tau_g \propto a^{-(4+z)}$, $z = 1$ or 2.

• $\tau_q \propto (m_q a)^{-p}$, $p = 1$ or 2.

• Imaginary time:
  • static quantities

• size $L_S = N_S a$, $L_4 = N_4 a$;

• dimension of spacetime = 4

• critical slowing down

• especially dire with sea quarks

• thermodynamics: $T = 1/N_4 a$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S)[\bullet]$$

$$= \text{Tr}\{\bullet e^{-\hat{H}/T}\} / \text{Tr}\{e^{-\hat{H}/T}\}$$
Sea Quarks

• Staggered quarks, with rooted determinant, $O(a^2)$.

• Wilson quarks, $O(a)$:
  
  • tree or nonperturbatively $O(a)$ improved $\Rightarrow O(a^2)$;

  • twisted mass term—auto $O(a)$ improvement $\Rightarrow O(a^2)$.

• Ginsparg-Wilson (domain wall or overlap), $O(a^2)$:
  
  • $D\gamma_5 + \gamma_5 D = 2aD^2$ implemented w/ sign($D_w$).
• Many numerical simulations with sea quarks are called (perhaps misleadingly) “full QCD.”

• $n_f = 2$: with same mass, omitting strange sea;

• $n_f = 3$: may (or may not) imply 3 of same mass;

• $n_f = 2+1$: strange sea + 2 as light as possible for up and down;

• $n_f = 2+1+1$: add charmed sea to 2+1.

• “Full QCD” can also mean $m_{\text{val}} = m_{\text{sea}}$, or $D_{\text{val}} = D_{\text{sea}}$. 
Correlators Yield Masses & Matrix Elements

• Two-point functions for masses $\pi(t) = \bar{\psi}_u \gamma_5 S \psi_d$:

$$\langle \pi(t) \pi^\dagger(0) \rangle = \sum_n |\langle 0 | \hat{\pi} | \pi_n \rangle|^2 \exp(-m_{\pi_n} t)$$

• Two-point functions for decay constants:

$$\langle J(t) \pi^\dagger(0) \rangle = \sum_n \langle 0 | \hat{J} | \pi_n \rangle \langle \pi_n | \hat{\pi}^\dagger | 0 \rangle \exp(-m_{\pi_n} t)$$

• Three-point functions for form factors, mixing:

$$\langle \pi(t) J(u) B^\dagger(0) \rangle = \sum_{mn} \langle 0 | \hat{\pi} | \pi_m \rangle \langle \pi_n | \hat{J} | B_m \rangle \langle B_m | \hat{B}^\dagger | 0 \rangle \times \exp[-m_{\pi_n} (t - u) - m_{B_m} u]$$
Standard Model: 19 Parameters or 28

• Gauge couplings: $\alpha_s$, $\alpha_{\text{QED}}$, $\alpha_W = (m_W/v)^2/\pi$;

• Lepton masses: $m_e$, $m_\mu$, $m_\tau$; $m_{\nu1}$, $m_{\nu2}$, $m_{\nu3}$;

• Quark masses: $m_u e^{i\theta}$, $m_d$, $m_s$, $m_c$, $m_b$, $m_t$; “Instability” → “renormalization.”

• CKM: $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $\exp(i\delta_{\text{KM}})$; $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\delta_{\text{PMNS}}$, $\phi_1$, $\phi_2$;

• EWSB: $v = 246$ GeV, $\lambda = (m_H/v)^2/2$. “Infinite $\lambda$” → “triviality.”

• Need lattice QCD, lattice Yukawa.
2+1 Sea Quarks!

HPQCD, MILC, Fermilab Lattice, hep-lat/0304004

- \( a = 0.12 \) & 0.09 fm;
- \( O(a^2) \) improved: asqtad;
- FAT7 smearing;
- \( 2m_l < m_q < m_s \);
- \( \pi, K, \Upsilon(2S-1S) \) input.
Predictions

Fermilab Lattice, MILC, HPQCD, 
hep-ph/0408306, hep-lat/0411027, hep-lat/0506030

- Semileptonic form factor for $D \to Kl\nu$
- Mass of $B_c$ meson
- Charmed-meson decay constants
Outline

• Introduction

• Chiral Symmetry Breaking

• Hadron Spectrum

• QCD Parameters

• Flavor Physics

• Thermodynamics

• Summary & Challenges
Chiral Symmetry Breaking
The hadron spectrum has a striking feature:

- \( m_\pi = 135 \text{ MeV} \) but \( m_Q = 770 \text{ MeV} \), \( m_p = 938 \text{ MeV} \), etc.

Nambu applied lessons from superconductivity, noting (4 years before quarks) that the pion’s small mass could be arise from a \textit{spontaneously} broken axial symmetry (moderated with a small amount of explicit breaking).

QCD explained the origin: if up and down quark masses are neglected, the Lagrangian has an \( SU_L(2) \times SU_R(2) \) chiral symmetry, which provides candidate axial symmetry.

(If so, pions break \( SU_L(2) \times U_Y(1) \): without terascale EWSB, \( W^\pm \) and \( Z \) would have masses around 100 MeV.)
• In the 20th century, we were already confident that QCD was a good theory of the strong interactions, based on, e.g., its explanation of the SLAC deep-inelastic scattering experiments.

• Because QCD was (considered) right, and since Nambu’s picture was (considered) right, it was believed that QCD must drive spontaneous chiral symmetry breaking.

• But does it?

• Goldstone formula: \[ m^2_\pi \langle \bar{\psi} \psi \rangle = 0 \], if \( \langle \bar{\psi} \psi \rangle \neq 0 \), then \( m_\pi = 0 \).

• What is it?
Chiral Condensate

e.g., H. Fukaya et al. [JLQCD], arXiv:0911.5555

\( m_u, m_d \rightarrow 0, \ m_s \text{ physical} \)

\[
\langle \bar{\psi} \psi \rangle_{\overline{\text{MS}}}^{\text{MS}} (2 \text{ GeV}) = [242 \pm 4_{\text{stat}}^{+19}_{-18_{\text{syst}}} \text{ MeV}]^3
\]

• At the hadronic level, the spontaneous breaking of chiral symmetry allows the nucleon mass to be nonzero [Nambu], even when \( m_u = m_d = 0 \).

• In nature, \( m_u \) & \( m_d \) are small, so the physical picture of chiral symmetry is:
  
  • dominantly spontaneously broken (Nambu’s mechanism);

  • small corrections from explicit breaking (chiral perturbation theory).
Hadron Spectrum
Why Compute Hadron Masses?

• Show that the QCD Lagrangian generates hadron masses.

• Understand more deeply Nature’s only known mechanism for generating masses.

• At short distances, the potential (force) is Coulombic.

• At large distances, the potential (force) rises linearly (flattens at a positive value).

G. Bali, hep-ph/0001312
QCD postdicts the low-lying hadron masses!

- \( a = 0.12 \& 0.09 \text{ fm} \);
- \( O(a^2) \) improved: asqtad;
- FAT7 smearing;
- \( 2m_l < m_q < m_s \);
- \( \pi, K, \Upsilon(2S-1S) \) input.
QCD postdicts the low-lying hadron masses!

- $a = 0.091$ fm;
- NP $O(a)$ Wilson;
- no smearing;
- $m_q \approx 1.3 m_l$;
- $\pi, K, \Omega$ input
• $a = 0.125, 0.085, \& 0.065$ fm;

• tree $O(a)$ Wilson;

• $6\times$ stout smearing;

• $2m_l < m_q < 1.7m_s$;

• $\pi, K, \Xi$ input.

QCD postdicts the low-lying hadron masses!
Now, quark masses are MeV not GeV!

\[ m = \frac{E_0}{c^2} \]
QCD Parameters: $\alpha_s$ and Quark Masses
Light Quark Masses

- The nonzero pion (kaon) mass is very sensitive to the light (strange) masses.

- Chiral perturbation theory predicts ratios of masses, but not the overall scale.

<table>
<thead>
<tr>
<th>Lattice QCD</th>
<th>MILC</th>
<th>RBC</th>
<th>BMW</th>
<th>HPQCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{m}_u(2\text{ MeV})$</td>
<td>$1.9 \pm 0.2$</td>
<td>$2.24 \pm 0.35$</td>
<td>$2.15 \pm 0.11$</td>
<td></td>
</tr>
<tr>
<td>$\bar{m}_d(2\text{ MeV})$</td>
<td>$4.6 \pm 0.3$</td>
<td>$4.65 \pm 0.35$</td>
<td>$4.79 \pm 0.14$</td>
<td></td>
</tr>
<tr>
<td>$\bar{m}_s(2\text{ MeV})$</td>
<td>$88 \pm 5$</td>
<td>$97.6 \pm 6.2$</td>
<td>$95.5 \pm 1.9$</td>
<td>$92.4 \pm 1.5$</td>
</tr>
</tbody>
</table>

- These are small—up & down masses are 4 & 9 times electron mass.

- The up mass is far from 0: the strong CP problem is indeed a problem.
Strong CP Problem

- Quark masses arise from Yukawa couplings, \( m = vy/\sqrt{2} \), and from low-energy QCD instantons (tunneling between classical vacua):
  - observable CP violation \( \propto \Theta = \theta_{\text{QCD}} - \arg \det y < 10^{-11} \).

- If \( y \) has a zero mode, then its phase can be anything and, thus, chosen so that \( \Theta = 0 \); no CP violation arises.

- Though \( m_u \) is small, lattice QCD calculations show no evidence for an instanton effect big enough to allow a zero mode in \( y \).

- A non-Standard symmetry (Peccei-Quinn) is then the least implausible explanation for the cancellation. Consequence: weird particles called axions.
Heavy Quark Masses

- The charmonium correlator also yields impressive precision on the charm mass [Bochkarev & de Forcrand, hep-lat/9505025]; analogous to determination from $e^+e^-$ by Chetyrkin et al.:

  - lattice + PT: $m_c(m_c) = 1.268(9)$ GeV [arXiv:0807.1687];

  - $e^+e^-$ + PT: $m_c(m_c) = 1.279(13)$ GeV [arXiv:0907.2110].

- Similarly, $m_b(m_b) = 4.164(23)$ GeV [HPQCD, arXiv:1004.4285], cf. $m_b(m_b) = 4.163(16)$ GeV [Chetyrkin et al. $e^+e^-$, arXiv:0907.2110].

- Even more stunning [HPQCD, arXiv:0910.3102]: $m_c/m_s = 11.85(16)$, whence $m_s(2$ GeV$) = 92.4(1.5)$ MeV.
Strong Coupling $\alpha_s$

- In lattice gauge theory, the bare coupling $g_0^2(1/a)$ is an input. Aim is to relate this to $\alpha_s(m_Z) = g_s^2(m_Z)/4\pi$. Alas, conversion in PT does not converge.

- Two main strategies:
  - compute a short-distance lattice quantity (e.g., small Wilson loop, Creutz ratio of small Wilson loops, ...); compare MC with (lattice) PT $\rightarrow \alpha_s(1/a)$;
  - compute short-distance continuum quantity (e.g., Schrödinger functional, quarkonium correlator, Adler function); compare MC with PT $\rightarrow \alpha_s(2m_Q)$. 

Friday, October 21, 2011
Results for $\alpha_s$ (all $n_f = 2+1$):

  
  \[ \alpha_s(m_Z) = 0.1205(8)(5)(+0/–17) | \alpha_s(m_Z) = 0.1181(3)(+13/–4); \]

  
  \[ \alpha_s(m_Z) = 0.1183(8) | 0.1192(11); \]

- Charmonium correlator [HPQCD + Karlsruhe, arXiv:0805.2999]:
  
  \[ \alpha_s(m_Z) = 0.1174(12) \] [update in arXiv:1004.4285];

- Bethke’s world average (without lattice | with HPQCD WL) [arXiv:0908.1135]:
  
  \[ \alpha_s(m_Z) = 0.1186(11) | 0.1184(7). \]
QCD of hadrons = QCD of partons

Bethke, arXiv:0908.1135
Flavor Physics
Weak Interactions

• At energies probed by the Tevatron and the LHC, left- and right-handed quarks are different:

\[
\begin{pmatrix}
u \\ d
\end{pmatrix}_L \begin{pmatrix}c \\ s
\end{pmatrix}_L \begin{pmatrix}t \\ b
\end{pmatrix}_L \quad \text{aka } Q^i_L
\]

\begin{pmatrix}
u_R \\ d_R \\ c_R \\ s_R \\ t_R \\ b_R
\end{pmatrix}_R \quad \text{aka } U^i_R \quad \text{aka } D^i_R

9 fields: 3 doublets and 6 singlets under SU_L(2)×U_Y(1).

• The electroweak interactions treat all three “generations” same, and the fields can be transformed so the SU_L(2)×U_Y(1) gauge fields don’t couple generations to each other—“weak eigenstate basis.”
Identity from Higgs and Yukawa

- Whatever breaks electroweak symmetry has a weak-SU(2) doublet, $\Phi$, so it can have Yukawa interactions

$$y_{ij}^u \bar{Q}_L^i \Phi U_R^j + y_{ij}^d \bar{Q}_L^i \Phi^* D_R^j + h.c. =$$

$$y_{ij}^u (\bar{U} \bar{D})_L^i \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix} U_R^j + y_{ij}^d (\bar{U} \bar{D})_L^i \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} D_R^j + h.c.$$  

where indices label generations.

- Spontaneous symmetry breaking driven by $\Phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$:
  - generates masses for the quarks (as noted above on strong $CP$ problem).
CKM Matrix

- Mass and weak eigenstates of quarks are related by unitary transformations.

- Observable part of these rotations is the CKM matrix.

- 4 parameters: $|V_{us}|, |V_{cb}|, |V_{ub}|, i\delta_{KM}$—as fundamental as electron mass.

- Unitarity relations, e.g., $V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb} = 0$:
  
  - triangle in the complex plane.

- Probed by many measurements + corresponding QCD.
Unitarity Triangle


$\chi^2/\text{d.o.f.} = 2.6$
$p$-value = 1.7%

$\bar{\eta}$

$S_{\psi K}$

$\epsilon_K + |V_{cb}|$

$\left| \frac{V_{ub}}{V_{cb}} \right|$

$\frac{\Delta M_s}{\Delta M_d}$

$\text{BR}(B \rightarrow \tau \nu) + |V_{cb}|$
**B-Meson Averages from Lattice QCD**

inclusive $|V_{cb}|$ is $\sim 2\sigma$ higher

inclusive $|V_{ub}|$ is $\sim 1\sigma$ higher
π- & K-Meson Averages from Lattice QCD

plots from latticeaverages.org
Lessons

• Lattice QCD plays a crucial role for neutral-meson mixing ($K$, $B$, $B_s$).

• Lattice QCD plays a key role in $|V_{us}|$, $|V_{cs}|$, $|V_{ub}/V_{cb}|$, $|V_{cb}|$.

• Suite of experiments, pQCD, and lQCD shows that CKM flavor violation and KM CP violation predominates.

• Still room for new physics: tension at 2–3$\sigma$ level:
  
  • confidence level of global fit improves more, if NP in kaon mixing [LLV];
  
  • $\epsilon_K$ band uses corrections of ASK, Ligeti, Nierste [hep-ph/0201071].
Tension in $D_s \rightarrow l\nu$?

$\text{BR} \propto |f_{D_s}V_{cs}|^2$

- Earlier 3.8$\sigma$, now <2$\sigma$.
- New physics, e.g., leptoquark?
- w/ $A_{LQ} \sim +0.1A_{SM}$?
- Dobrescu & ASK
  arXiv:0803.0512
Tension in $D_s \rightarrow l\nu$?

ASK, arXiv:0912.0543

- Gray: running lQCD avg.
- Yellow: running expt avg.
- Orange: BaBar, Belle.
- Red: CLEO.
- Green (right y axis): running deviation in $\sigma$.
- $\sigma$ is mostly exptl stats.
Tension in $B \rightarrow \tau\nu$?

$\text{BR} \propto |f_B V_{ub}|^2$

- New physics, e.g., charged Higgs of MSSM?
- $w/ A_{\text{MSSM}} \sim -1.1 A_{\text{SM}}$?
Tension in $b \rightarrow u$?

- Right-handed currents could explain different “$V_{ub}$” from exclusive, inclusive, $B \rightarrow \tau \nu$.
- Best fit is $\sim-15\%$ RH current.

- Denote couplings $V_{ubR}$ & $V_{ubL}$
- Yellow: $B \rightarrow \tau \nu$: $|V_{ubR} - V_{ubL}|^{2}$
- Ochre: $B \rightarrow \pi l \nu$: $|V_{ubR} + V_{ubL}|^{2}$
- Black: $B \rightarrow X_{ul} \nu$: $|V_{ubR}|^{2} + |V_{ubL}|^{2}$
- Green CKM unitarity.
Thermodynamics
QCD Phase Diagram

CBM Collaboration
QCD Phase Transition ($\mu = 0$)

- Temperature $T = 1/N_T a$.
- Smooth crossover to phase, in which the grand canonical average becomes:
  - chirally symmetric;
  - deconfined.
- Same transition temperature: also for other observables, e.g., susceptibilities peak.
Quarks and gluons vs. hadrons

• The thermal average is

\[ \langle \bullet \rangle = \frac{\text{Tr} \left[ \bullet e^{-\hat{H}/T} \right]}{\text{Tr} e^{-\hat{H}/T}} \]

which is as “inclusive” as possible.

• Parton-hadron duality in scattering & decay seems to work once \( E > 2 \) GeV; at such \( T \) exchange trace over hadronic states for trace over partons.

• Does not contradict “chiral symmetry restoration” or “deconfinement” at lower temperatures: average includes a state and its chiral partner & and includes states with lots of hadrons.
Quark Masses are Key

- Explicit $\chi_{SB}$ softens the transition.
- Quark masses are small, but ...
- ... if even smaller, the $\mu = 0$ transition would be second order, or even first order.
- Implications for the early universe.

from de Forcrand & Philipsen
arXiv:0808.1096
Equation of State ($\mu \neq 0$)

- Studies limited to $\mu \approx 0$.
- Curvature a matter of controversy.
- Models and qualitative arguments suggest top picture, whence a critical point for some $\mu \neq 0$.
- Several groups find the opposite:
  - a cutoff effect?
Summary and Challenges
Summary: from “QCD should work this way”

to “QCD does work this way”

• Quantitative
  • Precise $\alpha_s, m_c, m_b$:
  • Precise $m_s, m_d, m_u$:
  • Hadron masses:
  • CKM $V_{cb}, V_{ub}, V_{td}/V_{ts} \oplus B_K, f_B$:
  • Chiral condensate $\langle \bar{q}q \rangle$:
  • Smooth crossover:

• Qualitative
  • $\text{QCD}_{\text{hadrons}} = \text{QCD}_{\text{partons}}$
  • strong CP is a problem
  • Your mass $= E_0/c^2$;
  • Nobel$^{\text{KM}} \oplus$ BSM hints;
  • $\text{QCD}$ breaks chiral symmetry;
  • Cooling universe.
Challenges

- Particle physics:
  - 1% precision for flavor physics;
  - reliable moments for the nucleon’s gluon distribution;
  - non-QCD gauge theories of electroweak symmetry breaking.

- Nuclear physics:
  - larger chemical potential;
  - excited states;
  - multi-hadron states, mixing, and, soon enough, nuclei—
Excited Baryons


- Future applications to glueball spectra and mixing.
Atomic Nuclei from QCD


- The simplest nucleus is the deuteron, $pn$.

- Barely bound: fine tuning of QCD parameters.

- Do similar dibaryons exist? Conjectured $H = \Lambda\Lambda$.

- Recent lattice QCD calculations, with slightly unphysical quark masses, suggest that the $H$ is indeed bound.